INDICES AND LOGARITHMS

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Laws of Indices

$$a^p a^q = a^{p+q}$$

$$(ab)^p = a^p b^p$$

$$a^p / a^q = a^{p-q}$$

$$(a/b)^p = a^p / b^p$$

$$(a^p)^q = a^{pq}$$

Some Properties of Indices

$$a^0 = 1$$

$$a^{-p} = 1/a^p$$

$$a^{1/q} = \sqrt[q]{a}$$

$$a^{p/q} = \sqrt[q]{a^p}$$

Proof of the properties

To prove: $a^0 = 1$

Let $a^0 = c.$

Then $a^p \cdot c = a^p \cdot a^0 = a^p$

$\Rightarrow \quad a^0 = 1.$

Proof of the properties

To prove: $a^{-p} = 1/a^p$

Let $a^{-p} = d$.

Then $a^p \cdot d = a^p \cdot a^{-p} = a^0 = 1$

$\Rightarrow \quad a^{-p} = 1/a^p$.

Proof of the properties

To prove: $a^{1/q} = \sqrt[q]{a}$

Let $a^{1/q} = f$.

Then $f^{q} = \left(a^{1/q}\right)^{q} = a^{1} = a$

$\Rightarrow \quad a^{1/q} = \sqrt[q]{a}.$

Proof of the properties

To prove: $a^{p/q} = \sqrt[q]{a^p}$

$$a^{p/q} = \left(a^p\right)^{1/q} = \sqrt[q]{a^p}.$$

Definition of Logarithms

The log of x to the base a, written as $\log_a x$ is the value y such that $a^y = x$, i.e.,

$$\log_a x = y \quad \Leftrightarrow \quad a^y = x,$$

where $a > 0$ and $a \neq 1$,
$x > 0$.

Particular Cases:

$\log_a 1 = ?$ $\log_a 1 = 0$

$\log_a\left(a^m\right) = ?$ and $a^{\log_a b} = ?$

$\log_a\left(a^m\right) = m$ $a^{\log_a b} = b$

Common Logarithm (a = 10):

$\log_{10} x$ or simply $\log x$

Natural Logarithm (a = e = 2.71828…):

$\log_e x$ or simply $\ln x$

Some Properties of Logarithms

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \left(x/y \right) = \log_a x - \log_a y$$

$$\log_a x^n = n \log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Proof of the properties

To prove: $\log_a xy = \log_a x + \log_a y$

Let $\log_a x = u$ and $\log_a y = v$

Then $a^u = x$ and $a^v = y$

$$\Rightarrow \quad xy = a^u a^v = a^{u+v}$$

$$\Rightarrow \quad \log_a xy = \log_a a^{u+v} = u + v = \log_a x + \log_a y$$

Proof of the properties

To prove: $\log_a x^n = n\log_a x$

Let $\log_a x = u$

Then $a^u = x$

$\Rightarrow \quad x^n = \left(a^u\right)^n = a^{nu}$

$\Rightarrow \quad \log_a x^n = \log_a a^{nu} = nu = n\log_a x$

Proof of the properties

To prove: $\log_a(x/y) = \log_a x - \log_a y$

$$\log_a(x/y) = \log_a x + \log_a(1/y) = \log_a x + \log_a y^{-1}$$
$$= \log_a x - \log_a y$$

Proof of the properties

To prove: $\log_a x = \dfrac{\log_b x}{\log_b a}$

Let $\log_a x = u$

Then $a^u = x$

$\Rightarrow \quad \log_b a^u = \log_b x$

$\Rightarrow \quad u \log_b a = \log_b x \quad \Rightarrow \quad u = \dfrac{\log_b x}{\log_b a}$

Exercises

1. If $\log_2 12 = 3.58$, then what is $\log_2 \frac{1}{24}$?

2. Compute $\frac{\log_5 1000}{\log_5 100}$.