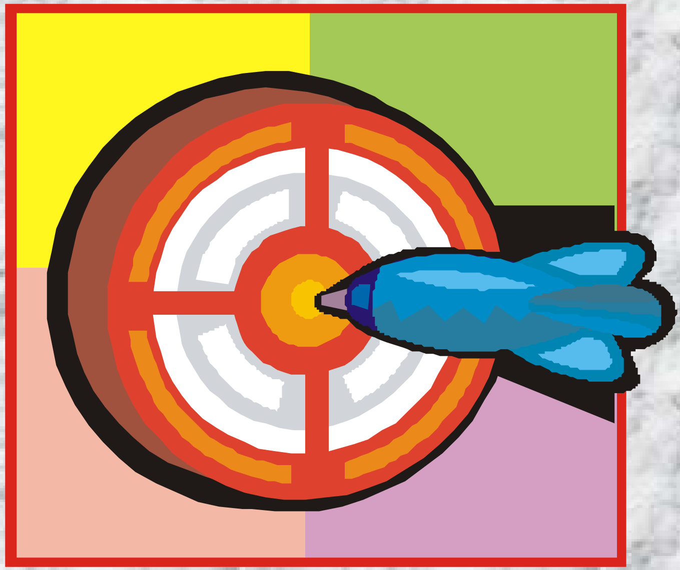
Mathematics

Session

Cartesian Coordinate Geometry

and

Straight Lines



Session Objectives

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1. **Definition of straight line – locus**
2. **Slope of a line**
3. **Angle between two lines**
4. **Intercepts of a line on the axes**
5. **Slope, intercept form**
6. **Point, slope form**
7. **Two-point form**
8. **Intercepts form**
9. **Normal form**
10. **Parametric or distance form**

Locus definition of a straight line

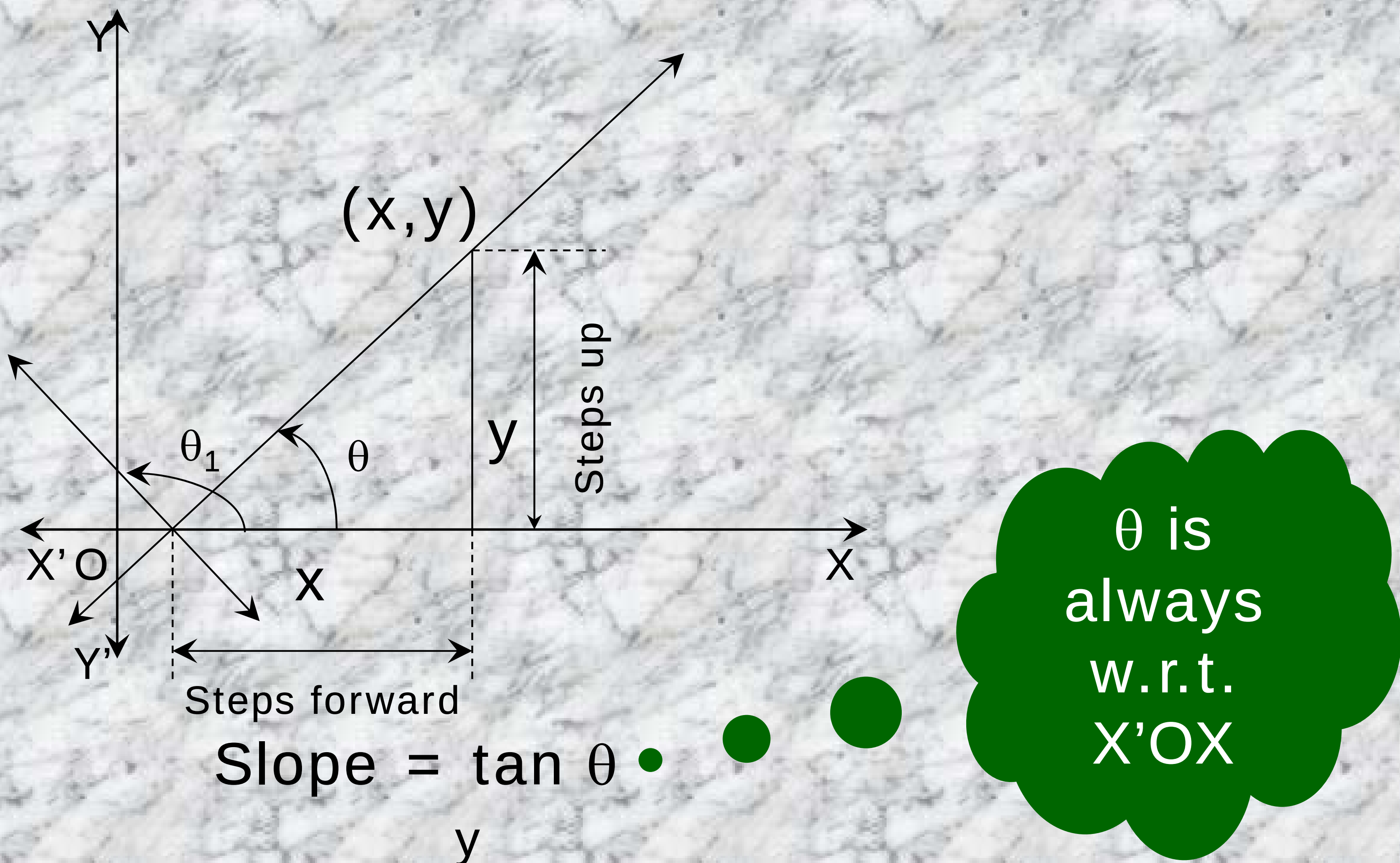
A straight line is the locus of a point whose coordinates satisfy a linear equation

Slope - Concept

How many steps forward?

Even More Slope!!

Slope



Slope = tan θ

$$= \frac{y}{x}$$

Slope

slope +ve $\Leftrightarrow$ θ is acute

slope -ve $\Leftrightarrow$ θ is obtuse

$\theta = 0° \Leftrightarrow$ slope = 0

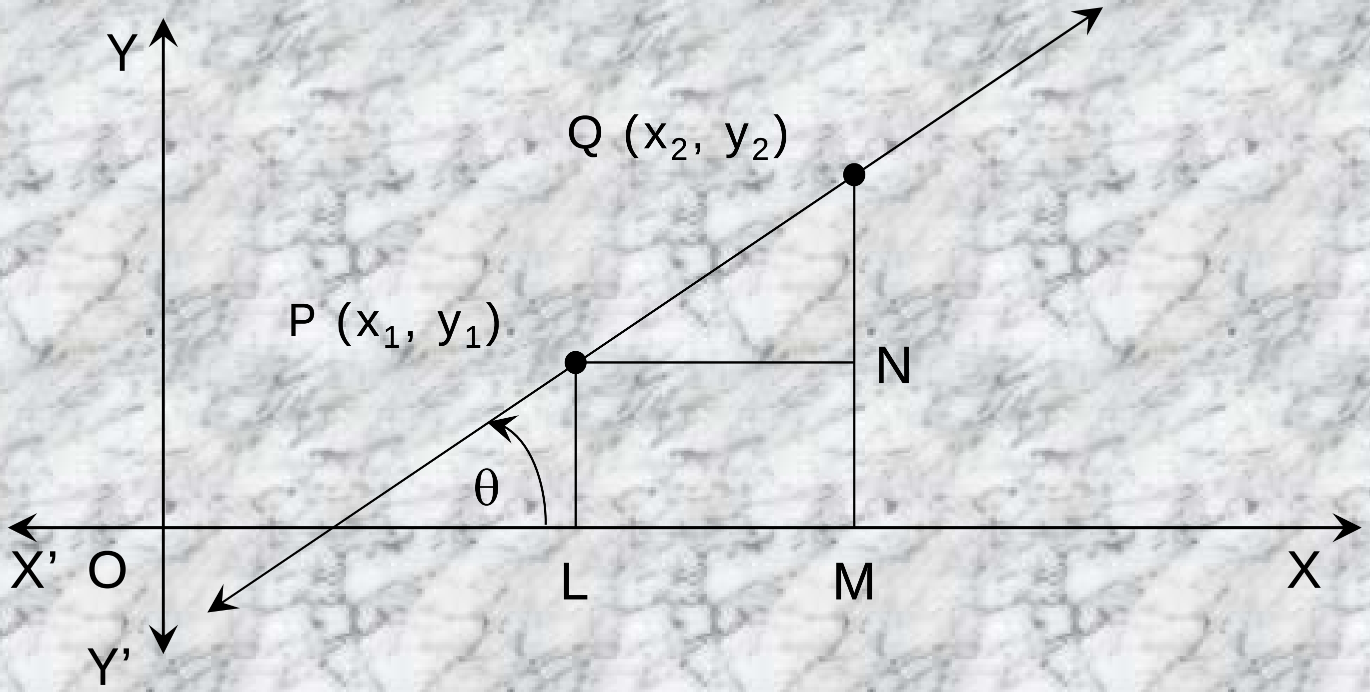
$\theta = 90° \Leftrightarrow$ slope = ?

Infinite?

Not infinite. It is not defined.

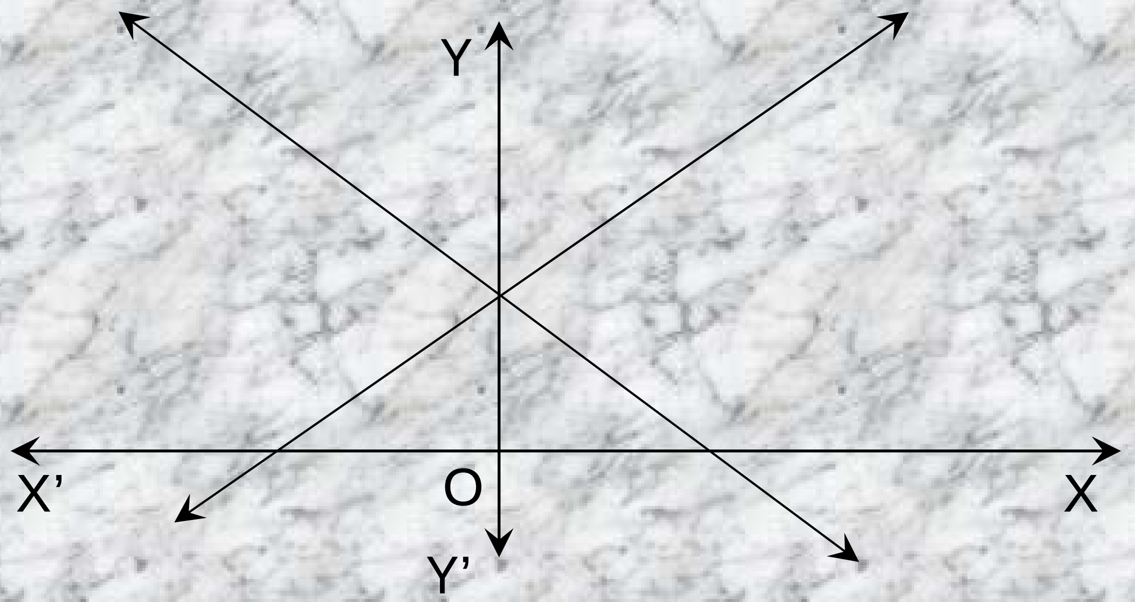
Slope is usually denoted by m

Slope in terms of points on a line



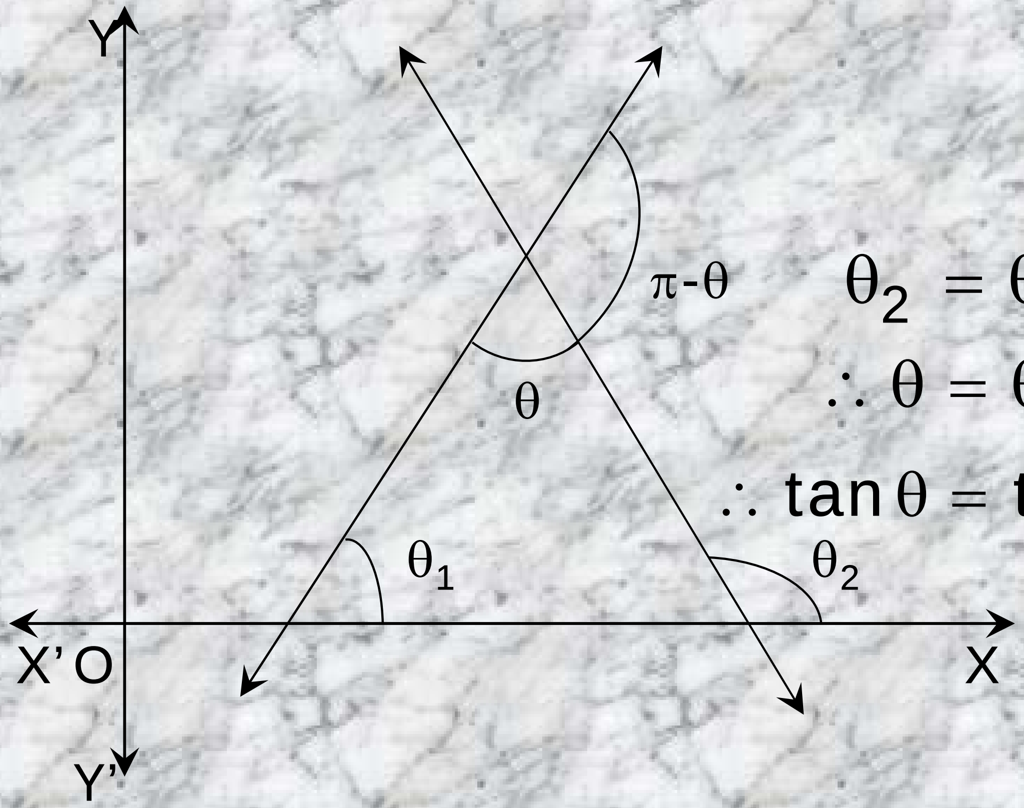
$$\tan\theta = \frac{QN}{PN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{difference of ordinates}}{\text{difference of abcissae}}$$

Slope of reflection in either axis



Slope of a line = m ⇔ slope of reflection = -m

Angle between two lines



$$\theta_2 = \theta + \theta_1$$

$$\therefore \theta = \theta_2 - \theta_1$$

$$\therefore \tan\theta = \tan(\theta_2 - \theta_1)$$

$$\therefore \tan\theta = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

Also $\tan(\pi - \theta) = -\tan\theta = -\dfrac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$

$$\therefore \tan\theta = \pm\frac{m_2 - m_1}{1 + m_1 m_2}$$

Parallel lines

$$\tan\theta = 0$$

$$\therefore \frac{m_2 - m_1}{1 + m_1 m_2} = 0$$

$$\therefore m_1 = m_2$$

Perpendicular lines

$$\cot\theta = 0$$

$$\therefore \frac{1 + m_1 m_2}{m_2 - m_1} = 0$$

$$\therefore m_1 m_2 = -1$$

Illustrative example

Let A (6, 4) and B (2, 12) be two points. Find the slope of a line perpendicular to AB.

Solution :

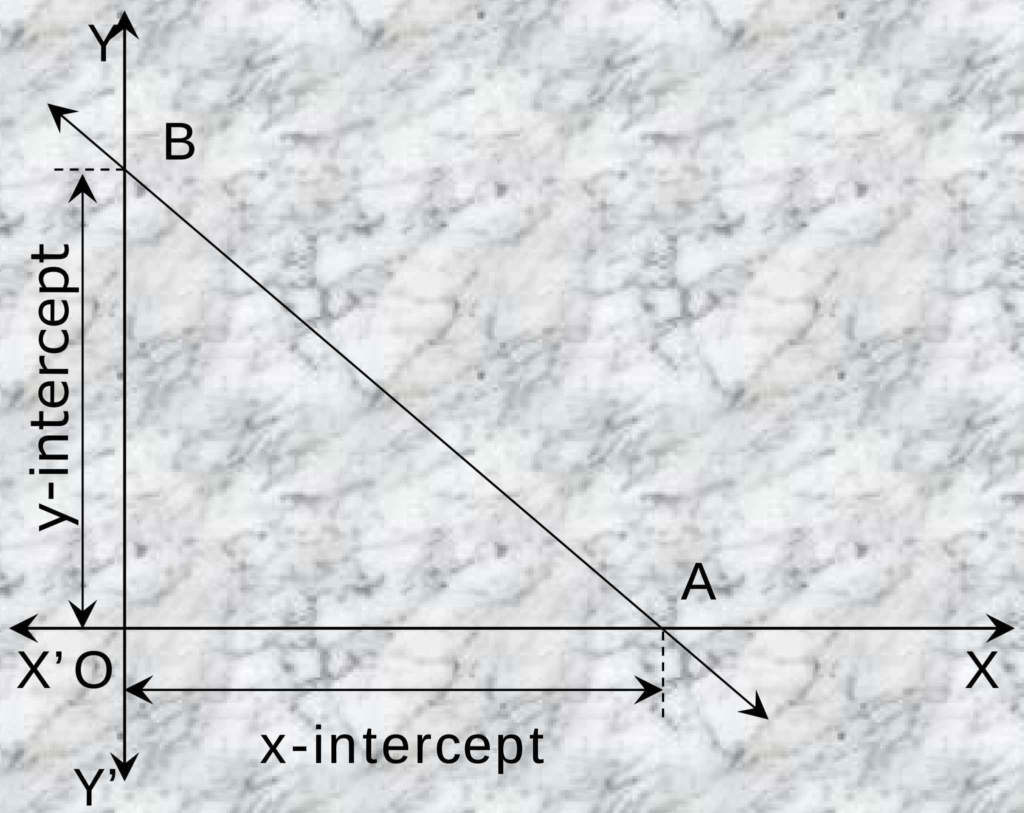
$$m_1 = \frac{12 - 4}{2 - 6}$$

$$\therefore \; m_1 = -2$$

$$m_1 m_2 = -1$$

$$\therefore \; m_2 = \frac{1}{2}$$

Intercepts on x axis, y axis

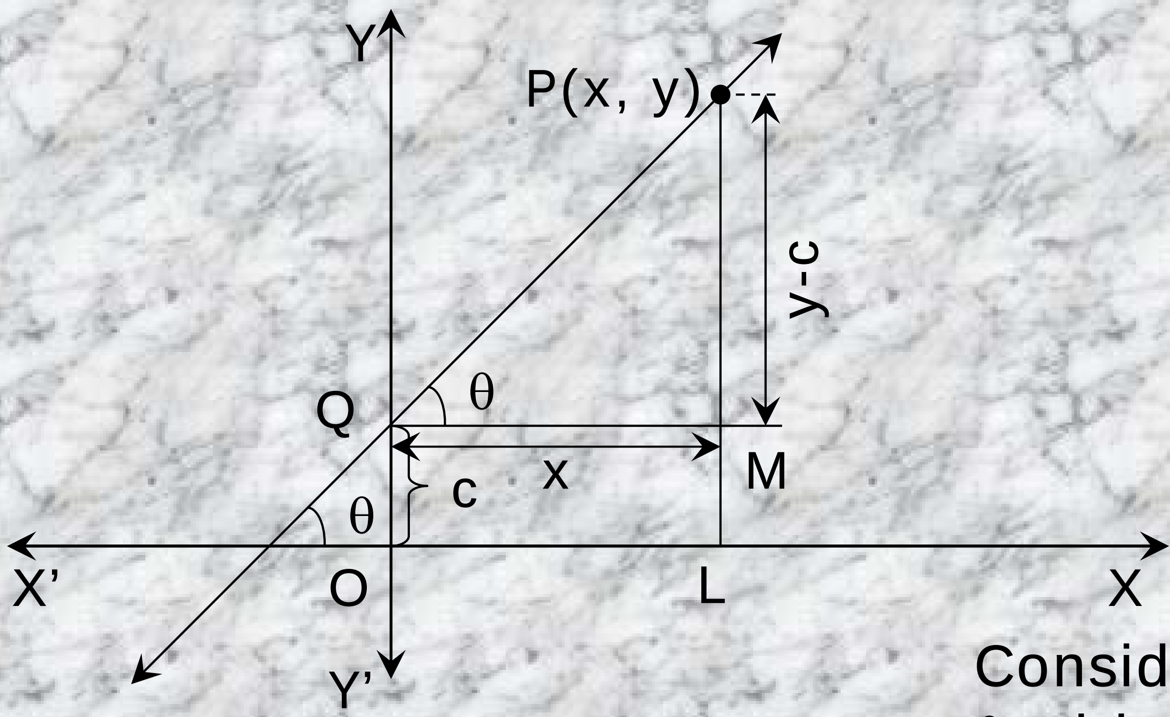


Consider a line cutting the axes in A and B

OA = x-intercept

OB = y-intercept

Slope intercept form



Consider a line making an angle θ with the x-axis and an intercept c with the y-axis

Consider a point P (x, y) on it

$$\text{Slope} = m = \tan\theta = \frac{PM}{QM} = \frac{y - c}{x}$$

$$y = mx + c$$

Coefficient of y = 1

Illustrative example

Find the equation of a line which makes an angle of $\tan^{-1}(3)$ with the x-axis and cuts off an intercept of 4 units with the negative direction of the y-axis.

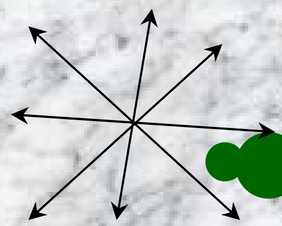
Solution :

Slope $m = \tan\theta = 3$, y-intercept $c = -4$

$\therefore$ the required equation is $y = 3x - 4$.

Locus definition of a straight line

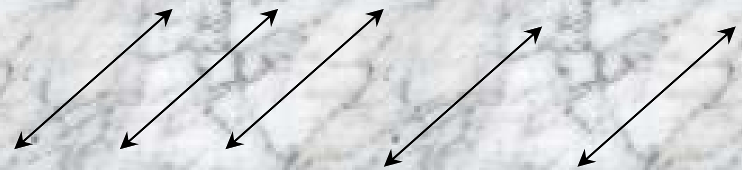
Condition 1: A point on the line is given



Any number of lines may pass through a given point.

Condition2: Direction of the line is given

Any number of lines can lie in a certain direction.



Point slope form

Consider a line passing through P (x_1, y_1) and having a slope m.

Consider any point Q (x, y) on it.

$$\text{slope } m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m\left(x - x_1\right)$$

BUT ONLY ONE
straight line can pass
through a given point
in a given direction

Illustrative example

Find the equation of the perpendicular bisector of the line segment joining the points A (-2, 3) and B (6, -5)

Solution :

Slope of AB = $\dfrac{3+5}{-2-6} = -1$

Slope of perpendicular = 1

Perpendicular bisector will pass through midpoint of AB which is (2, -1)

$\therefore$ the required equation is y+1 = x-2 or y = x-3

Two point form

Consider a line passing through P (x_1, y_1) and Q (x_2, y_2).

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Using point slope form,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Illustrative example

Find the equation of the medians of the triangle ABC whose vertices are A (2, 5), B (-4, 9) and C (-2,-1) through A

Solution :

Let the midpoints of BC, CA and AB be D, E and F respectively

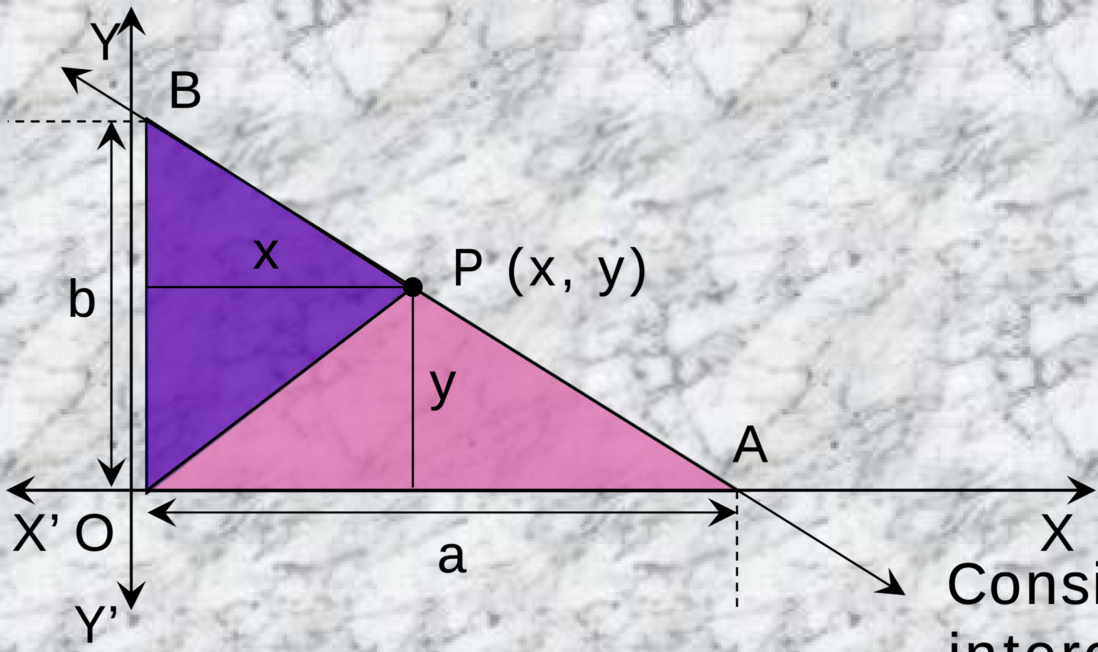
By section formula,

$D \equiv (-3, 4)$, $E \equiv (0, 2)$ and $F \equiv (-1, 7)$

Using two point form,

$$AD \equiv y - 5 = \frac{4-5}{-3-2}\left(x-2\right) \equiv x - 5y + 23 = 0$$

Intercepts form



Consider a line making intercepts a and b on the axes.

Consider a point P (x, y) on it.

Area of Δ OPB + Area of Δ OPA = Area of Δ OAB

$$\therefore \frac{1}{2}bx + \frac{1}{2}ay = \frac{1}{2}ab \qquad \frac{x}{a} + \frac{y}{b} = 1$$

Illustrative example

Find the equation of a line which passes through (22, -6) and is such that the x-intercept exceeds the y- intercept by 5.

Solution :

Let the y-intercept = c.

$\therefore$ the x-intercept = c+5

$\therefore$ Intercept form of line is given by $\frac{22}{c+5} - \frac{6}{c} = 1$

As this passes through (22,-6)

$$\frac{22}{c+5} - \frac{6}{c} = 1$$

Solution Cont.

$\therefore\ c^2-11c+30 = 0$

$\therefore\ (c-5)(c-6) = 0$

$\therefore\ c = 5$ or $c = 6$

$\therefore$ the required equation is

$$\frac{x}{10}+\frac{y}{5}=1 \text{ or } \frac{x}{11}+\frac{y}{6}=1$$

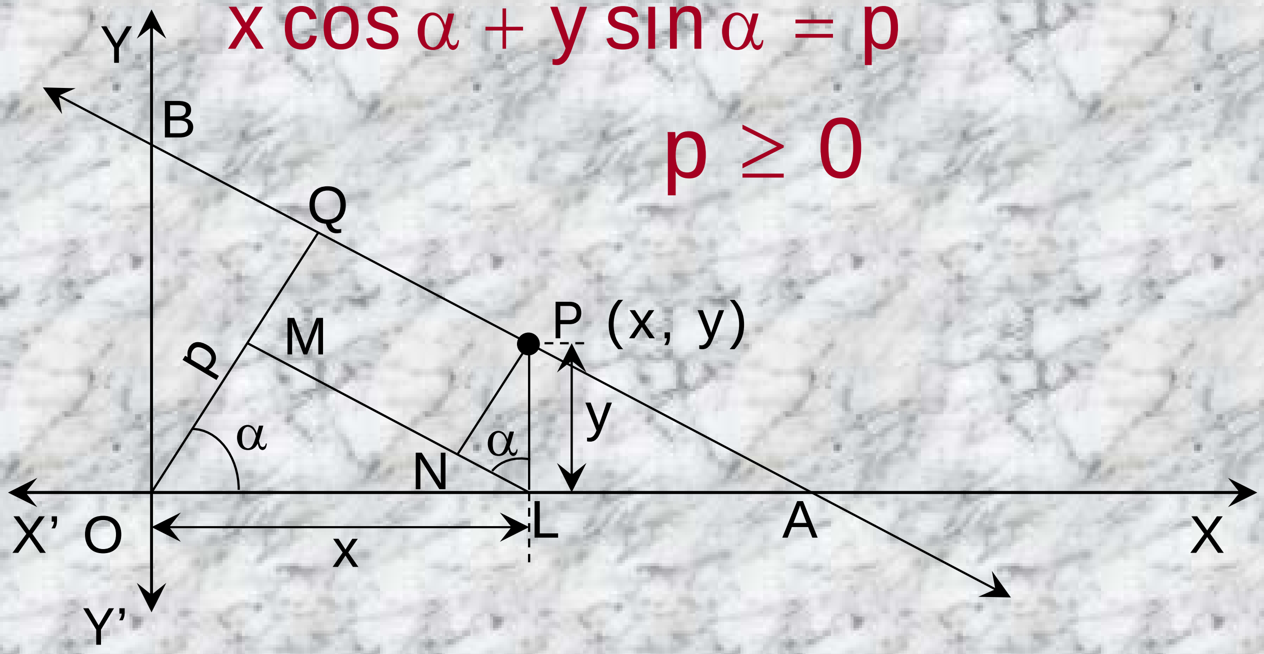
Rearranging,

$x+2y-10 = 0$ or $6x+11y-66 = 0$

Normal form

$$x\cos\alpha + y\sin\alpha = p$$

$$p \geq 0$$



Consider a line meeting the axes at A and B, at a distance p = OQ from the origin making an angle α with the x-axis.

Consider a point P (x, y) on this line.

Draw PL ⊥ OX, LM ⊥ OQ and PN ⊥ LM. ∠ PLN = α

p = OQ = OM + MQ = OM + NP = xcos α + ysin α

Illustrative example

The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150° with the positive direction of y-axis. Find the equation of the line.

Solution :

$p = 7$ and $\alpha = 30°$

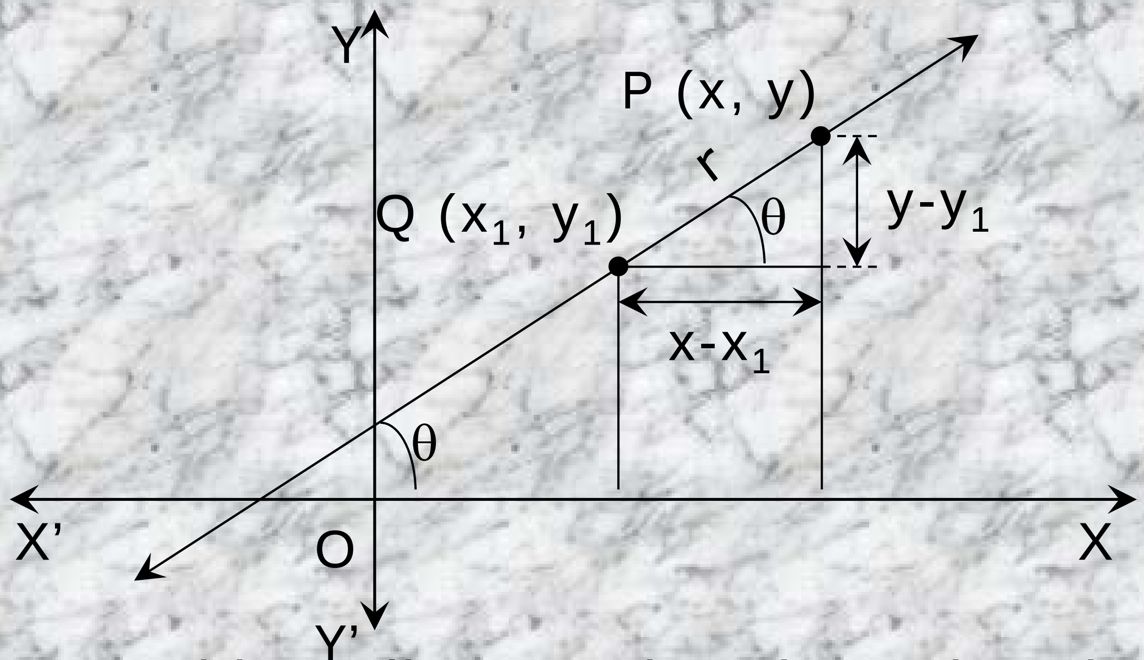
Therefore, the required equation is :

$x \cos 30° + y \sin 30° = 7$

$$\therefore \frac{\sqrt{3}x}{2} + \frac{y}{2} = 7$$

$$\therefore \sqrt{3}x + y - 14 = 0$$

Distance or parametric form



Consider a line passing through Q (x_1, y_1) and making an angle θ with the X'OX.

Consider a point P (x, y) on this line at a distance r from Q.

$$\cos\theta = \frac{x - x_1}{r}, \ \sin\theta = \frac{y - y_1}{r}$$

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

Distance or parametric form

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

Can also be written as

$$x = x_1 + r\cos\theta$$
$$y = y_1 + r\sin\theta$$

Illustrative example

The slope of a straight line through A (3, 2) is 3/4. Find the coordinates of the points on the line that are 5 units away from A.

Solution

The equation of the line is :

$x = 3 + r\cos\theta$, $y = 2 + r\sin\theta$

$\theta = \tan^{-1}(3/4) \Rightarrow \sin\theta = 3/5$, $\cos\theta = 4/5$.

$r = \pm 5$,

$x = 3 \pm 5\cos\theta$, $y = 2 \pm 5\sin\theta \Rightarrow x = 3 \pm 4$, $y = 2 \pm 3$

The required coordinates are (7, 5) and (-1, -1)

Class Exercise - 1

Trace the straight lines whose equations are as follows.

(i) $x + 2y + 3 = 0$

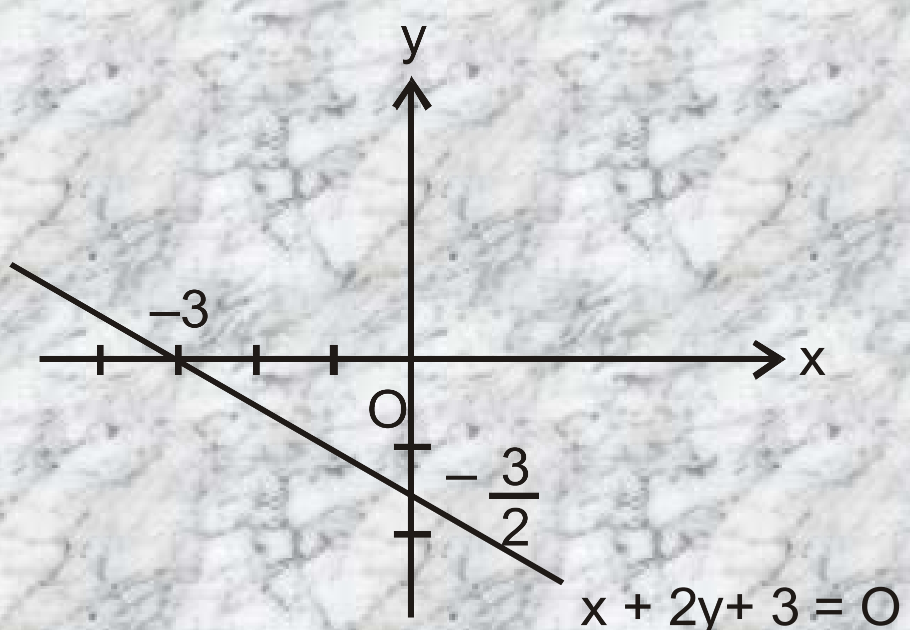
(ii) $2x - 3y + 4 = 0$

Solution

$x + 2y = -3$

x-intercept = -3 i.e. line pass through (0,-3/2)

y-intercept = -3/2 i.e. line pass through (-3,0)

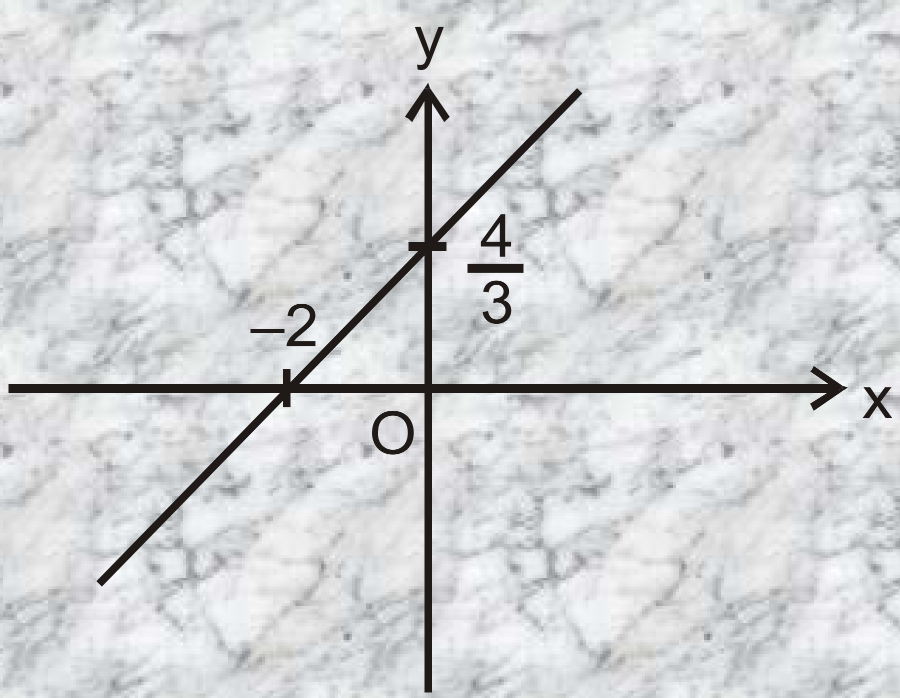


Solution

$2x - 3y + 4 = 0$

$2x - 3y = -4$

$$\frac{x}{-2} + \frac{y}{\frac{4}{3}} = 1$$ Intercept form



Class Exercise - 2

Find the equation of the lines passing through the following points.

(i) (0, –a) and (b, 0)

(ii) $\left(at_1, \dfrac{a}{t_1}\right)$ and $\left(at_2, \dfrac{a}{t_2}\right)$

Solution

(i) $y - 0 = \frac{0-(-a)}{b-0}(x-b)$

$$y = \frac{a}{b}(x-b)$$

$$ax - by - ab = 0$$

(ii) $y - \frac{a}{t_1} = \left(\frac{\frac{a}{t_2} - \frac{a}{t_1}}{at_2 - at_1}\right)(x - at_1) = \frac{-1}{t_1 t_2}(x - at_1)$

$$t_1 t_2 y - at_2 = -x + at_1$$

$$x + t_1 t_2 y = a(t_1 + t_2)$$

Class Exercise - 3

Find the equation of the line cutting off an intercept of –3 from axis of Y and inclined at an angle θ to positive X-axis, where $\tan \theta = 3/5$.

Solution :

Using slope intercept form

$y = \tan \theta \, x - 3$

$= 3/5 \, x - 3$

or $3x - 5y - 15 = 0$

Class Exercise - 4

(i) Find the equation of the line which passes through (1, 2) and the sum of the intercepts on axis is 6.

(ii)Find the equation of the line through (3, 2) so that the segment of the line intercepted between the axis is bisected at this point.

(iii)The length of the perpendicular from the origin to a line is 5 and the line makes an angle of 120° with the positive direction of Y-axis. Find the equation of the line.

Solution 4(i)

Let the equation of line in intercept form be

$\frac{x}{a}+\frac{y}{b}=1$, if it passes through (1, 2), then

$\frac{1}{a}+\frac{2}{b}=1$, also a + b = 6

$\Rightarrow \frac{1}{a}+\frac{2}{6-a}=1 \Rightarrow 6-a+2a=a(6-a)$

$\Rightarrow 6+a=6a-a^2 \Rightarrow a^2-5a+6=0 \Rightarrow (a-3)(a-2)=0$

$\Rightarrow a=3 \text{ or } 2$ Corresponding b = 3 or 4

Hence, equations become

$\frac{x}{3}+\frac{y}{3}=1 \text{ or } \frac{x}{2}+\frac{y}{4}=1$

Solution 4(ii)

Let x-intercept and y-intercept of the line be a and b respectively i.e. line passes through (a, 0) and (0, b)

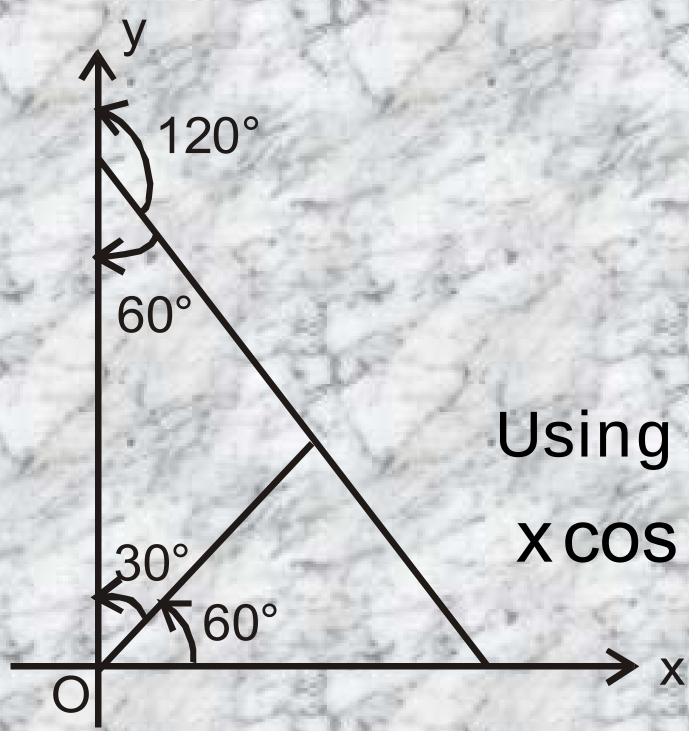
As segment joining (a, 0) and (0, b) is bisected by (3, 2)

$$\therefore \frac{a+0}{2} = 3 \text{ and } \frac{0+b}{2} = 2$$

a = 6 and b = 4 Equation of line becomes

$$\frac{x}{6} + \frac{y}{4} = 1 \text{ or } 2x + 3y = 12$$

Solution 4(iii)



Using normal form

$$x\cos\alpha + y\sin\alpha = p,$$

the equation of line becomes

x cos60° + y sin60° = 5 or

$$\frac{x}{2} + y\,\frac{\sqrt{3}}{2} = 5$$

or $x + \sqrt{3}y = 10$

Class Exercise - 5

A straight line is drawn through the point P(3, 2) and is inclined at an angle of 60° with the positive X-axis. Find the coordinates of points on it at a distance of 2 from P.

Solution

Using parameter form of line, i.e.

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

$$\frac{x-3}{\cos 60°} = \frac{y-2}{\sin 60°} = \pm 2 \qquad \frac{x-3}{\left(\frac{1}{2}\right)} = \frac{y-2}{\left(\frac{\sqrt{3}}{2}\right)} = 2$$

$$\frac{x-3}{\left(\frac{1}{2}\right)} = \frac{y-2}{\left(\frac{\sqrt{3}}{2}\right)} = -2 \qquad x = 4,\ y = 2 + \sqrt{3} \text{ or } x = 2,\ y = 2 - \sqrt{3}$$

Hence required points are

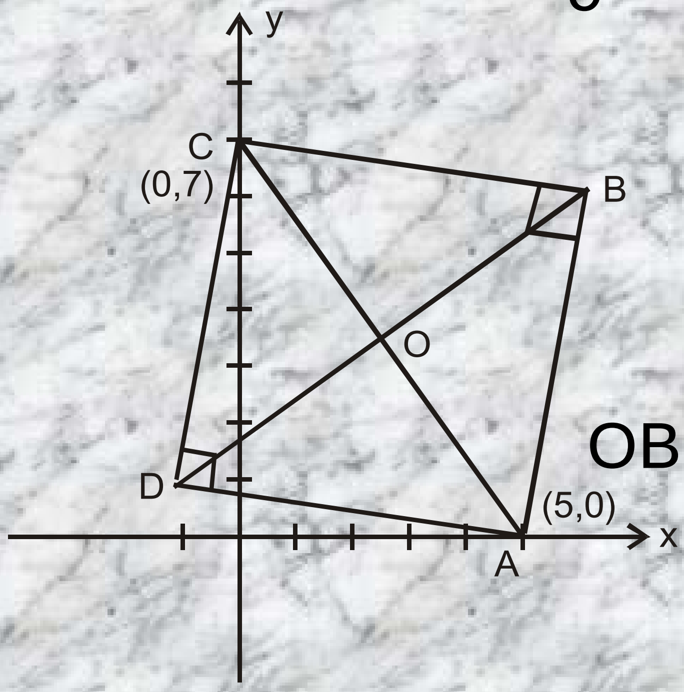
$$\left(4, 2 + \sqrt{3}\right) \text{ and } \left(2, 2 - \sqrt{3}\right)$$

Class Exercise - 6

One diagonal of a square is the portion of the line 7x + 5y = 35 intercepted by the axis. Obtain the extremities of the other diagonal.

Solution :

$$7x + 5y = 35 \text{ or } \frac{x}{5} + \frac{y}{7} = 1$$



Coordinates of O $\left(\frac{5}{2}, \frac{7}{2}\right)$

$$OB = OD = \frac{AC}{2} = \frac{\sqrt{5^2 + 7^2}}{2} = \frac{\sqrt{25 + 49}}{2} = \frac{\sqrt{74}}{2}$$

Solution

Equation of BD = $$\frac{x-\frac{5}{2}}{\cos\theta}=\frac{y-\frac{7}{2}}{\sin\theta}=r$$

where $$\tan\theta=\frac{-1}{\text{Slope of AC}}=\frac{-1}{\frac{7-0}{(0-5)}}=\frac{5}{7}$$

$$\frac{x-\frac{5}{2}}{\cos\theta}=\frac{y-\frac{7}{2}}{\sin\theta}=r$$

$$\tan\theta=\frac{-1}{\text{Slope of AC}}=\frac{-1}{\frac{7-0}{(0-5)}}=\frac{5}{7}$$

Solution

$$\cos\theta = \frac{7}{\sqrt{74}} \quad \sin\theta = \frac{5}{\sqrt{74}}$$

Equation of diagonal BD:

$$\frac{x-\frac{5}{2}}{\frac{7}{\sqrt{74}}} = \frac{y-\frac{7}{2}}{\frac{5}{\sqrt{74}}} = \pm\frac{\sqrt{74}}{2} \qquad x = \frac{5}{2}+\frac{7}{2},\ y = \frac{7}{2}+\frac{5}{2}$$

$$\text{or} \quad x = \frac{5}{2}-\frac{7}{2},\ y = \frac{7}{2}-\frac{5}{2} \qquad B\left(\frac{5}{2}+\frac{7}{2}, \frac{7}{2}+\frac{5}{2}\right), D\left(\frac{5}{2}-\frac{7}{2}, \frac{7}{2}-\frac{5}{2}\right)$$

B (6, 6), D (–1, 1)

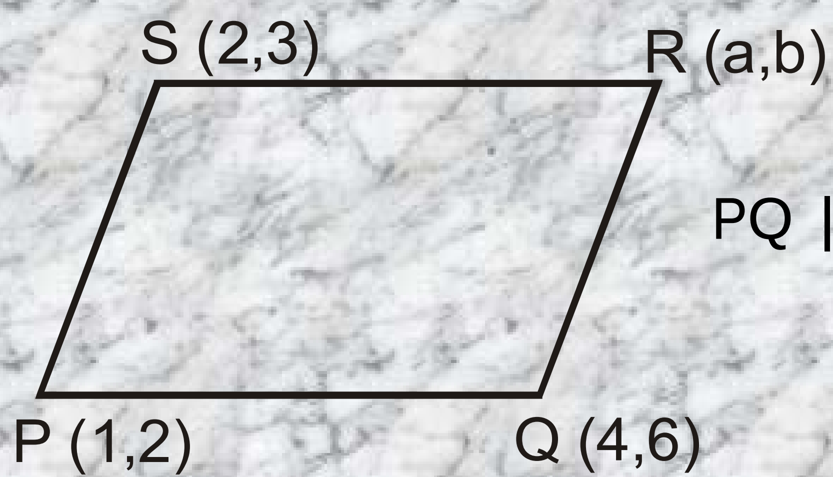
Class Exercise - 7

If P(1, 2), Q(4, 6), R(a, b) and S(2, 3) are the vertices of a parallelogram PQRS in order, then

(a) a = 5, b = 7 (b) a = 7, b = 5

(c) a = –5, b = 7 (d) a = –7, b = 5

Solution :



PQ || RS   Slope of PQ = Slope of RS

Solution

$$\Rightarrow \frac{6-2}{4-1} = \frac{b-3}{a-2} \Rightarrow 4a-8 = 3b-9$$

$$\Rightarrow 4a-3b+1=0 \quad \ldots(i)$$

PS || QR   Slope of PS = Slope of QR

$$\Rightarrow \frac{3-2}{2-1} = \frac{b-6}{a-4} \Rightarrow a-b+2=0 \quad \ldots(ii)$$

Solving (i) and (ii), we get

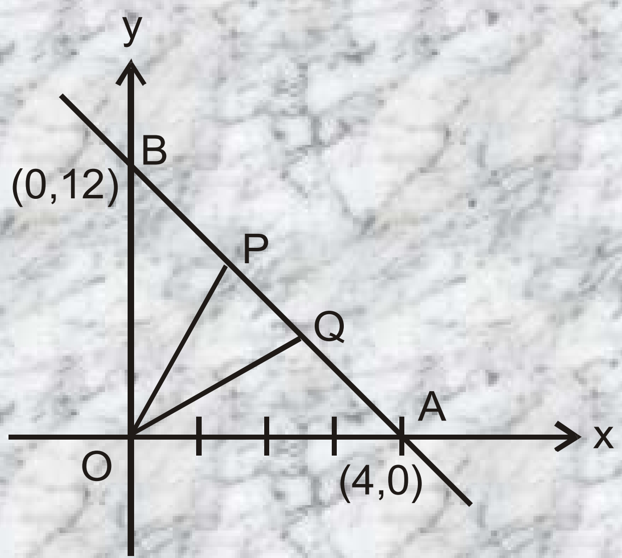
a = 5, b = 7

Class Exercise - 8

Find the equations of the lines which passes through the origin and trisect the portion of the straight line $3x + y = 12$ which is intercepted between the axes of coordinates.

Solution :

Let P be the point dividing AB in 2 : 1,



$$P \equiv \left(\frac{4+2.0}{3}, \frac{1.0+2.12}{3}\right) \equiv \left(\frac{4}{3}, 8\right)$$

Solution

And Q be the point dividing AB in the ratio 1 : 2, then

$$Q \equiv \left(\frac{1.0 + 2.4}{3}, \frac{1.12 + 2.0}{3} \right) \equiv \left(\frac{8}{3}, 4 \right)$$

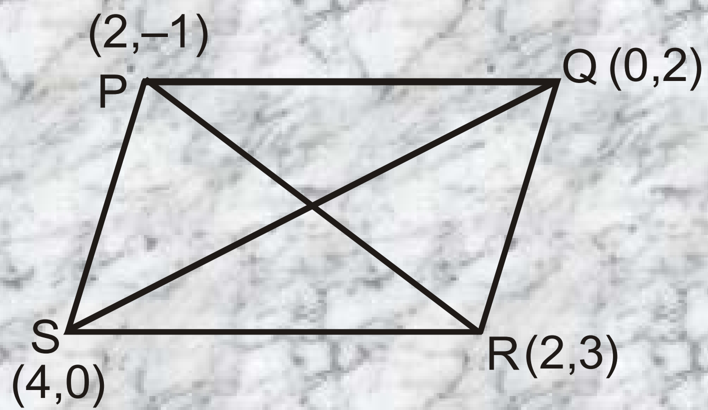
Equation of $OP \equiv y = \frac{8}{\frac{4}{3}} x \Rightarrow y = 6x$

Equation of $OQ \equiv y = \frac{4}{\frac{8}{3}} x \Rightarrow y = \frac{3}{2} x$

Class Exercise - 9

Prove that the points (2, –1), (0, 2), (2, 3) and (4, 0) are the vertices of a parallelogram.

Solution :



Slope of PQ = $\frac{2-(-1)}{0-2} = \frac{3}{-2} = \frac{-3}{2}$

Solution

Slope of RS = $\frac{0-3}{4-2} = \frac{-3}{2}$

Slope of PS = $\frac{0-(-1)}{4-2} = \frac{1}{2}$

Slope of QR = $\frac{3-2}{2-0} = \frac{1}{2}$

As slope of PQ = Slope of RS and
Slope of PS = Slope of QR

PQRS is a parallelogram

Class Exercise - 10

(i) Find the equation of line passing through the point (–4, –3) and perpendicular to the straight line joining (1, 3) and (2, 7).

(ii) Find the equation to the straight line drawn at right angle to the straight line $x/a - y/b = 1$ through the point where it meets the axis of x.

Solution - 10(i)

Slope of the required line is

$$\frac{-1}{\left(\frac{7-3}{2-1}\right)} = \frac{-1}{4}$$

Equation of required line by point slope form is given by

$$\left(y-(-3)\right) = \frac{-1}{4}\left(x-(-4)\right)$$

$$y+3 = \frac{-1}{4}(x+4) \quad 4y + 12 = -x - 4$$

$$x + 4y + 16 = 0$$

Solution - 10(ii)

$$\frac{x}{a}-\frac{y}{b}=1$$

meets X-axis at (a, 0) any line perpendicular to

$\frac{x}{a}-\frac{y}{b}=1$ is given by $\frac{x}{b}+\frac{y}{a}=c$

it passes through (a, 0) then $\frac{a}{b}+0=c$

$$\Rightarrow \frac{x}{b}+\frac{y}{a}=\frac{a}{b}$$

$$\Rightarrow ax+by=a^2$$

Class Exercise - 11

Show that the equations to the straight lines passing through the points (3, –2) and inclined at 60° to the line $\sqrt{3}x + y = 1$ are y + 2 = 0 and $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Solution

Let the slope of the required line is m, then

$$\left|\frac{m-\left(-\sqrt{3}\right)}{1+m\left(-\sqrt{3}\right)}\right| = \tan 60^\circ = \sqrt{3}$$

$$\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3} \Rightarrow \frac{m+\sqrt{3}}{1-\sqrt{3}m} = \pm\sqrt{3}$$

$$m+\sqrt{3} = \sqrt{3}-3m \text{ or } m+\sqrt{3} = -\sqrt{3}+3m$$

$\Rightarrow m = 0$ or $\sqrt{3}$ Equation of lines are given by

$$\frac{y+2}{x-3} = 0 \text{ or } \frac{y+2}{x-3} = \sqrt{3} \quad \Rightarrow y+2=0 \text{ or } y-\sqrt{3}x+2+3\sqrt{3}=0$$

Class Exercise - 12

Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle; keeping the origin fixed, the same line has intercepts p and q, then

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

Solution - Method I

Equation of line in old reference is

$$\frac{x}{a} + \frac{y}{b} = 1$$

if the axis of coordinates is rotated at an angle α

$$x = X\cos\alpha - Y\sin\alpha \quad y = X\sin\alpha + Y\cos\alpha$$

$$\frac{X\cos\alpha - Y\sin\alpha}{a} + \frac{X\sin\alpha + Y\cos\alpha}{b} = 1$$

$$X\left(\frac{\cos\alpha}{a} + \frac{\sin\alpha}{b}\right) + Y\left(\frac{\cos\alpha}{b} - \frac{\sin\alpha}{a}\right) = 1 \quad \ldots(i)$$

In new frame equation is $\frac{X}{p} + \frac{Y}{q} = 1 \quad \ldots(ii)$

Solution Cont.

$$X\left(\frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}\right)+Y\left(\frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}\right)=1\ \ldots\text{(i)}$$

$$\frac{X}{p}+\frac{Y}{q}=1\ \ldots\text{(ii)}$$

As (i) and (ii) represent same line

$$\frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}=\frac{1}{p}\text{ and }\frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}=\frac{1}{q}$$

Squaring and adding, we get

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

Solution Method II

$$\frac{x}{a}+\frac{y}{b}=1 \quad \ldots(i)$$

$$\frac{X}{p}+\frac{Y}{q}=1 \quad \ldots(ii)$$

Now both equation represents the

same line with different axes

Hence distance of origin from both lines is same

$$\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \quad \text{or} \quad \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

Class Exercise - 13

Equations

$(b - c)x + (c - a)y + (a - b) = 0$

and $\left(b^3 - c^3\right)x + \left(c^3 - a^3\right)y + a^3 - b^3 = 0$

will represent the same line if

(a) $b = c$

(b) $c = a$

(c) $a + b + c = 0$

(d) All of these

Solution

$$\frac{b-c}{b^3-c^3}=\frac{c-a}{c^3-a^3}=\frac{a-b}{a^3-b^3}=\frac{1}{k}\,(\text{say})$$

$$b^3-c^3=k(b-c)$$

$$\Rightarrow (b-c)\left(b^2+c^2+bc\right)=k(b-c)$$

Similarly, c = a or $c^2+a^2+ca=k$

and a = b or $a^2+b^2+ab=k$

$$b^2+c^2+bc=c^2+a^2+k \qquad b^2-a^2=c(a-b)$$

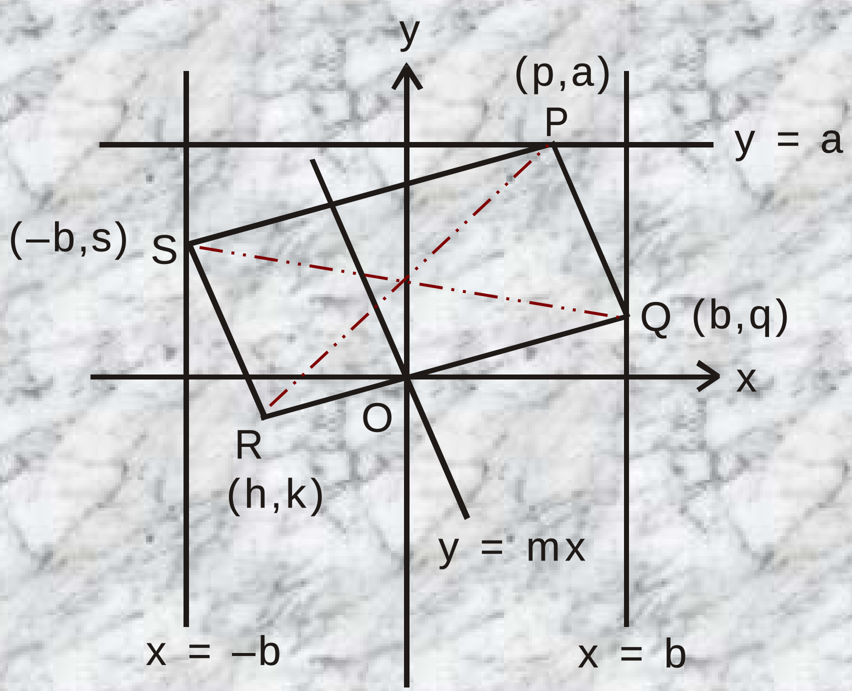
$$(b-a)(b+a+c)=0 \quad \Rightarrow b=a \text{ or } a+b+c=0$$

Hence either a = b or b = c or c = a or a + b + c = 0

Class Exercise - 14

A rectangle PQRS has its side PQ parallel to the line $y = mx$ and vertices P, Q and S on the lines $y = a$, $x = b$ and $x = -b$, respectively. Find the locus of the vertex R.

Solution



As PQRS is a rectangle Diagonals bisect each other i.e.

$$\left(\frac{h+p}{2}, \frac{k+a}{2}\right) \equiv \left(0, \frac{s+q}{2}\right)$$

$$\Rightarrow p = -h, \; q + s = k + a$$

$$\text{Slope of PQ} = \frac{a-q}{p-b} = m$$

Solution

$p = -h,\ q + s = k + a$

Slope of PQ $= \dfrac{a - q}{p - b} = m$

$\Rightarrow q = a + m(h + b)$ as $p = -h$

Slope of PS $= \dfrac{a - s}{p + b} = -\dfrac{1}{m}$

$\Rightarrow s = a + \dfrac{1}{m}(b - h)$

As q + s = k + a we get

$a + m(h + b) + a + \dfrac{1}{m}(b - h) = k + a$

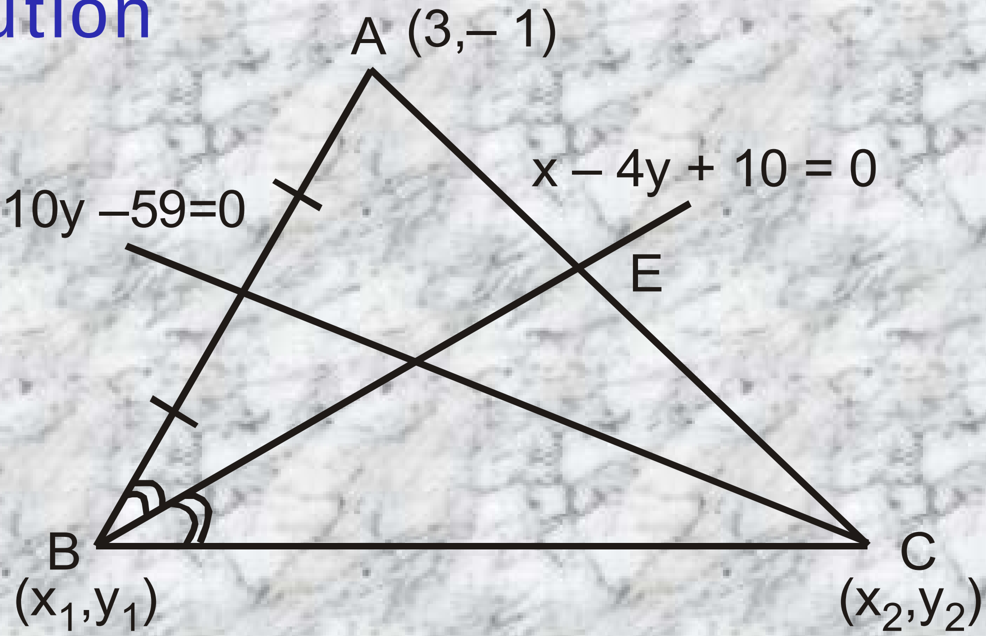
$(m^2 - 1)h - mk + (m^2 + 1)b + ma = 0$

Hence locus is $(m^2 - 1)x - my + am + (m^2 + 1)b = 0$

Class Exercise 15

Find equations of the sides of the triangle having (3, –1) as a vertex, $x - 4y + 10 = 0$ and $6x + 10y - 59 = 0$ being the equations of an angle bisector and a median respectively drawn from different vertices.

Solution



(x_1, y_1) lies on $x - 4y + 10 = 0$

$$x_1 - 4y_1 + 10 = 0 \quad \ldots(i)$$

Also $\left(\frac{x_1+3}{2}, \frac{y_1-1}{2}\right)$ lies on $6x + 10y - 59 = 0$

$$\therefore 3(x_1+3) + 5(y_1-1) - 59 = 0$$

Solution

$$x_1 - 4y_1 + 10 = 0 \quad \ldots(i)$$

$$3x_1 + 5y_1 - 55 = 0 \quad \ldots(ii)$$

Solving (i) and (ii), we get

$$17y_1 - 85 = 0 \Rightarrow y_1 = 5, x_1 = 10$$

Now (x_2, y_2) lies on $6x + 10y - 59 = 0$

$$6x_2 + 10y_2 - 59 = 0 \quad \ldots(iii)$$

$$\tan\angle CBE = \tan\angle EBA$$

$$\Rightarrow \frac{\frac{y_2 - 5}{x_2 - 10} - \frac{1}{4}}{1 + \frac{1}{4}\left(\frac{y_2 - 5}{x_2 - 10}\right)} = \frac{\frac{1}{4} - \frac{6}{7}}{1 + \frac{1}{4}.\frac{6}{7}}$$

Solution

$$\Rightarrow \frac{4y_2 - 20 - x_2 + 10}{4x_2 - 40 + y_2 - 5} = \frac{7-24}{28+6} = \frac{-17}{34} = \frac{-1}{2}$$

$$\Rightarrow 8y_2 - 2x_2 - 20 = -4x_2 - y_2 + 45$$

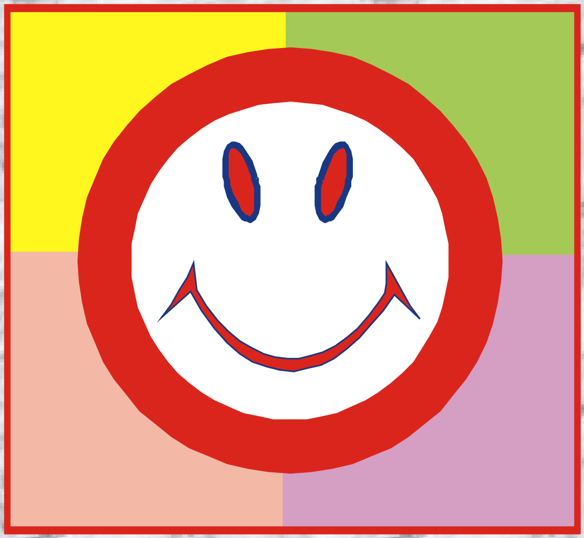
$$6x_2 + 10y_2 - 59 = 0 \quad \ldots\text{(iii)}$$

$$2x_2 + 9y_2 = 65 \quad \ldots\text{(iv)}$$

Solving (iii) and (iv), we get $x_2 = \frac{-7}{2}$, $y_2 = 8$

Hence using two point form equations of AB, BC and CA respectively are

6x – 7y = 25, 2x + 9y = 65, 18x + 13y = 41



Thank you