Geometry

Areas of Regular Polygons

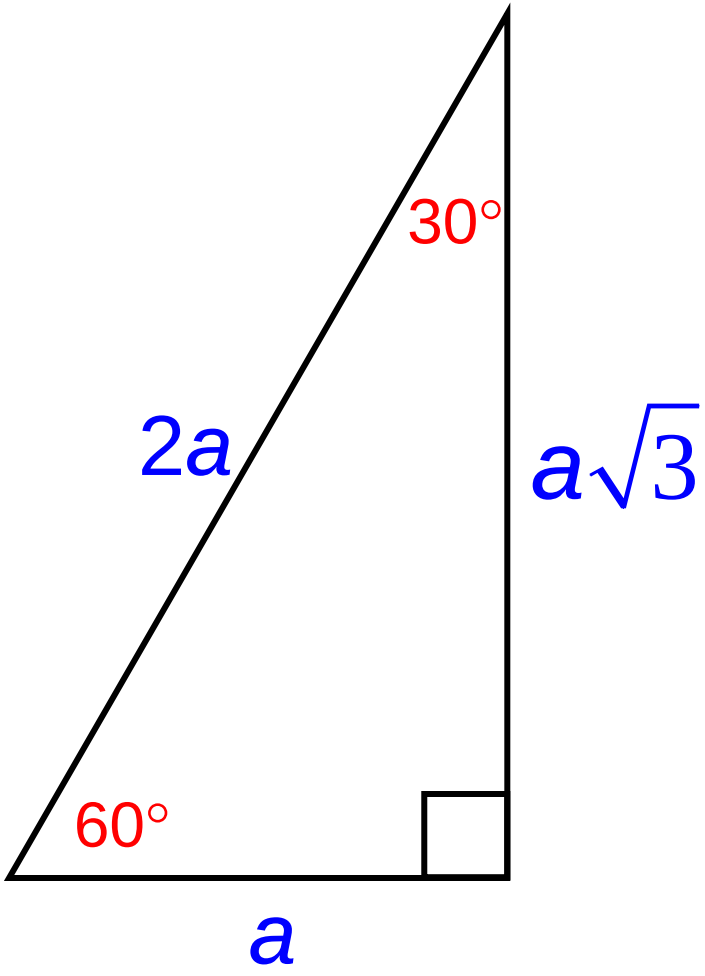
Goals

- Find the area of equilateral triangles.
- Know what an apothem is and be able to find its length.
- Use the apothem to find the area of a regular polygon.

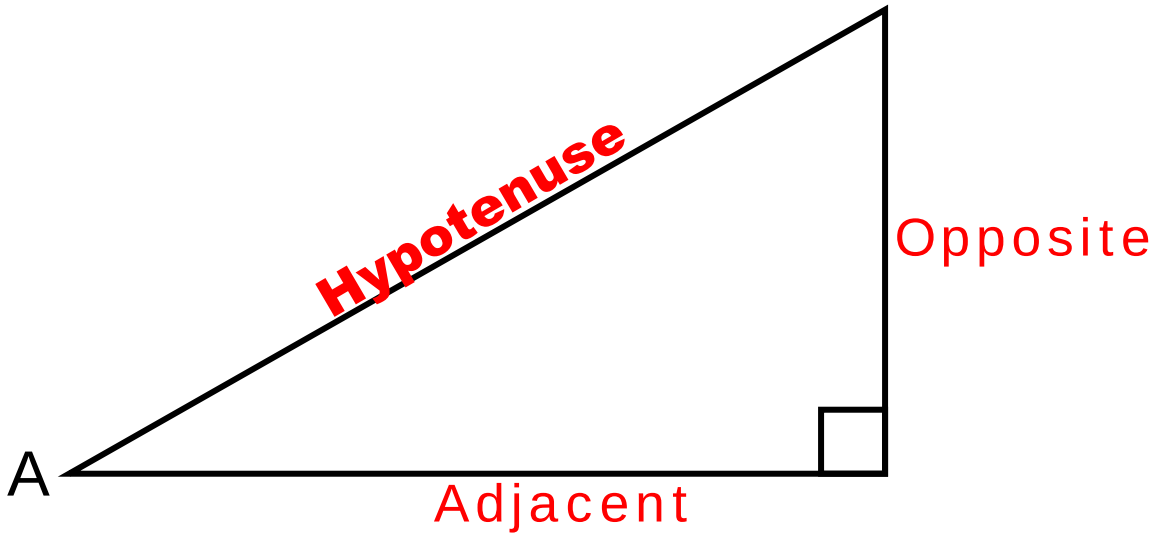
Quick Review

- 30-60-90 Triangles
- Right Triangle Trigonometry
- Area of a triangle

30-60-90 Triangle

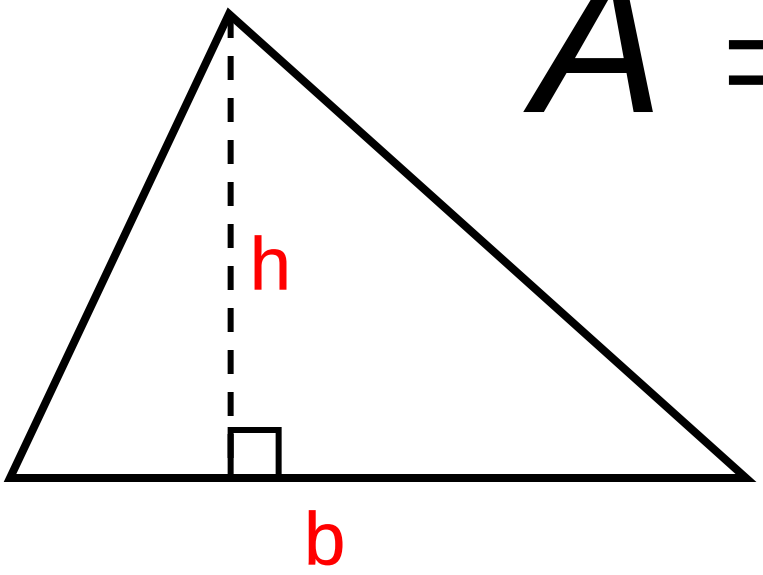


Trig Ratio Definition: Tangent



Tangent of $\angle A$ = $\frac{\text{Opposite}}{\text{Adjacent}}$

Area of any Triangle



$$A = \frac{1}{2}bh$$

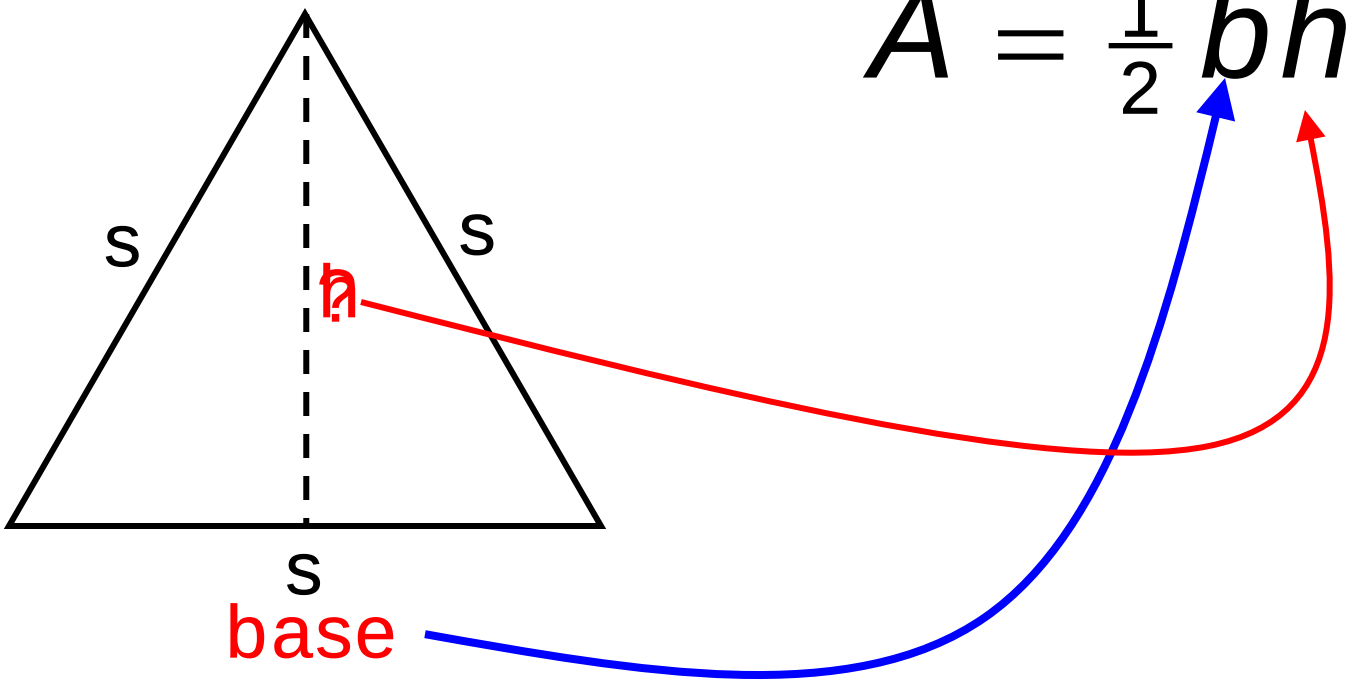
11.2

Area of Regular Polygons

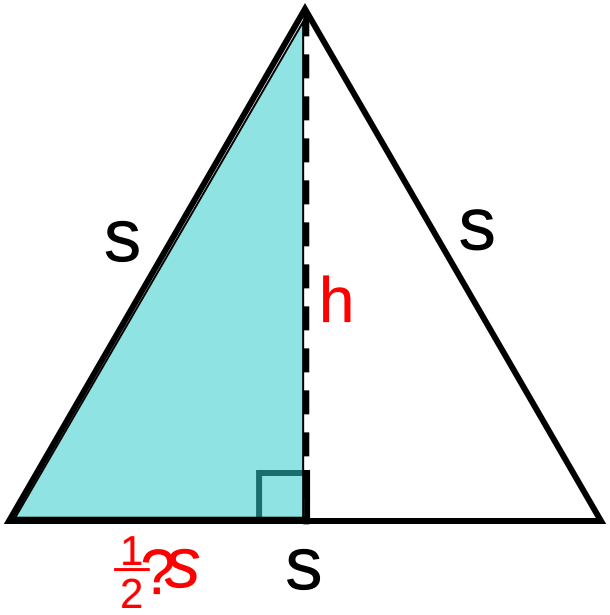
March 1, 2022

Area of an Equilateral Triangle

$$A = \frac{1}{2}bh$$

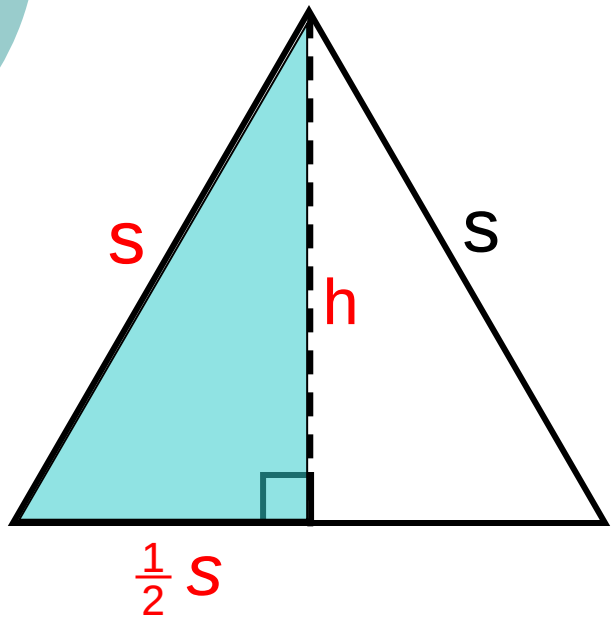


Finding h.

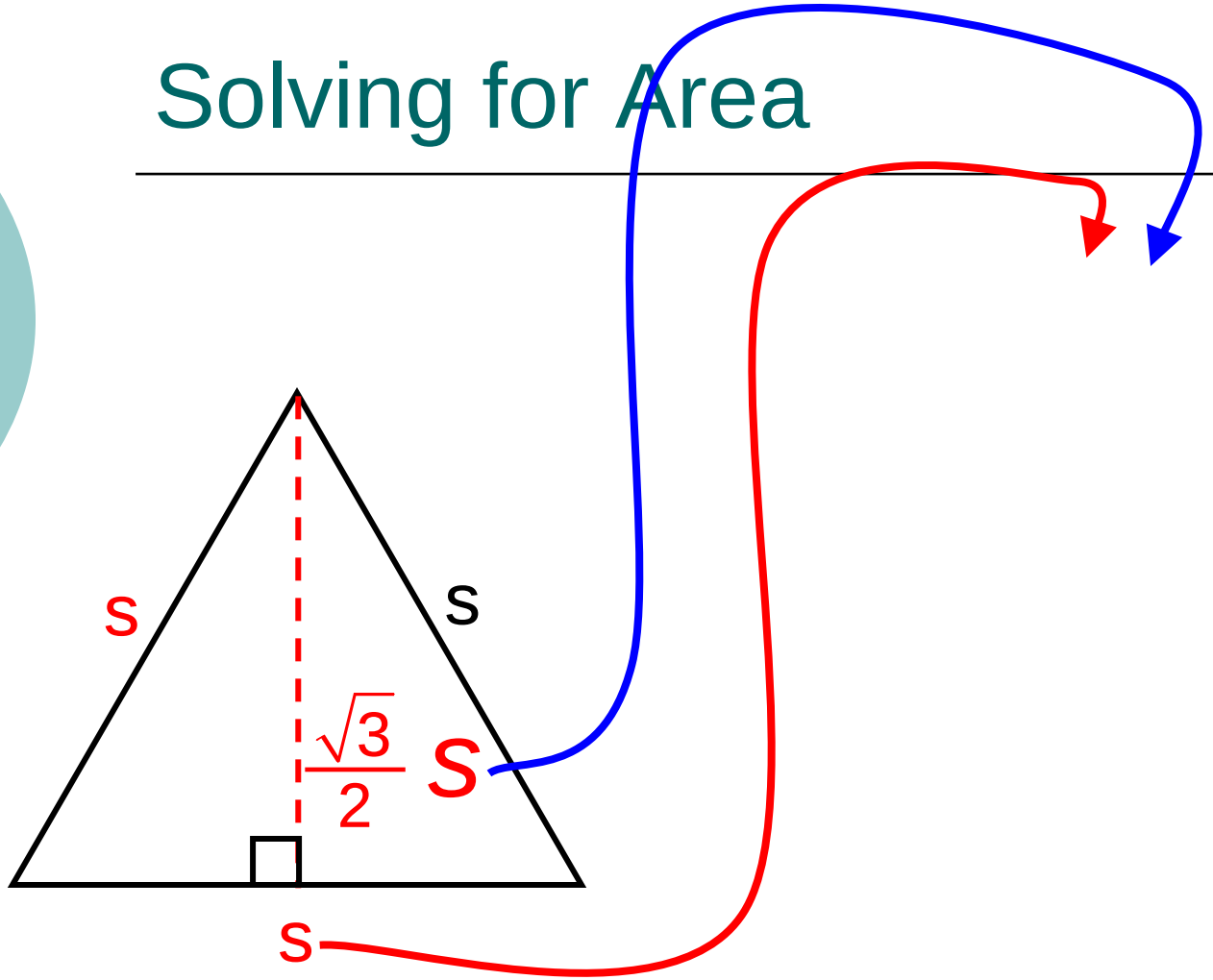


We can solve for h by using the Pythagorean Theorem.

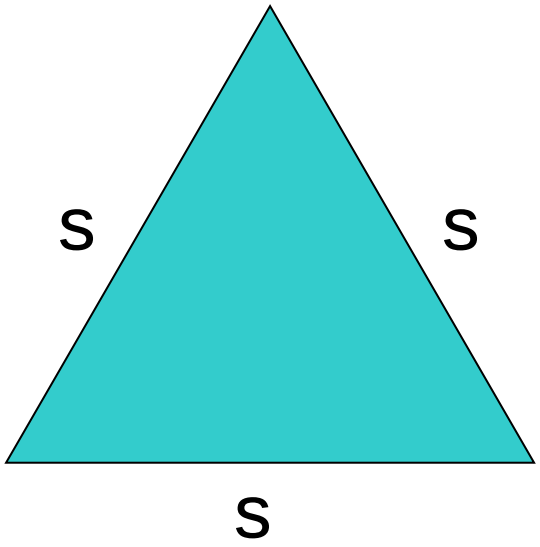
Finding h.



Solving for Area



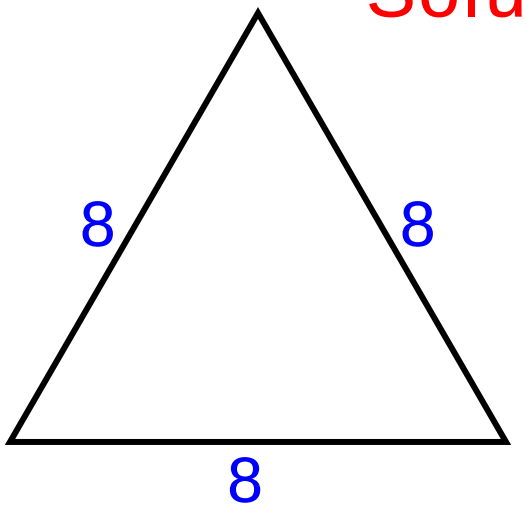
Area of an Equilateral Triangle



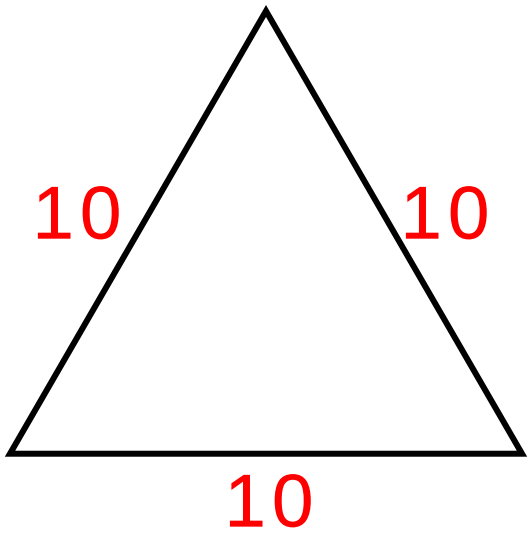
$$A = \frac{\sqrt{3}}{4} s^2$$

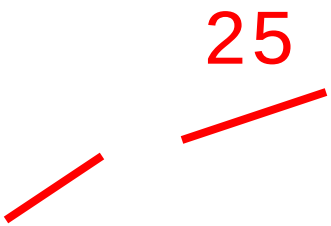
Example Find the area.

Solution:



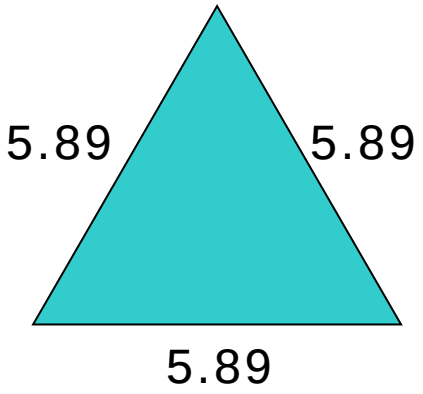
Your Turn Find the area.

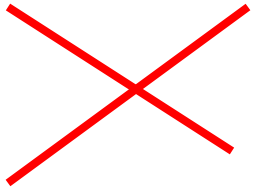




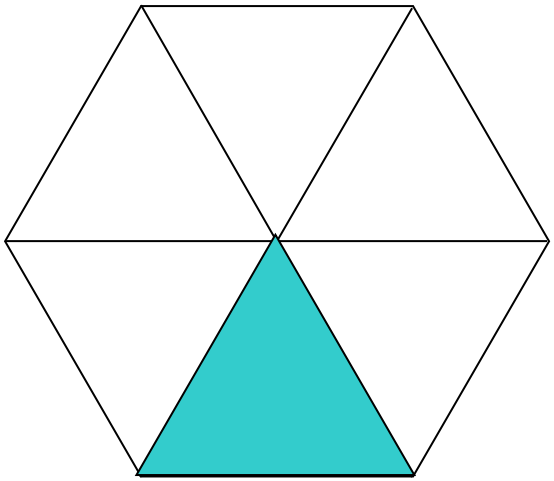
Example 2

The area of an equilateral triangle is 15. Find the length of the sides.





Area of a Regular Hexagon

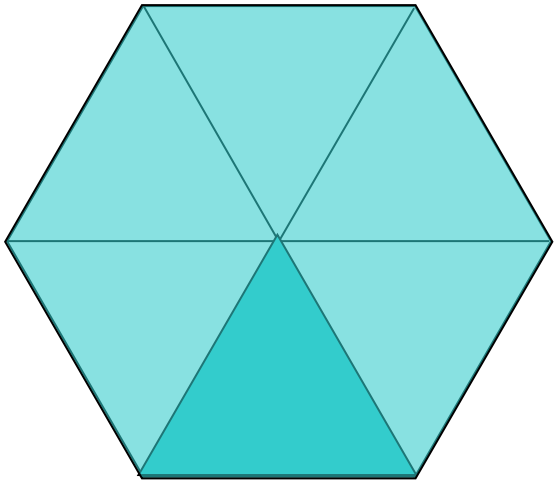


Divide the hexagon into six equilateral triangles.

Each triangle has an area of

$$A = \frac{\sqrt{3}}{4} s^2$$

Area of a Regular Hexagon



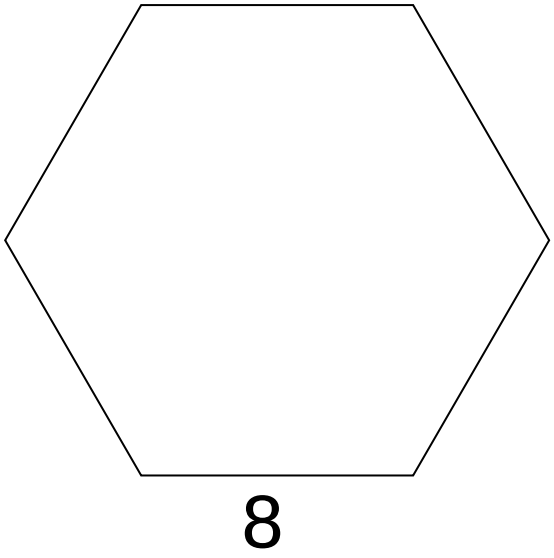
s

Multiply this by 6:

$$A = 6 \times \frac{\sqrt{3}}{4} s^2$$

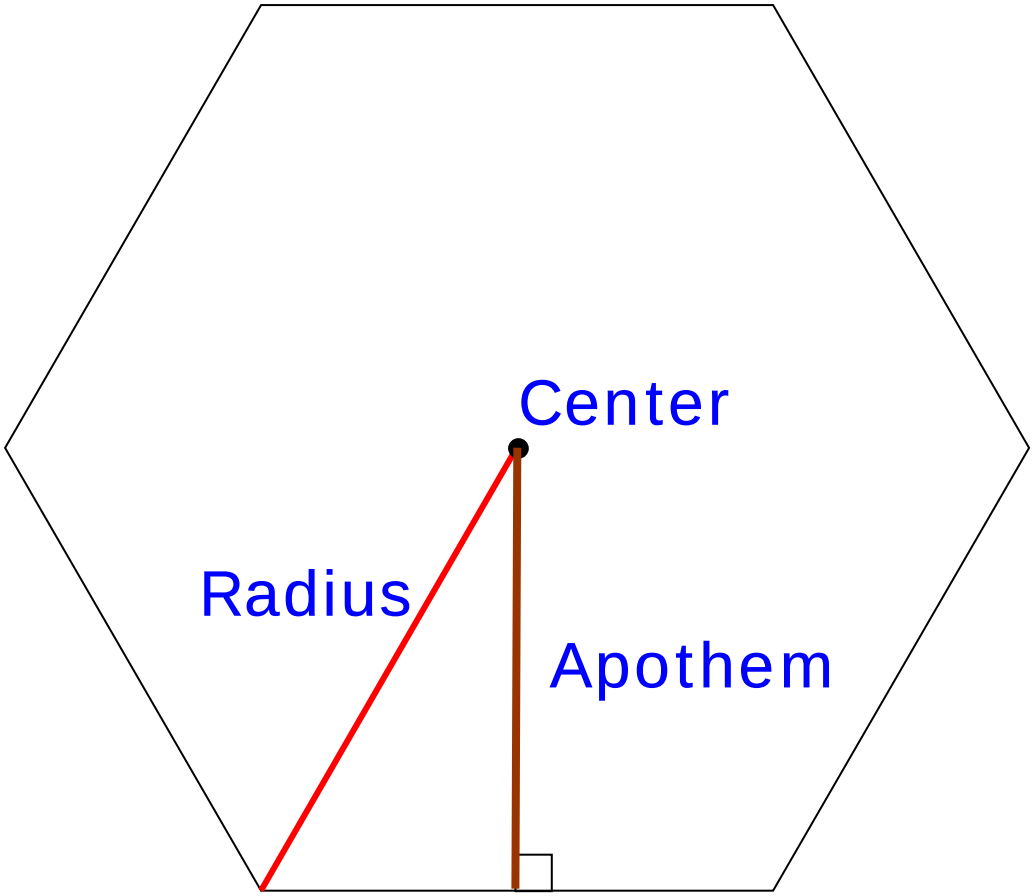
Example

Find the area of a regular hexagon with side length of 8.



32

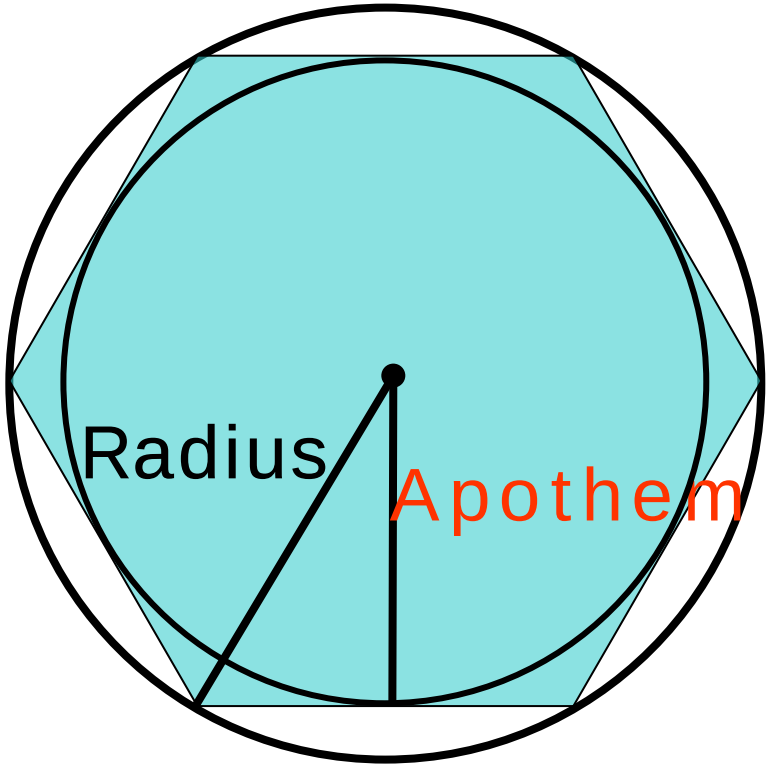
Segments in a regular polygon.



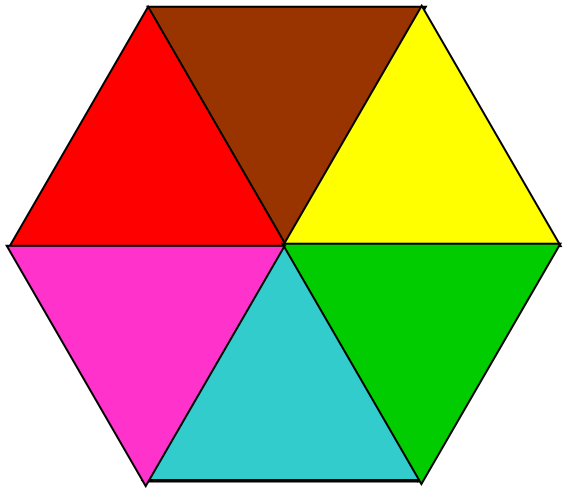
Apothem

- The perpendicular distance from the center of a regular polygon to one of its sides is called the apothem or short radius. It is the same as the radius of a circle inscribed in the polygon.
- Apothem is pronounced with the emphasis on the first syllable with the *a* pronounced as in apple (**A**-puh-thum).

Apothem



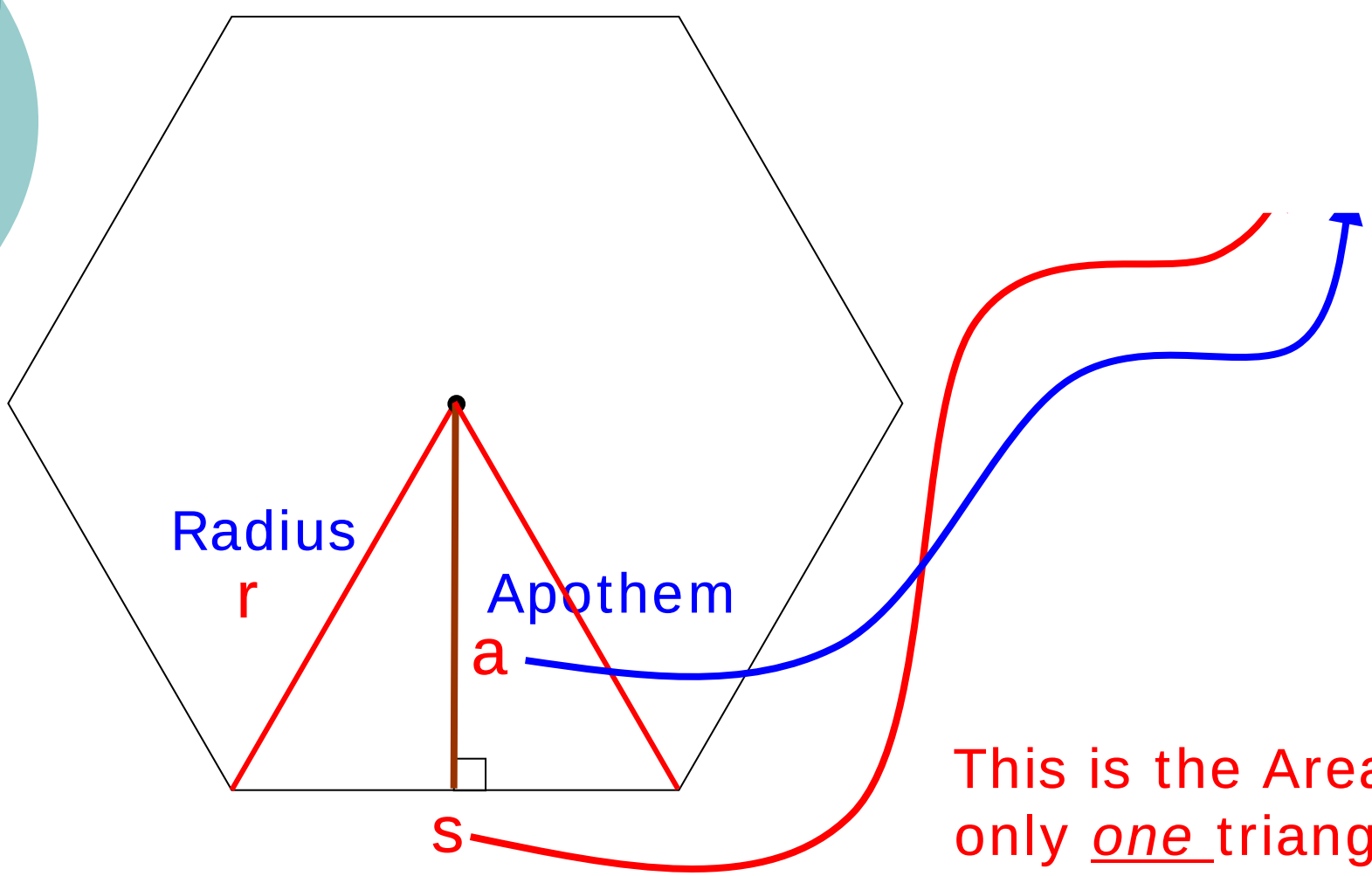
Another Way to Find the Area



The area of the hexagon is equal to the area of one triangle multiplied by the number of triangles, n.

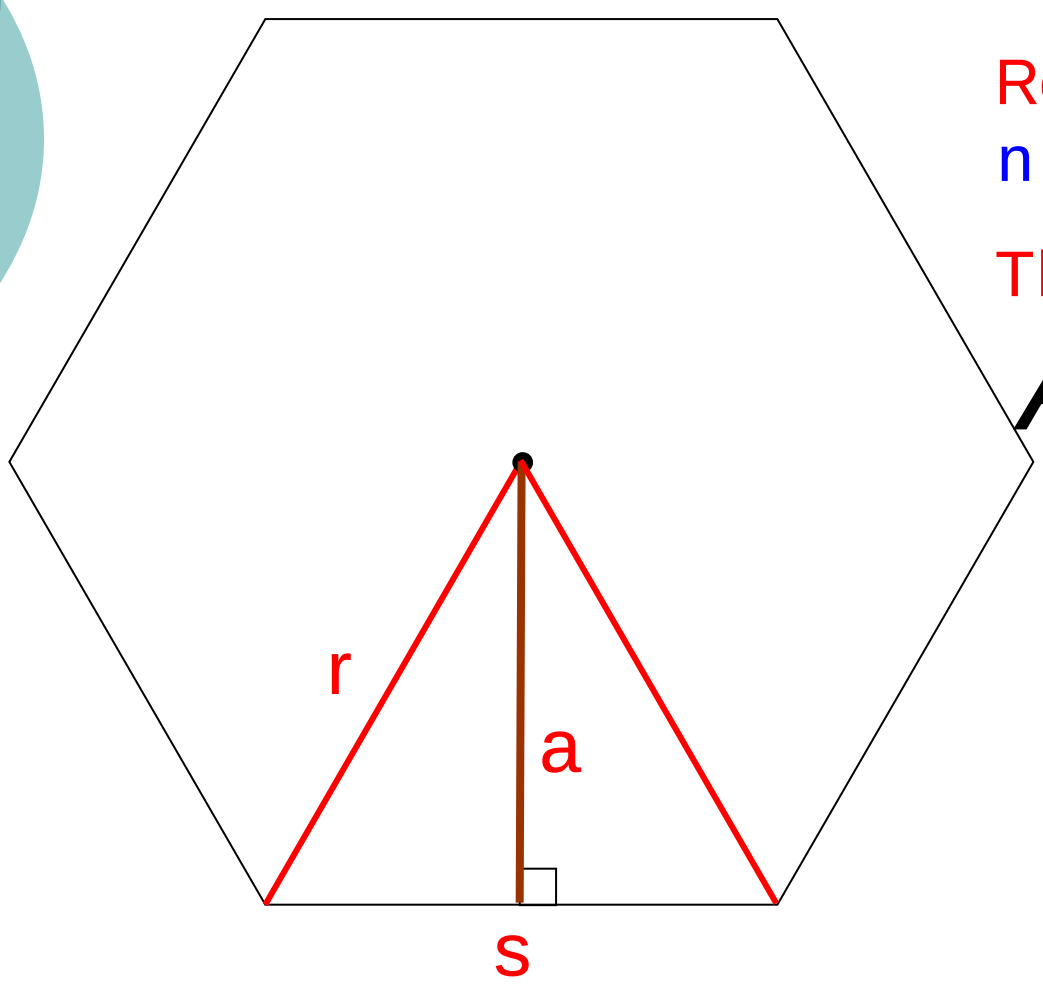
Area = (Area of one △) × (Number of △s)

Area of one triangle



This is the Area of only *one* triangle.

Area of one triangle



Remember, there are n triangles.

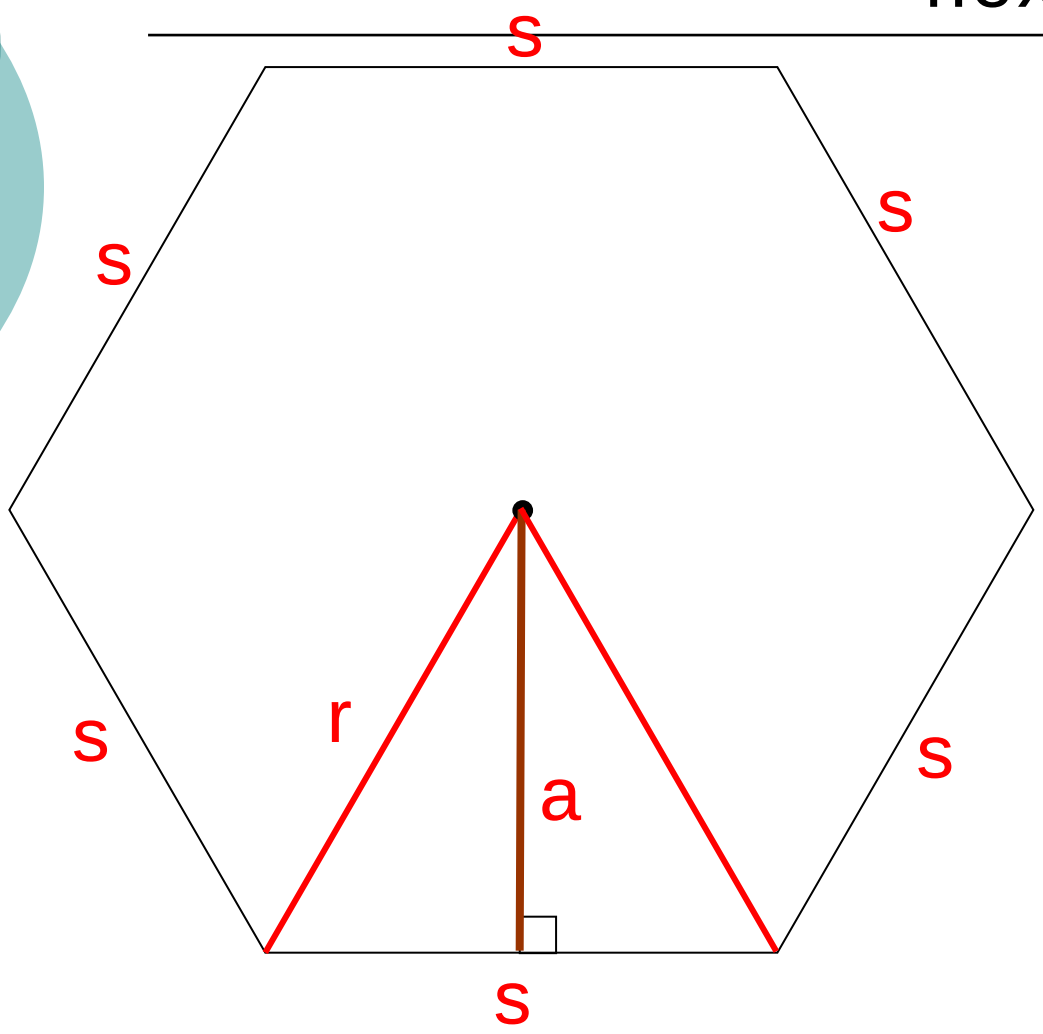
The total area then is

$$A = \frac{1}{2} sa \times n$$

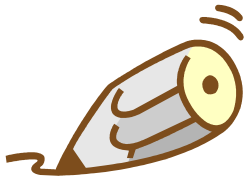
Perimeter

The perimeter of the hexagon is s × n.

$$p = s \times n$$



Area of a Regular Polygon



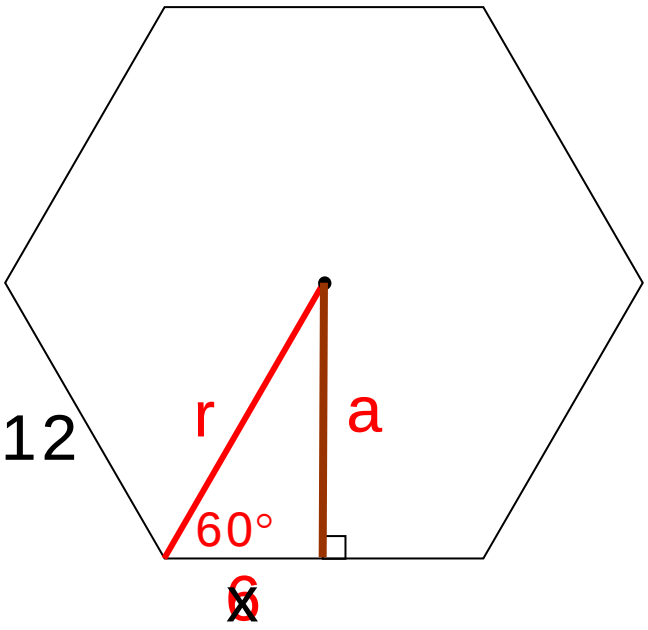
$$A = \frac{1}{2}ap$$

a = apothem

p = perimeter

This formula works for all regular polygons regardless of the number of sides.

Example



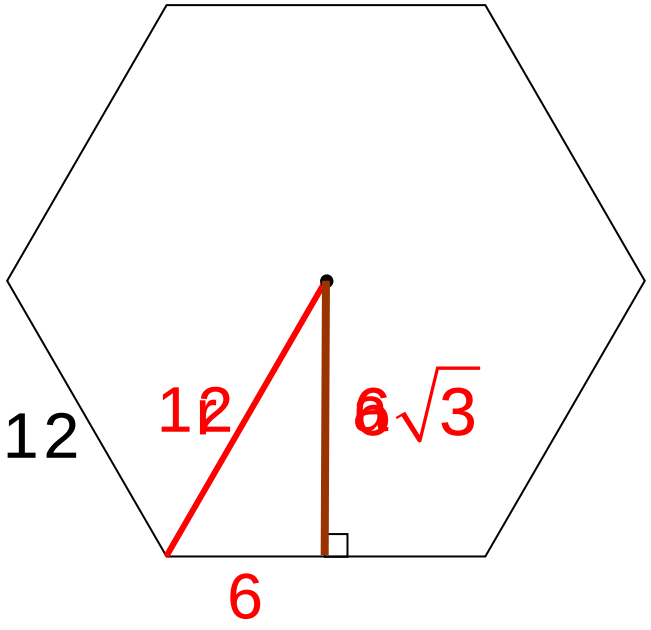
Find the area.

1. Draw a radius and an apothem.
2. What kind of triangle is formed?

   30°-60°-90°
3. What is the length of the segment marked x?

   6

Example Find the area.



4. So what is r?

12

5. And what is a?

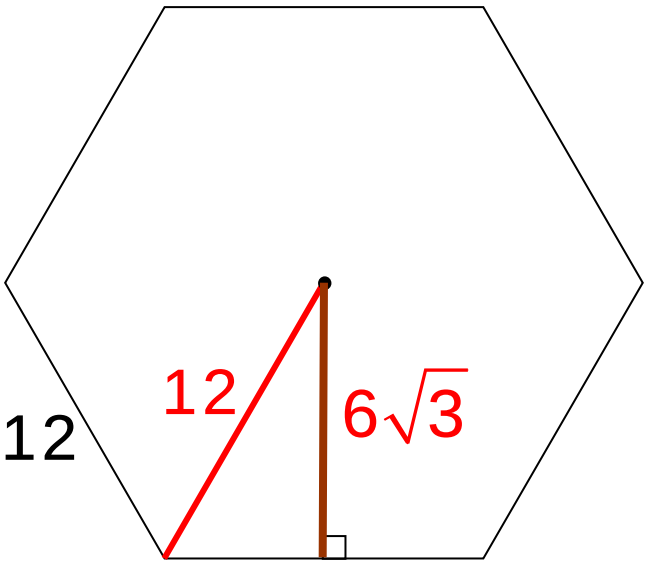
$6\sqrt{3}$

6. The perimeter is?

72 (6×12)

Example

Find the area.



The apothem is

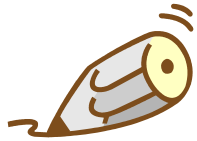
$$6\sqrt{3}$$

and the perimeter is 72.

The area is

Universal Formula

Another Very Useful Formula

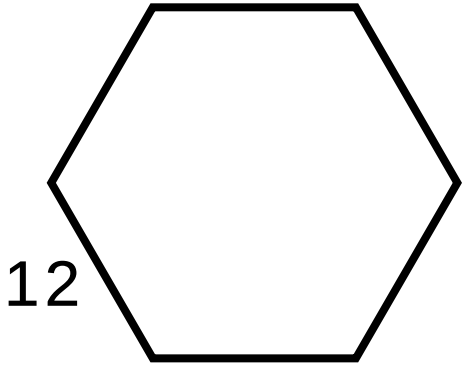


- Given the length of a side, s, of a regular polygon with n sides:

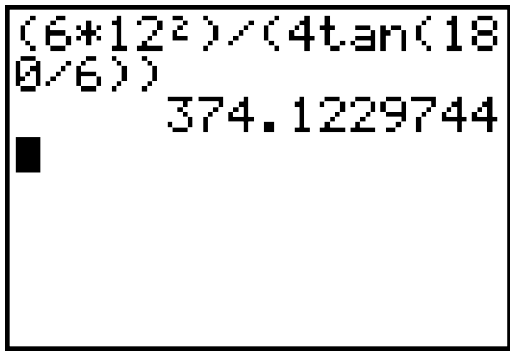
$$A = \frac{ns^2}{4\tan(180/n)}$$

- n = the number of sides
- s = the length of a side

Previous Example Again



$$A = \frac{ns^2}{4\tan(180/n)}$$

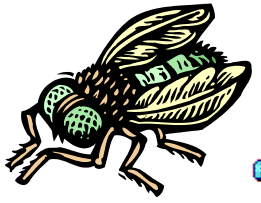


(graphing calculator)

Notice!

- In a regular hexagon, the radius is always equal to the length of a side.
- This is because we divide the hexagon into equilateral triangles.
- A hexagon is the *only* shape where this is true.

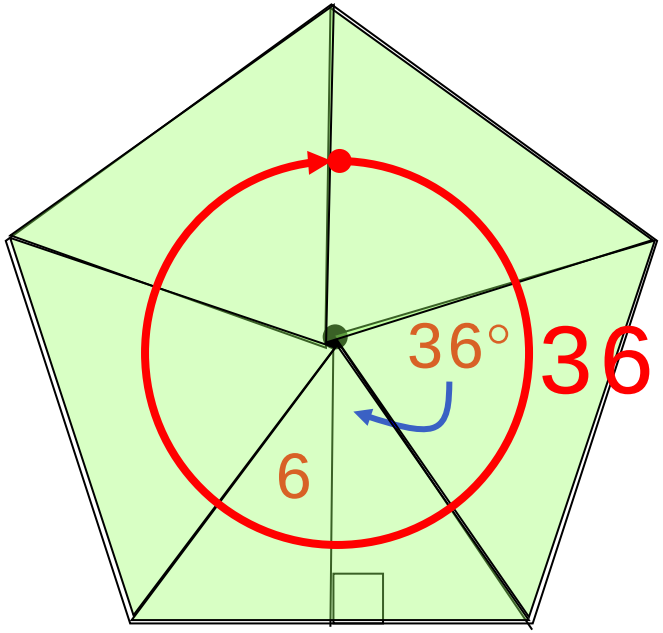
The Fly in the Ointment…



- If the polygon is anything other than an equilateral triangle, a square, or a regular hexagon, finding the apothem and the radius can be very challenging.
- Use what you know about 30-60-90 triangles, 45-45-90 triangles, and even trig to solve the problem.

A harder example

Find the area of the regular pentagon.



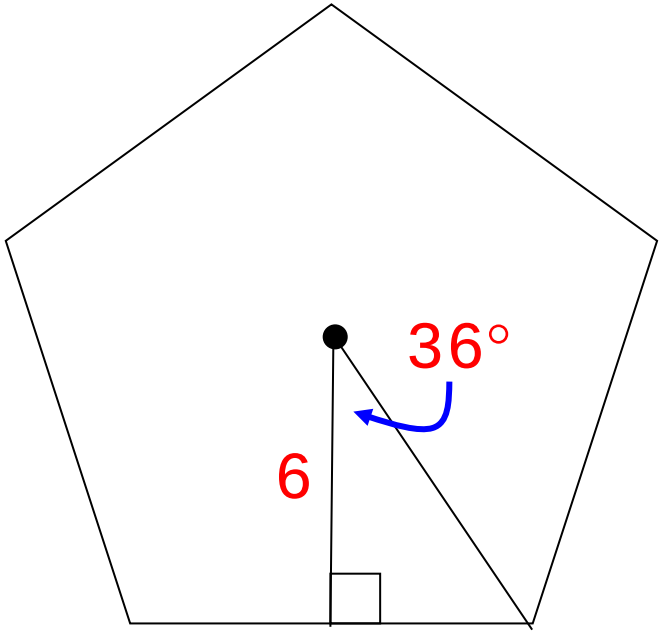
Where did 36° come from?

Each central angle measures 1/5 of 360°, or 72°.

The apothem bisects the central angle. Half of 72° is 36°.

A harder example

Find the area of the regular pentagon.



What is the apothem?

6

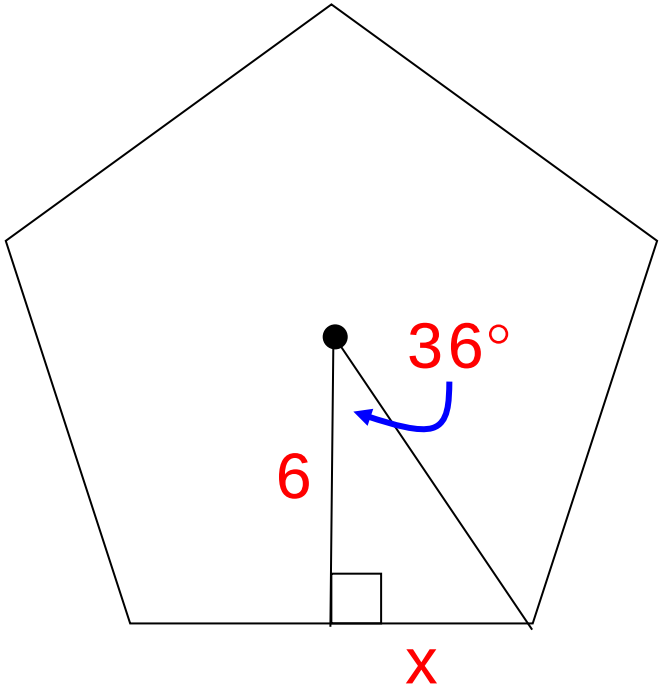
What is the perimeter?

Don't know.

Let's find it.

A harder example

Find the area of the regular pentagon.



What trig function can be used to find x?

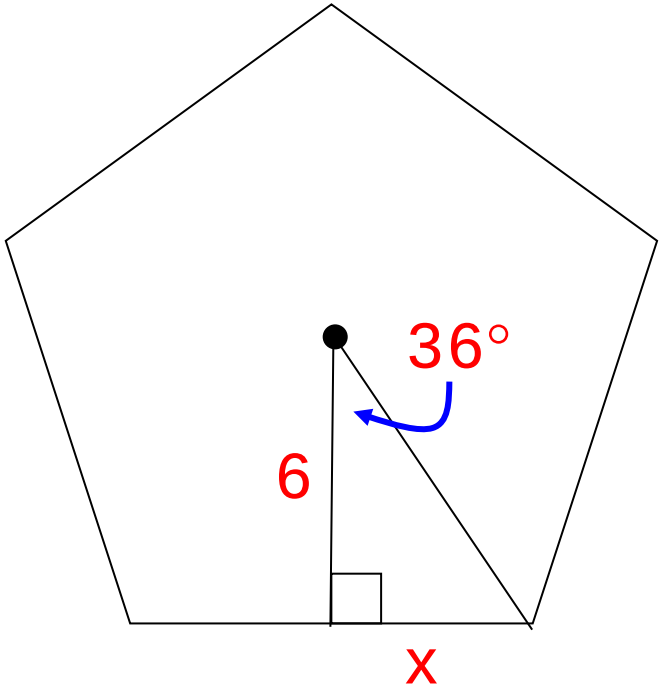
TANGENT

(SOHCAHTOA)

Equation:

$$\tan 36 = \frac{x}{6}$$

A harder example

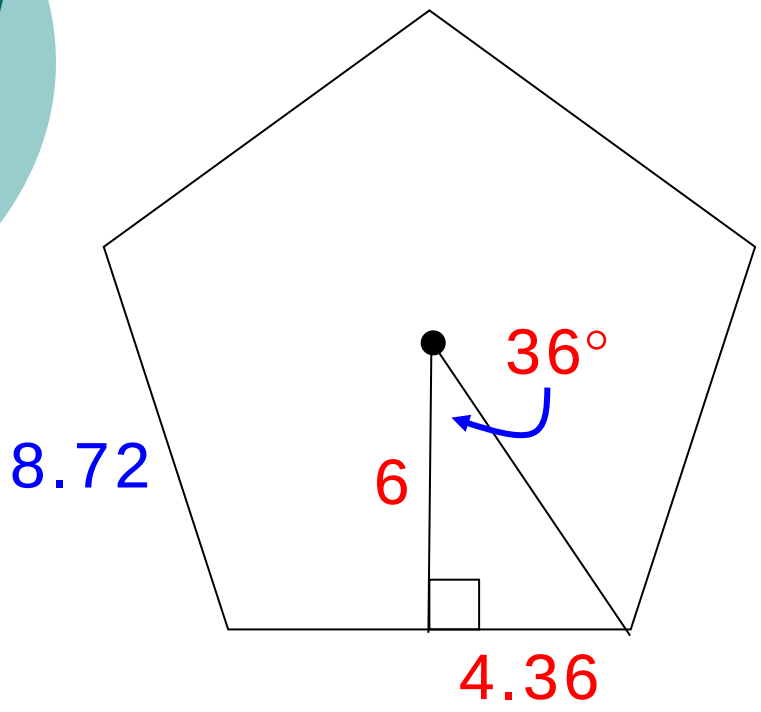


Solve the equation:

$$\tan 36 = \frac{x}{6}$$

Use a scientific calculator or use the table on page 845.

A harder example



x = 4.36

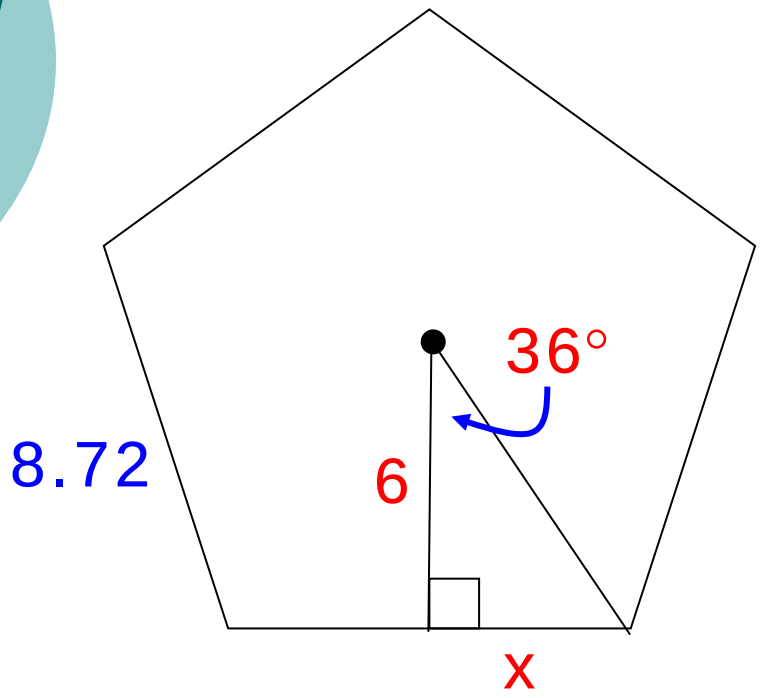
One side of the pentagon measures?

8.72 (2 × 4.36)

The perimeter is

43.59 (5 × 8.72)

A harder example



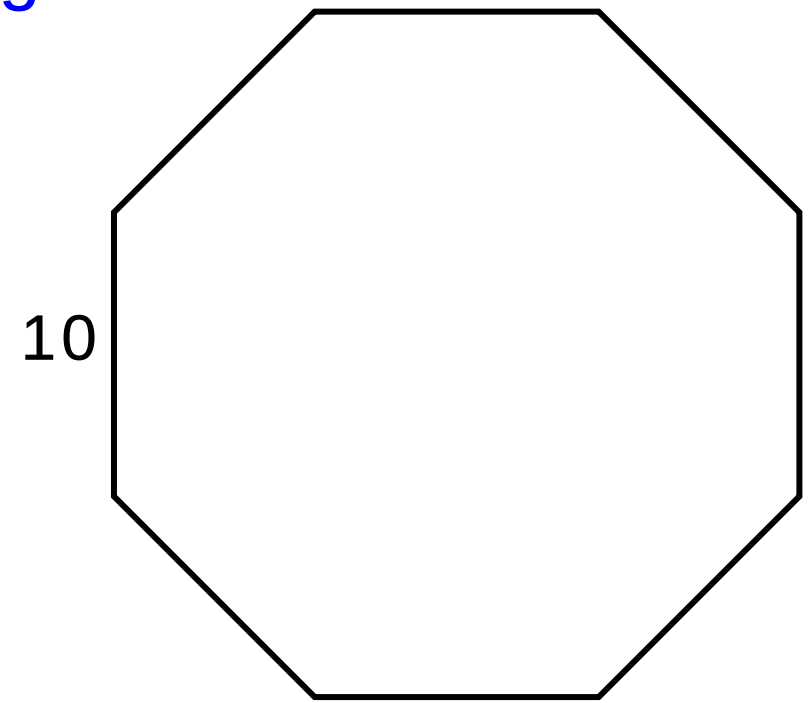
The area is:

Final Example

Find the area of a regular octagon if the length of the sides is 10.

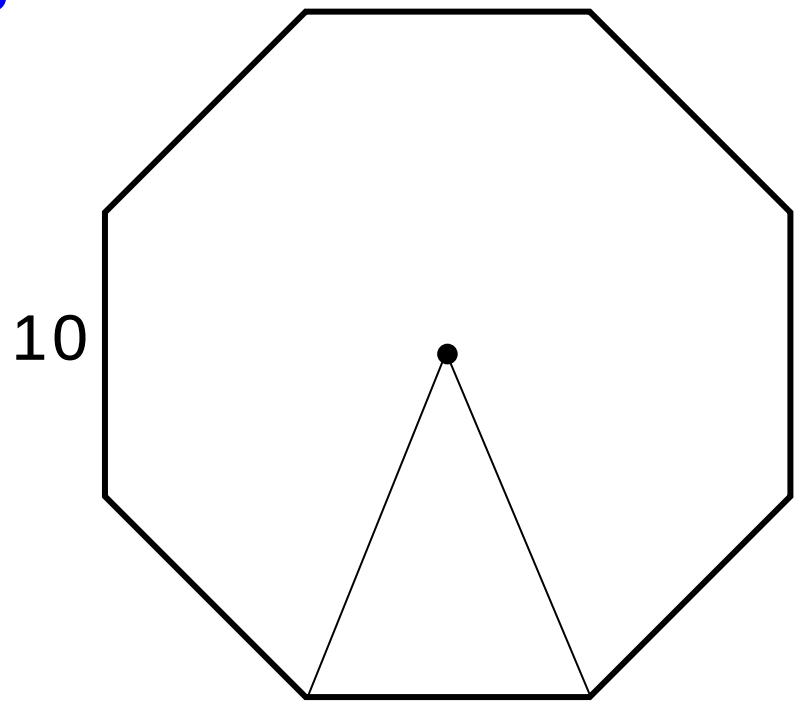
Step 1

- Draw a regular octagon with side length 10.



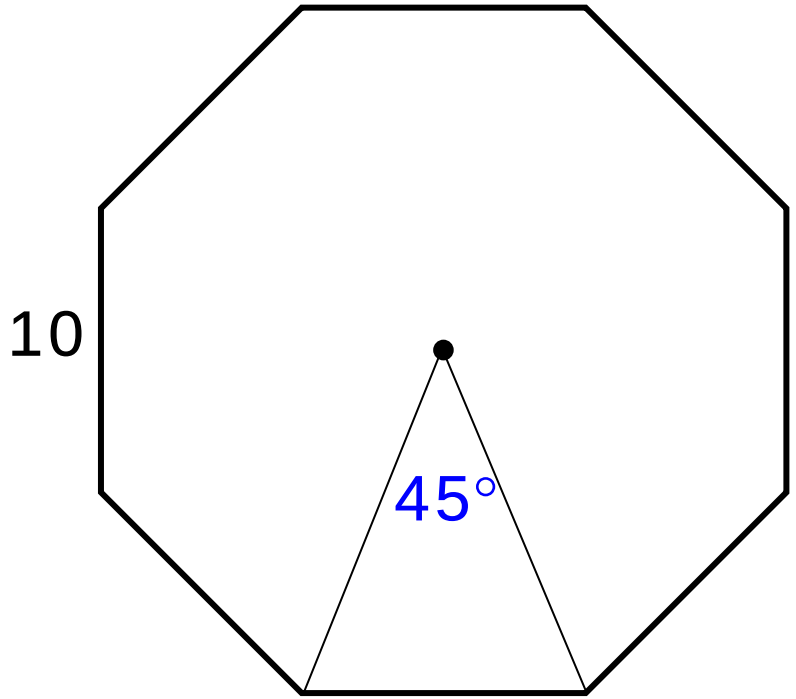
Step 2

- Locate the center and draw a central angle.



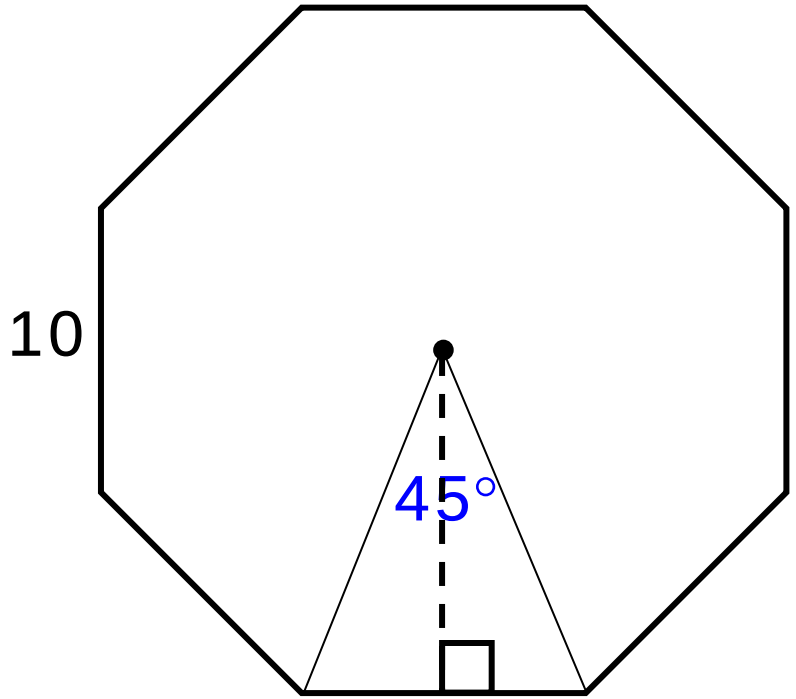
Step 3

- Determine the measure of the central angle.



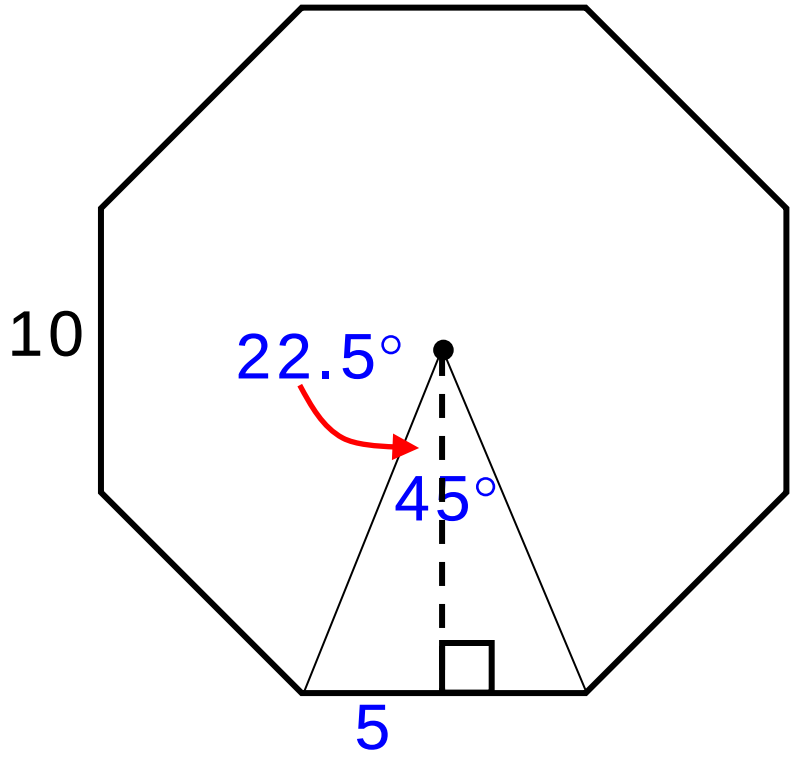
Step 4

- Draw the apothem.



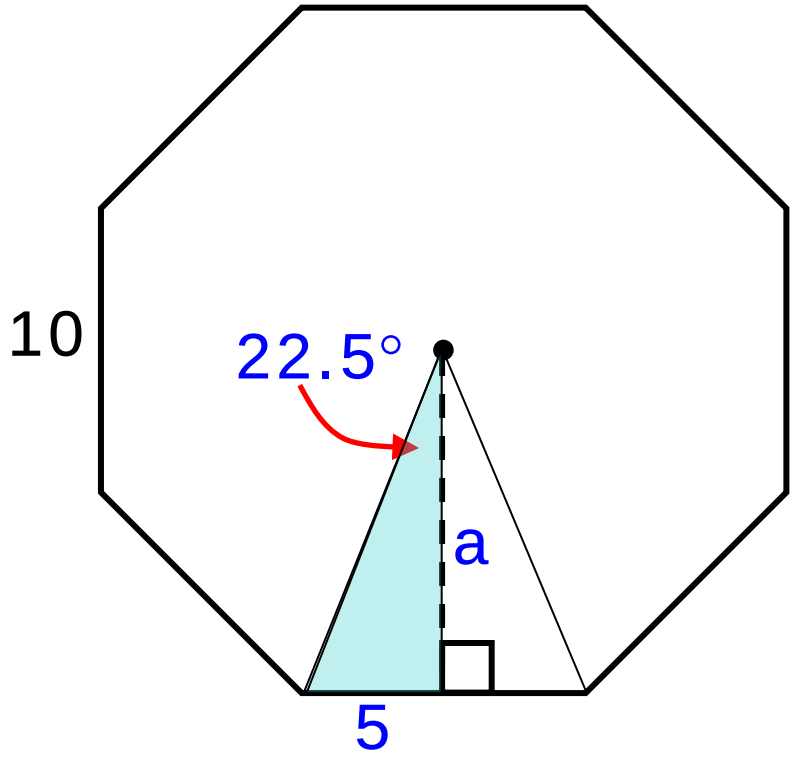
Step 5

- The apothem bisects the angle and the side. Write their measures.



Step 6

- Use a trig function to find the apothem.

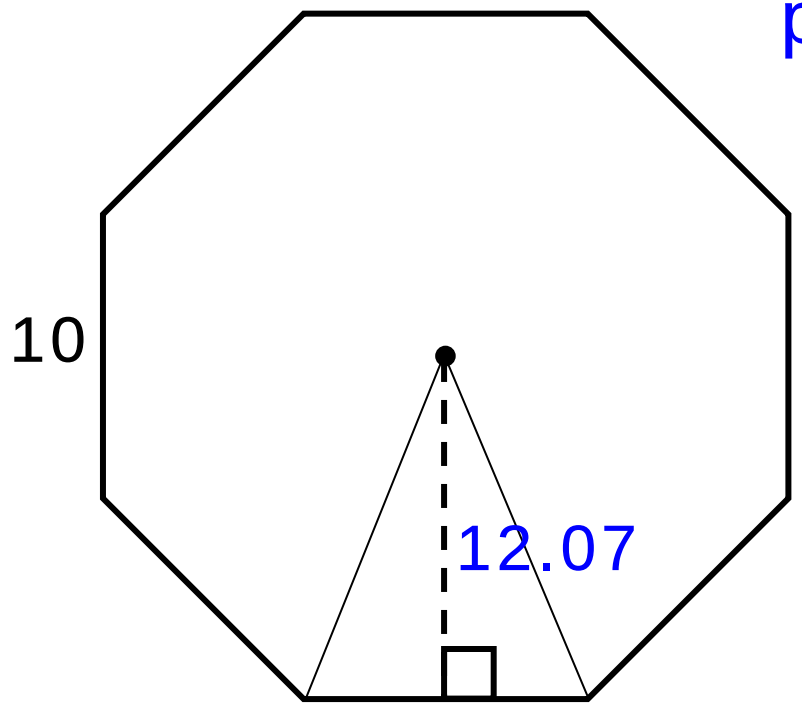


Step 7

- Find the perimeter.

$p = 10 \times 8$

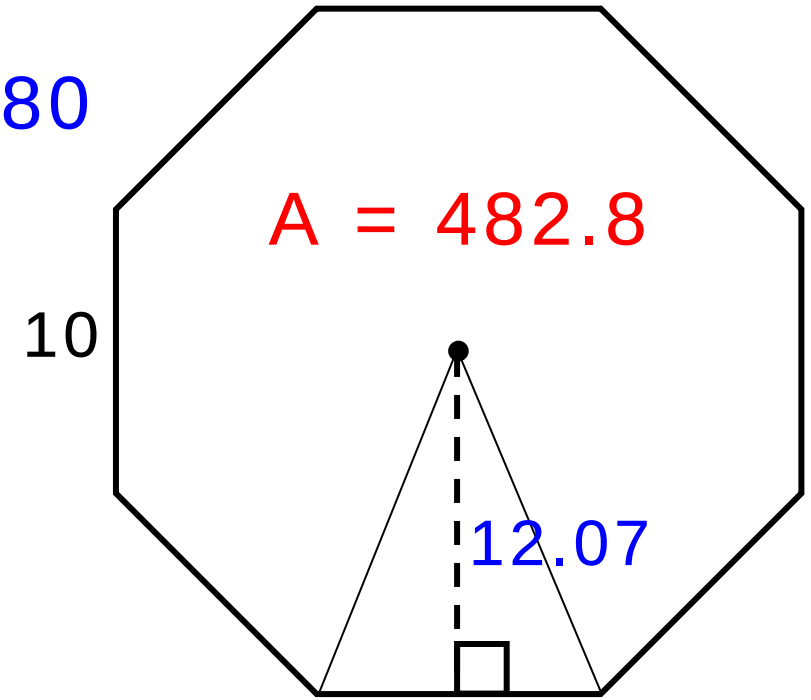
$p = 80$



Step 8

- Find the area.

p = 80



Using the area formula:

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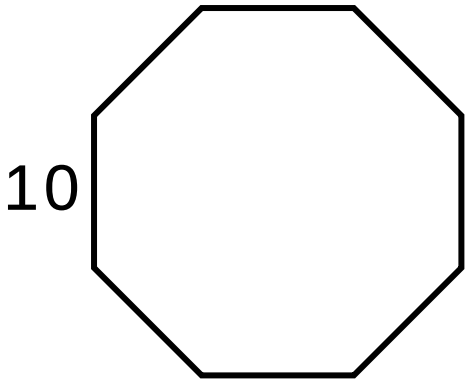

$$A = \frac{ns^2}{4\tan(180/n)}$$

Summary

- The area of any regular polygon can be found be dividing the shape into congruent triangles, finding the area of one triangle, then multiplying by the number of triangles.
- Or, multiply the length of the apothem by the perimeter and divide that by 2.

Practice Problems