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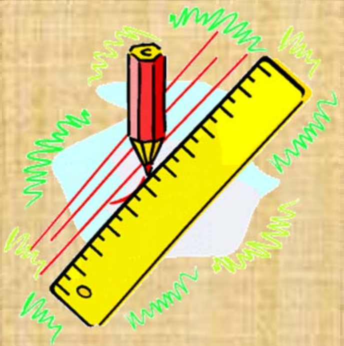
Mathematical Similarity



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Working out Scale Factor

Give one distance from the map and the corresponding actual distance we can work out the scale of the map.

Example :

The map distance from Ben Nevis to Ben Doran is 2cm.
The real-life distance is 50km.
What is the scale of the map.

	Map		Real Distance
	2	⇨	50km
	1	⇨	50 ÷ 2 = 25km
Scale Factor	1	:	250 000

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Working out Scale Factor

Example :

The actual length of a Olympic size swimming pool is 50m.
On the architect's plan it is 10cm.
What is the scale of the plan.

	Plan		Real Distance
	10	⇨	50m
	1	⇨	5
Scale Factor	1	:	500

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Similar Triangles 1

Learning Intention

1. To explain how the scale factor applies to similar triangles.

Success Criteria

1. Understand how the scale factor applies to similar triangles.
2. Solve problems using scale factor.

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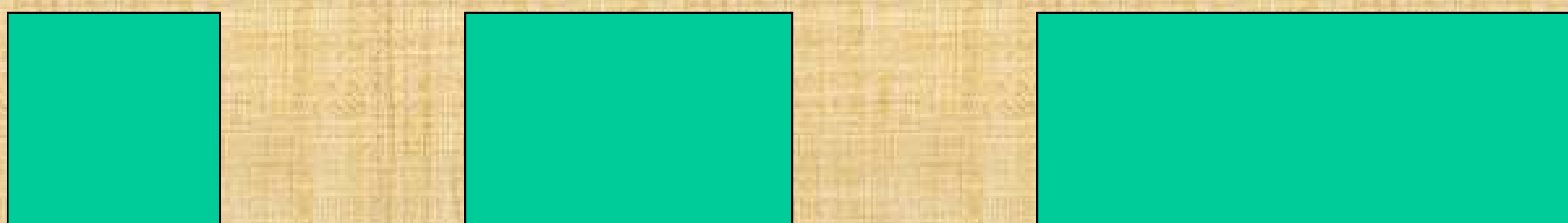
Conditions for similarity

Two shapes are similar only when:

- Corresponding sides are in proportion **and**
- Corresponding angles are equal



All rectangles are **not** similar to one another since only condition 2 is true.

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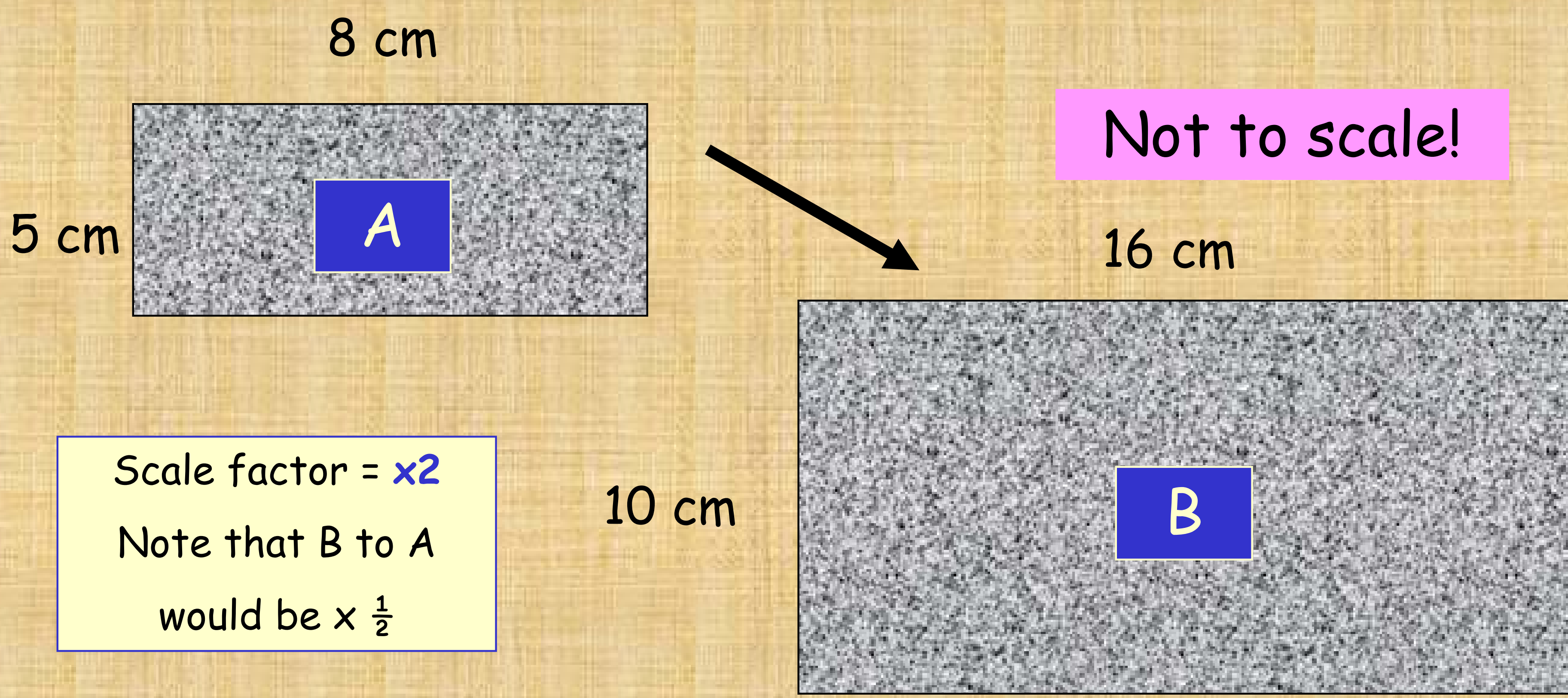
If two objects are similar then one is an enlargement of the other

The rectangles below are **similar**:

Find the scale factor of enlargement that maps A to B



Scale factor = **x2**

Note that B to A

would be x $\frac{1}{2}$

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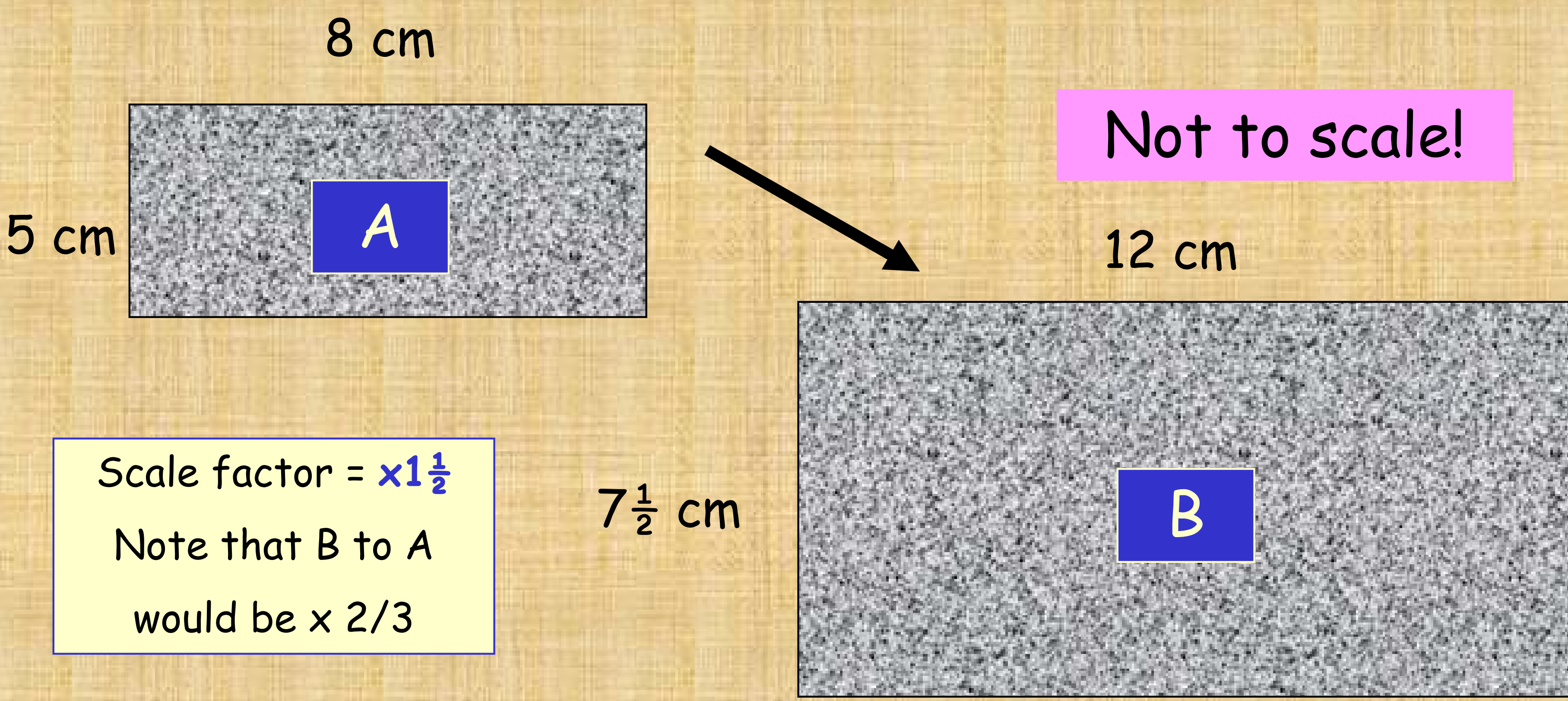
If two objects are similar then one is an enlargement of the other

The rectangles below are **similar**:

Find the scale factor of enlargement that maps A to B



Scale factor = **x1½**

Note that B to A

would be x 2/3

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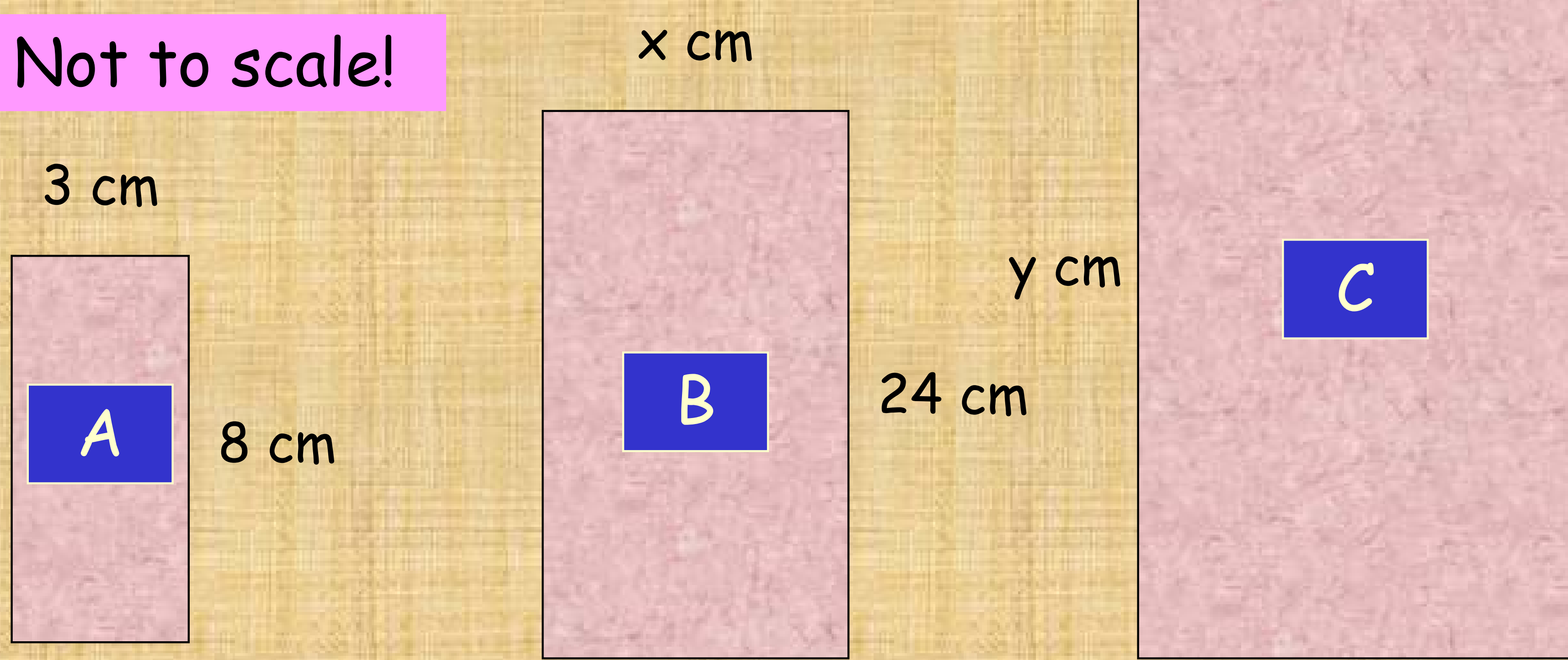
If we are told that two objects are similar and we can find the scale factor of enlargement then we can calculate the value of an unknown side.

The 3 rectangles are similar. Find the unknown sides

Not to scale!



Comparing corresponding sides in A and B: 24/8 = **3** so x = 3 x 3 = **9 cm**

Comparing corresponding sides in A and C: $13\frac{1}{2}$ /3 = **$4\frac{1}{2}$** so y = $4\frac{1}{2}$ x 8 = **36 cm**

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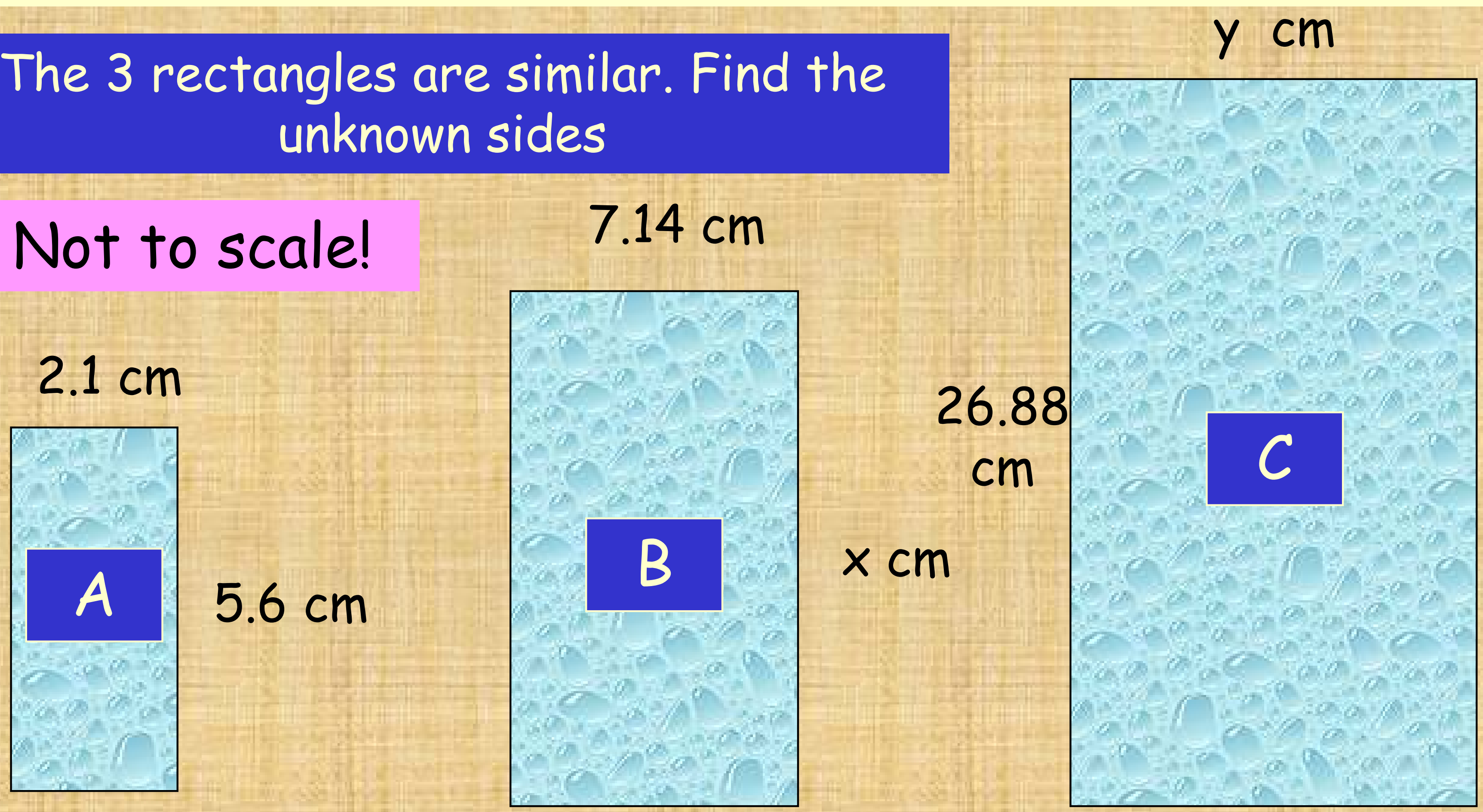
If we are told that two objects are similar and we can find the scale factor of enlargement then we can calculate the value of an unknown side.

The 3 rectangles are similar. Find the unknown sides

Not to scale!



Comparing corresponding sides in A and B: 7.14/2.1 = 3.4 so x = 3.4 x 5.6 = 19.04 cm

Comparing corresponding sides in A and C: 26.88/5.6 = 4.8 so y = 4.8 x 2.1 = 10.08 cm

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Similar Triangles

Similar triangles are important in mathematics and their application can be used to solve a wide variety of problems.

The two conditions for similarity between shapes as we have seen earlier are:

- Corresponding sides are in proportion **and**
- Corresponding angles are equal

Triangles are the exception to this rule.

only the second condition is needed

Two triangles are similar if their

- Corresponding angles are equal

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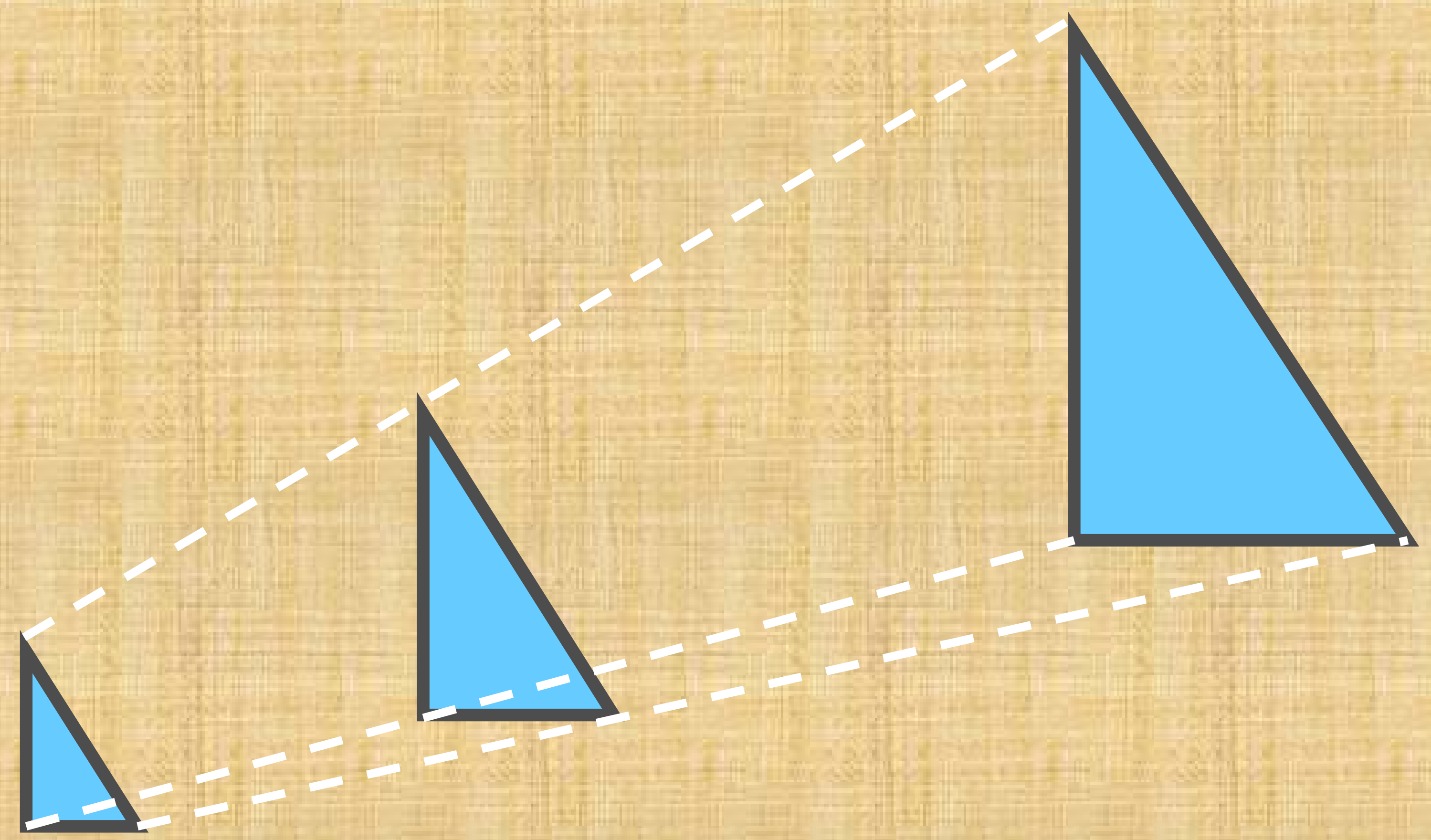
Similar Triangles



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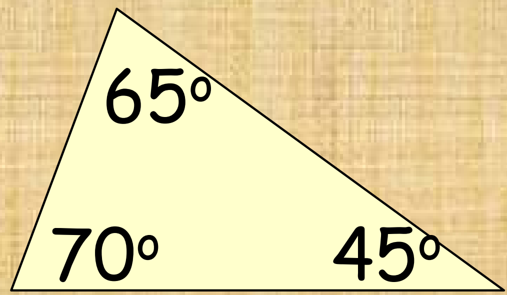
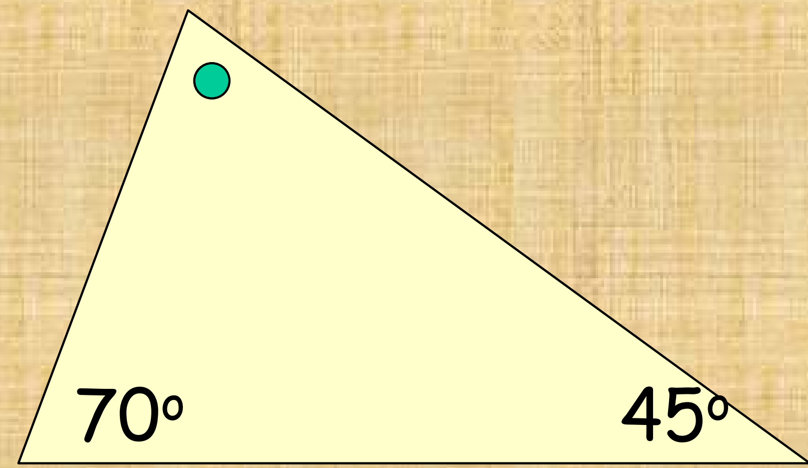
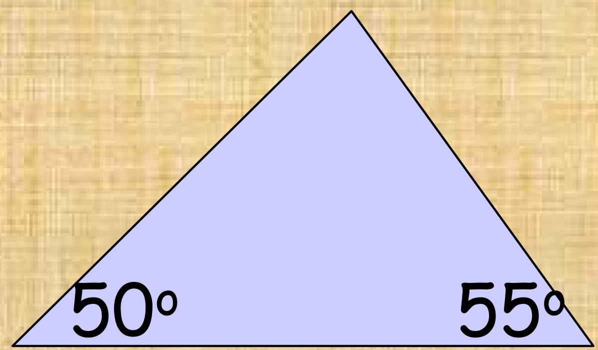
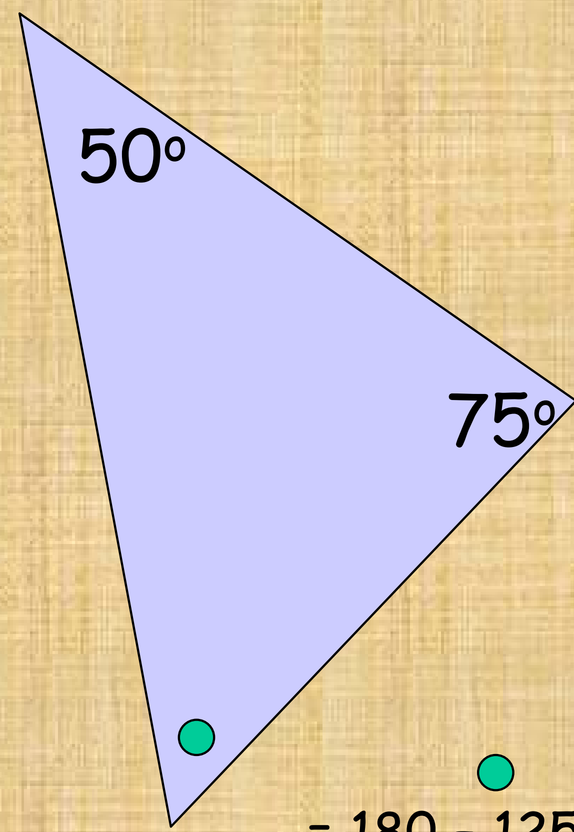
These two triangles are similar since they are equiangular.









These two triangles are similar since they are equiangular.

If 2 triangles have 2 angles the same then they must be equiangular

= 180 – 125 = 55

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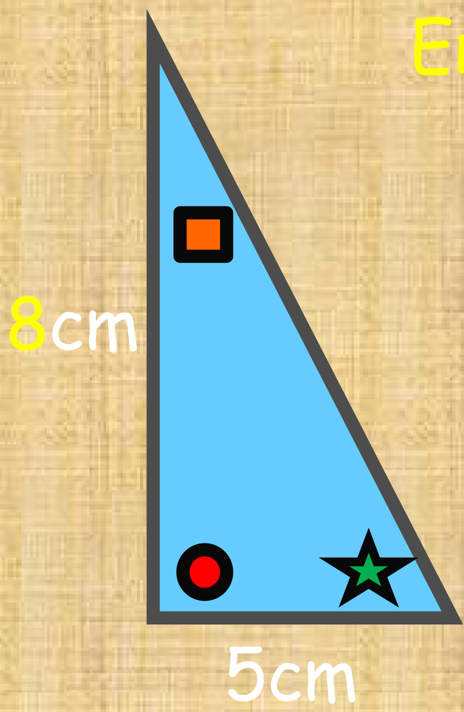
Scale factors

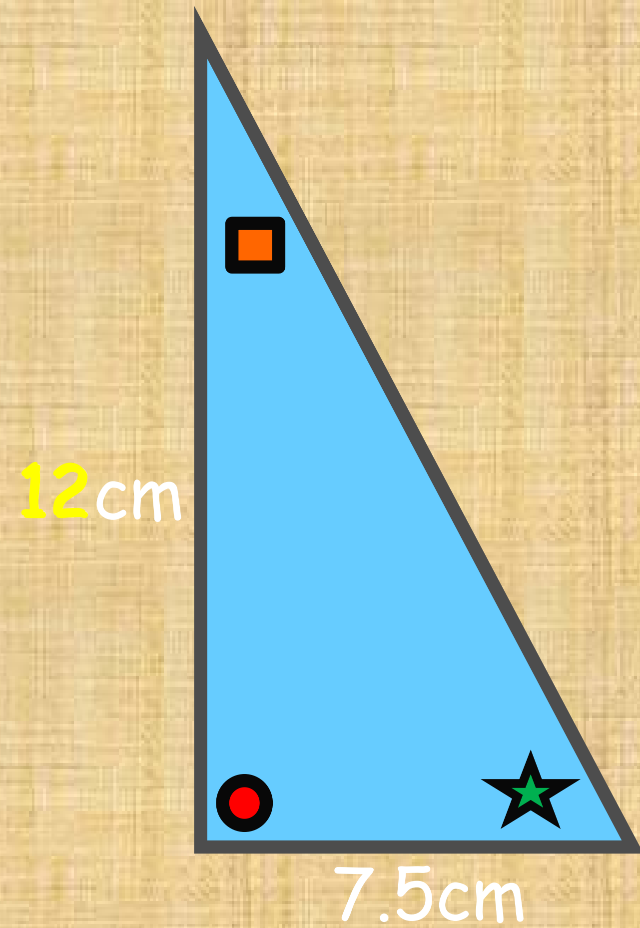


Enlargement Scale factor?

ESF = ___ = $\frac{3}{2}$

Reduction Scale factor?

RSF = ___ = $\frac{2}{3}$



Can you see the relationship between the two scale factors?

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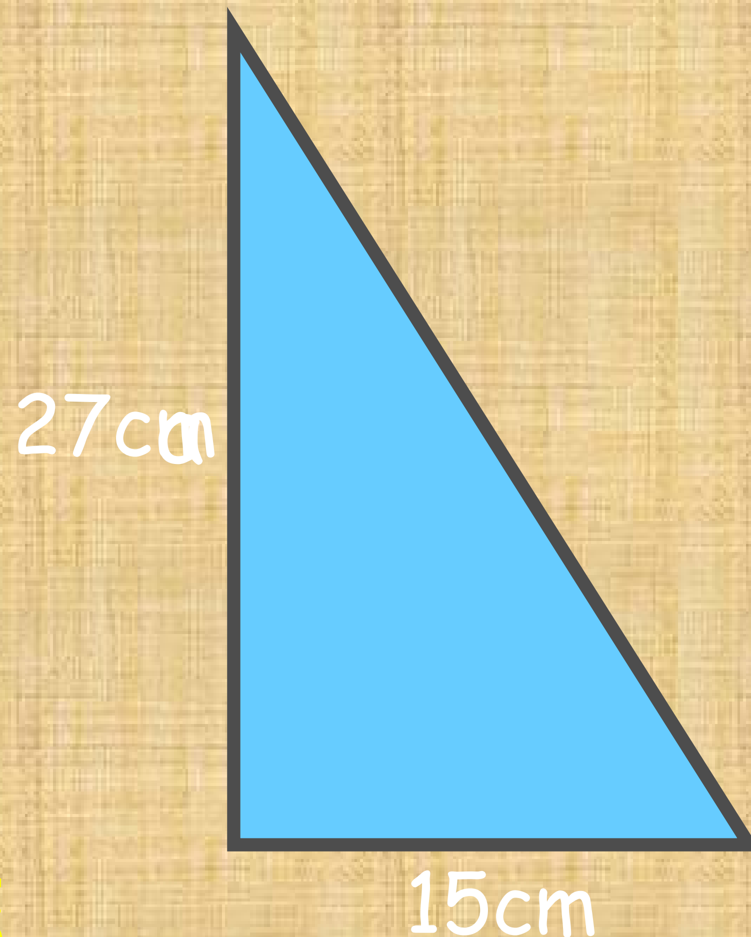
Scale factors

Find a given ESF = 3

$$ESF = 3 = \frac{a}{9}$$

By finding the RSF
Find the value of b.

$$RSF = \frac{1}{3} = \frac{b}{15} = \frac{5}{15}$$



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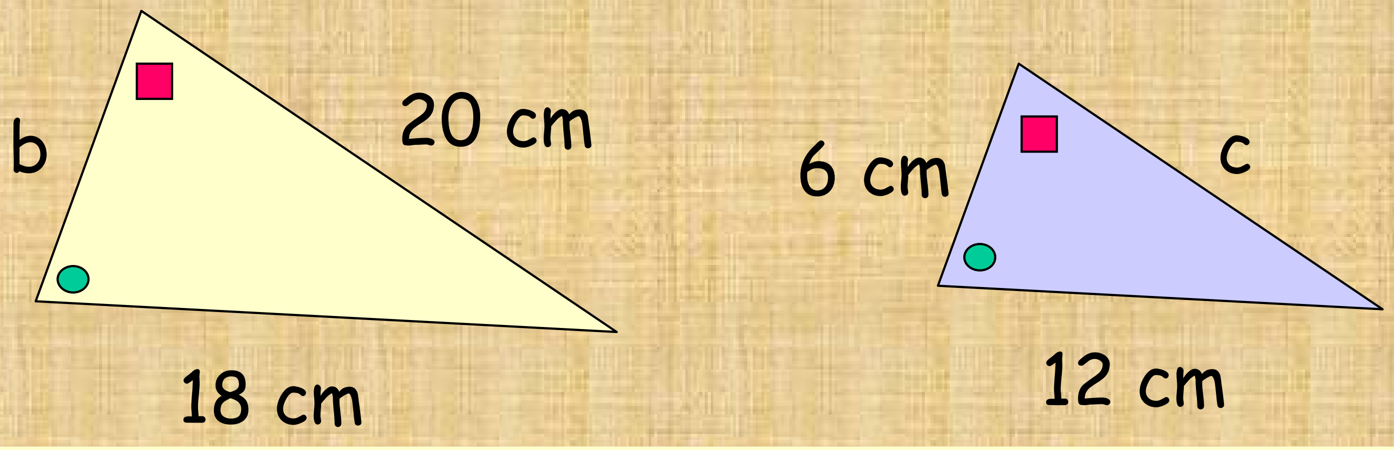
Finding Unknown sides (1)



Since the triangles are equiangular they are similar.

So comparing corresponding sides.

$$\frac{b}{6} = \frac{18}{12}$$

$$\frac{c}{20} = \frac{12}{18}$$

$$b = \frac{6x18}{12} = 9 \text{ cm}$$

$$c = \frac{20x12}{18} = 13.3 \text{ cm}$$

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Starter Questions

Q1. Find the roots to 1 decimal place

$$1 - 7x - x^2 = 0$$

Q2. A freezer is reduced by 20% to £200 in a sale. What was the **original** price.

Q3. Calculate $3\frac{3}{4} \times 1\frac{1}{3}$

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Similar Triangles 2

Learning Intention

1. To explain how the scale factor applies to similar triangles with algebraic terms.

Success Criteria

1. Understand how the scale factor applies to similar triangles with algebraic terms.

2. Solve problems using scale factor that contain algebraic terms.

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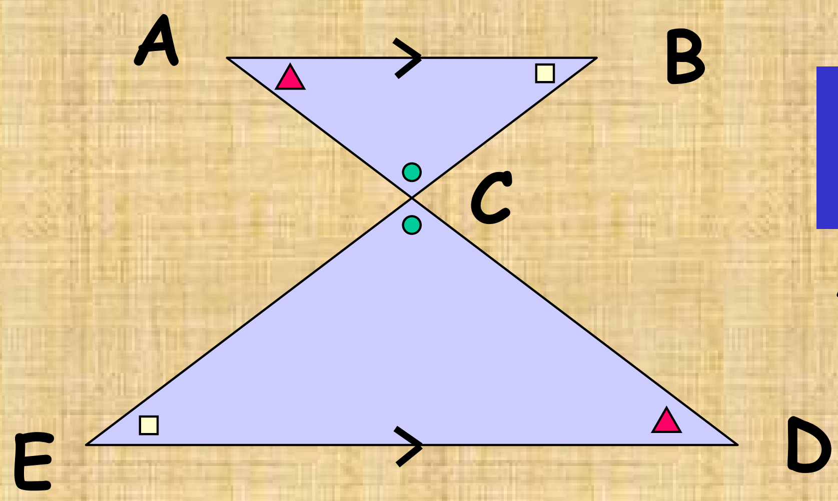
Determining similarity



Triangles ABC and DEC are similar. Why?

Angle ACB = angle ECD (Vertically Opposite)

Angle ABC = angle DEC (Alt angles)

Angle BAC = angle EDC (Alt angles)

Since ABC is similar to DEC we know that corresponding sides are in proportion

AB→DE BC→EC AC→DC

The order of the lettering is important in order to show which pairs of sides correspond.

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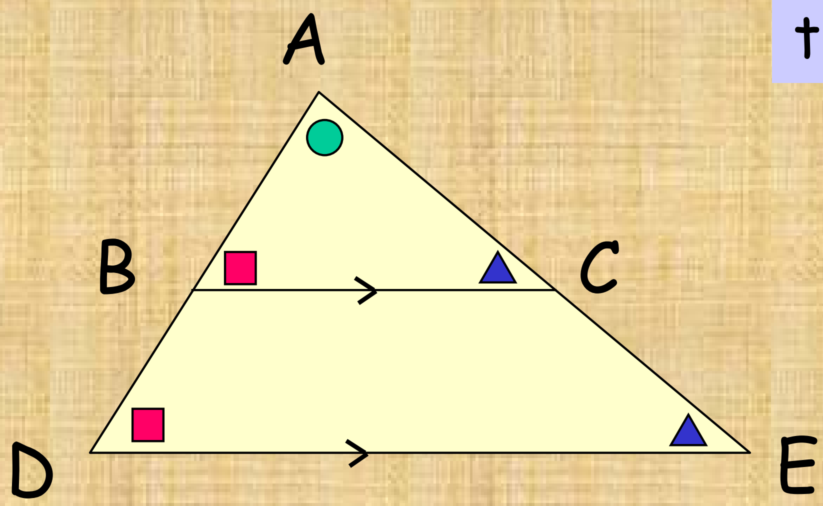


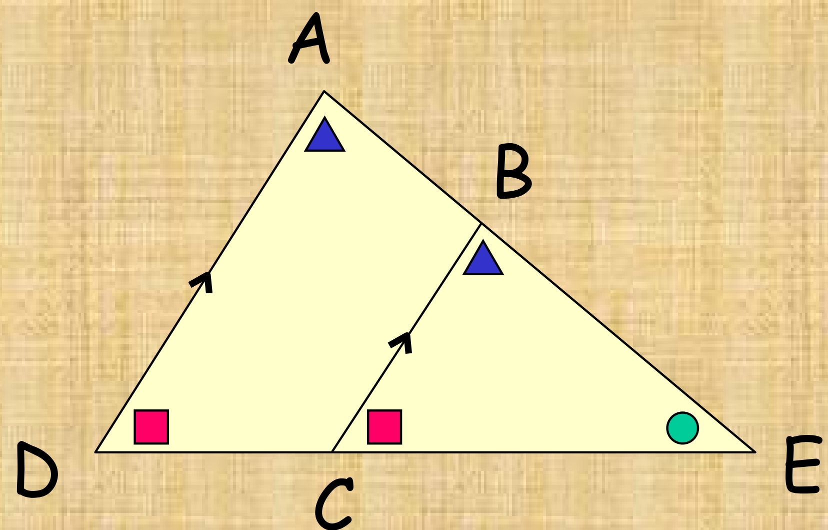
If BC is parallel to DE, explain why triangles ABC and ADE are similar

Angle BAC = angle DAE (common to both triangles)

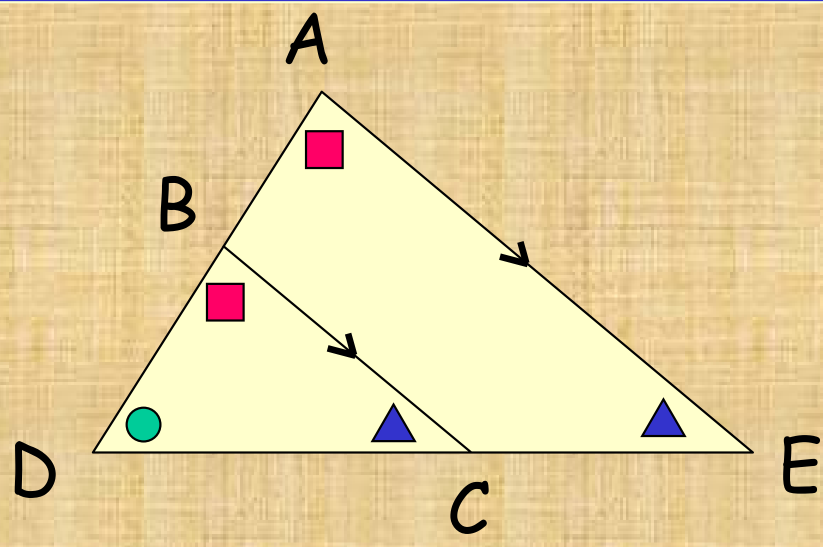
Angle ABC = angle ADE (corresponding angles between parallels)

Angle ACB = angle AED (corresponding angles between parallels)

A line drawn parallel to any side of a triangle produces 2 similar triangles.



Triangles EBC and EAD are similar



Triangles DBC and DAE are similar

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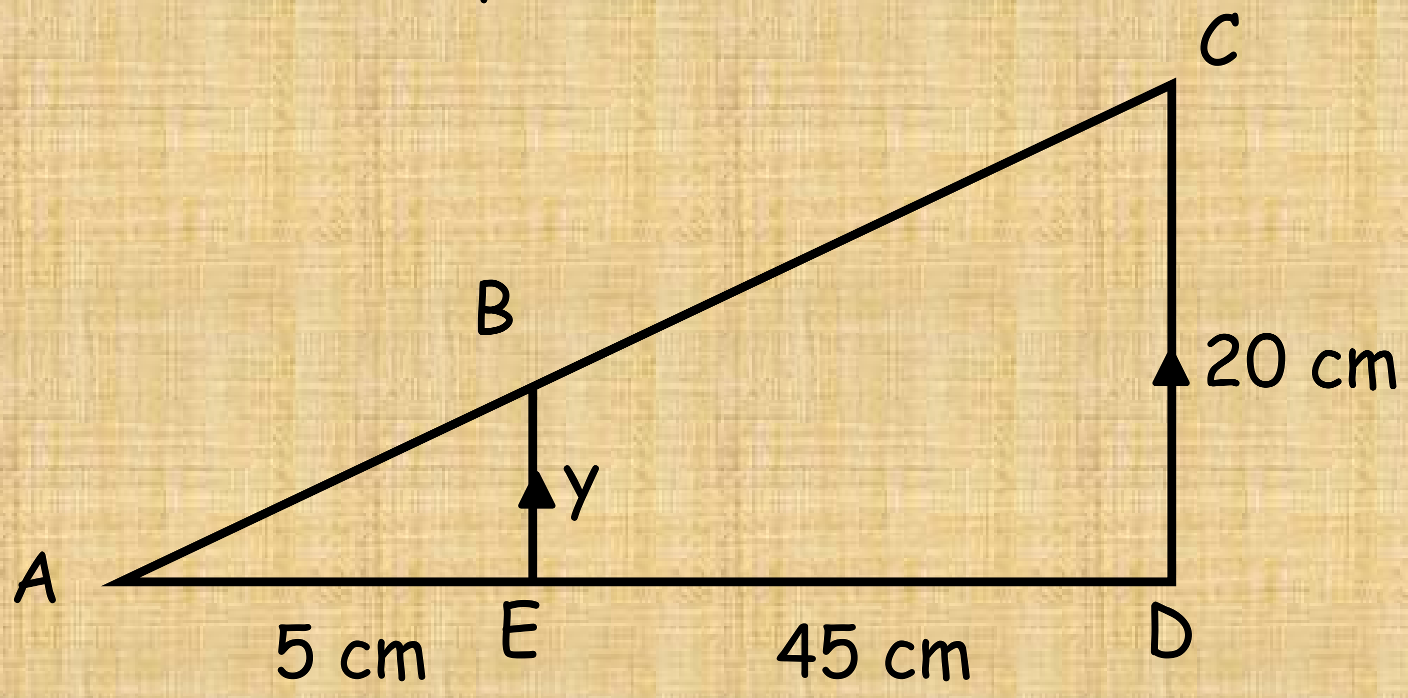
The two triangles below are similar:

Find the distance y.



$$RSF = \frac{AE}{AD} = \frac{5}{50} = \frac{y}{20}$$

$$y = \frac{5 \times 20}{50} = 2\text{ cm}$$

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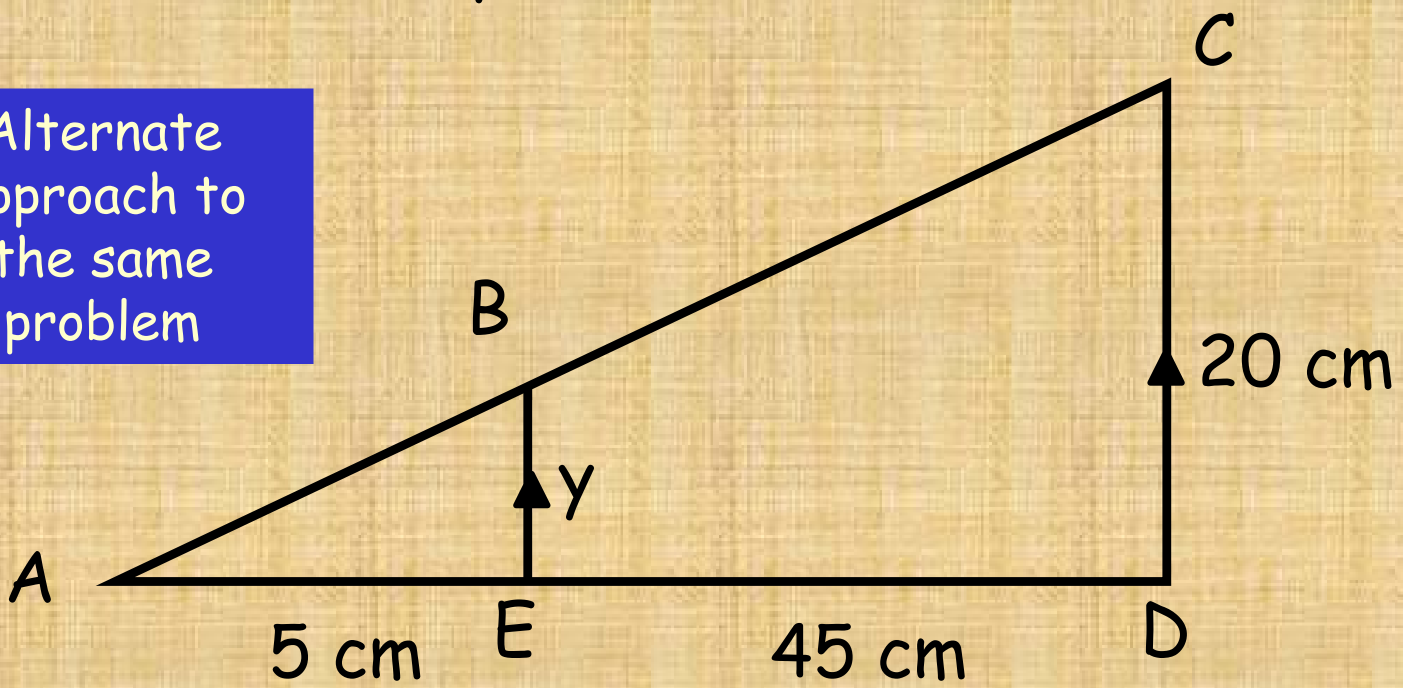
The two triangles below are similar:

Find the distance y.

Alternate approach to the same problem



$$ESF = \frac{AD}{AE} = \frac{50}{5} = \frac{20}{y}$$

$$y = \frac{5 \times 20}{50} = 2 \text{ cm}$$

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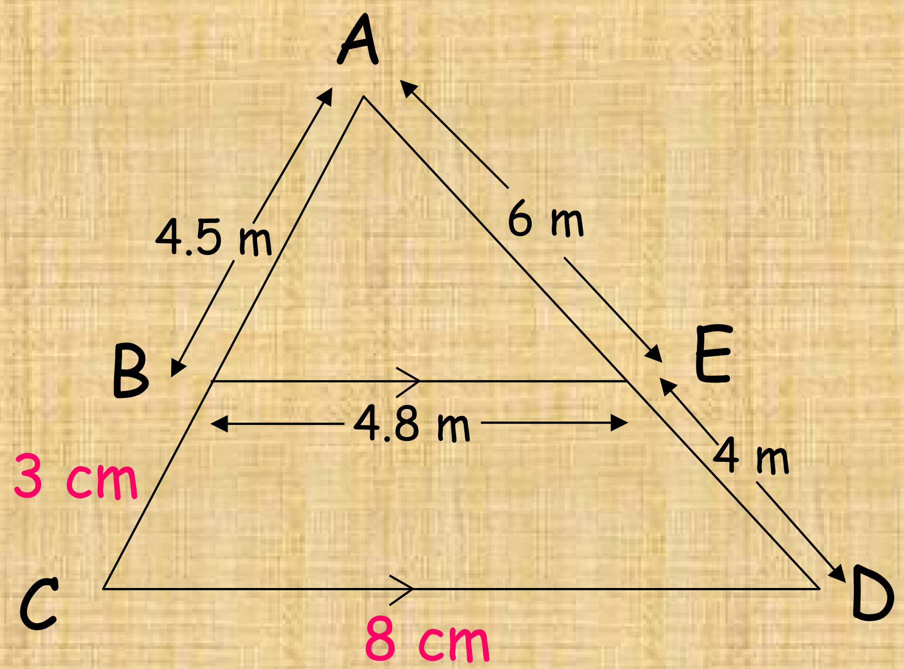
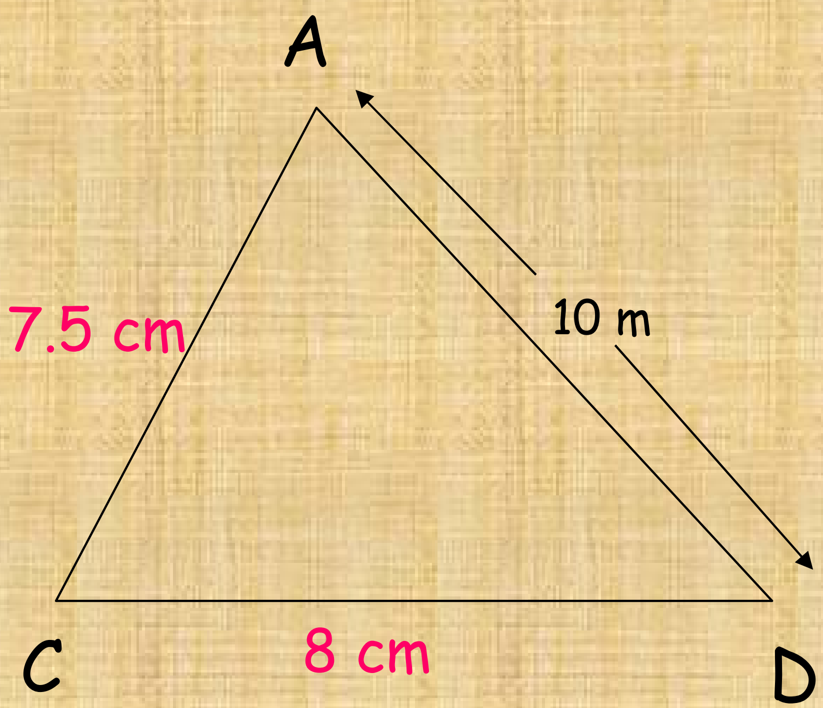
In the diagram below BE is parallel to CD and all measurements are as shown.

(a) Calculate the length CD

(b) Calculate the perimeter of the Trapezium EBCD





(a) $SF = \frac{10}{6} = \frac{CD}{4.8}$

$CD = \frac{4.8 \times 10}{6} = 8 \text{ cm}$

(b) $SF = \frac{10}{6} = \frac{AC}{4.5}$

$AC = \frac{4.5 \times 10}{6} = 7.5 \text{ cm}$

So perimeter

= 3 + 8 + 4 + 4.8

= 19.8 cm

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Similar Triangles 3

Learning Intention

1. To explain how the scale factor applies to similar triangles with harder algebraic terms.

Success Criteria

1. Understand how the scale factor applies to similar triangles with harder algebraic terms.
2. Solve problems using scale factor that contain harder algebraic terms.

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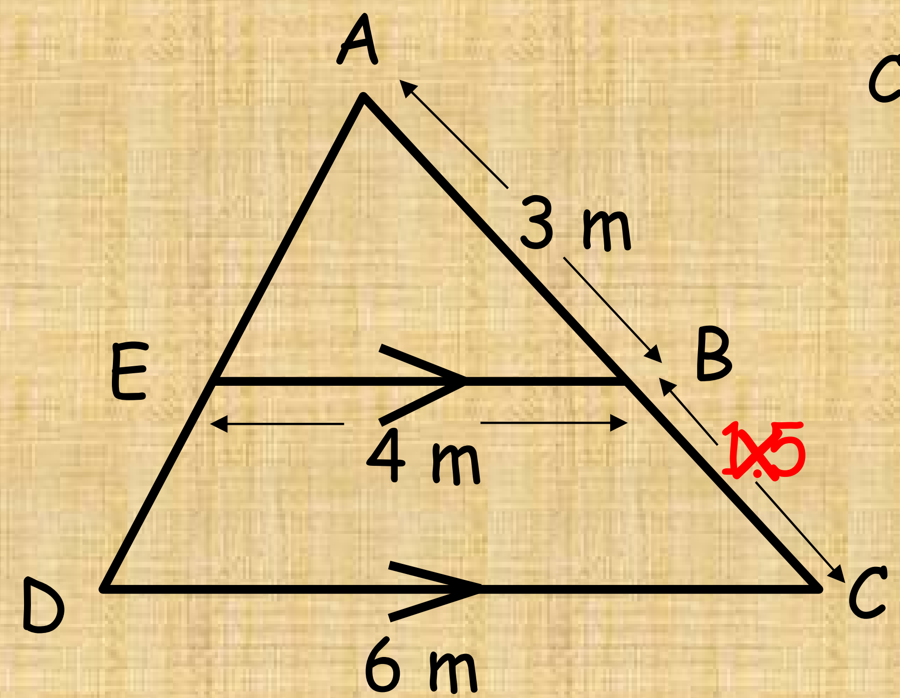
In a pair of similar triangles the ratio of the corresponding sides is constant, always producing the same enlargement or reduction.

Find the values of x given that the triangles are similar.



Corresponding sides are in proportion

$$\frac{AB}{AC} = \frac{EB}{DC}$$

$$\frac{3}{3 + x} = \frac{4}{6}$$

$$3 \times 6 = 4(3 + x)$$

$$18 = 12 + 4x$$

$$4x = 6$$

$$x = 1.5$$

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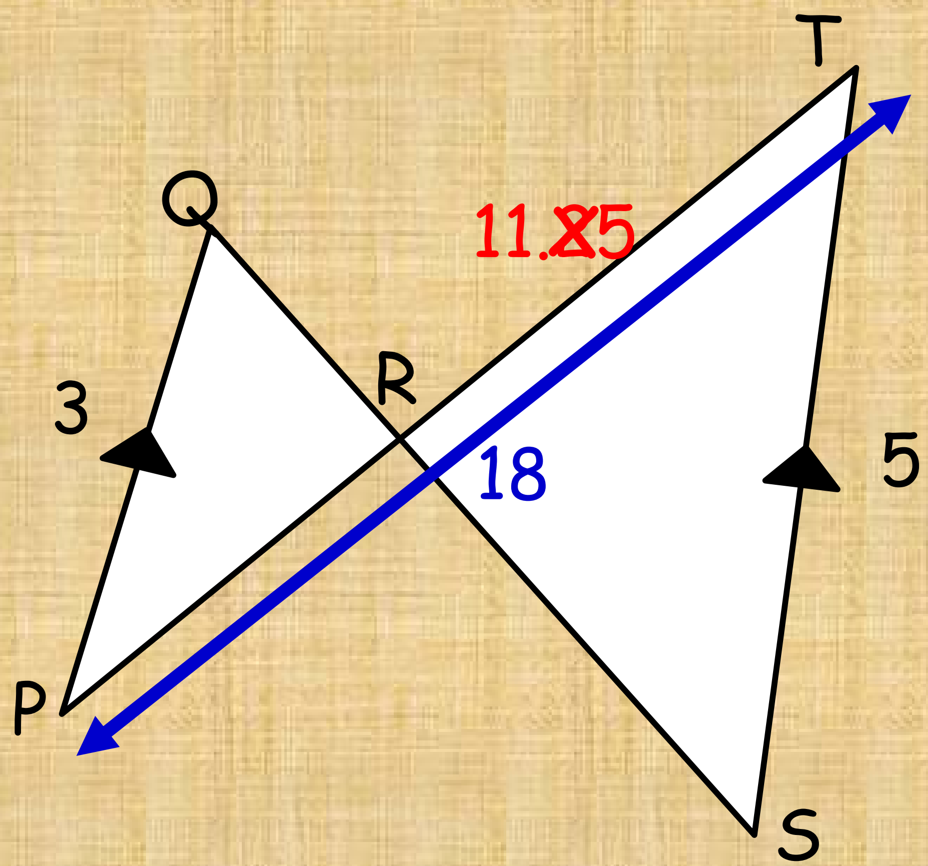
In a pair of similar triangles the ratio of the corresponding sides is constant, always producing the same enlargement or reduction.

Find the values of x given that the triangles are similar.



Corresponding sides are in proportion

$$\frac{PQ}{ST} = \frac{PR}{PT}$$

$$\frac{3}{5} = \frac{18 - x}{x}$$

$$3x = 5(18 - x)$$

$$3x = 90 - 5x$$

$$8x = 90$$

$$x = 11.25$$

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Similar Figures

Learning Intention

1. To explain how the scale factor applies to other similar figures.

Success Criteria

1. Understand how the scale factor applies to other similar figures.
2. Solve problems using scale factor.

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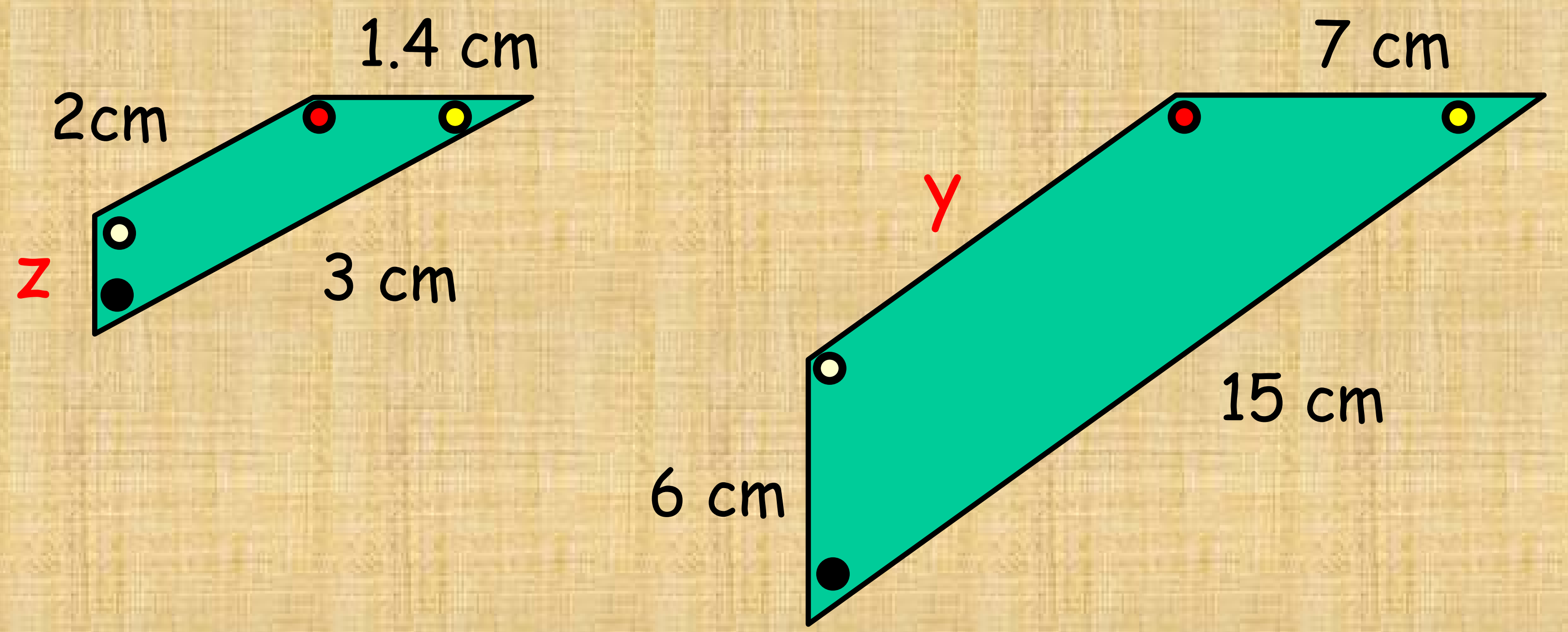
Scale Factor applies to ANY SHAPES that are mathematically similar.



Given the shapes are similar, find the values y and z ?

Scale factor = ESF = $\frac{15}{3}$ = 5

y is 2 x 5 = 10

Scale factor = RSF = $\frac{1}{5}$ = 0.2

z is 6 x 0.2 = 1.2

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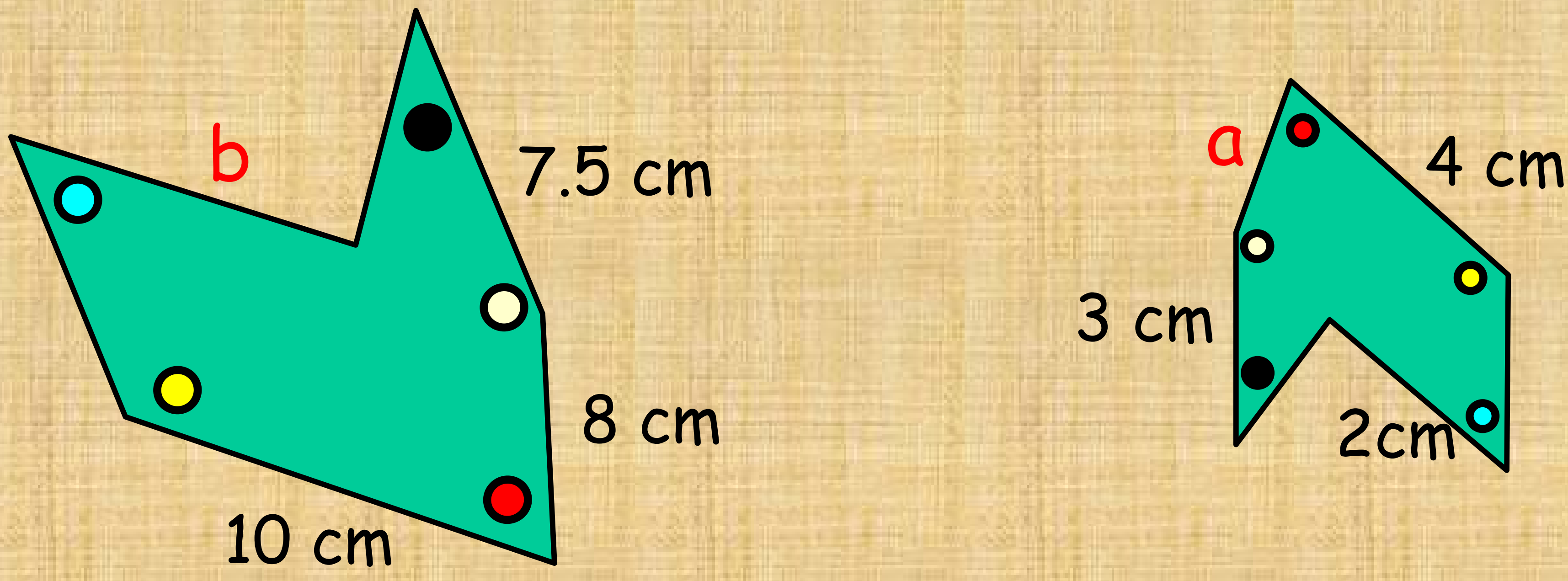
Scale Factor applies to ANY SHAPES that are mathematically similar.



Given the shapes are similar, find the values a and b ?

Scale factor = RSF = $\frac{4}{10}$ = 0.4

a is 8 × 0.4 = 3.2

Scale factor = ESF = $\frac{10}{4}$ = 2.5

b is 2 × 2.5 = 5

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Area of Similar Shape

Learning Intention

1. To explain how the scale factor applies to area.

Success Criteria

1. Understand how the scale factor applies to area.

2. Solve area problems using scale factor.

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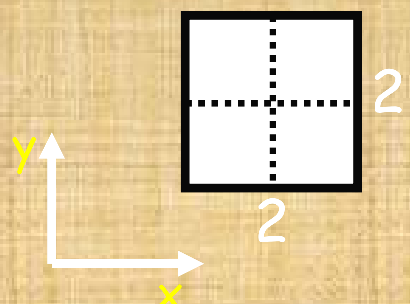
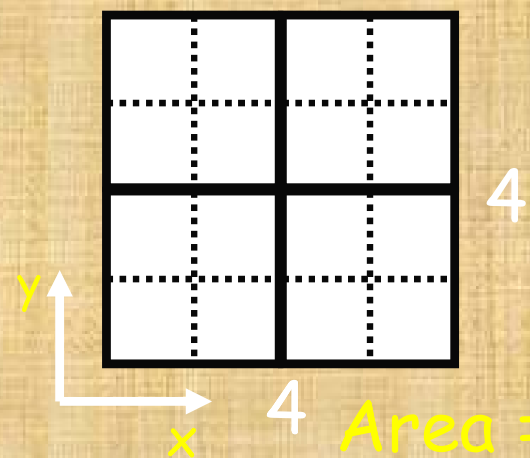
Area of Similar Shape

Draw an area with sides 2 units long.



Area = 2 x 2 = 4

Draw an area with sides 4 units long.



Area = 4 x 4 = 16

It should be quite clear that second area is four times the first.

The scaling factor in 2D (AREA) is $(SF)^2$.

For this example we have this case SF = 2 $(2)^2 = 4$.

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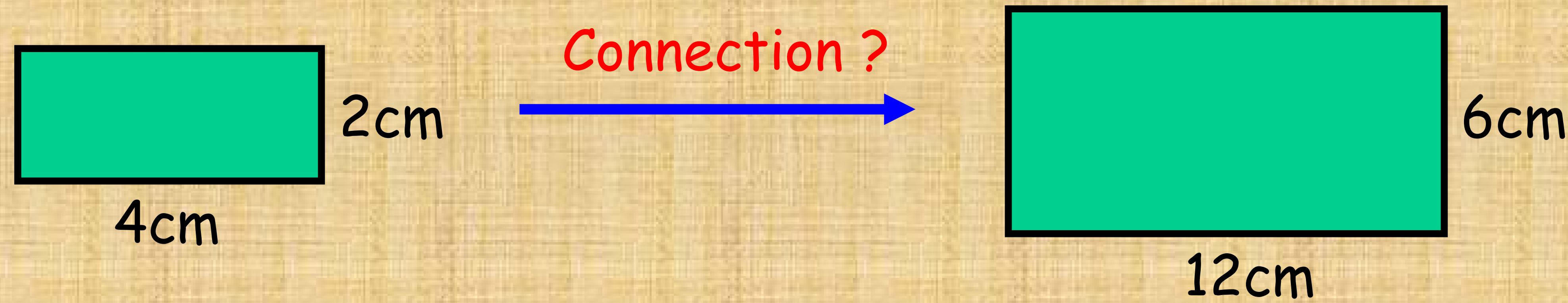
Another example of similar area ?

Work out the area of each shape
and try to link AREA and SCALE FACTOR



Small Area = $4 \times 2 = 8cm^2$

Large Area = $12 \times 6 = 72cm^2$

Scale factor = ESF = $\frac{12}{4} = 3$

Large Area = $(3)^2 \times 8 = 9 \times 8 = 72cm^2$

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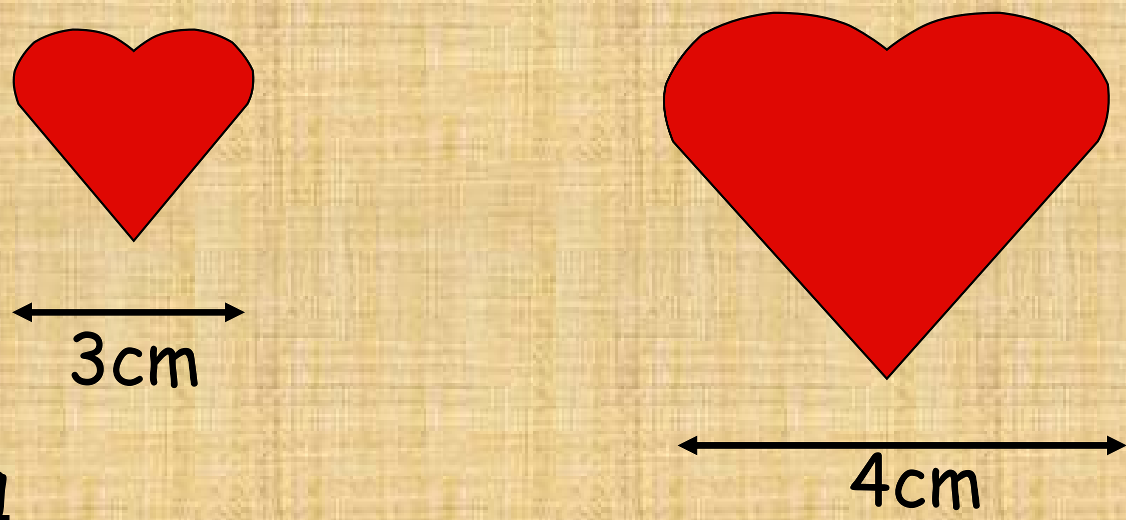
Example The following two shapes are said to be similar. If the smaller shape has an area of $42cm^2$. Calculate the area of the larger shape.



Working

$$ESF = \frac{4}{3}$$

$$\text{So area S.F} = \left(\frac{4}{3}\right)^2$$

$$\text{Area of } 2^{nd} \text{ shape} = \left(\frac{4}{3}\right)^2 \times 42 = \frac{16}{9} \times 42 = 74.67cm^2$$

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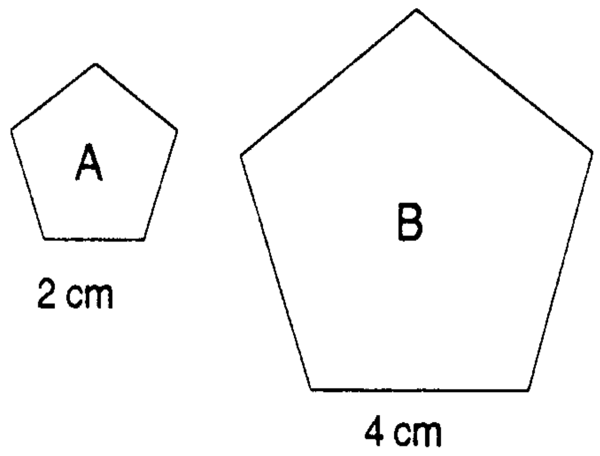
Questions

1.



Shapes A and B are SIMILAR.

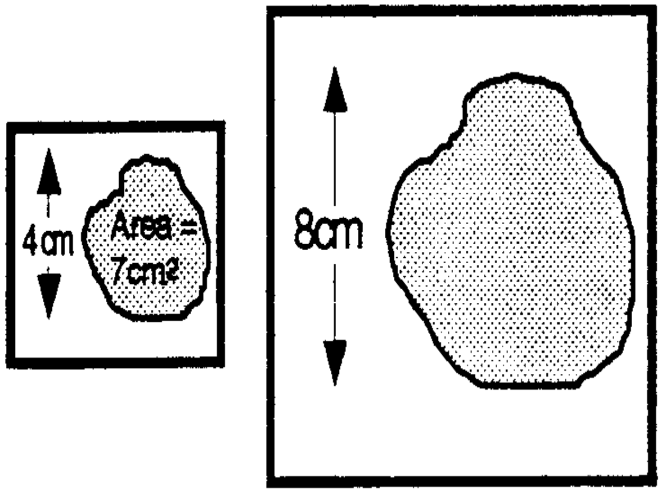
If shape A has an area of 5·2 cm^2, calculate the area of shape B. (Its NOT 10·4cm^2) !!

2.

The 2 photographs of an ink blot are similar.

The area of the small one is 7cm^2.

Calculate the area of the larger one.



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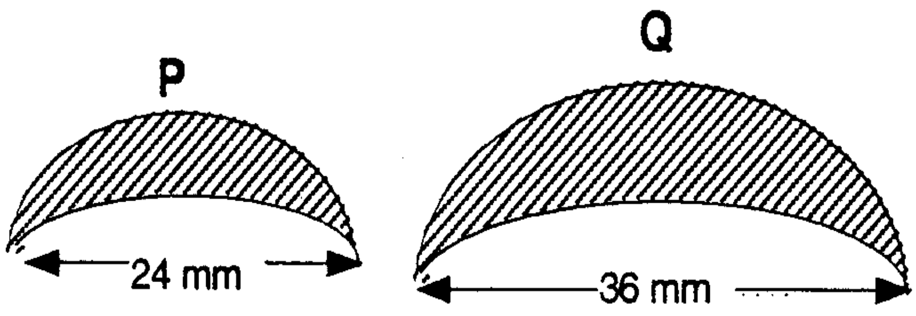
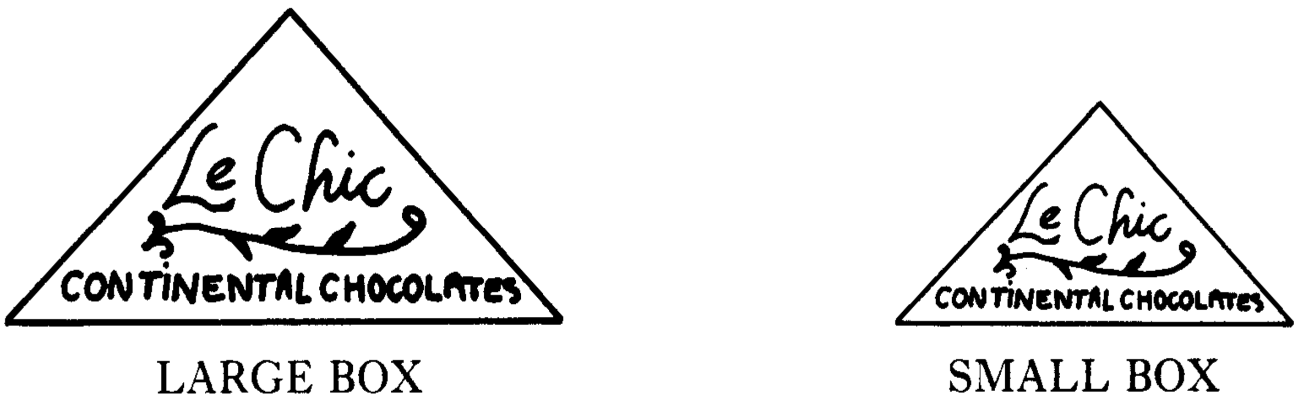
3. Shape Q is an enlargement of shape P
If the area of shape Q is 45 mm2,
calculate the area of shape P.



4. The small box of Le Chic chocolates is **similar** in shape to the large box but its edges are **half as long**.



The area of the face shown of the large box is 320 cm^2.

What is the area of the corresponding face of the small box?

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Surface Area of Similar Solids

Learning Intention

1. To explain how the scale factor applies to surface area.

Success Criteria

1. Understand how the scale factor applies to surface area.
2. Solve surface area problems using scale factor.

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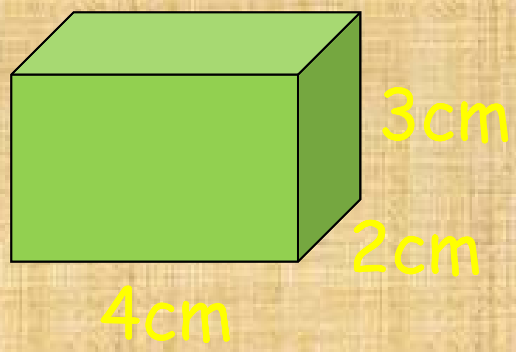
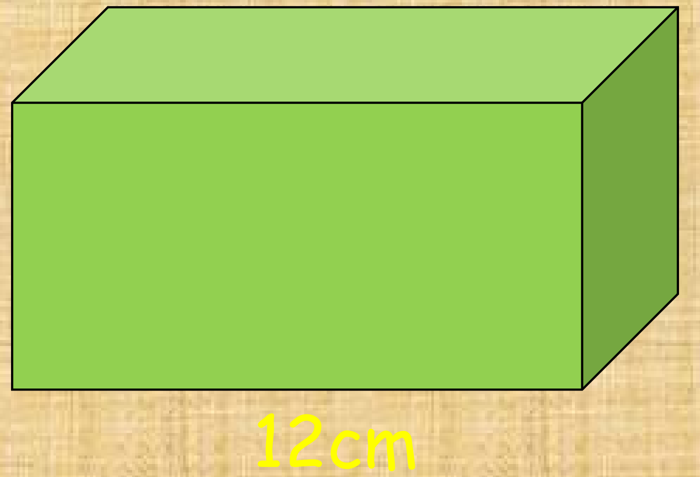
The same rule applies when dealing with Surface Area

Example : Work out the surface area of the larger cuboid.





Surface Area of small cuboid : $2(2 \times 3) + 2(4 \times 3) + 2(2 \times 4) = 52cm^2$

Surface Area of large cuboid :

$$ESF = \frac{12}{4} = 3$$

$$(3)^2 \times 52 = 468\ cm^2$$

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Volumes of Similar Solids

Learning Intention

1. To explain how the scale factor applies to volume.

Success Criteria

1. Understand how the scale factor applies to 3D – volume.
2. Solve volume problems using scale factor.

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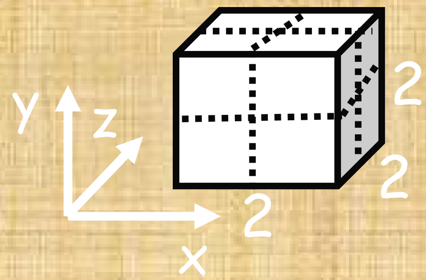
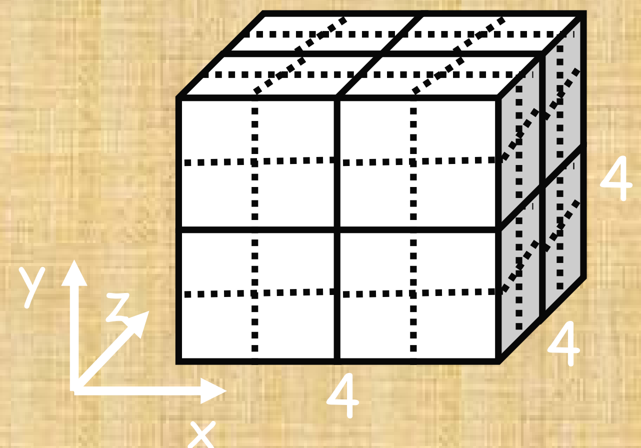
Volumes of Similar Solids

Draw a cube with sides 2 units long.



Volume = 2 x 2 x 2 = 8

Draw a cube with sides 4 units long.



Volume = 4 x 4 x 4 = 64

Scale factor = ESF = $\frac{4}{2}$ = 2

Using our knowledge from AREA section, $(SF)^2$.

For VOLUME the scale factor is $(SF)^3 = (2)^3 = 8$

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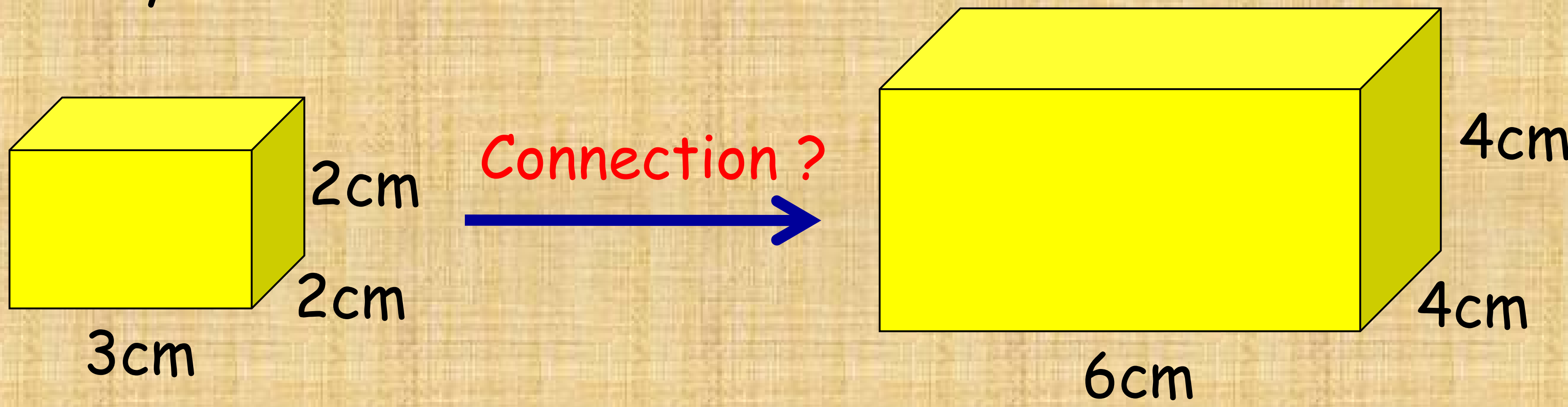
Another example of similar volumes ?

Work out the volume of each shape
and try to link volume and scale factor



$V = 3 \times 2 \times 2 = 12cm^3$

$V = 6 \times 4 \times 4 = 96cm^3$

Scale factor = ESF = $\frac{6}{3}$ = 2

Large Volume = $(2)^3 \times 12 = 8 \times 12 = 96cm^3$

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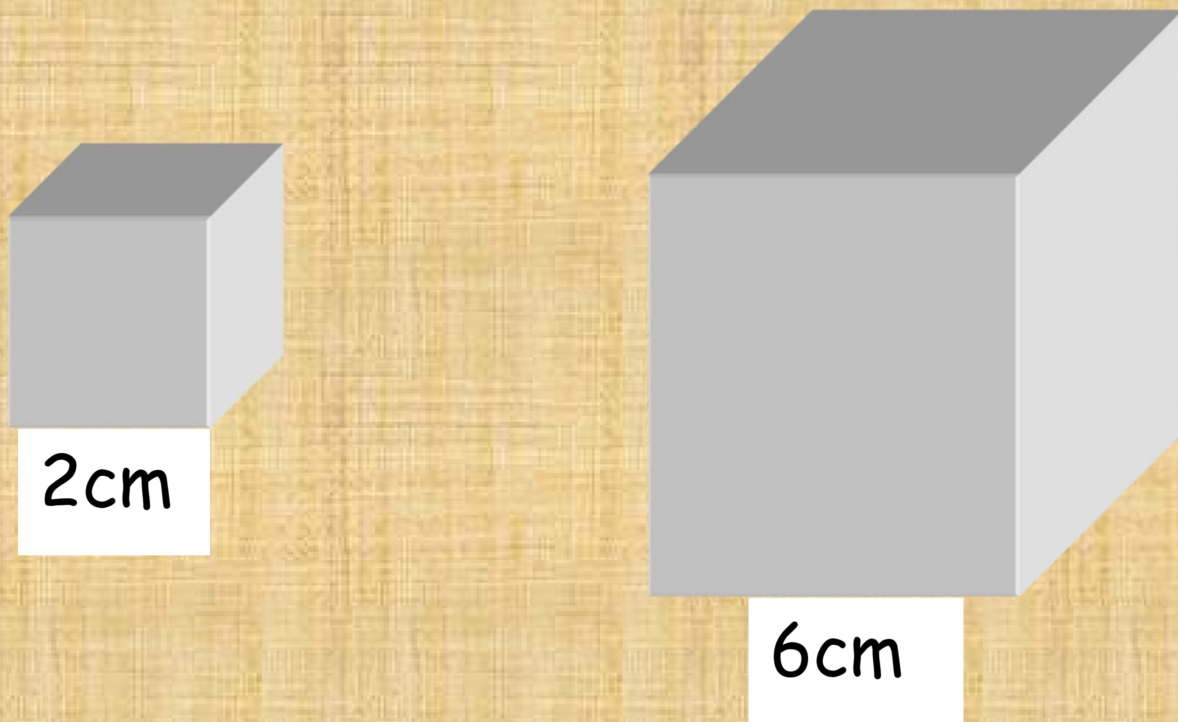
Given that the two boxes are similar, calculate the volume of the large box if the small box has a volume of 15ml



$$\text{ESF} = \frac{6}{2} = 3$$

So volume of large box = $3^3 \times 15$ = = 405 ml

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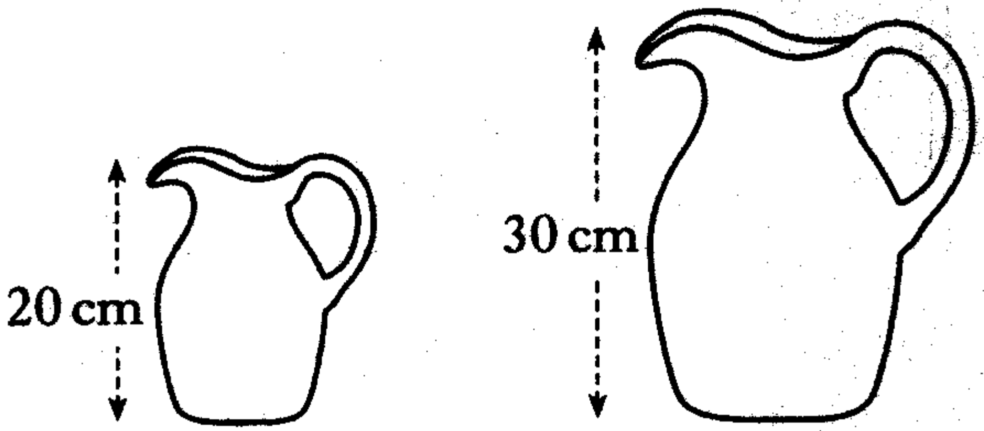
Example

The diagram below shows two jugs which are mathematically similar.



The volume of the smaller jug is 0·8 litre.

Find the volume of the larger jug.

$$\text{ESF} = \frac{30}{20} = \frac{3}{2}$$

So volume of large jug = $\left(\frac{3}{2}\right)^3 \times 0.8$ = $\frac{27}{8} \times 0.8$ = 2.7 litres

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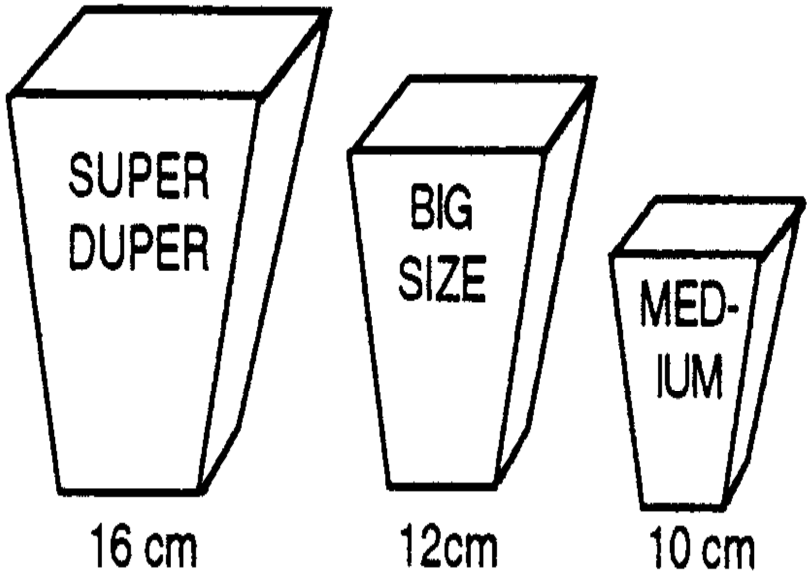
Shown are three SIMILAR Popcorn Packets

(a) Write down and simplify the ratio of the sides of the Super Duper : Big, and the Super Duper : Medium.

(b) The Super Duper box needs 1600 cm^2 of cardboard to make it. How much will the Big Box need ?



(c) The Big one holds 2700 cm^3 of Popcorn. How much will the Medium one hold ?

(a) SD : B = 4 : 3   SD : M = 8 : 5

(b) RSF = $\frac{3}{4}$ $\text{Area}_B = \left(\frac{3}{4}\right)^2 \times 1600 =$ 900cm^2

(c) RSF = $\frac{B}{M} = \frac{5}{6}$ $\text{Volume}_B = \left(\frac{5}{6}\right)^3 \times 2700 =$ 1562.5cm^3

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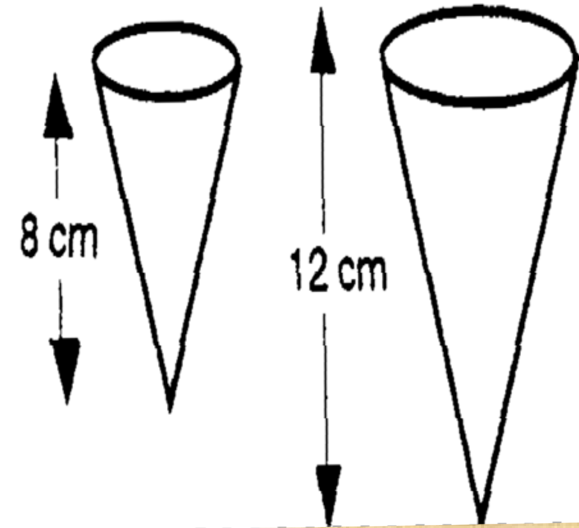
Walls make two sizes of "Cornettos" which are similar.

The small one costs 40p. What should the big one cost ?



$$\text{ESF} = \frac{12}{8} = \frac{3}{2}$$

So volume of large jug $= \left(\frac{3}{2}\right)^3 \times 40 = \frac{27}{8} \times 40 = £1.35$

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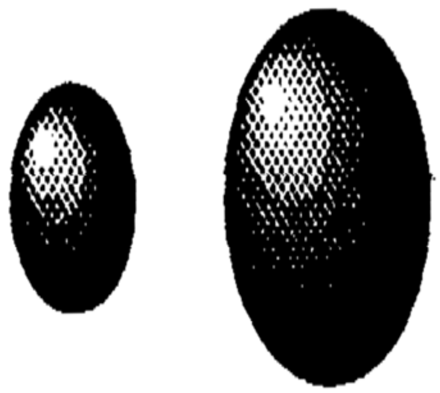
The ratio of the volumes of 2 spheres is 1 : 10
What is the ratio of their surface areas
(answer to 1 decimal place in the form 1 : ?)



$$(SF)^3 = \frac{10}{1} = 10$$

$$SF = 2.1544$$

So surface area ratio = $(SF)^2 = 2.1544^2 = 4.64$

Ratio of their surface area is 1 : 4.6 (to 1 d.p.)