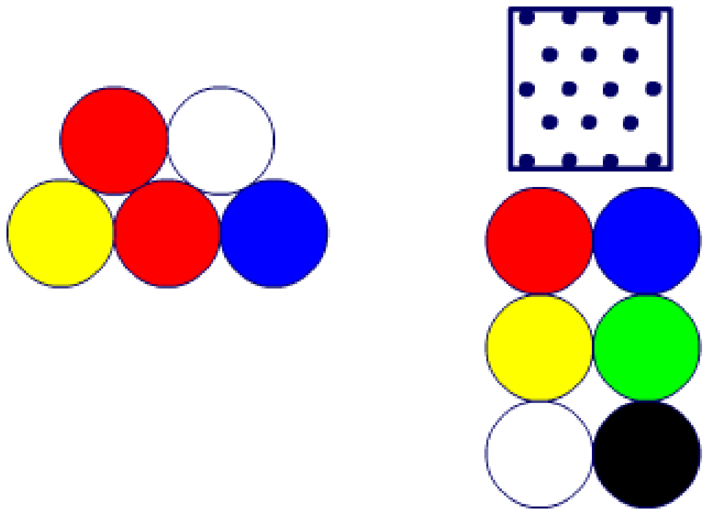
KS3 Mathematics

N4 Powers and roots

Contents

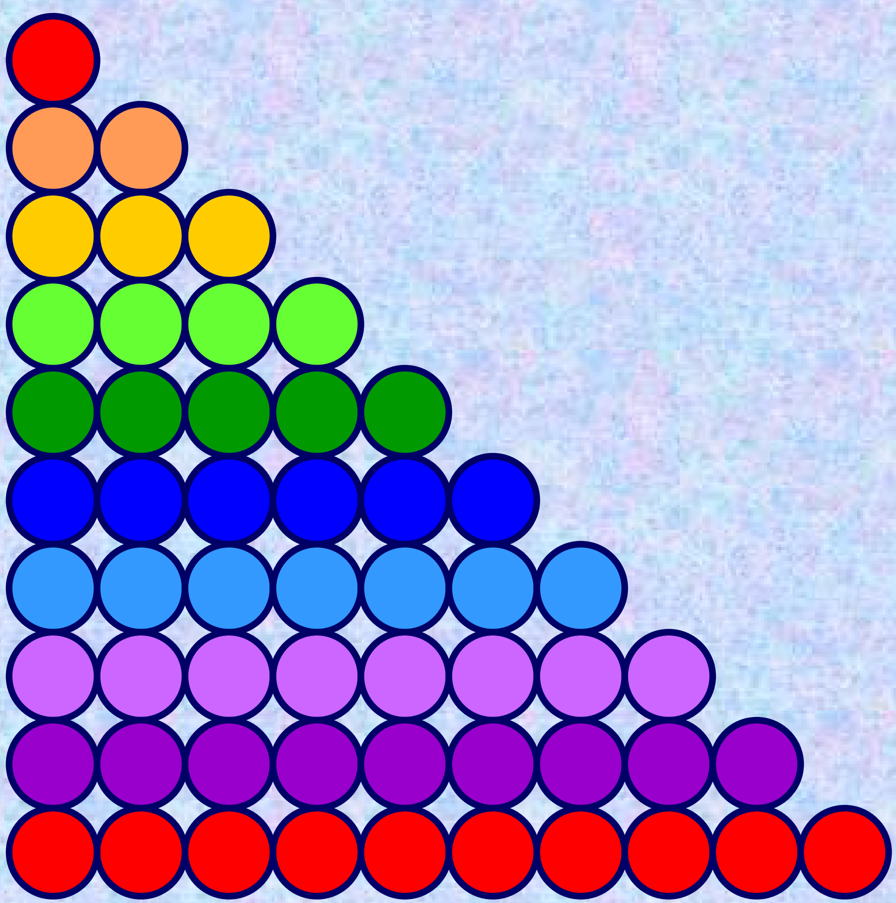
N4 Powers and Roots

Making triangles



Triangular numbers

The tenth triangular number is 55.



1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55

Making squares

Square numbers

When we multiply a number by itself we say that we are **squaring** the number.

To square a number we can write a small ^{2} after it.

For example, the number 3 multiplied by itself can be written as

Three squared

$$3 \times 3 \quad \text{or} \quad 3^2$$

The value of three squared is 9.

The result of any *whole* number multiplied by itself is called a **square number**.

Square numbers

Here are the first 10 square numbers:

$1^2 = 1 \times 1 = \mathbf{1}$

+ 3

$2^2 = 2 \times 2 = \mathbf{4}$

+ 5

$3^2 = 3 \times 3 = \mathbf{9}$

+ 7

$4^2 = 4 \times 4 = \mathbf{16}$

+ 9

$5^2 = 5 \times 5 = \mathbf{25}$

+ 11

$6^2 = 6 \times 6 = \mathbf{36}$

+ 13

$7^2 = 7 \times 7 = \mathbf{49}$

+ 15

$8^2 = 8 \times 8 = \mathbf{64}$

+ 17

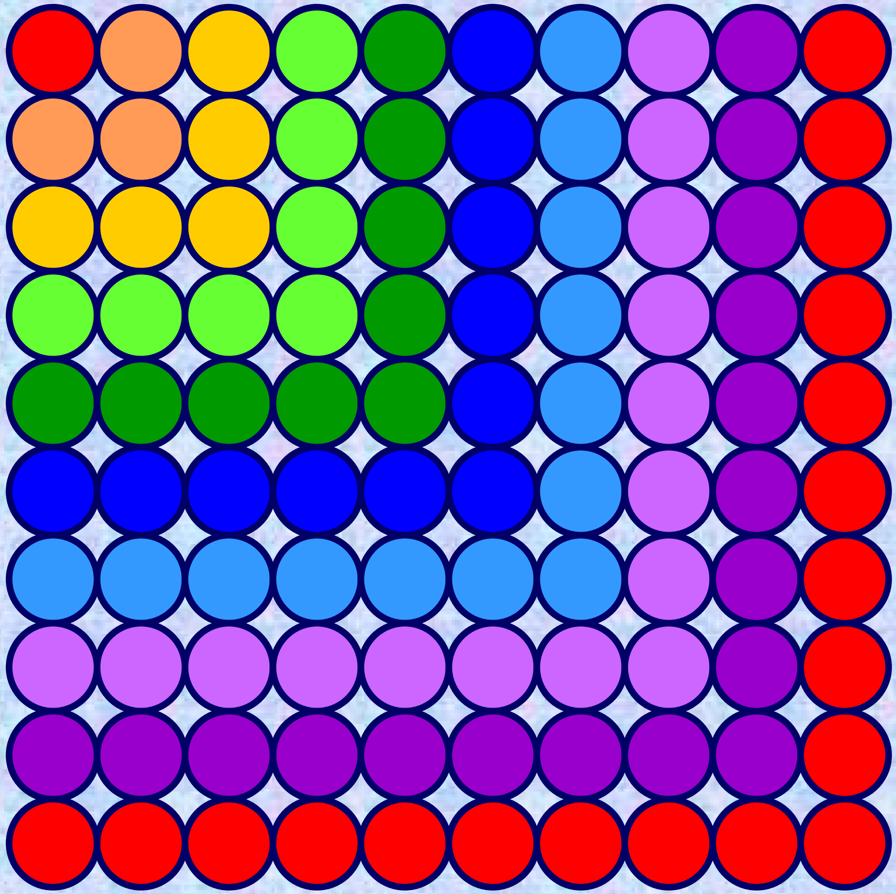
$9^2 = 9 \times 9 = \mathbf{81}$

+ 19

$10^2 = 10 \times 10 = \mathbf{100}$

Adding consecutive odd numbers

The tenth square number is 100.



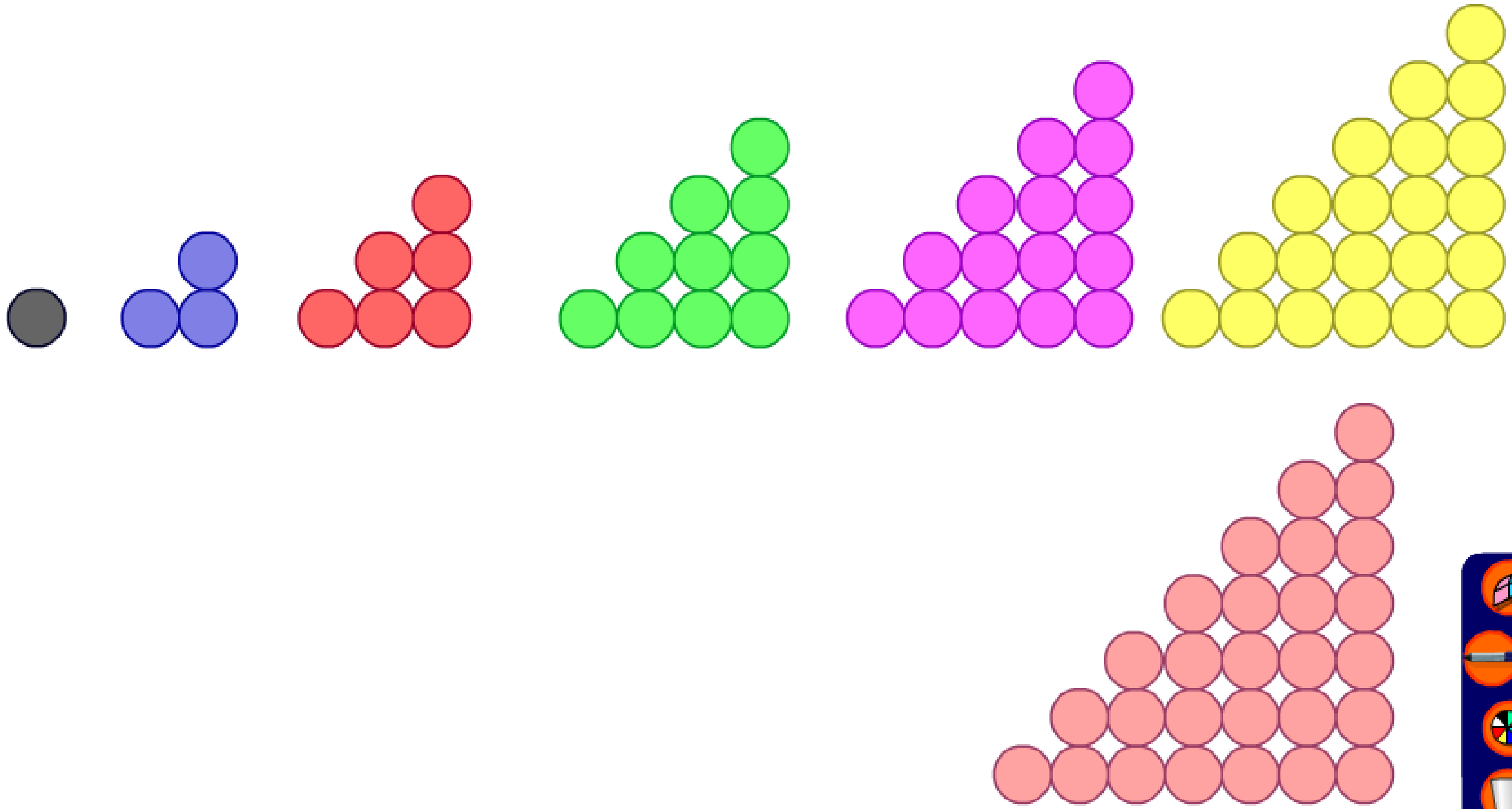
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100

Making square numbers

There are several ways to generate a sequence of square numbers.

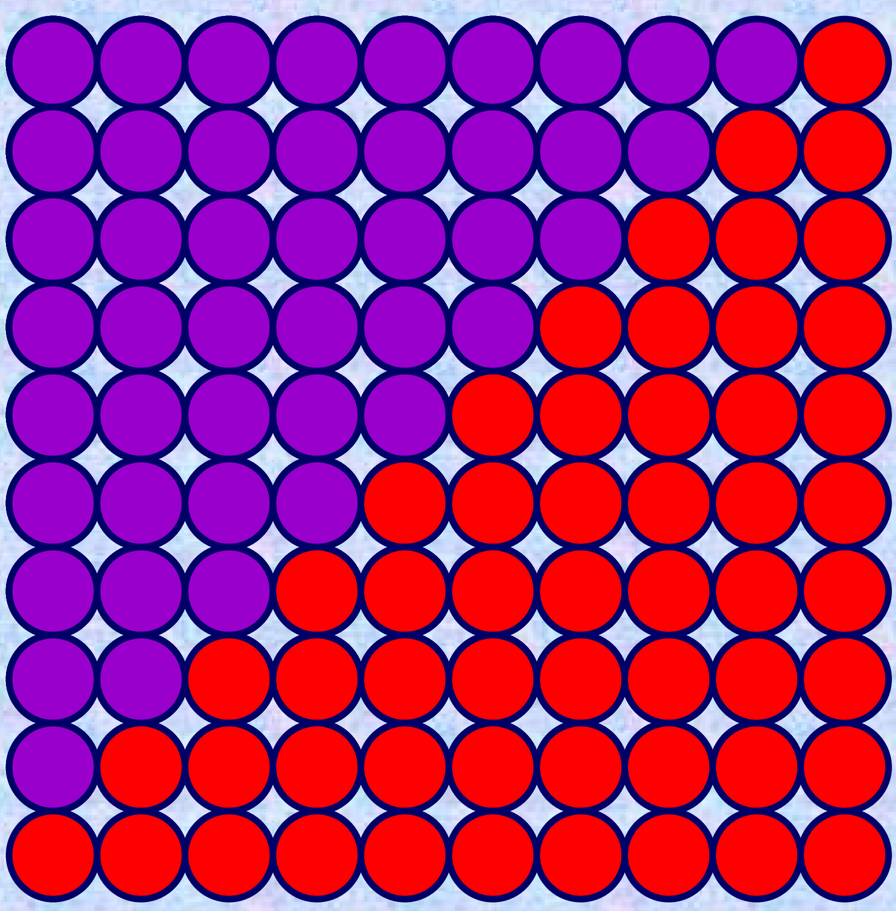
- We can multiply a whole number by itself.
- We can add consecutive odd numbers starting from 1.
- We can add together two consecutive triangular numbers.

Adding consecutive triangular numbers



Adding two consecutive triangular numbers

We can make square numbers by adding two consecutive triangular numbers.



45 + 55 = 100

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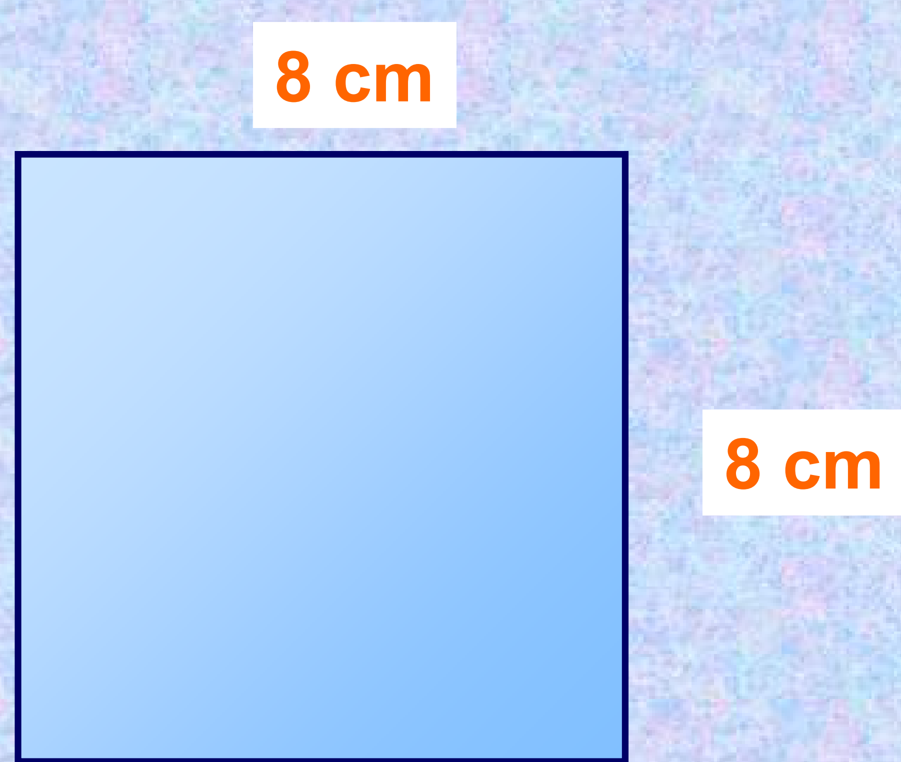
N4 Powers and Roots

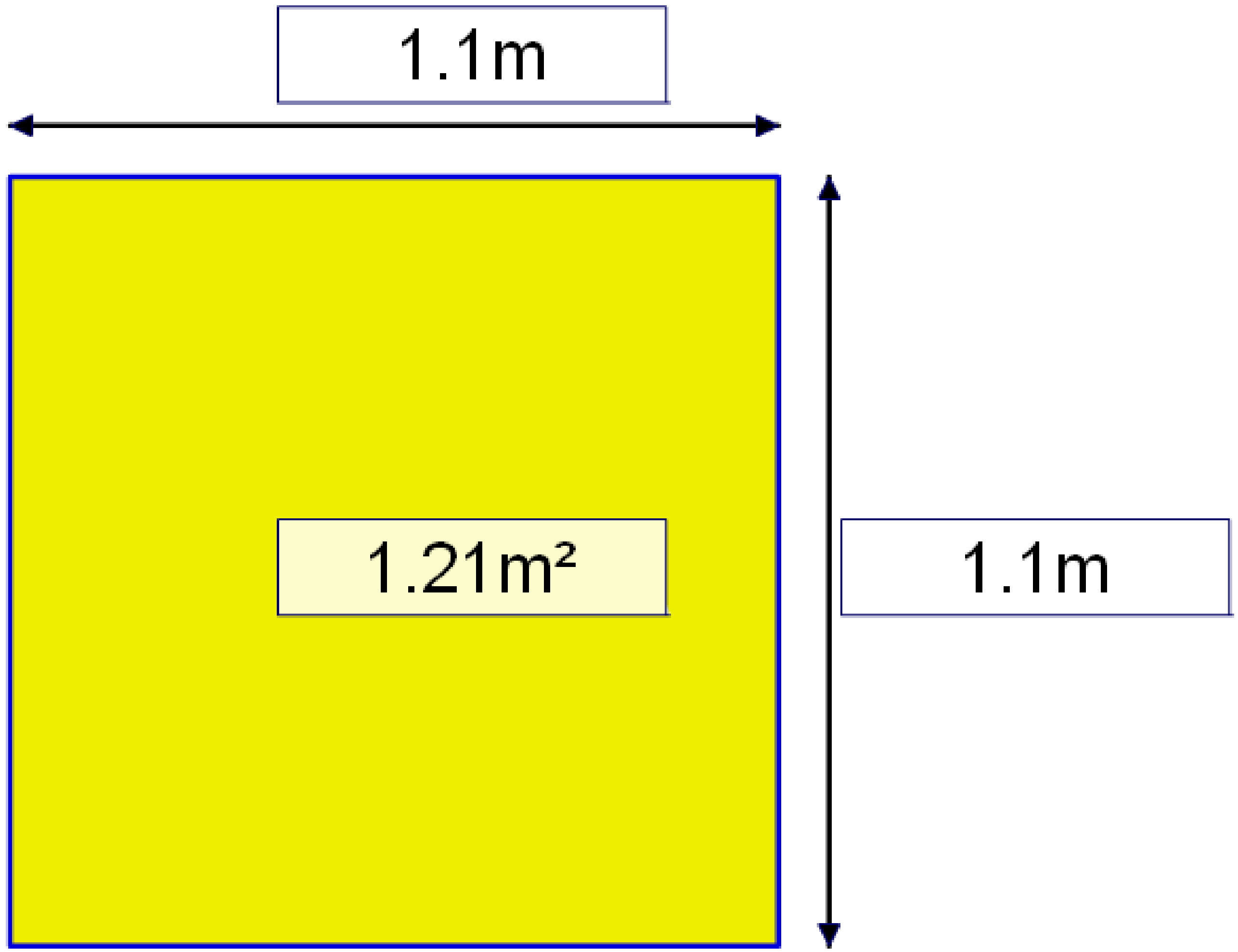
Square roots

The area of this square is 64 cm^2.



What is the length of the sides?

Square roots



Square roots

Finding the **square root** is the inverse of finding the square:

squared

8 **64**

square rooted

We write

$$\sqrt{64} = 8$$

The square root of 64 is 8.

Square roots

We can easily find the square root of a square number.

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

$\sqrt{100} = 10$

The product of two square numbers

The product of two square numbers is always another square number.

For example,

$$4 \times 25 = 100$$

because

$$2 \times 2 \times 5 \times 5 = 2 \times 5 \times 2 \times 5$$

and

$$(2 \times 5)^2 = 10^2$$

We can use this fact to help us find the square roots of larger square numbers.

Using factors to find square roots

If a number has factors that are square numbers then we can use these factors to find the square root.

For example,

Find $\sqrt{400}$

$$\sqrt{400} = \sqrt{4 \times 100}$$
$$= 2 \times 10$$
$$= 20$$

Find $\sqrt{225}$

$$\sqrt{225} = \sqrt{9 \times 25}$$
$$= 3 \times 5$$
$$= 15$$

Finding square roots of decimals

If a number can be made be dividing two square numbers then we can find its square root.

For example,

Find $\sqrt{0.09}$

$\sqrt{0.09} = \sqrt{9 \div 100}$

$= 3 \div 10$

$= 0.3$

Find $\sqrt{1.44}$

$\sqrt{1.44} = \sqrt{144 \div 100}$

$= 12 \div 10$

$= 1.2$

Approximate square roots

If a number cannot be written as a product or quotient of two square numbers then its square root cannot be found exactly.

Use the √ key on your calculator to find out √2.

The calculator shows this as 1.414213562

This is an approximation to 9 decimal places.

The number of digits after the decimal point is infinite.

Estimating square roots

What is √10?

10 lies between 9 and 16.

Therefore,

$$\sqrt{9} < \sqrt{10} < \sqrt{16}$$

So,

$$3 < \sqrt{10} < 4$$

10 is closer to 9 than to 16, so √10 will be about 3.2

Use the √ key on you calculator to work out the answer.

√10 = 3.16 (to 2 decimal places.)

Trial and improvement

Suppose our calculator does not have a √ key.

Find √40

$\sqrt{36} < \sqrt{40} < \sqrt{49}$

40 is closer to 36 than to 49, so √40 will be about 6.3

So,

$6 < \sqrt{40} < 7$

$6.3^2 = 39.69$ **too small!**

$6.4^2 = 40.96$ **too big!**

Trial and improvement

$6.33^2 = 40.0689$ **too big!**

$6.32^2 = 39.9424$ **too small!**

Suppose we want the answer to 2 decimal places.

$6.325^2 = 40.005625$ **too big!**

Therefore,

$$6.32 < \sqrt{40} < 6.325$$

$$\sqrt{40} = 6.32 \quad \text{(to 2 decimal places)}$$

Trial and improvement

Find $\sqrt{57}$ to 2 decimal places.

0.0		

Answer to 2 decimal places: 0.00

Negative square roots

$5 \times 5 = 25$ **and** $-5 \times -5 = 25$

Therefore, the square root of 25 is 5 or –5.

When we use the √ symbol we usually mean the positive square root.

We can also write ±√ to mean both the positive and the negative square root.

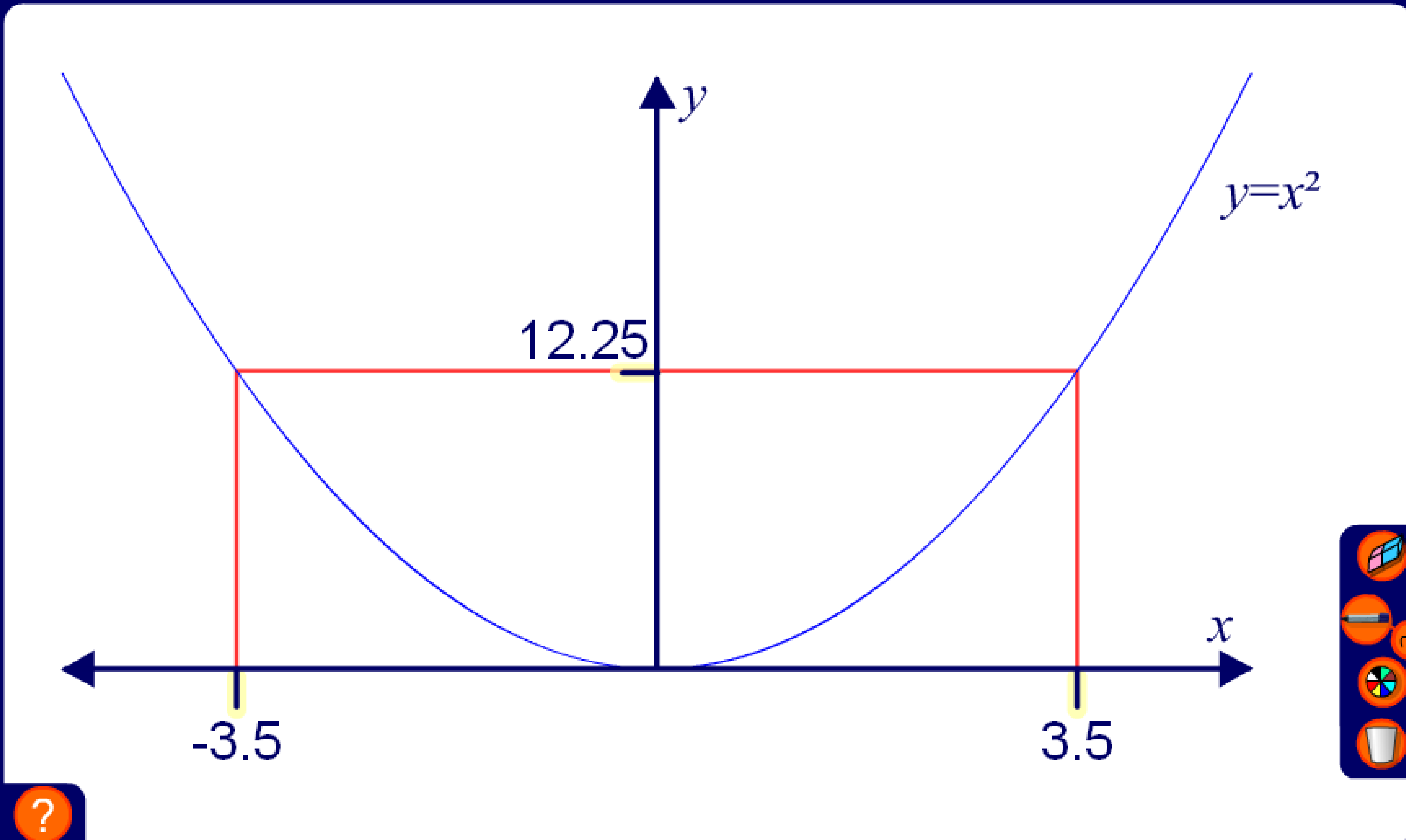
The equation,

$$x^2 = 25$$

has 2 solutions,

$x = 5$ **or** $x = -5$

Squares and square roots from a graph



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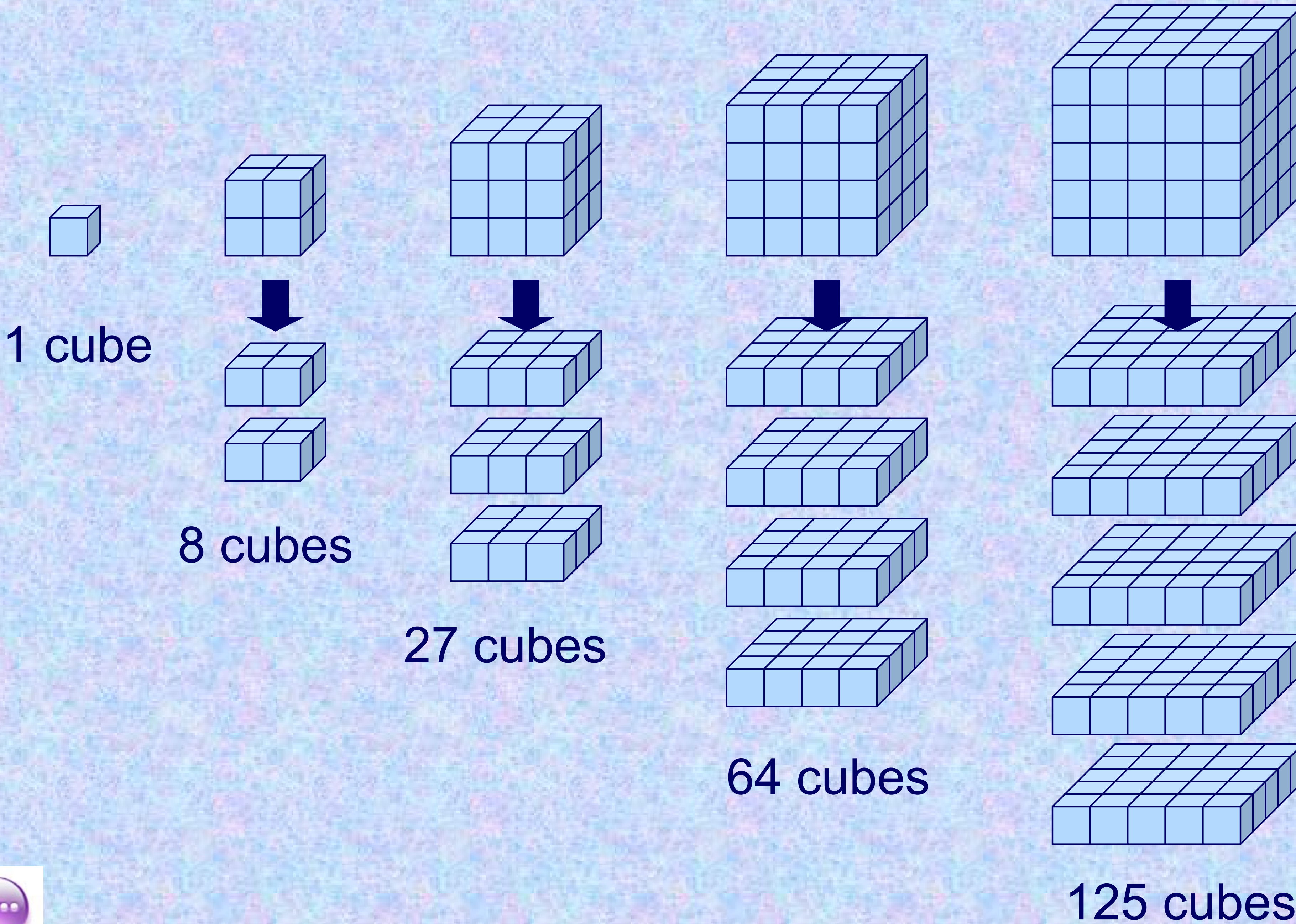
N4 Powers and Roots

Cubes



Cubes

The numbers 1, 8, 27, 64, and 125 are called:

Cube numbers

$1^3 = 1 \times 1 \times 1 = 1$ ‘1 cubed’ or ‘1 to the power of 3’

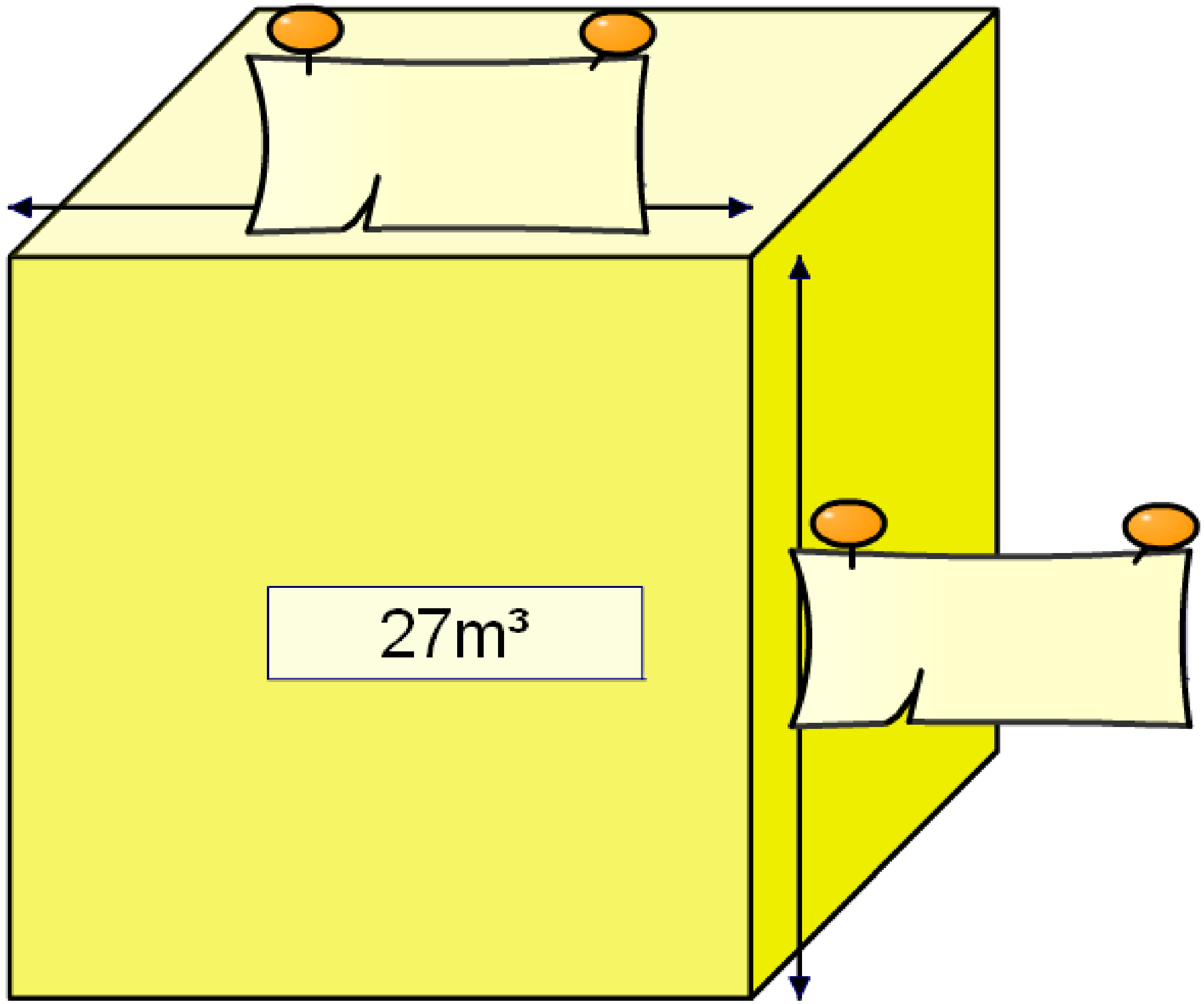
$2^3 = 2 \times 2 \times 2 = 8$ ‘2 cubed’ or ‘2 to the power of 3’

$3^3 = 3 \times 3 \times 3 = 27$ ‘3 cubed’ or ‘3 to the power of 3’

$4^3 = 4 \times 4 \times 4 = 64$ ‘4 cubed’ or ‘4 to the power of 3’

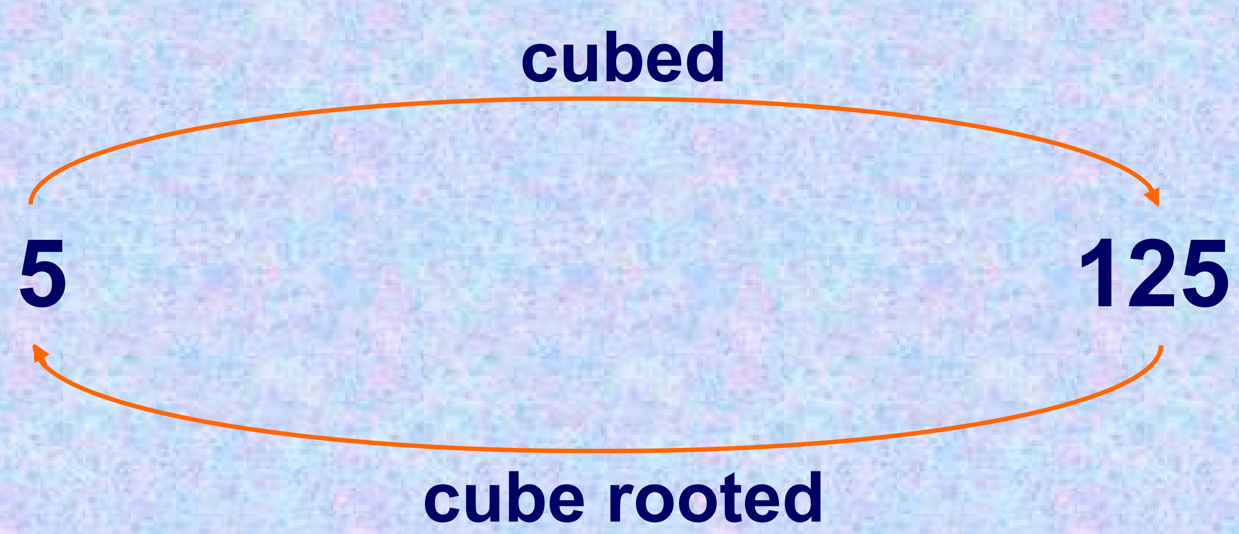
$5^3 = 5 \times 5 \times 5 = 125$ ‘5 cubed’ or ‘5 to the power of 3’

Cubes and cube roots



Cube roots

Finding the cube root is the inverse of finding the cube:



We write

$$\sqrt[3]{125} = 5$$

The cube root of 125 is 5.

Squares, cubes and roots

$\sqrt[3]{\square} = 30$

Squares 1
Square roots 2
Cubes 3
Cube roots 4

positive and negative

Contents

N4 Powers and Roots

Index notation

We use index notation to show repeated multiplication by the same number.

For example,

we can use index notation to write **2 × 2 × 2 × 2 × 2** as

Index or power

$$2^5$$

base

This number is read as ‘two to the power of five’.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Index notation

Evaluate the following:

$6^2 = 6 \times 6 = 36$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$(-5)^3 = -5 \times -5 \times -5 = -125$

When we raise a negative number to an odd power the answer is negative.

$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$

$(-1)^5 = -1 \times -1 \times -1 \times -1 \times -1 = -1$

$(-4)^4 = -4 \times -4 \times -4 \times -4 = 64$

When we raise a negative number to an even power the answer is positive.

Calculating powers

We can use the x^y key on a calculator to find powers.

For example,

to calculate the value of 7^4 we key in:

7 | x^y | 4 | =

The calculator shows this as 2401.

$7^4 = 7 \times 7 \times 7 \times 7 = 2401$

The first index law

When we multiply two numbers written in index form and with the same base we can see an interesting result.

For example,

$$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$$
$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$$
$$= 3^6 = 3^{(4+2)}$$

$$7^3 \times 7^5 = (7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7 \times 7)$$
$$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$$
$$= 7^8 = 7^{(3+5)}$$

When we **multiply** two numbers with the **same base** the indices are **added**.

The second index law

When we divide two numbers written in index form and with the same base we can see another interesting result.

For example,

$$4^5 \div 4^2 = \frac{\cancel{4} \times \cancel{4} \times 4 \times 4 \times 4}{\cancel{4} \times \cancel{4}} = 4 \times 4 \times 4 = 4^3 = 4^{(5-2)}$$

$$5^6 \div 5^4 = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}} = 5 \times 5 = 5^2 = 5^{(6-4)}$$

When we **divide** two numbers with the **same base** the indices are **subtracted**.

Zero indices

Look at the following division:

$6^4 \div 6^4 = 1$

Using the second index law

$6^4 \div 6^4 = 6^{(4-4)} = 6^0$

That means that

$6^0 = 1$

In fact, *any* number raised to the power of 0 is equal to 1.

For example,

$10^0 = 1$ $3.452^0 = 1$ $723\ 538\ 592^0 = 1$

Negative indices

Look at the following division:

$$3^2 \div 3^4 = \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times 3 \times 3} = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

Using the second index law

$$3^2 \div 3^4 = 3^{(2-4)} = 3^{-2}$$

That means that

$$3^{-2} = \frac{1}{3^2}$$

Similarly, $6^{-1} = \frac{1}{6}$ $7^{-4} = \frac{1}{7^4}$ and $5^{-3} = \frac{1}{5^3}$

Using algebra

We can write all of these results algebraically.

$$a^m \times a^n = a^{(m + n)}$$

$$a^m \div a^n = a^{(m - n)}$$

$$a^0 = 1$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n}$$

Using index laws

$$54^{-2} \div 54^{5} = 54^{-7}$$