KS4 Mathematics

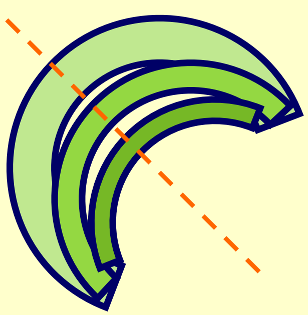
S6 Transformations

Contents

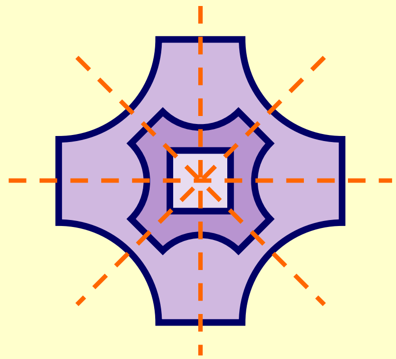
Reflection symmetry

If you can draw a line through a shape so that one half is the mirror image of the other then the shape has **reflection** or **line symmetry**.

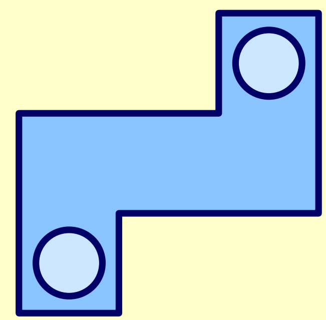
The mirror line is called a **line of symmetry**.



one line of symmetry



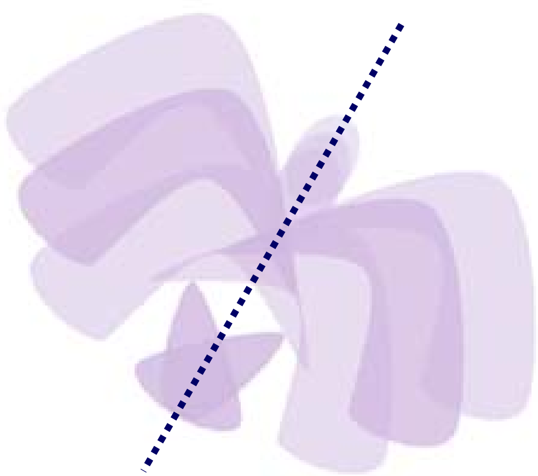
four lines of symmetry



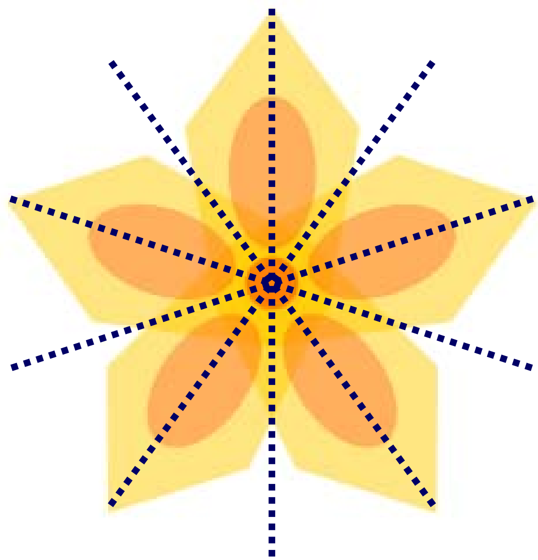
no lines of symmetry

Reflection symmetry

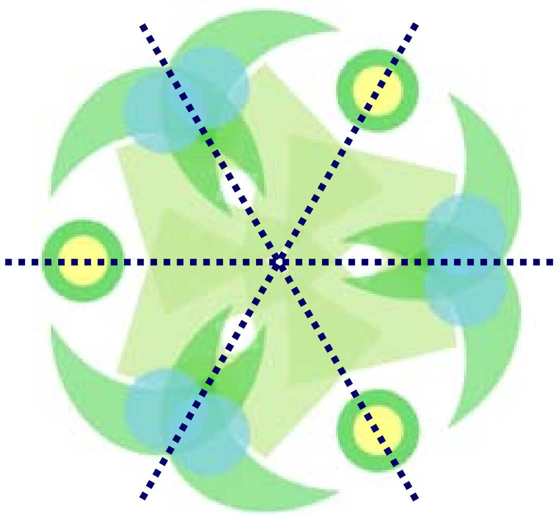
How many lines of symmetry do the following designs have?



one line of symmetry

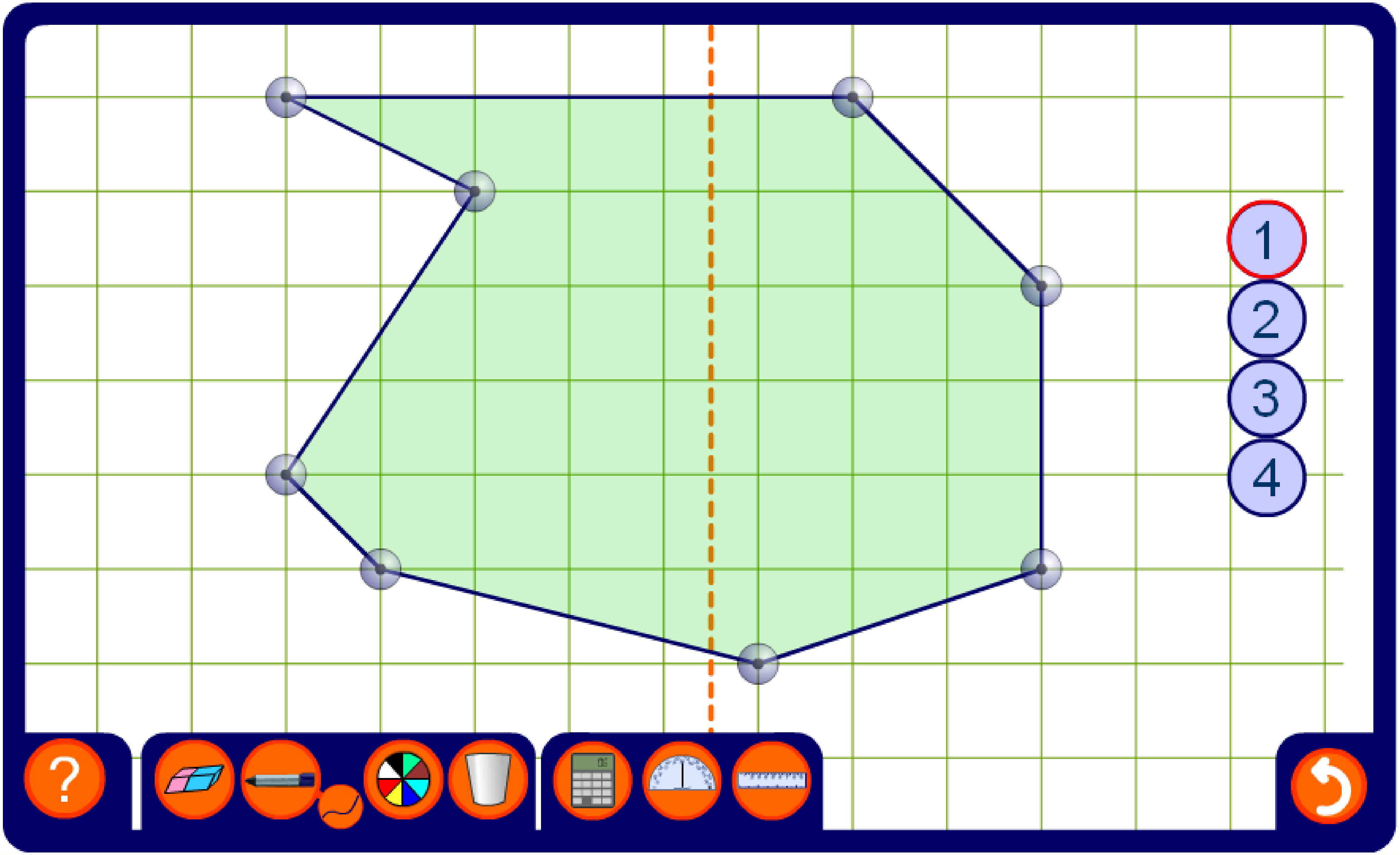


five lines of symmetry



three lines of symmetry

Make this shape symmetrical



Rotational symmetry

An object has **rotational symmetry** if it fits exactly onto itself when it is turned about a point at its centre.

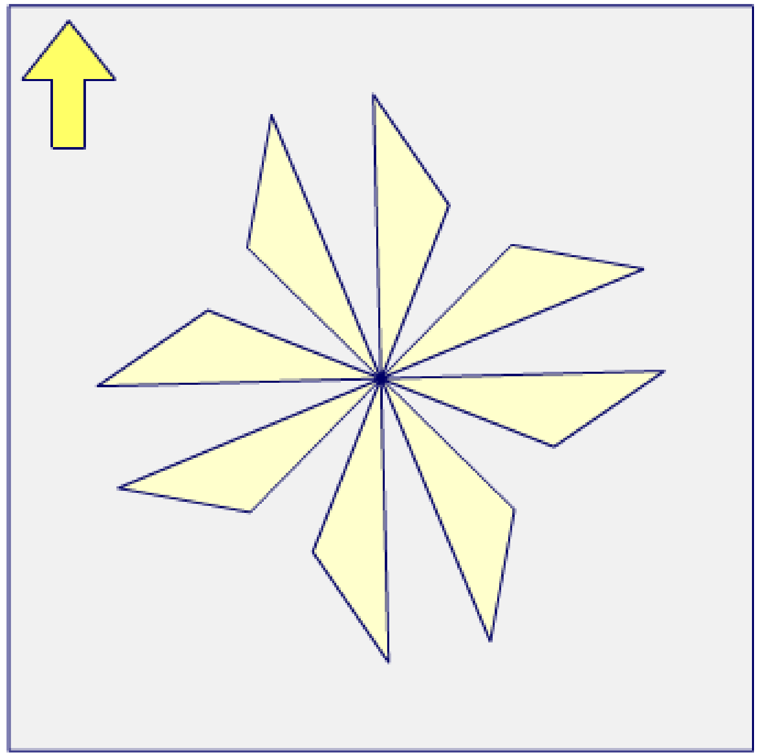
The **order of rotational symmetry** is the number of times the object fits onto itself during a 360° turn.

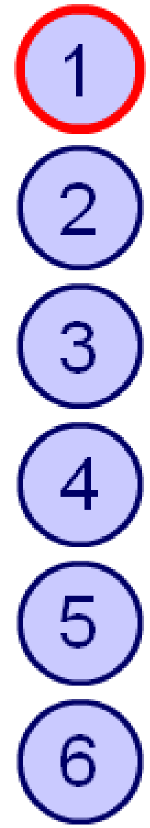
If the order of **rotational symmetry** is one, then the object has to be rotated through 360° before it fits onto itself again.

Only objects that have rotational symmetry of two or more are said to have rotational symmetry.

We can find the order of rotational symmetry using tracing paper.

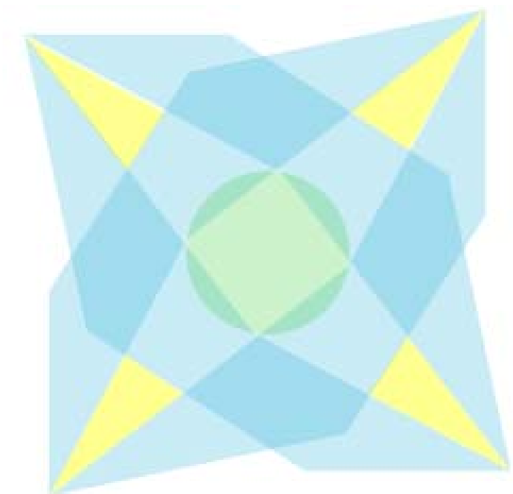
Finding the order of rotational symmetry



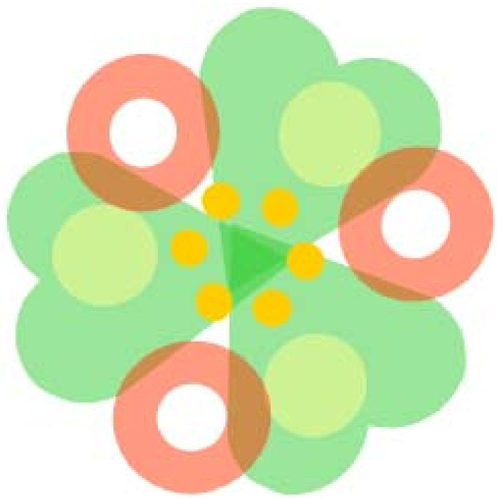


Rotational symmetry

What is the order of rotational symmetry for the following designs?



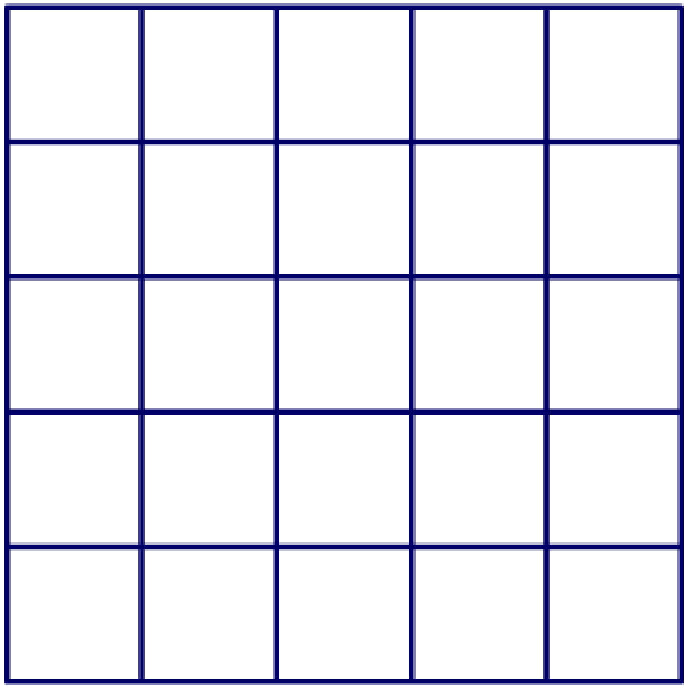
Rotational symmetry order 4



Rotational symmetry order 3

Rotational symmetry order 5

Reflection and rotational symmetry



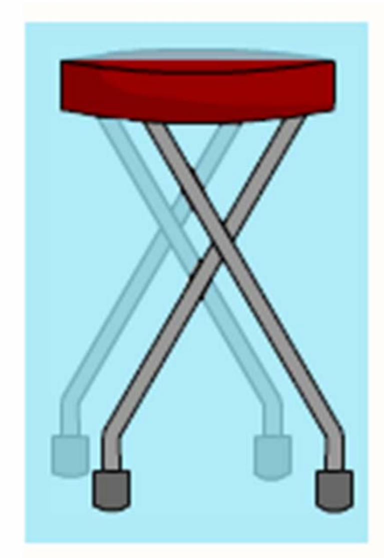
Try Again

Next Activity

Shade in 13 cells to make a pattern with 4 lines of symmetry and rotational symmetry of order 4.

Reflection symmetry in 3-D shapes

Sometimes a 3-D shape can be divided into two symmetrical parts.

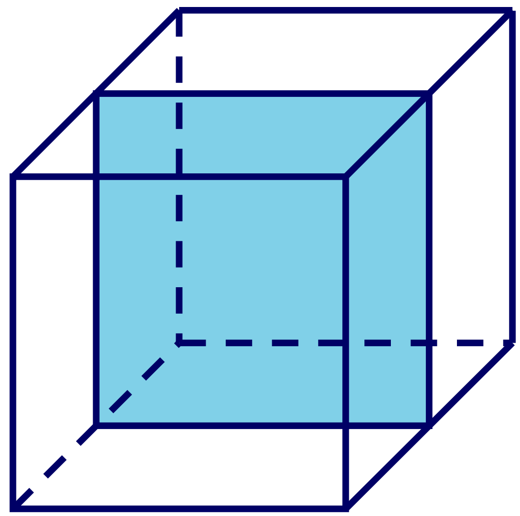


What is the shaded area called?

This shaded area is called a **plane of symmetry**.

Reflection symmetry in 3-D shapes

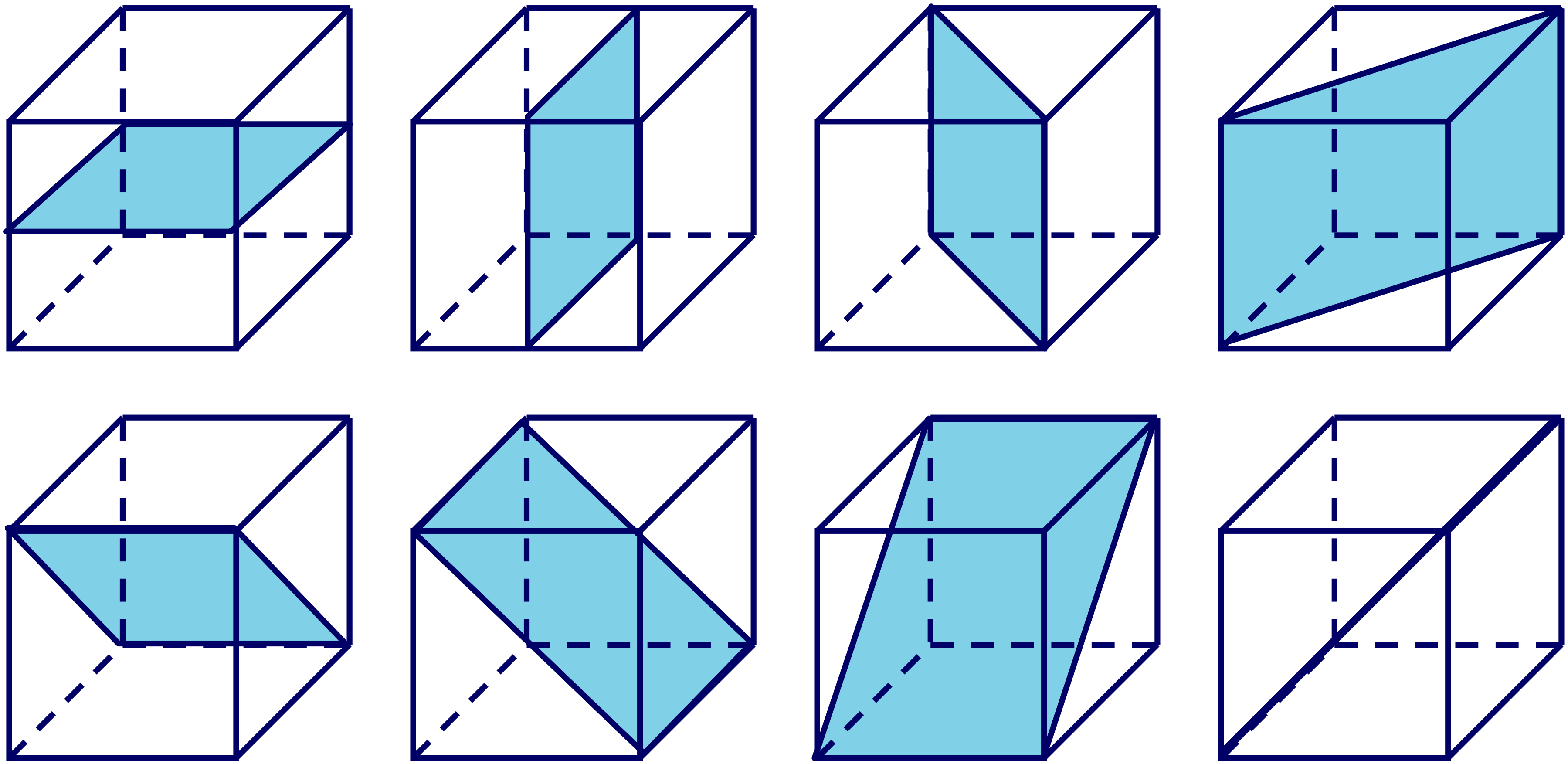
How many planes of symmetry does a cube have?



We can divide the cube into two symmetrical parts here.

Reflection symmetry in 3-D shapes

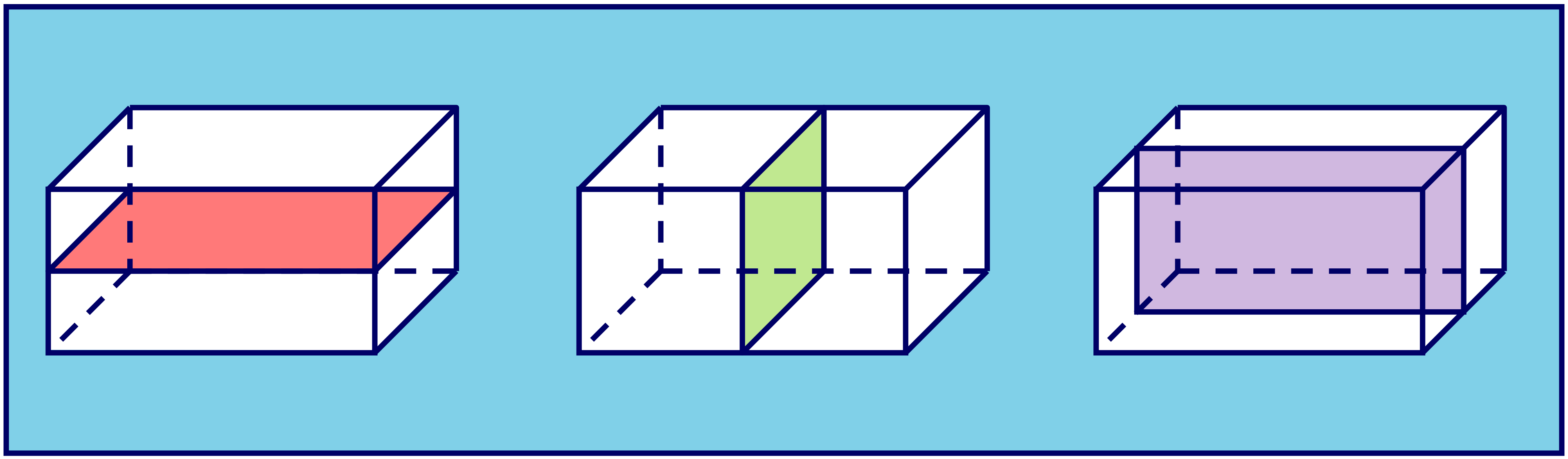
We can draw the other eight planes of symmetry for a cube, as follows:



Reflection symmetry in 3-D shapes

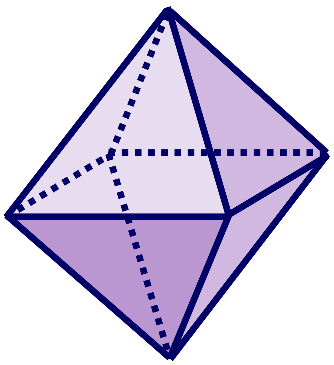
How many planes of symmetry does a cuboid have?

A cuboid has three planes of symmetry.

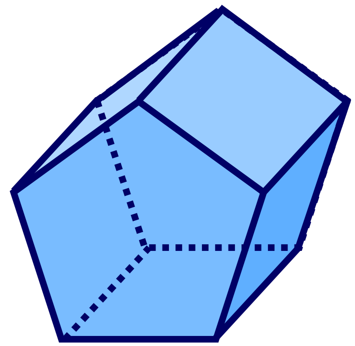


Reflection symmetry in 3-D shapes

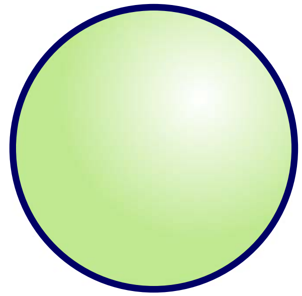
How many planes of symmetry do the following solids have?



A regular octahedron



A regular pentagonal prism

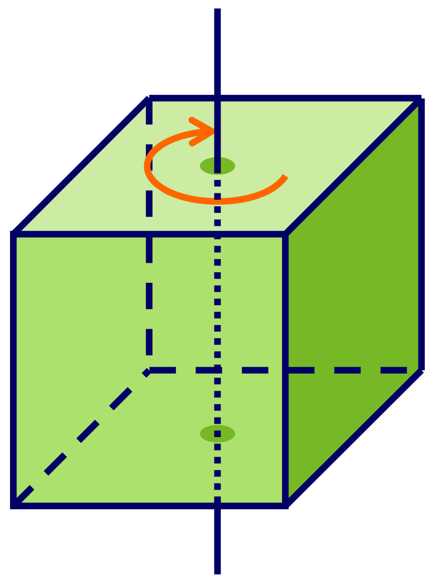


A sphere

Explain why any right prism will always have at least one plane of symmetry.

Rotational symmetry in 3-D shapes

Does a cube have rotational symmetry?



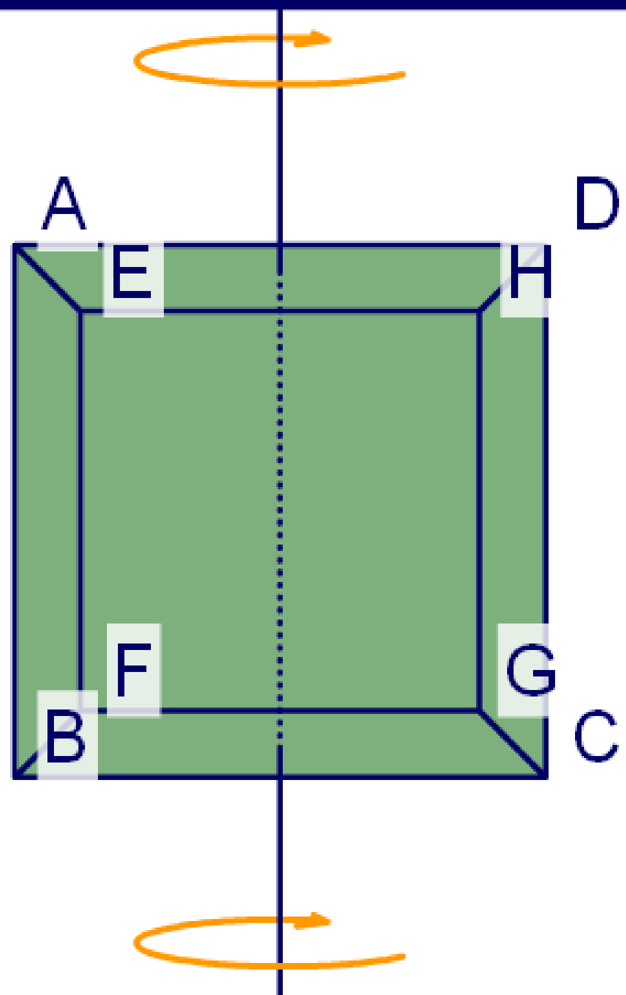
We can draw a line through the centre of the cube, here.

This line is called an **axis of symmetry**.

What is the order of rotational symmetry about this axis?

How many axes of symmetry does a cube have?

Rotational symmetry in 3-D shapes



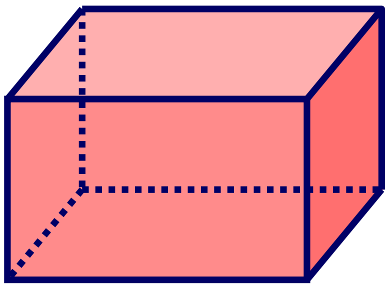
3 axes of rotational symmetry through its opposite faces

6 axes of rotational symmetry through its opposite edges

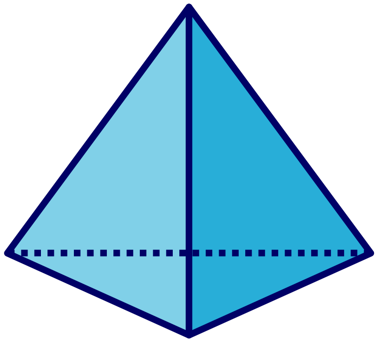
4 axes of rotational symmetry through its opposite vertices

Rotational symmetry in 3-D shapes

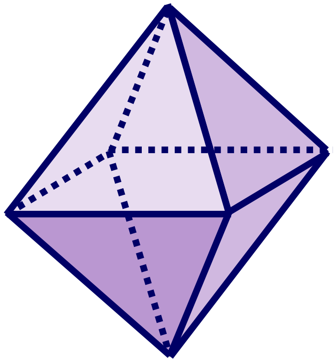
How many axes of symmetry do each of the following shapes have?



A cuboid



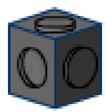
A tetrahedron

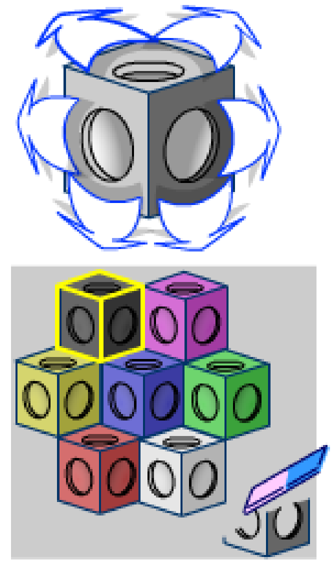


An octahedron

What is the order of rotational symmetry about each axis?

Symmetry in 3-D shapes





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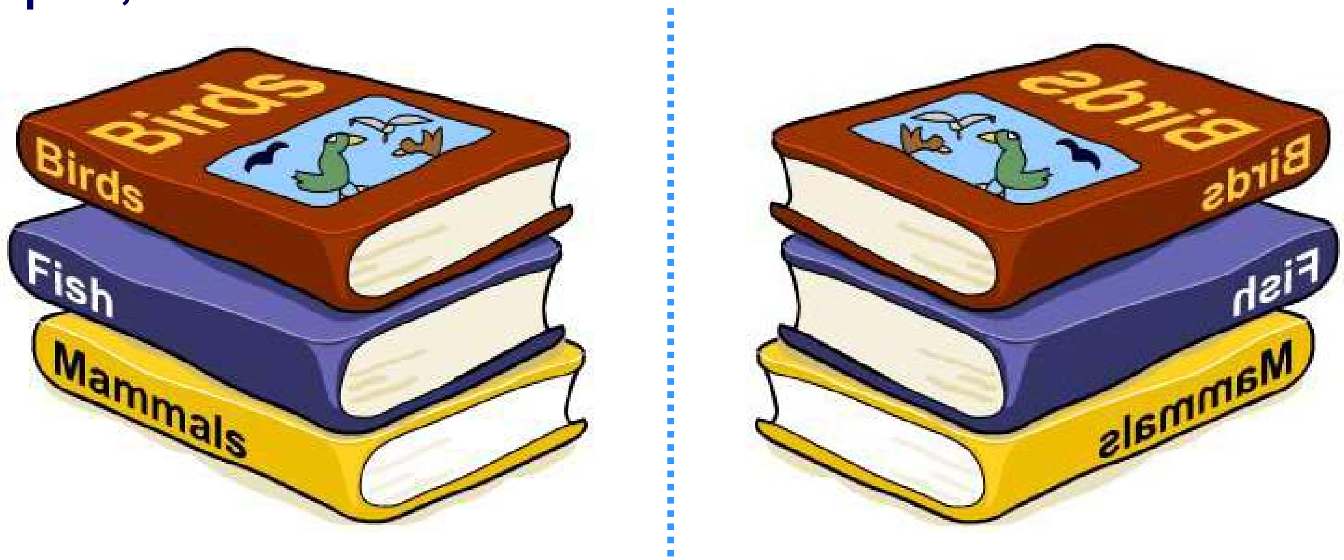
Reflection

An object can be **reflected** in a **mirror line** or **axis of reflection** to produce an image of the object.

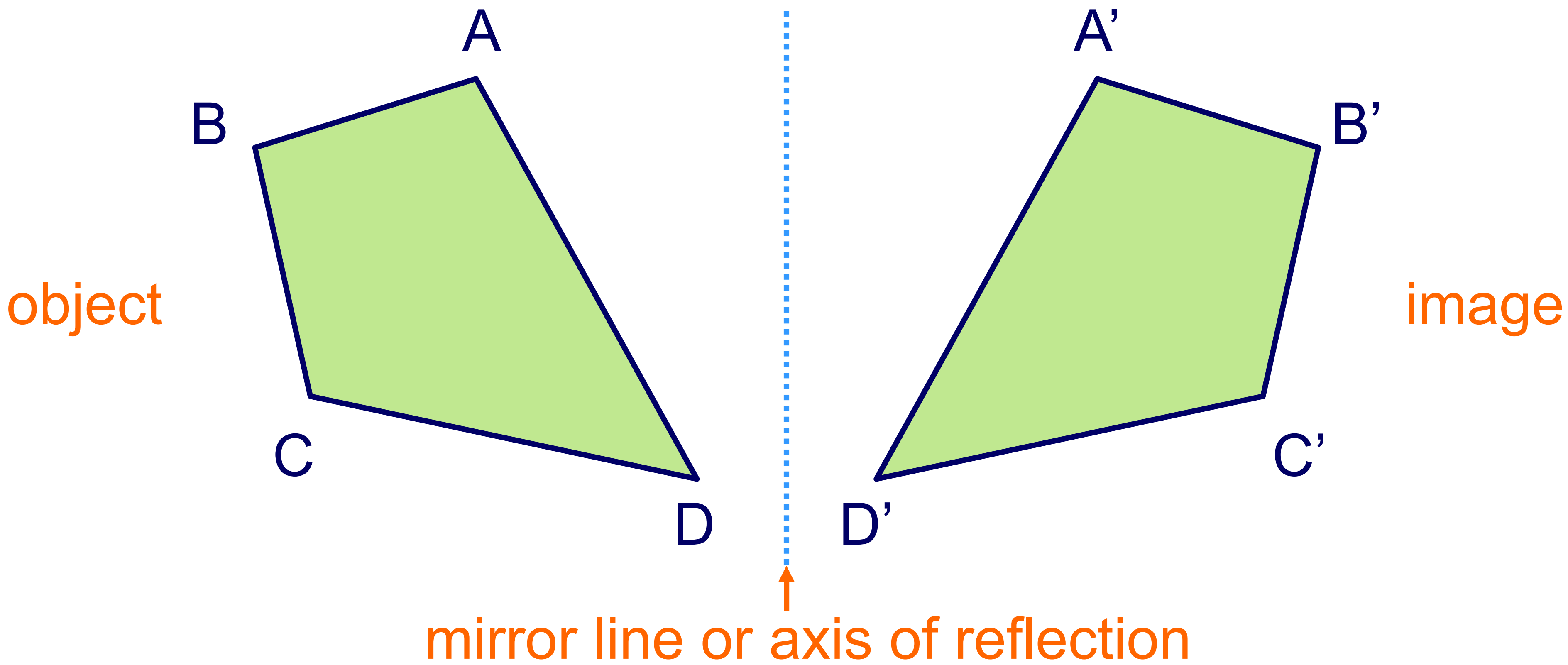
For example,



Each point in the image must be the same distance from the mirror line as the corresponding point of the original object.

Reflecting shapes

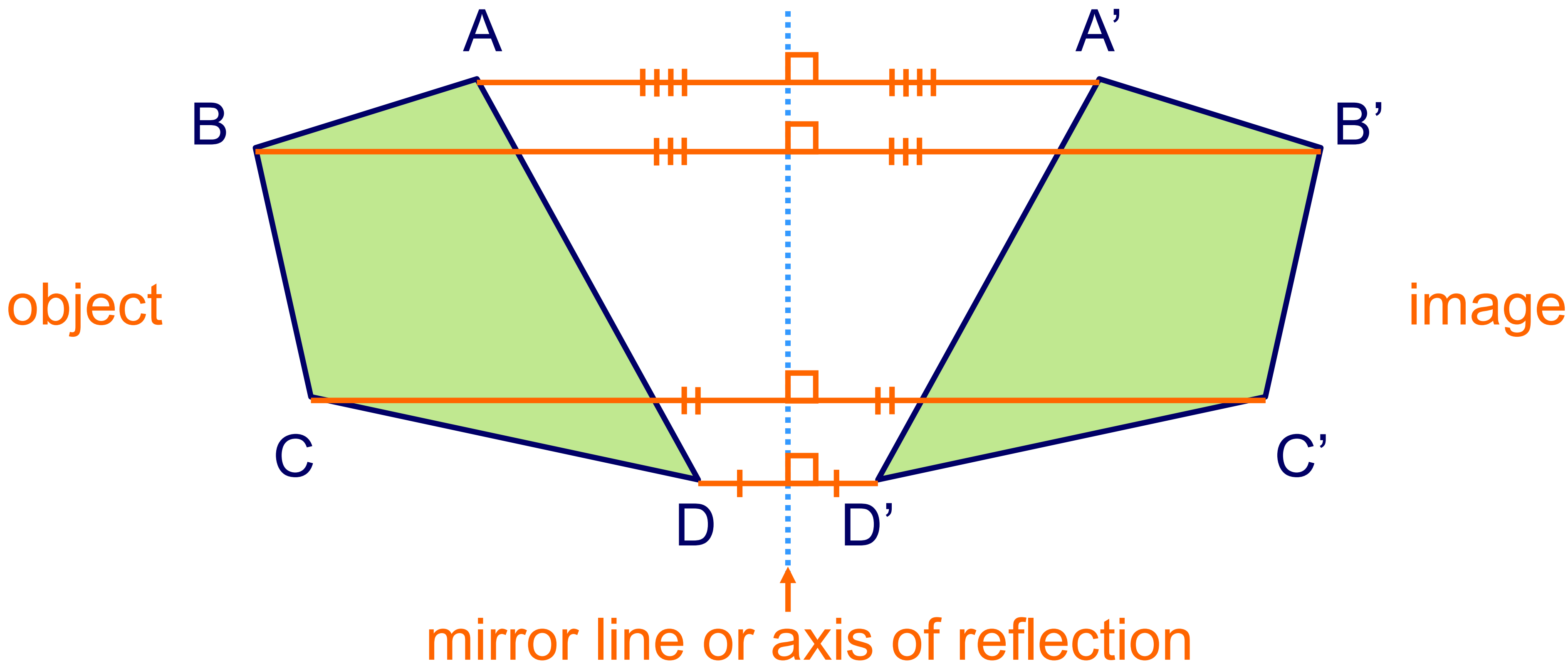
If we reflect the quadrilateral ABCD in a mirror line, we label the image quadrilateral A'B'C'D'.



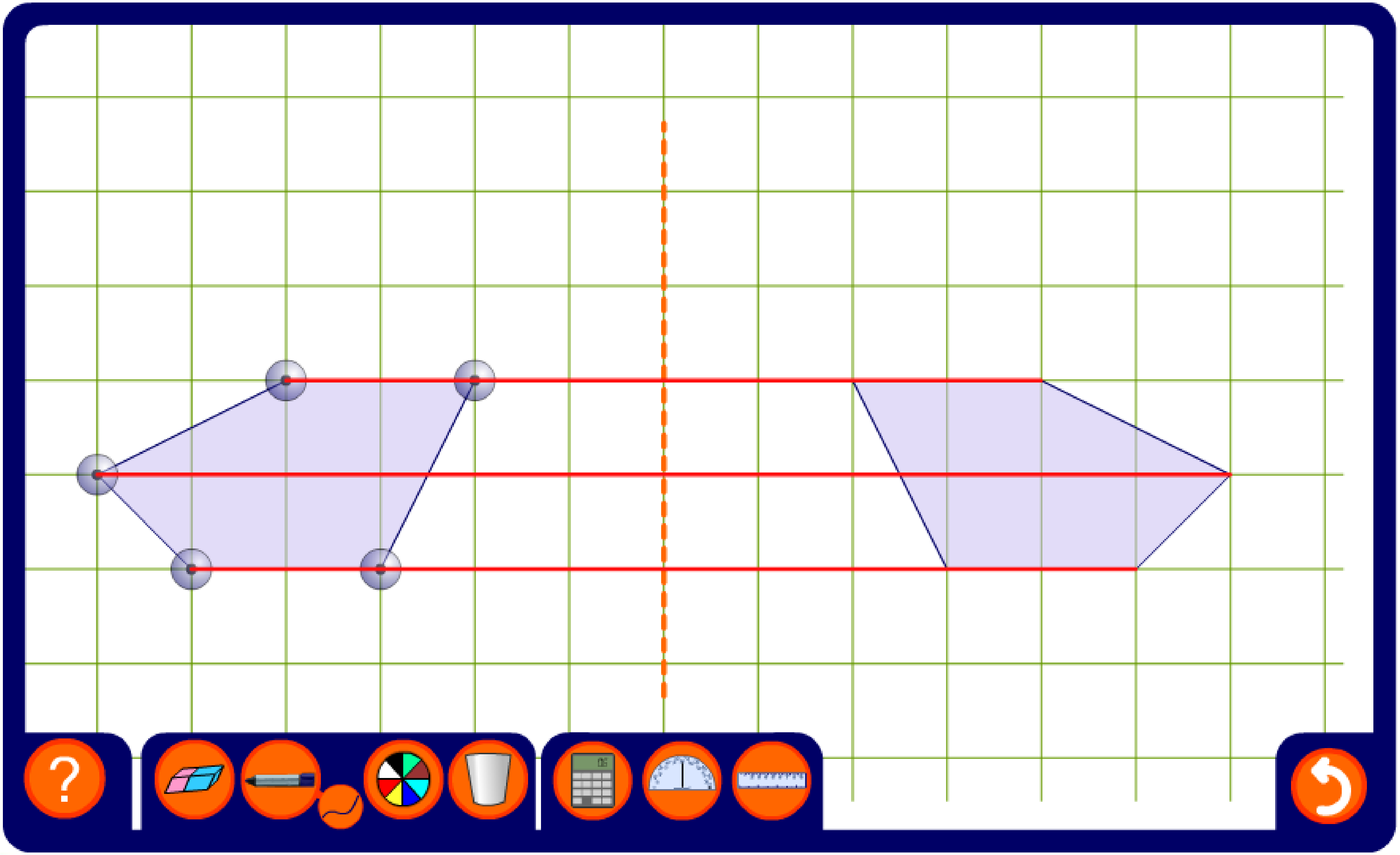
The image is **congruent** to the original shape.

Reflecting shapes

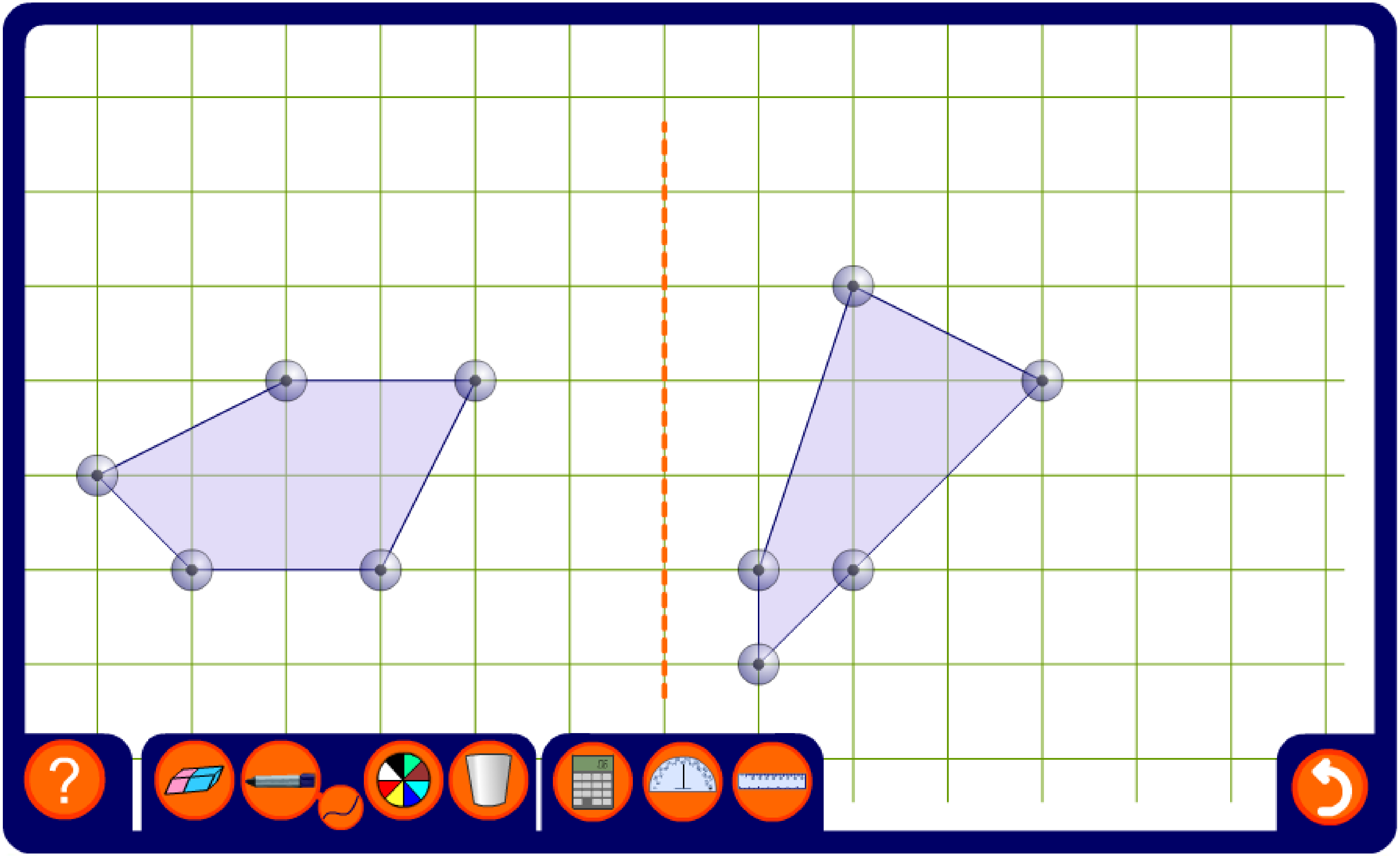
If we draw a line from any point on the object to its image, the line forms a **perpendicular bisector** to the mirror line.



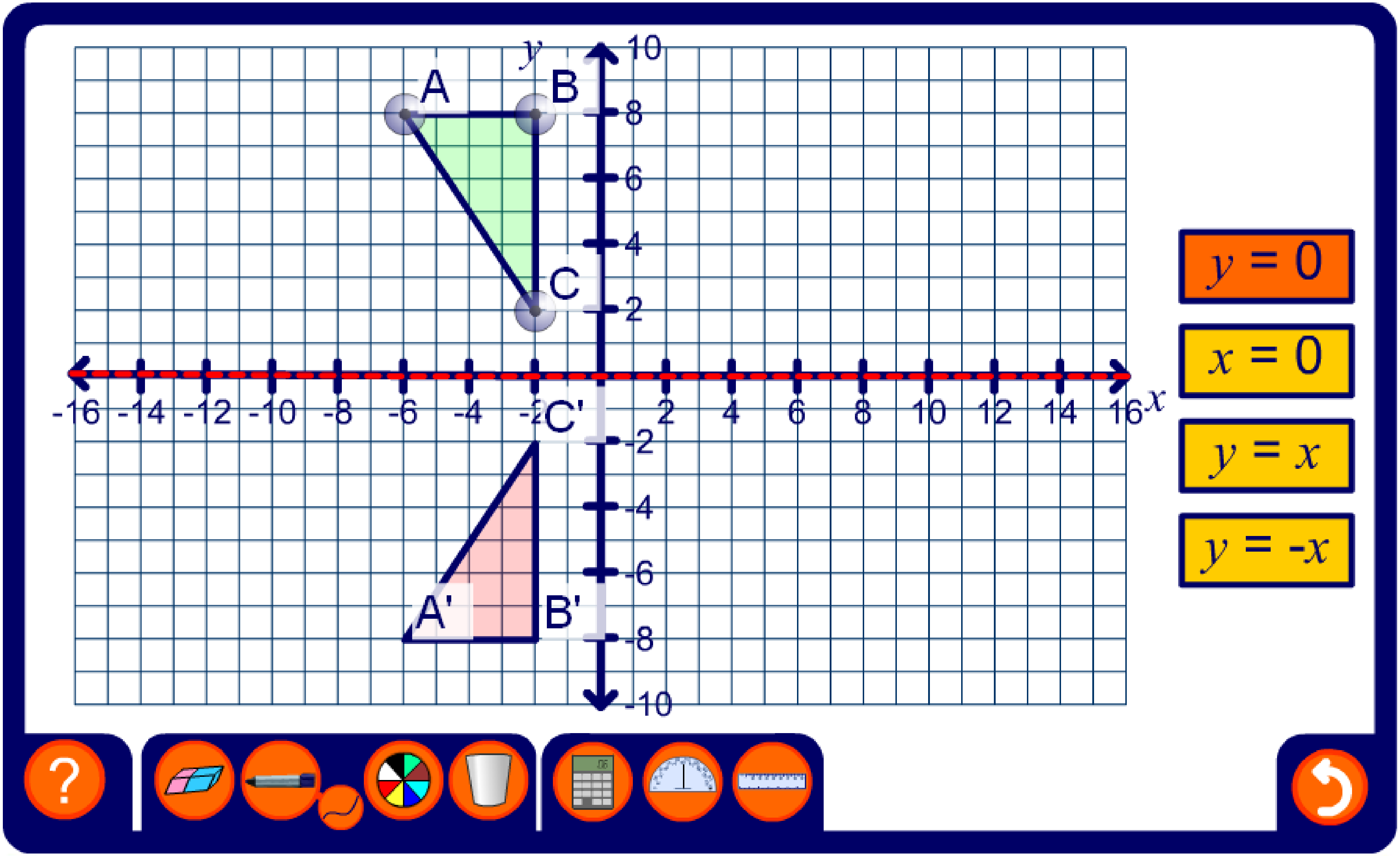
Reflecting shapes



Reflect this shape

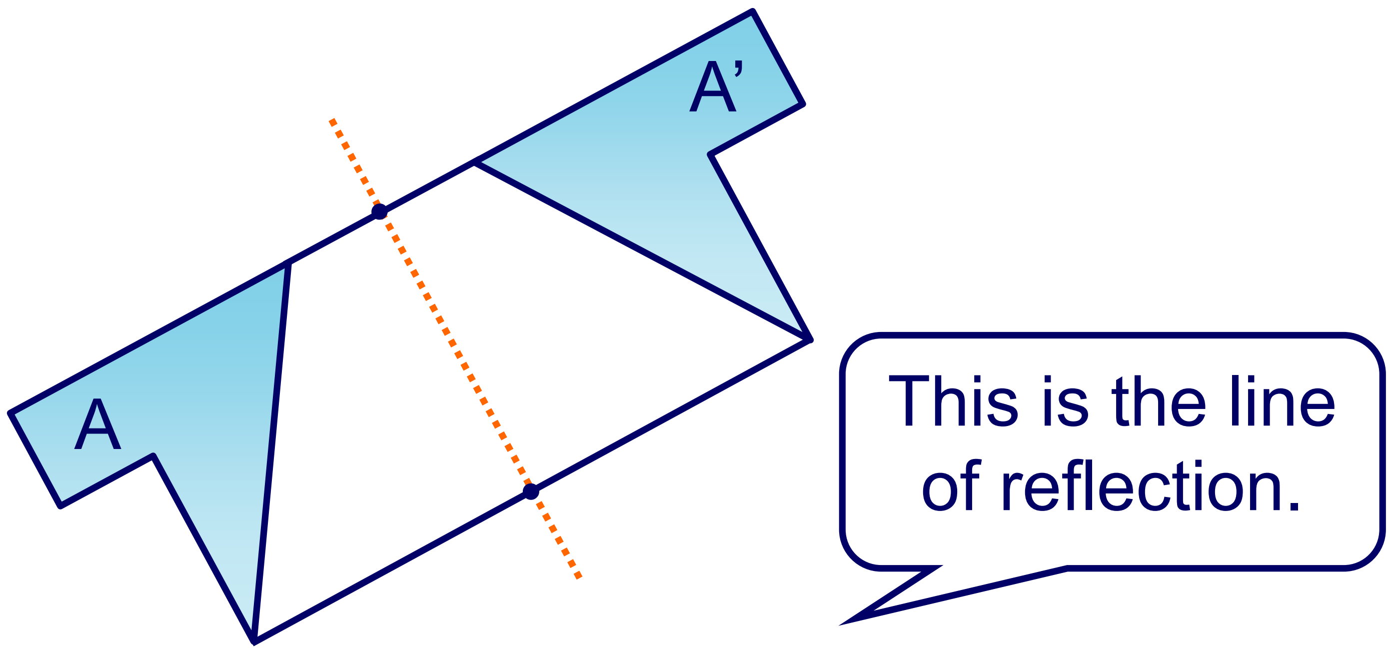


Reflection on a coordinate grid



Finding a line of reflection

Construct the line that reflects shape A onto its image A’.



- Draw lines from any two vertices to their images.
- Mark on the mid-point of each line.
- Draw a line through the mid points.

Contents

S6 Transformations

Describing a rotation

A **rotation** occurs when an object is turned around a fixed point.

To describe a rotation we need to know three things:

- The **angle** of the rotation.

  For example,

  ½ turn = 180°   ¼ turn = 90°   ¾ turn = 270°

- The **direction** of the rotation.

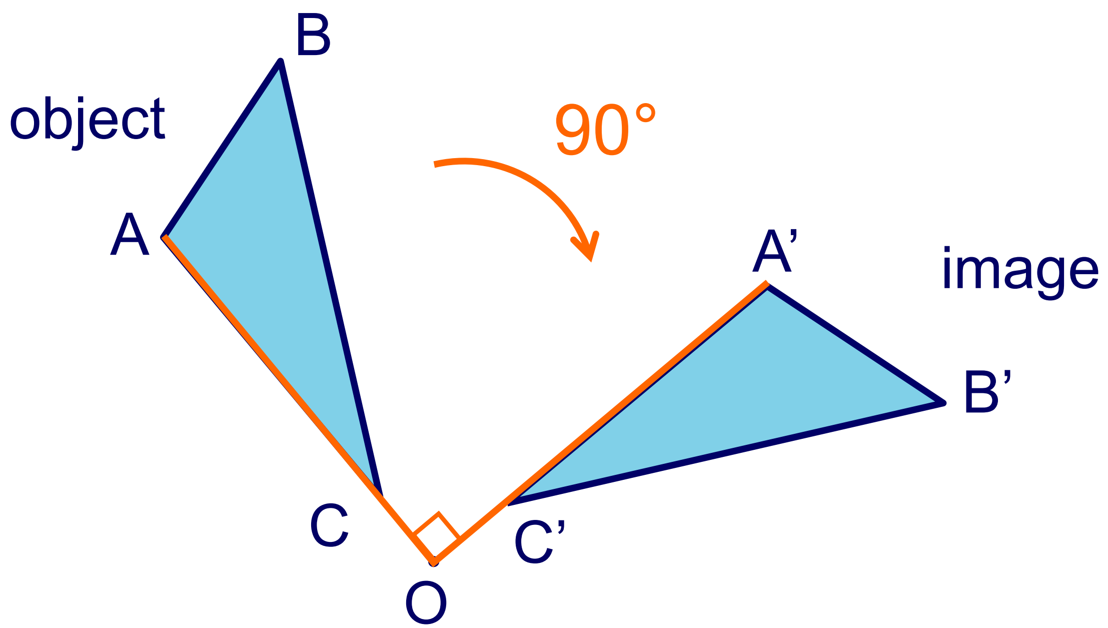
  For example, clockwise or anticlockwise.

- The **centre** of rotation.

  This is the fixed point about which an object moves.

Rotating shapes

If we rotate triangle ABC 90° clockwise about point O the following **image** is produced:



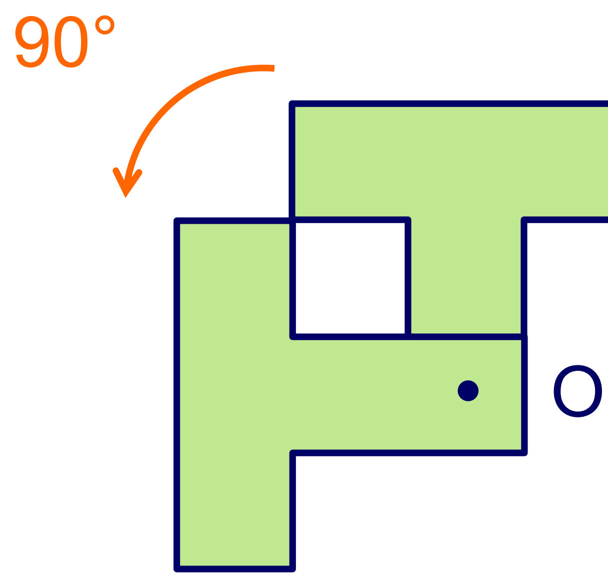
A is mapped onto A', B is mapped onto B' and C is mapped onto C'.

The image triangle A'B'C' is **congruent** to triangle ABC.

Rotating shapes

The centre of rotation can also be inside the shape.

For example,



Rotating this shape 90° anticlockwise about point O produces the following image.

Determining the direction of a rotation

Sometimes the direction of the rotation is not given.

If this is the case then we use the following rules:

A **positive** rotation is an **anticlockwise** rotation.

A **negative** rotation is an **clockwise** rotation.

For example,

A rotation of 60° = an anticlockwise rotation of 60°

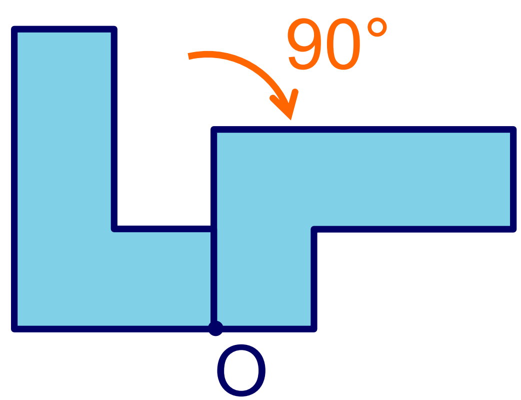
A rotation of –90° = an clockwise rotation of 90°

Explain why a rotation of 120° is equivalent to a rotation of –240°.

Inverse rotations

The inverse of a rotation maps the image that has been rotated back onto the original object.

For example, the following shape is rotated 90° clockwise about point O.



What is the inverse of this rotation?

Either, a 90° rotation anticlockwise,

or a 270° rotation clockwise.

Inverse rotations

The inverse of any rotation is either

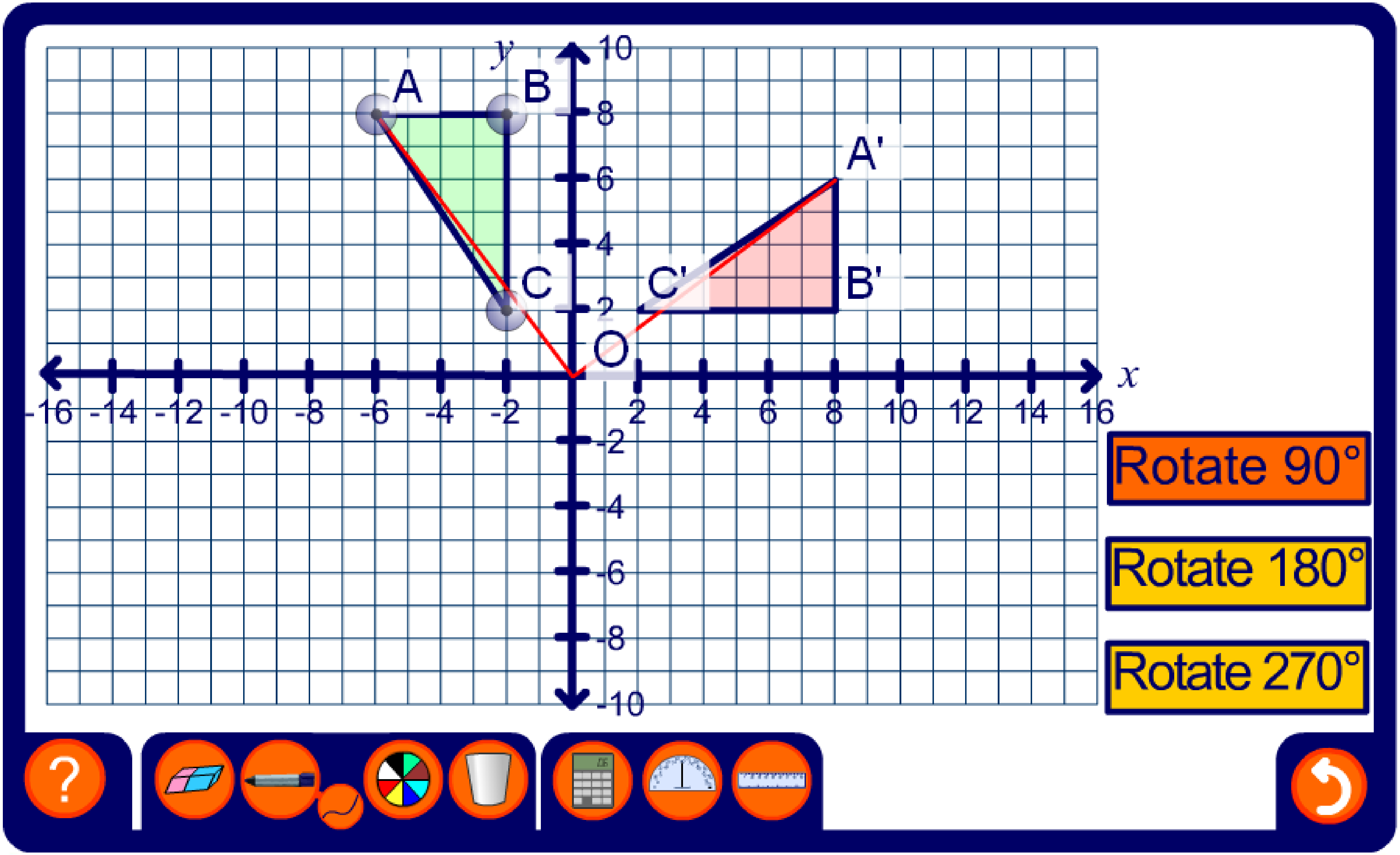
- A rotation of the *same* size, about the same point, but in the *opposite* direction, or
- A rotation in the *same* direction, about the same point, but such that the two rotations have a sum of 360°.

What is the inverse of a –70° rotation?

Either, a 70° rotation,

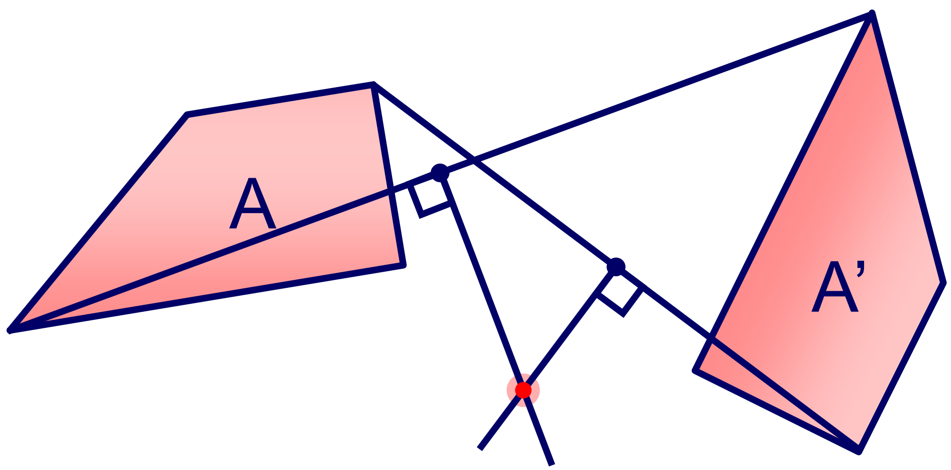
or a –290° rotation.

Rotations on a coordinate grid



Finding the centre of rotation

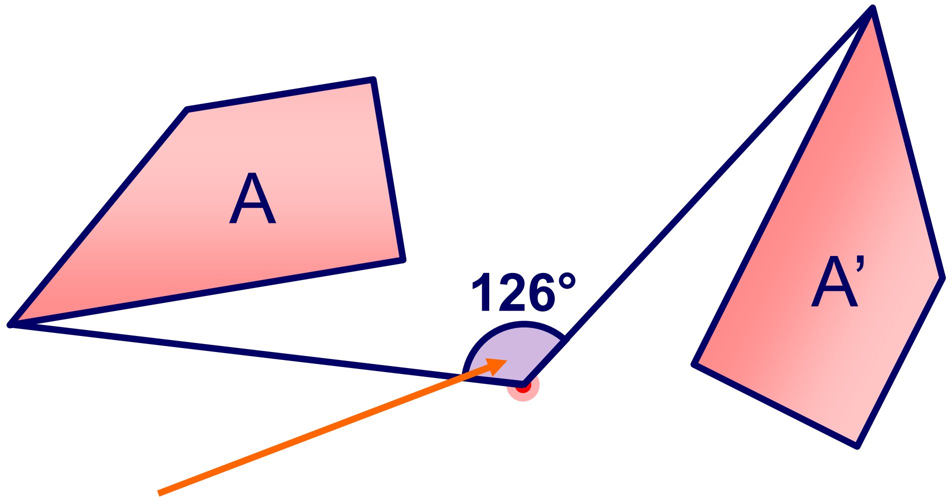
Find the point about which A is rotated onto its image A’.



- Draw lines from any two vertices to their images.
- Mark on the mid-point of each line.
- Draw perpendicular lines from each of the mid-points.
- The point where these lines meet is the centre of rotation.

Finding the angle of rotation

Find the angle of rotation from A to its image A'.



This is the angle of rotation

- Join one vertex and its image to the centre of rotation.
- Use a protractor to measure the angle of rotation.

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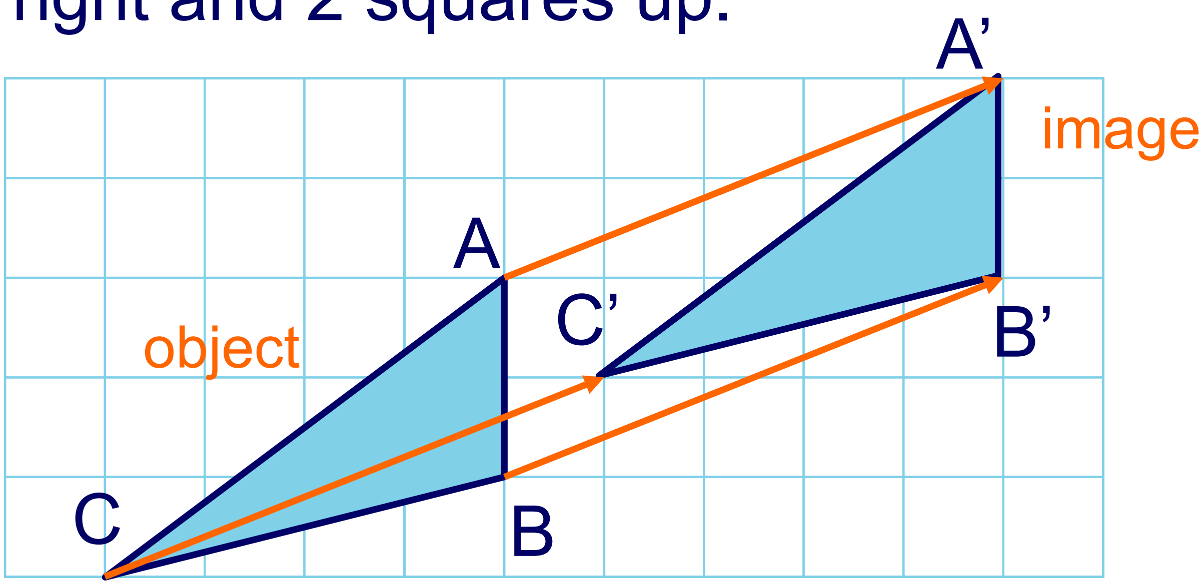
Translation

When an object is moved in a straight line in a given direction, we say that it has been **translated**.

For example, we can translate triangle ABC 5 squares to the right and 2 squares up.



Every point in the shape moves the same distance in the same direction.

We can describe this translation using the vector $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

Describing translations

When we describe a translation we always give the movement left or right first followed by the movement up or down.

We can also describe translations using **vectors**.

For example, the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ describes a translation 3 right and 4 down.

As with coordinates, positive numbers indicate movements up or to the right and negative numbers are used for movements down or to the left.

One more way of describing a translation is to give the direction as an angle and the distance as a length.

Describing translations

When a shape is translated the image is **congruent** to the original.

The orientations of the original shape and its image are the same.

An **inverse translation** maps the image that has been translated back onto the original object.

What is the inverse of a translation 7 units to the left and 3 units down?

The inverse is an equal move in the opposite direction.

That is, 7 units right and 3 units up.

Inverse translations

What is the inverse of the translation $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$?

This vector translates the object 3 units to the left and 4 units up.

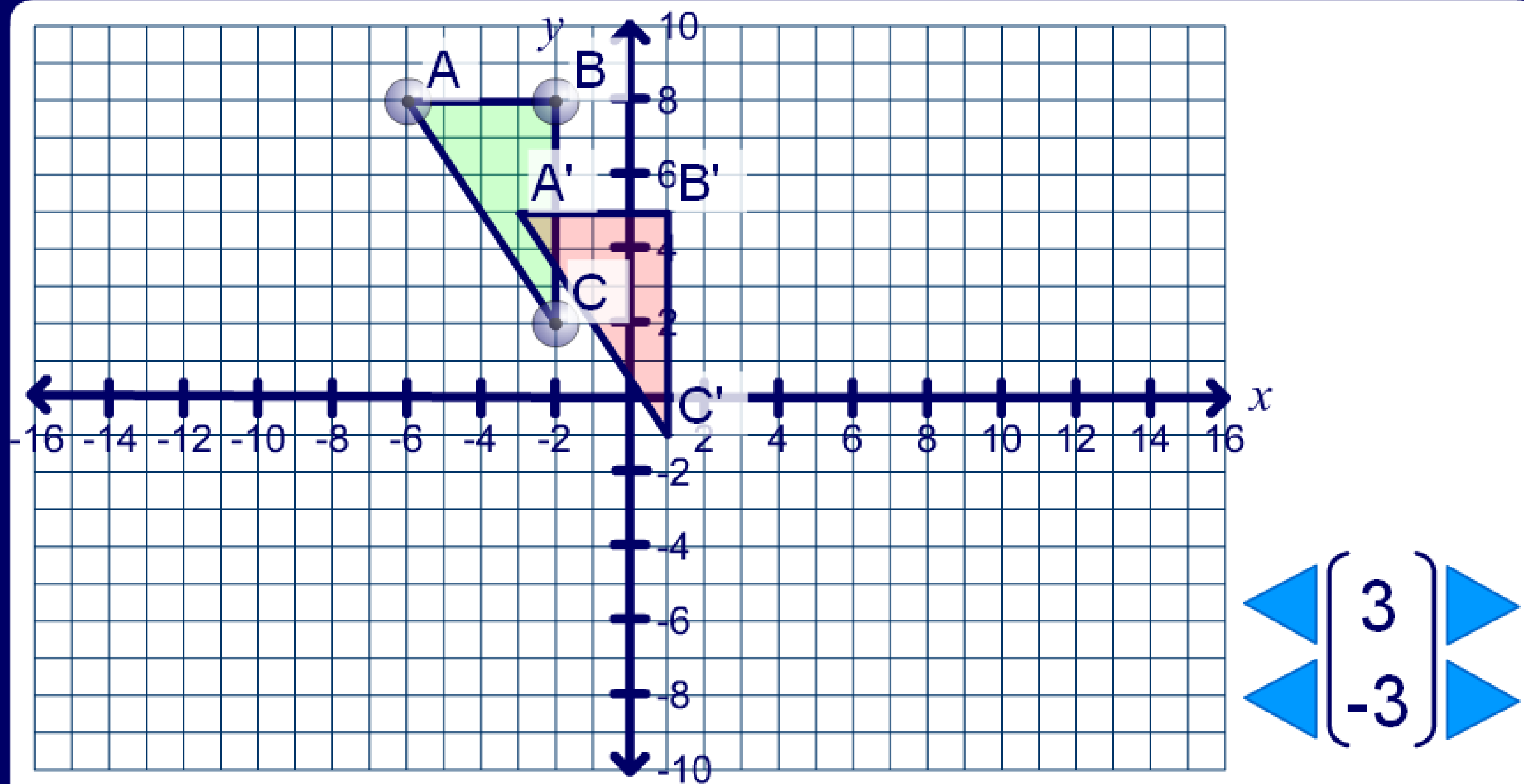
The inverse of this translation is a movement 3 units to the right and 4 units down.

The inverse of $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ is $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

In general,

The inverse of the translation $\begin{pmatrix} a \\ b \end{pmatrix}$ is $\begin{pmatrix} -a \\ -b \end{pmatrix}$

Translations on a coordinate grid



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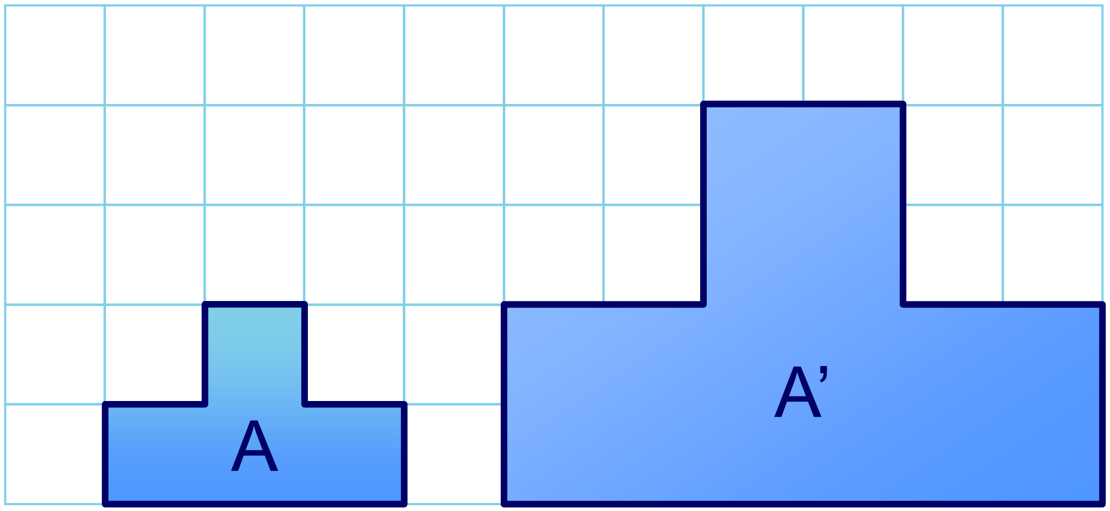
Enlargement



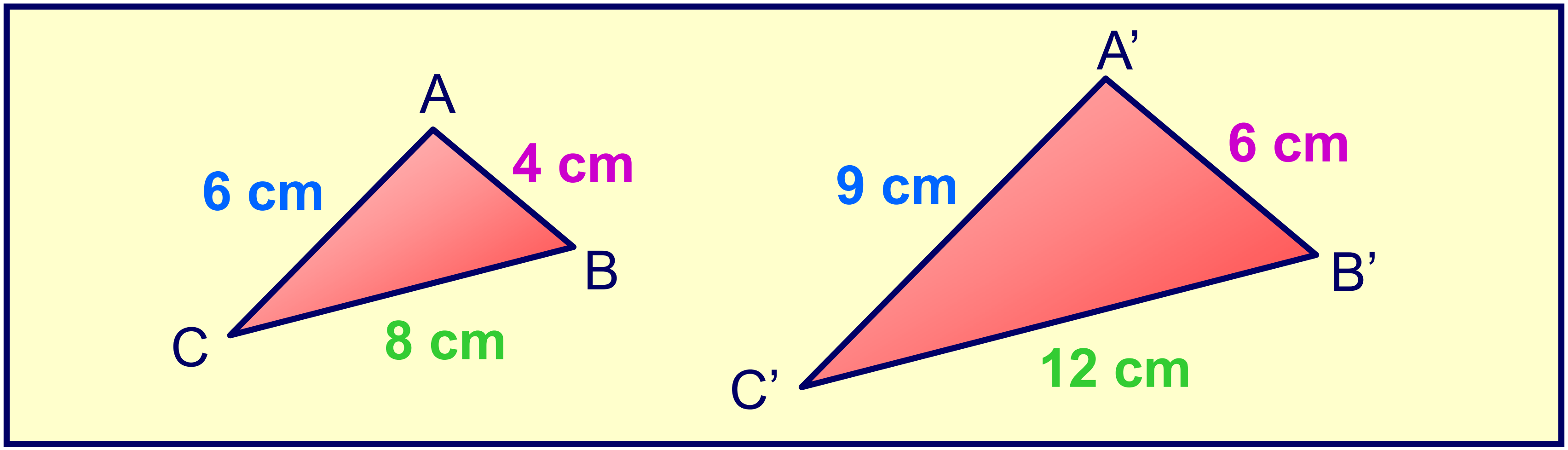
Shape A' is an **enlargement** of shape A.

The length of each side in shape A' is 2 × the length of each side in shape A.

We say that shape A has been enlarged by **scale factor** 2.

Enlargement

When a shape is **enlarged**, any length in the image divided by the corresponding length in the original shape (the object) is equal to the scale factor.



$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = \text{the scale factor}$$

$$\frac{6}{4} = \frac{12}{8} = \frac{9}{6} = 1.5$$

Congruence and similarity

Is an enlargement congruent to the original object?

Remember, if two shapes are congruent they are the same shape and size. Corresponding lengths and angles are equal.

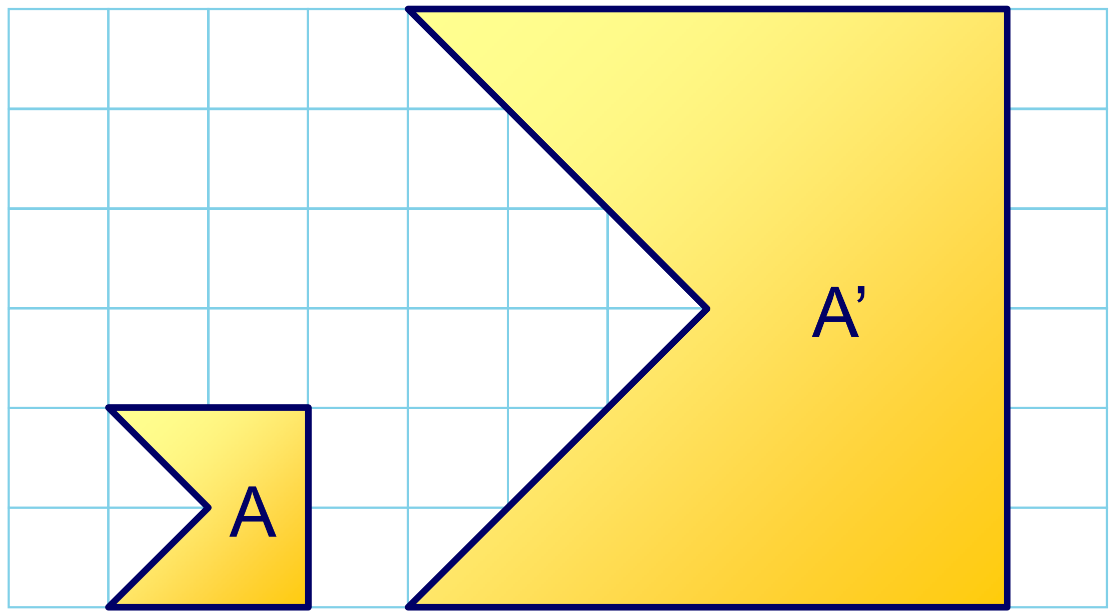
In an enlarged shape the corresponding angles are the same but the lengths are different. The object and its image are **similar**.

Reflections, rotations and translations produce images that are **congruent** to the original shape.

Enlargements produce images that are **similar** to the original shape.

Find the scale factor

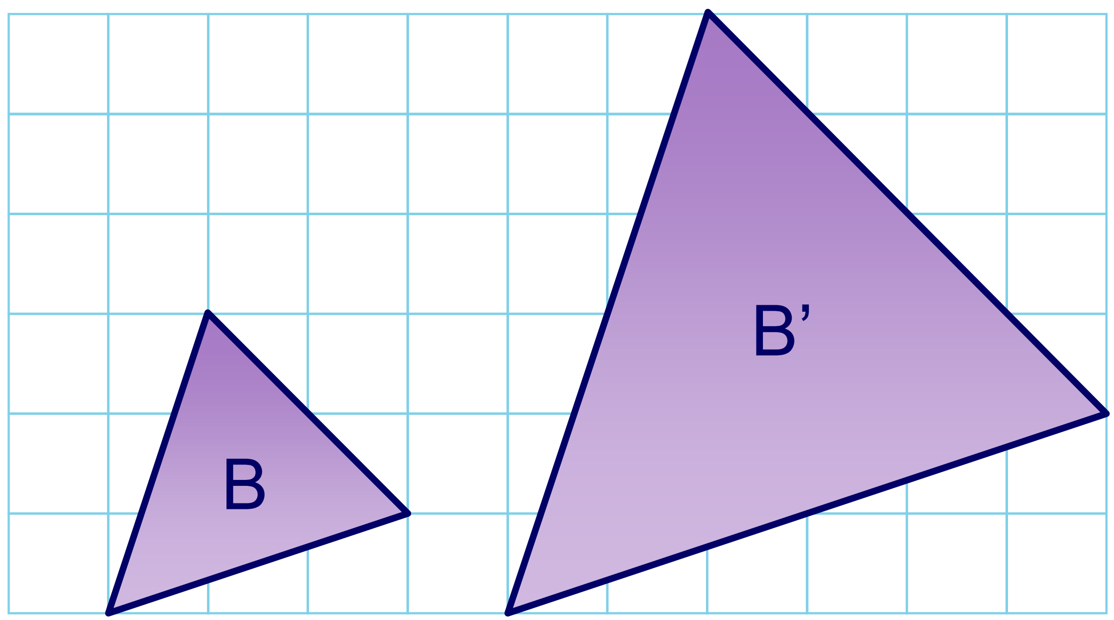
What is the scale factor for the following enlargements?



Scale factor = 3

Find the scale factor

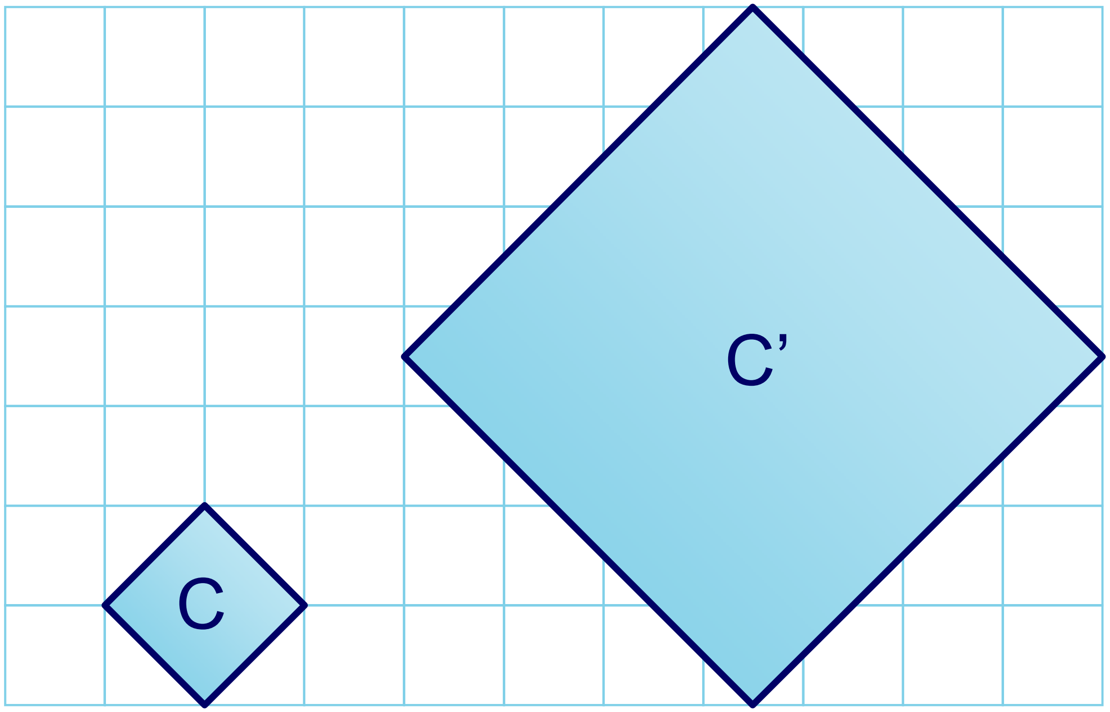
What is the scale factor for the following enlargements?



Scale factor = 2

Find the scale factor

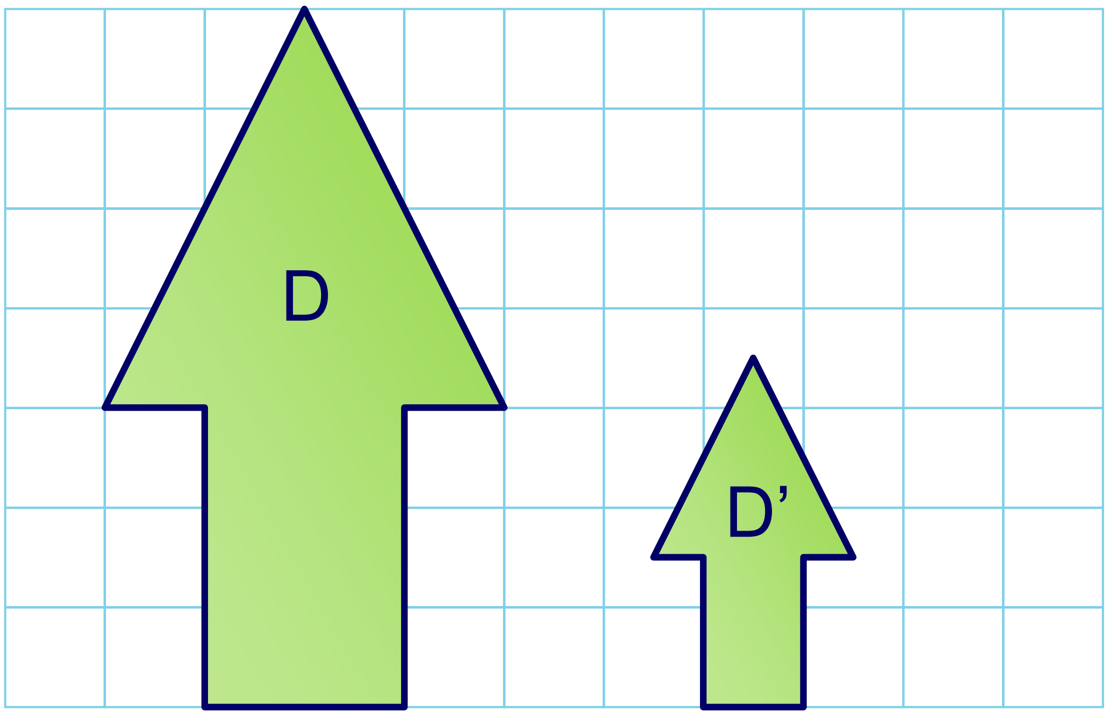
What is the scale factor for the following enlargements?



Scale factor = 3.5

Find the scale factor

What is the scale factor for the following enlargements?



Scale factor = 0.5

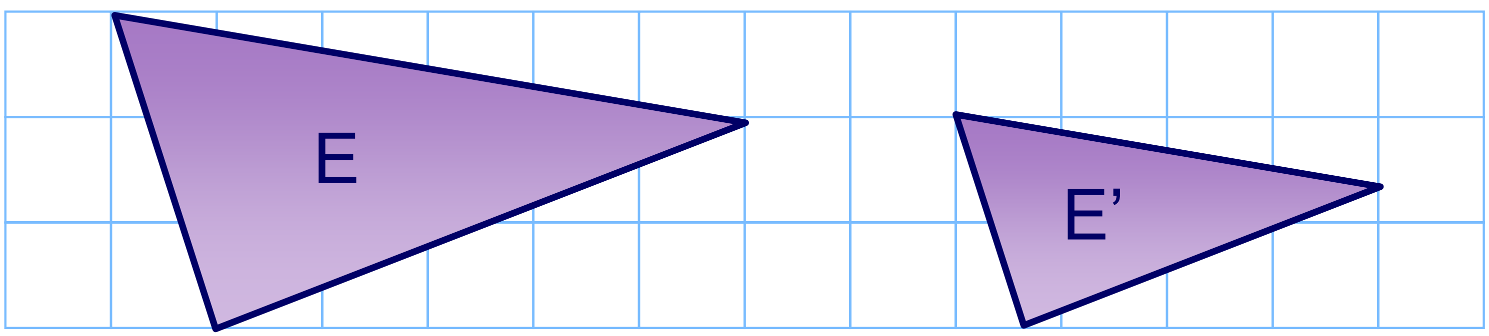
Scale factors between 0 and 1

What happens when the scale factor for an enlargement is between 1 and 0?

When the scale factor is between 1 and 0, the enlargement will be *smaller* than the original object.

Although there is a reduction in size, the transformation is still called an enlargement.

For example,

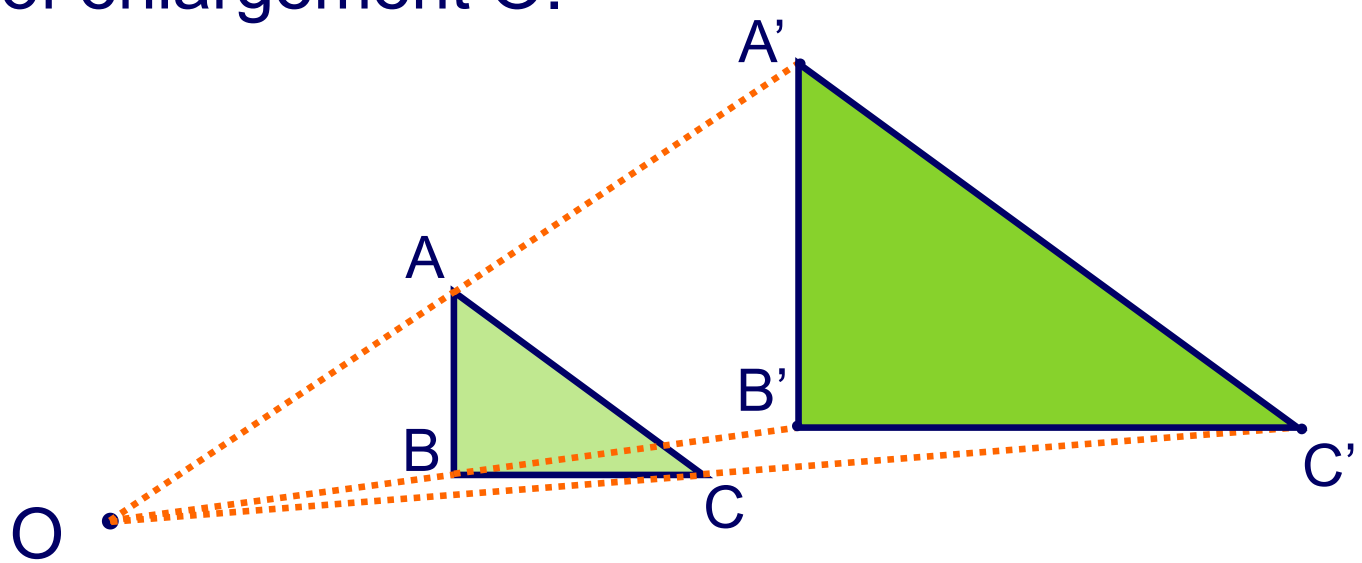


Scale factor = $\frac{2}{3}$

The centre of enlargement

To define an enlargement we must be given a **scale factor** and a **centre of enlargement**.

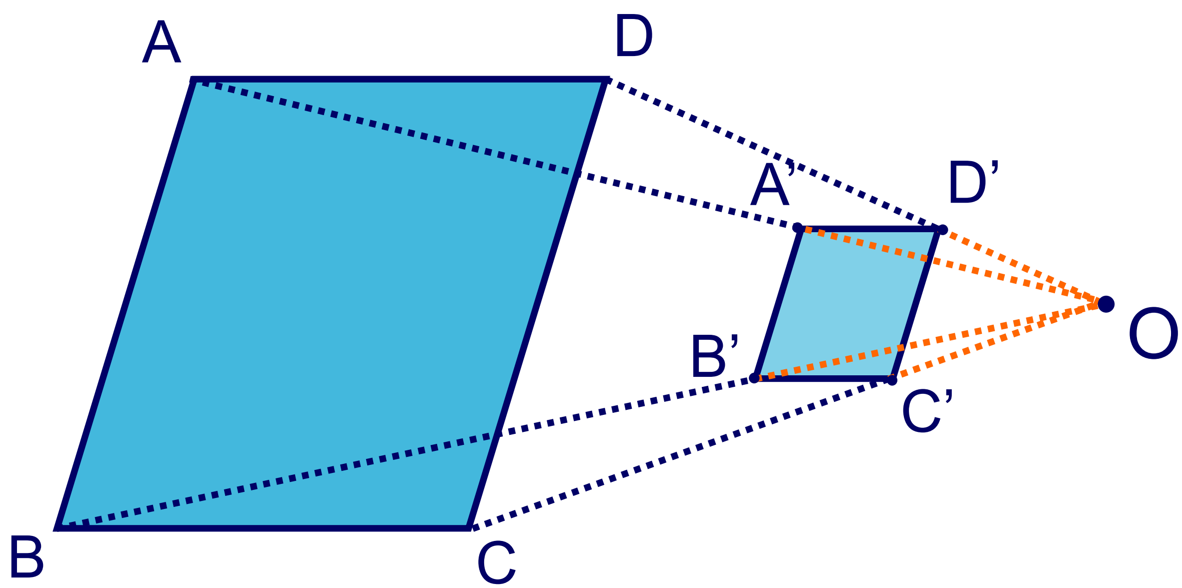
For example, enlarge triangle ABC by a scale factor of 2 from the centre of enlargement O.



$$\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC} = 2$$

The centre of enlargement

Enlarge quadrilateral ABCD by a scale factor of $\frac{1}{3}$ from the centre of enlargement O.

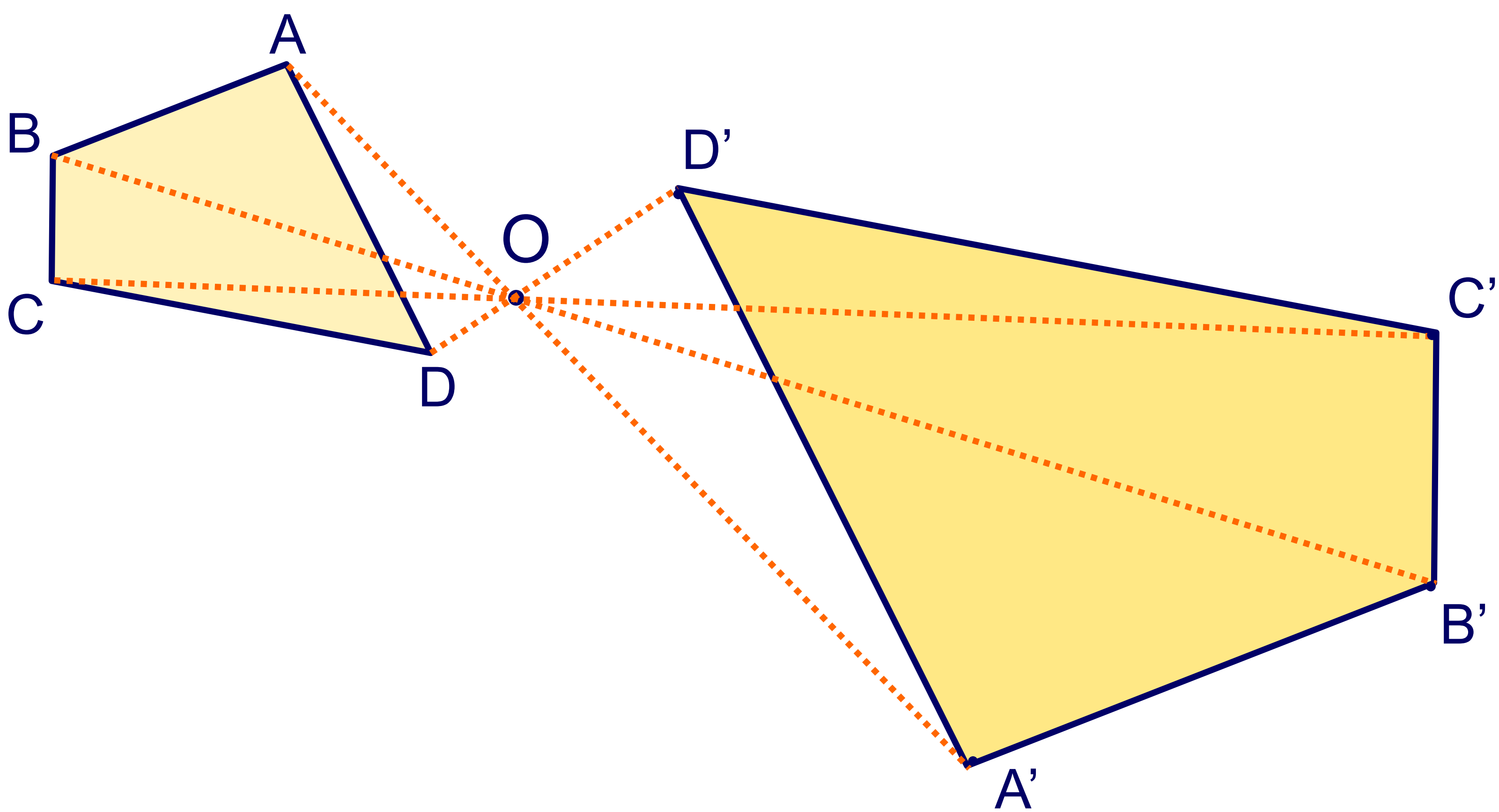


$$\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC} = \frac{OD'}{OE} = \frac{1}{3}$$

Negative scale factors

When the scale factor is negative the enlargement is on the opposite side of the centre of enlargement.

This example shows the shape ABCD enlarged by a scale factor of –2 about the centre of enlargement O.



Inverse enlargements

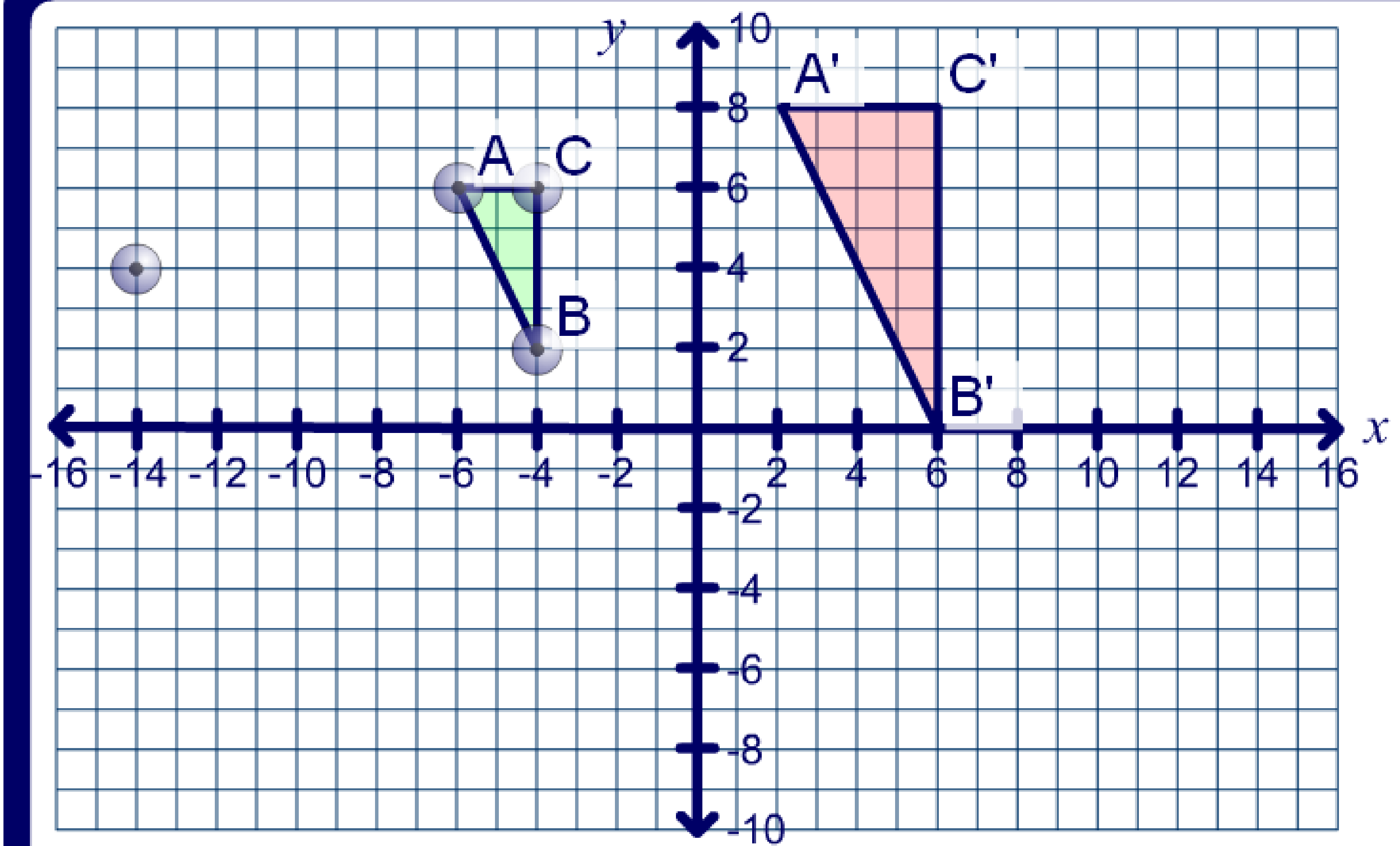
An **inverse enlargement** maps the image that has been enlarged back onto the original object.

In general, the inverse of an enlargement with a scale factor k is an enlargement with a scale factor $\frac{1}{k}$ from the same centre of enlargement.

What is the inverse of an enlargement of 0.2 from the point (1, 3)?

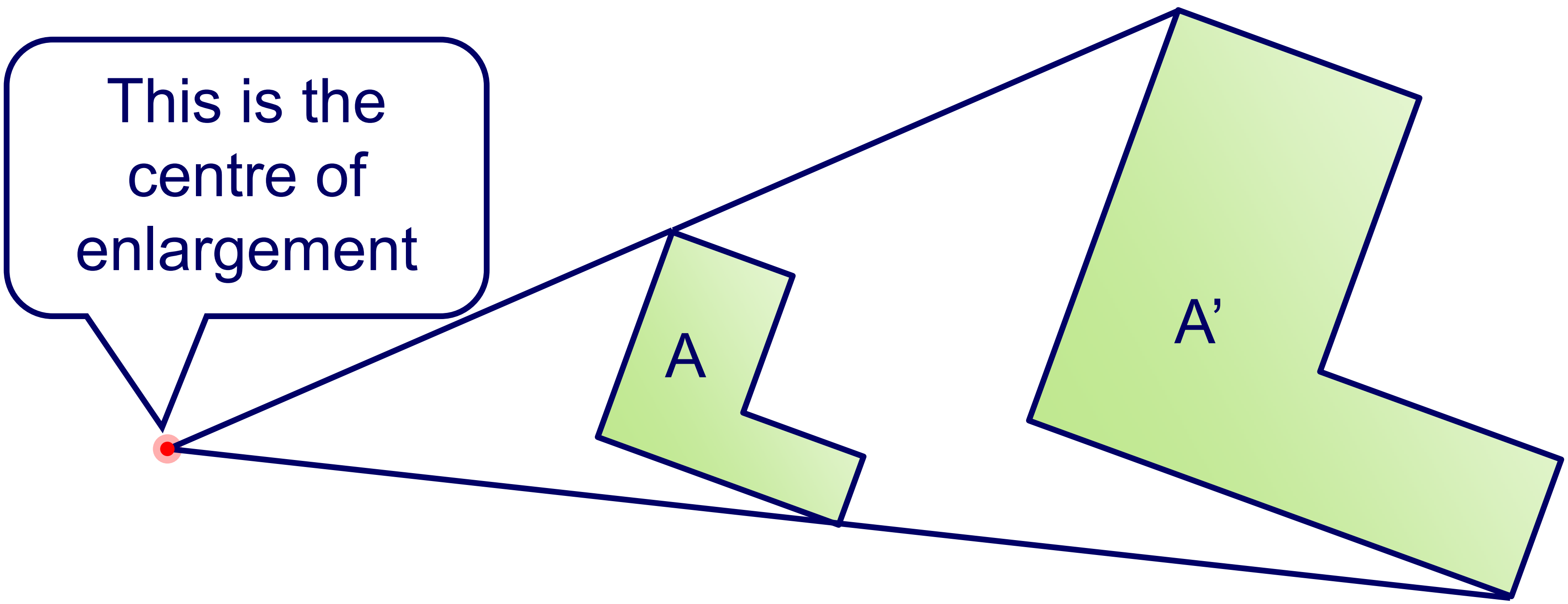
The inverse of an enlargement of 0.2 from the point (1, 3) is an enlargement of 5 from the point (1, 3).

Enlargement on a coordinate grid



Finding the centre of enlargement

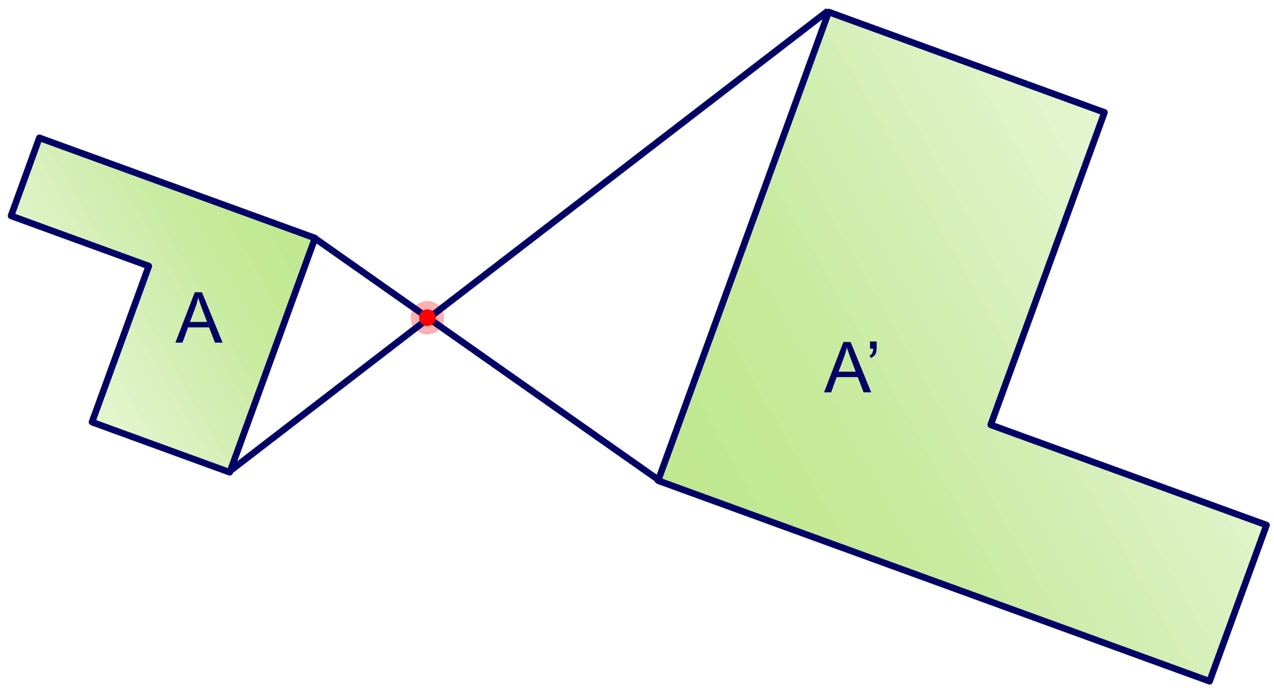
Find the centre the enlargement of A onto A'.



- Draw lines from any two vertices to their images.
- Extend the lines until they meet at a point.

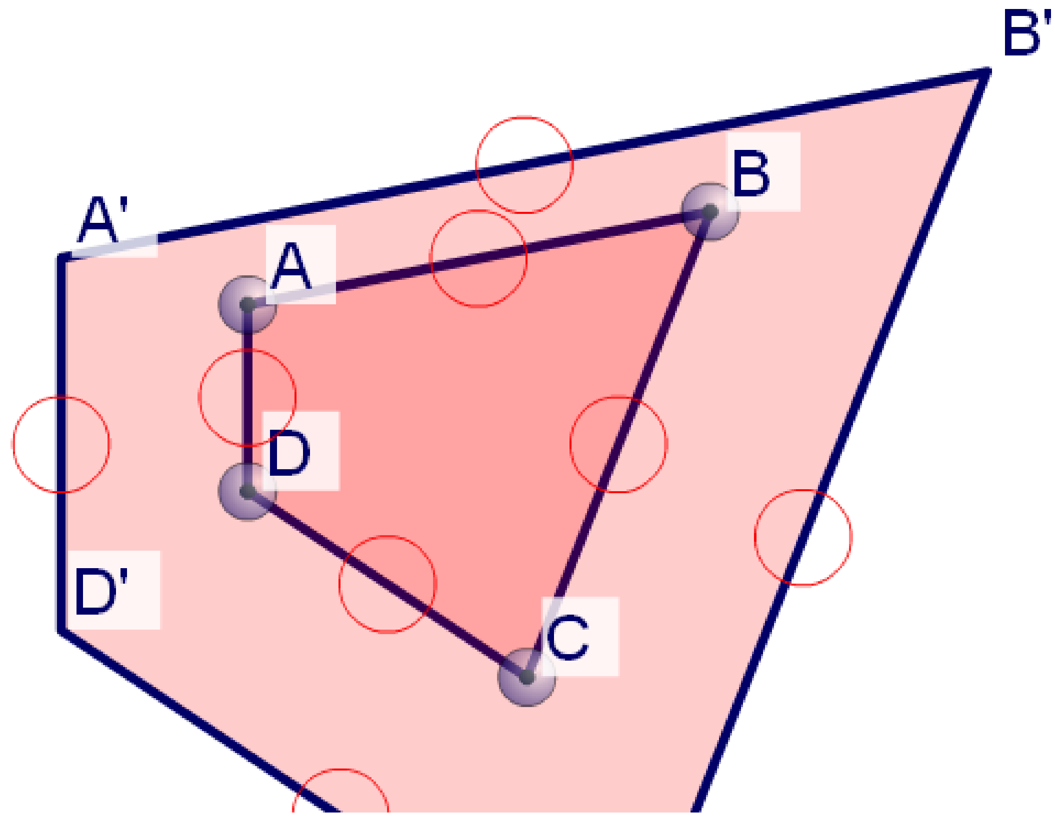
Finding the centre of enlargement

Find the centre the enlargement of A onto A'.



- Draw lines from any two vertices to their images.
- When the enlargement is negative the centre of enlargement is at the point where the lines intersect.

Describing enlargements



Positive

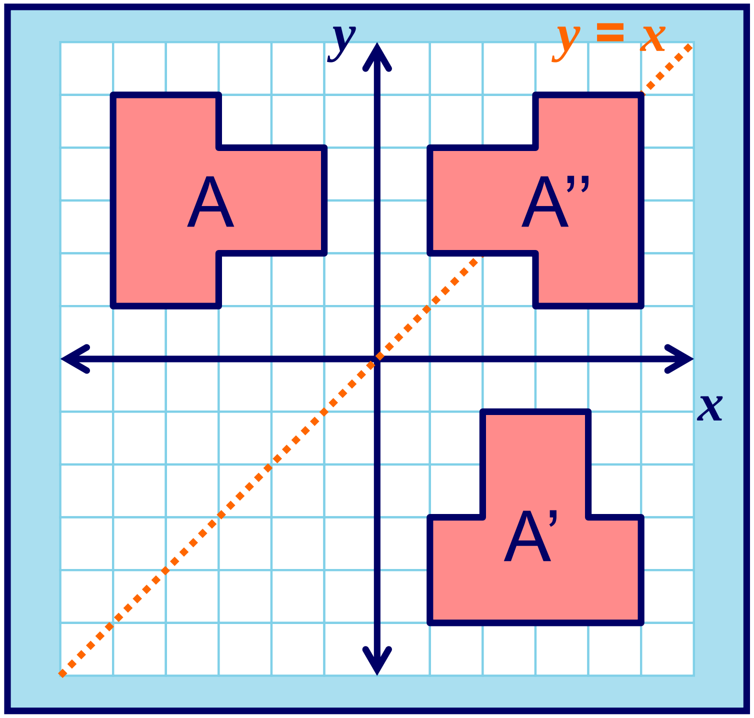
Positive and Negative

Contents

Combining transformations

When one transformation is followed by another, the resulting change can often be described by a single transformation.

For example, suppose we reflect shape A in the line $y = x$ to give its image A'.



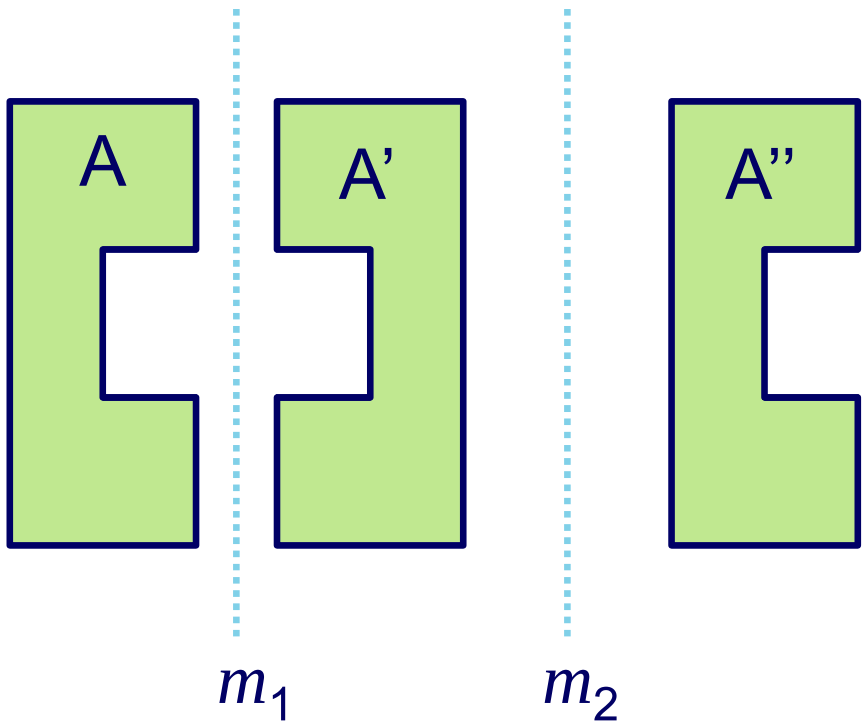
We then rotate A' through 90° about the origin to give the image A''.

What single transformation will map shape A onto A''?

We can map shape A onto shape A'' by a reflection in the y-axis.

Parallel mirror lines

Suppose we have two parallel mirror lines m_1 and m_2.



We can reflect shape A in mirror line m_1 to produce the image A'.

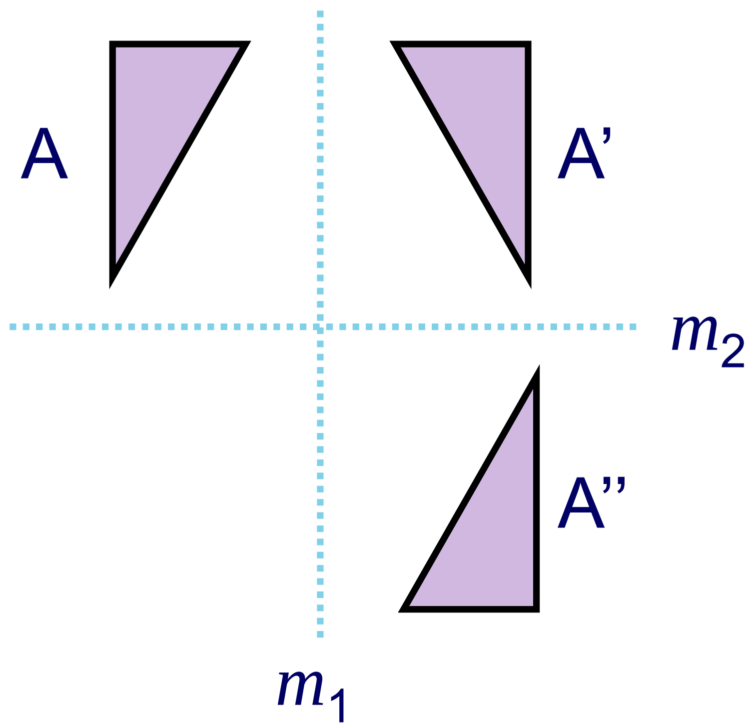
We can then reflect shape A' in mirror line m_2 to produce the image A''.

How can we map A onto A'' in a single transformation?

Reflecting an object in two parallel mirror lines is equivalent to a single translation.

Perpendicular mirror lines

Suppose we have two perpendicular mirror lines m_1 and m_2.



We can reflect shape A in mirror line m_1 to produce the image A'.

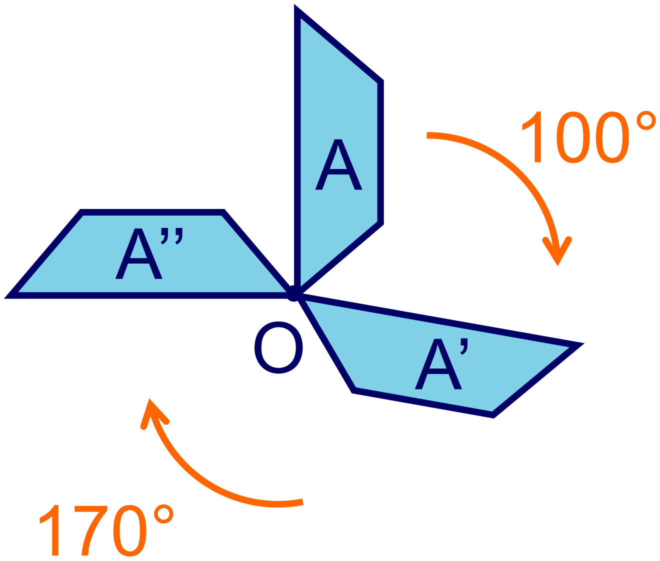
We can then reflect shape A' in mirror line m_2 to produce the image A''.

How can we map A onto A'' in a single transformation?

Reflection in two perpendicular lines is equivalent to a single rotation of 180°.

Combining rotations

Suppose shape A is rotated through 100° clockwise about point O to produce the image A'.



Suppose we then rotate shape A' through 170° clockwise about the point O to produce the image A''.

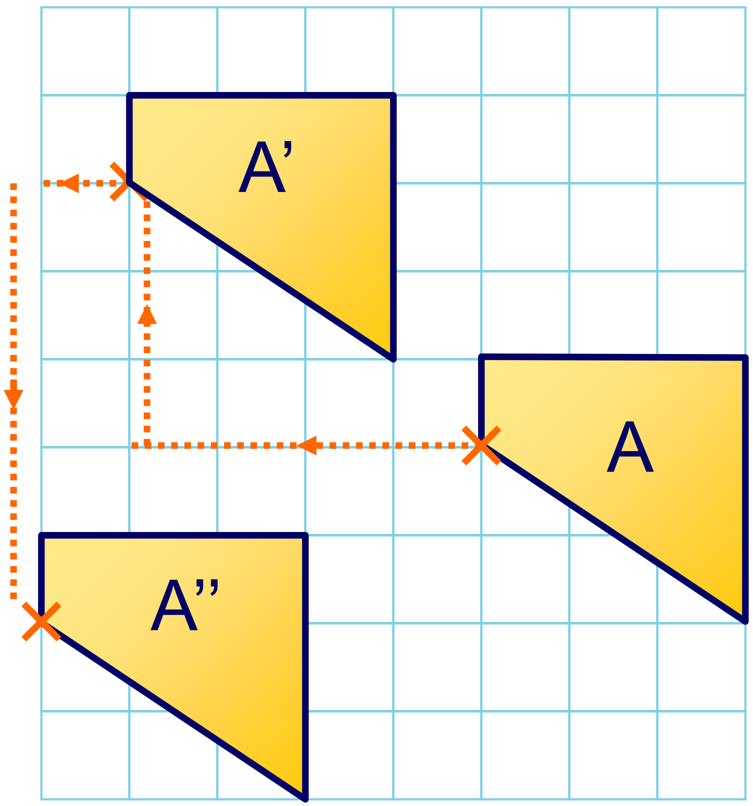
How can we map A onto A'' in a single transformation?

To map A onto A'' we can either rotate it 270° clockwise or 90° anti-clockwise.

Two rotations about the same centre are equivalent to a single rotation about the same centre.

Combining translations

Suppose shape A is translated 4 units left and 3 units up.



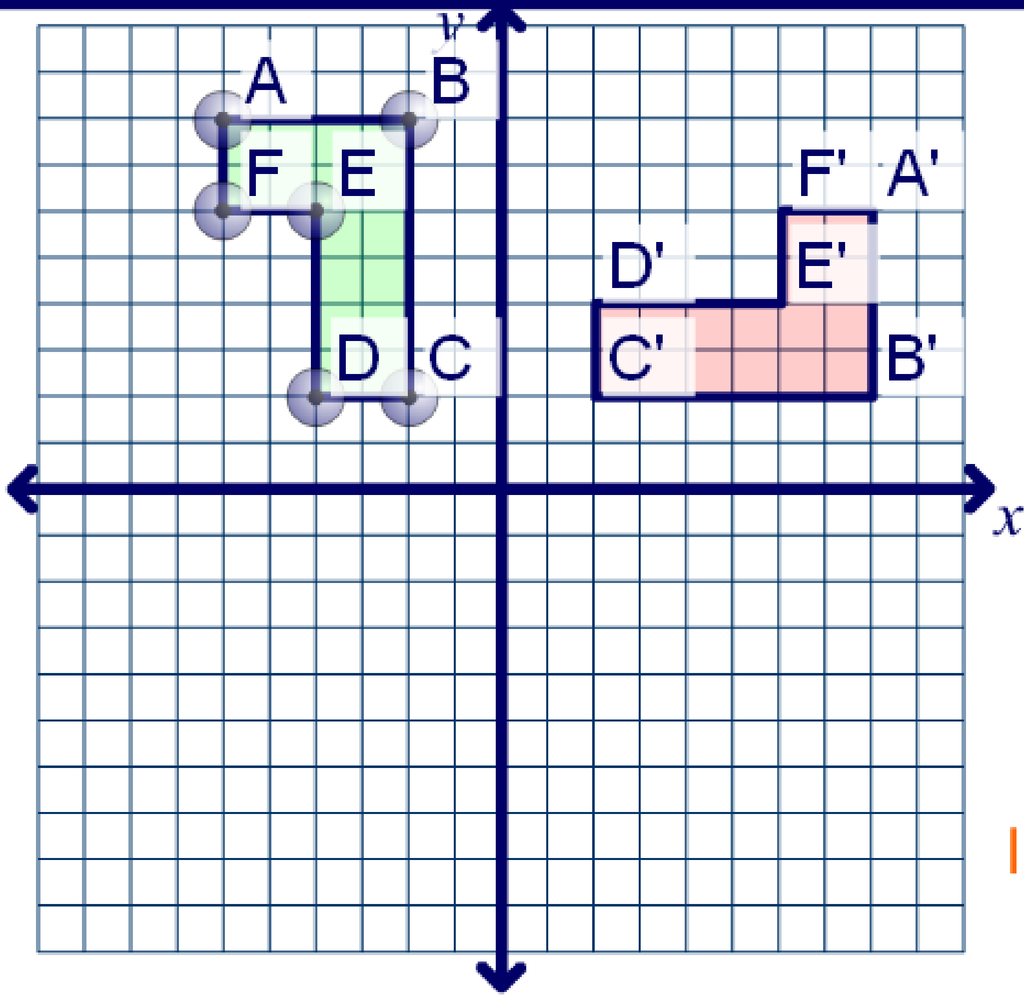
Suppose we then translate A' 1 unit to the left and 5 units down to give A''.

How can we map A to A'' in a single transformation?

We can map A onto A'' by translating it 5 units left and 2 units down.

Two or more consecutive translations are equivalent to a single translation.

Combining transformations



A reflect in the x-axis

B reflect in the y-axis

C reflect in $y = x$

D reflect in $x = 1$

E rotate 90° about (0, 3)

F rotate 90° about (0, 0)

G translate by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

H translate by $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

I enlargement -1 about (0, 0)

B followed by C

Go