AREA OF PART OF A CIRCLE

The Circle

- Isosceles Triangles in Circles
- Right angle in a Semi-Circle
- Tangent Line to a Circle
- Diameter Symmetry in a Circle
- Circumference of a Circle
- Length of an ARC of a Circle
- Area of a Circle
- Area of a SECTOR of a Circle
- Summary of Circle Chapter

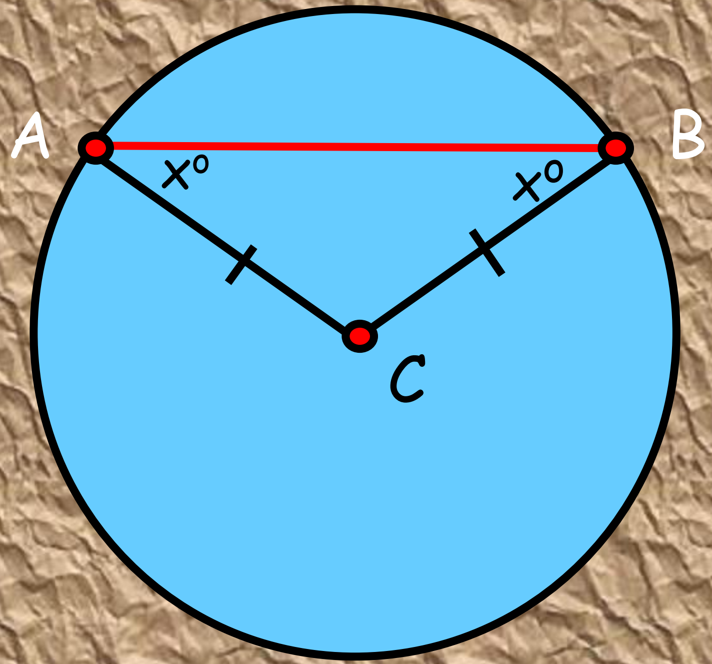
Isosceles triangles in Circles

Aim of Today's Lesson

To identify isosceles triangles within a circle.

Isosceles triangles in Circles

When two radii are drawn to the ends of a chord,
An isosceles triangle is formed.



Isosceles triangles in Circles

Special Properties of Isosceles Triangles

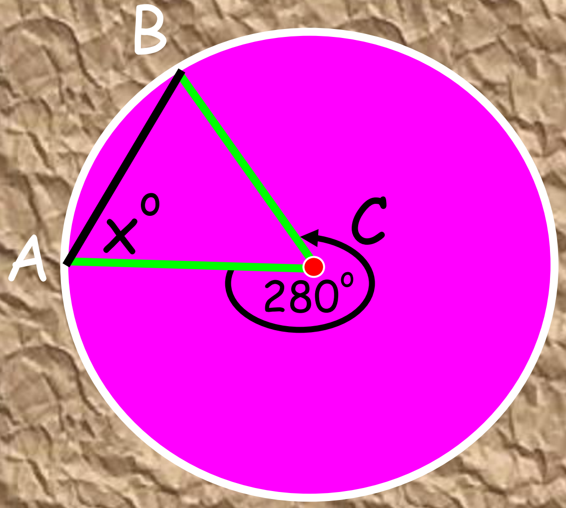
Two equal lengths

Two equal angles

Angles in any triangle sum to 180°

Isosceles triangles in Circles

Q. Find the angle x^o.



Solution

Angle at C is equal to:

$360^o - 280^o = 80^o$

Since the triangle is isosceles we have

$$2x^o + 80^o = 180^o$$

$$2x^o = 100^o$$

$$x^o = 50^o$$

Isosceles triangles in Circles

Semi-circle angle

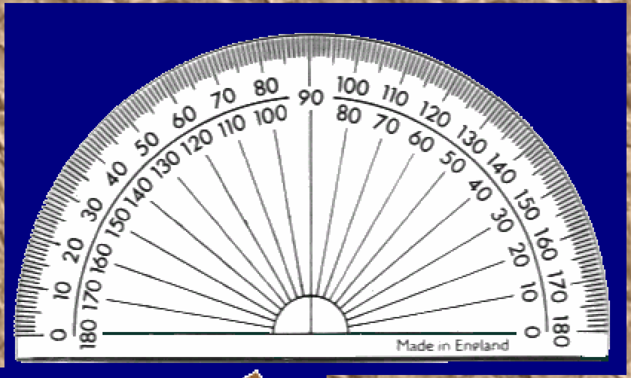
Aim of Today's Lesson

To find the angle in a semi-circle made by a triangle with hypotenuse equal to the diameter and the two smaller lengths meeting at the circumference.

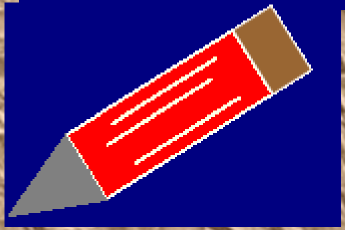
Semi-circle angle

<u>Tool-kit required</u>

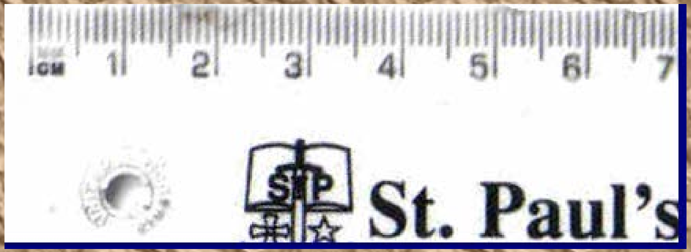
1. Protractor



2. Pencil

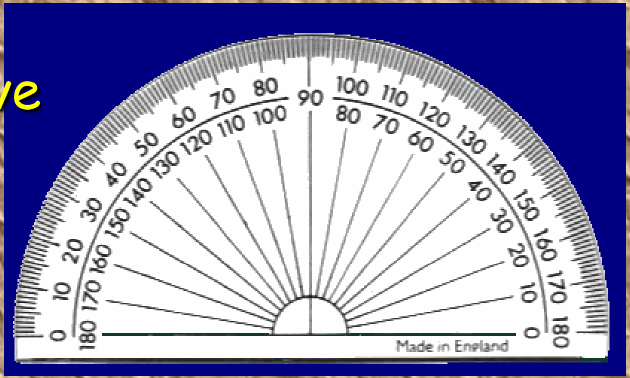


3. Ruler



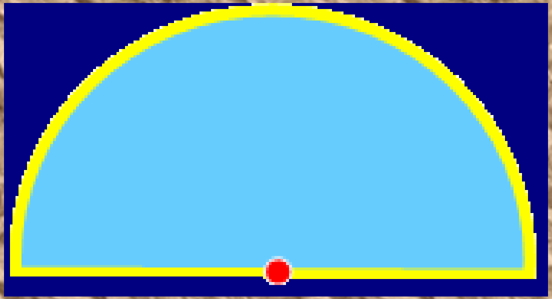
Semi-circle angle

1. Using your pencil trace round the protractor so that you have semi-circle.



2. Mark the centre of the semi-circle.

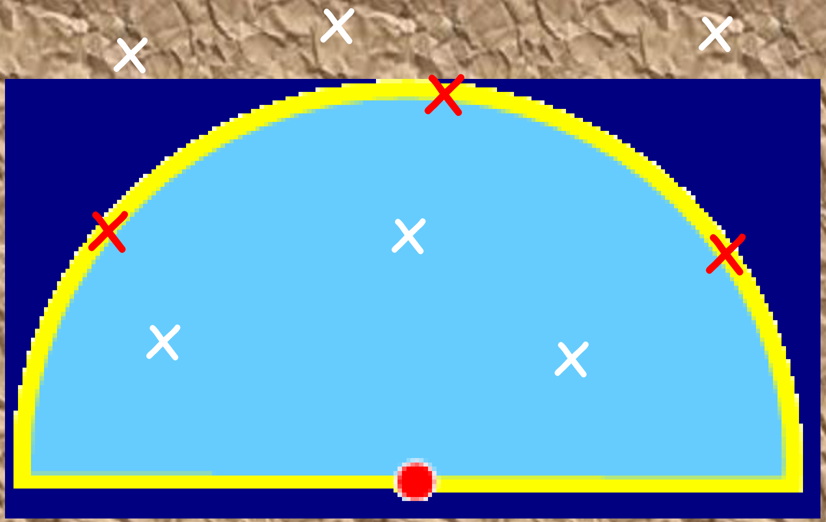
You should have something like this.



Semi-circle angle

Mark three points

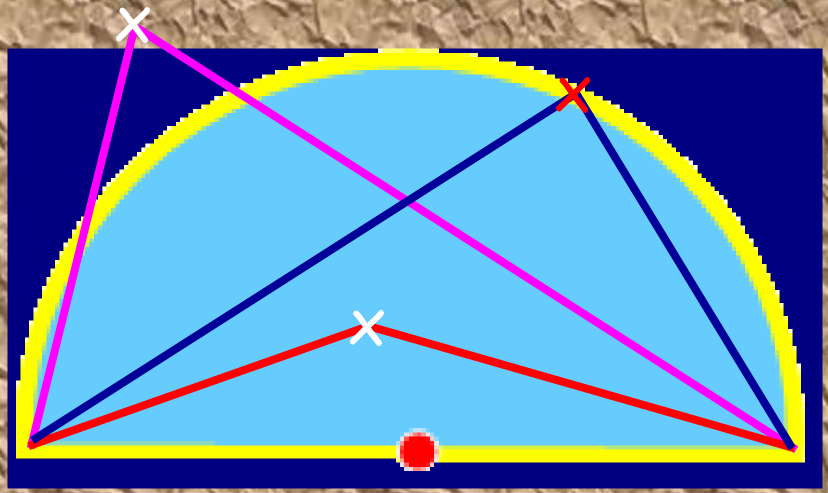
1. Outside the circle
2. On the circumference
3. Inside the circle



Semi-circle angle

For each of the points

Form a triangle by drawing a line from each end of the diameter to the point. Measure the angle at the various points.

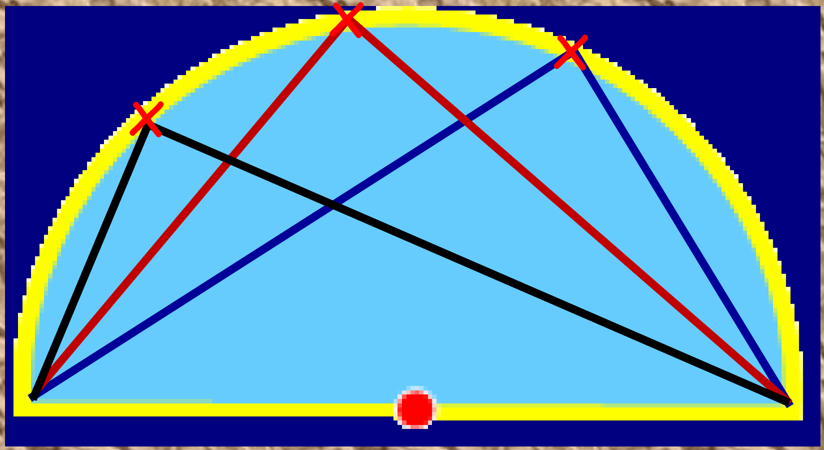


Log your results in a table.

Outside	Circumference	Inside

Semi-circle angle

Outside	Circumference	Inside
< 90°	= 90°	> 90°



Tangent line

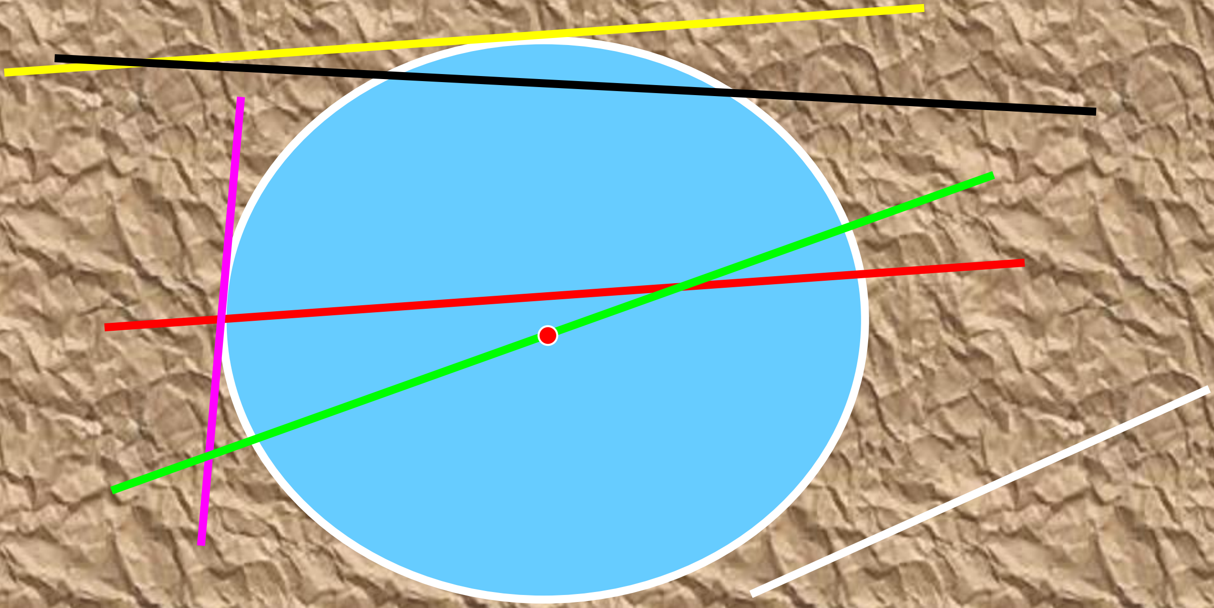
Aim of Today's Lesson

To understand what a tangent line is and its special property with the radius at the point of contact.

Tangent line

A tangent line is a line that touches a circle at only one point.

Which of the lines are tangent to the circle?

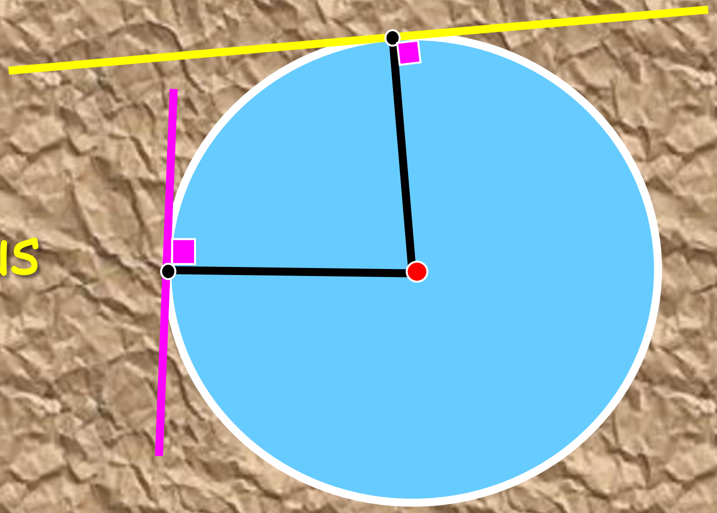


Tangent line

The radius of the circle that touches the tangent line is called the point of contact radius.

Special Property

The point of contact radius is always perpendicular (right-angled) to the tangent line.

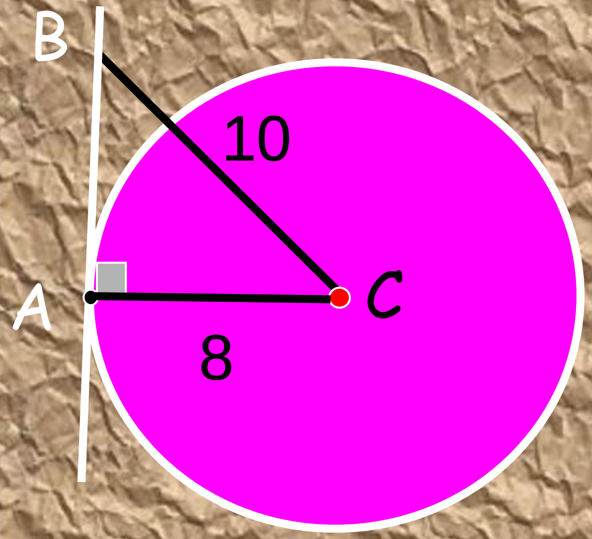


Tangent line

Q. Find the length of the tangent line between A and B.

Solution

Right-angled at A since AC is the radius at the point of contact with the Tangent.



By Pythagoras Theorem we have

$$a^2 + b^2 = c^2$$
$$a^2 + 8^2 = 10^2$$
$$a^2 = 10^2 - 8^2$$
$$a^2 = 100 - 64 = 36$$
$$a = \sqrt{36} = 6$$

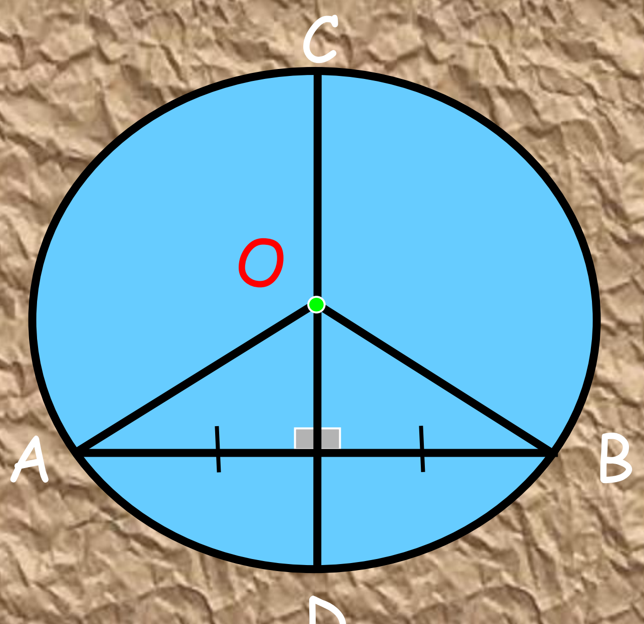
Tangent line

Diameter symmetry

Aim of Today's Lesson

To understand
some special properties
when a diameter bisects a chord.

Diameter symmetry



1. A line drawn through the centre of a circle through the midpoint a chord will ALWAYS cut the chord at right-angles
2. A line drawn through the centre of a circle at right-angles to a chord will ALWAYS bisect that chord.
3. A line bisecting a chord at right angles will ALWAYS pass through the centre of a circle.

Diameter symmetry

Q. Find the length of the chord A and B.

Solution

Radius of the circle is 4 + 6 = 10.

Since yellow line bisect AB and passes through centre O, triangle is right-angle.

By Pythagoras Theorem we have

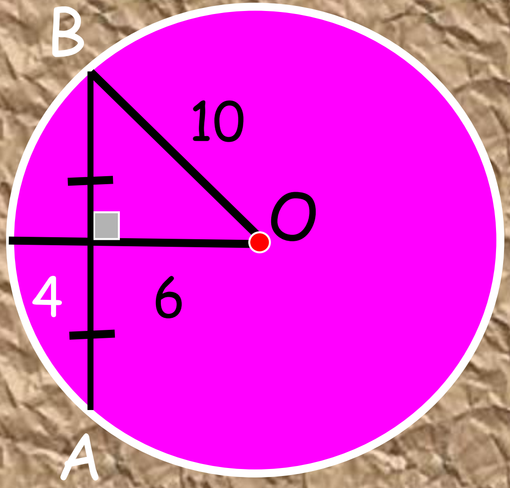
$$a^2 + b^2 = c^2$$

$$a^2 + 6^2 = 10^2$$

$$a^2 = 10^2 - 6^2$$

$$a^2 = 100 - 36 = 64$$

$$a = \sqrt{64} = 8$$



Since AB is bisected The length of AB is

$$length_{AB} = 2 \times 8 = 16$$

Diameter symmetry

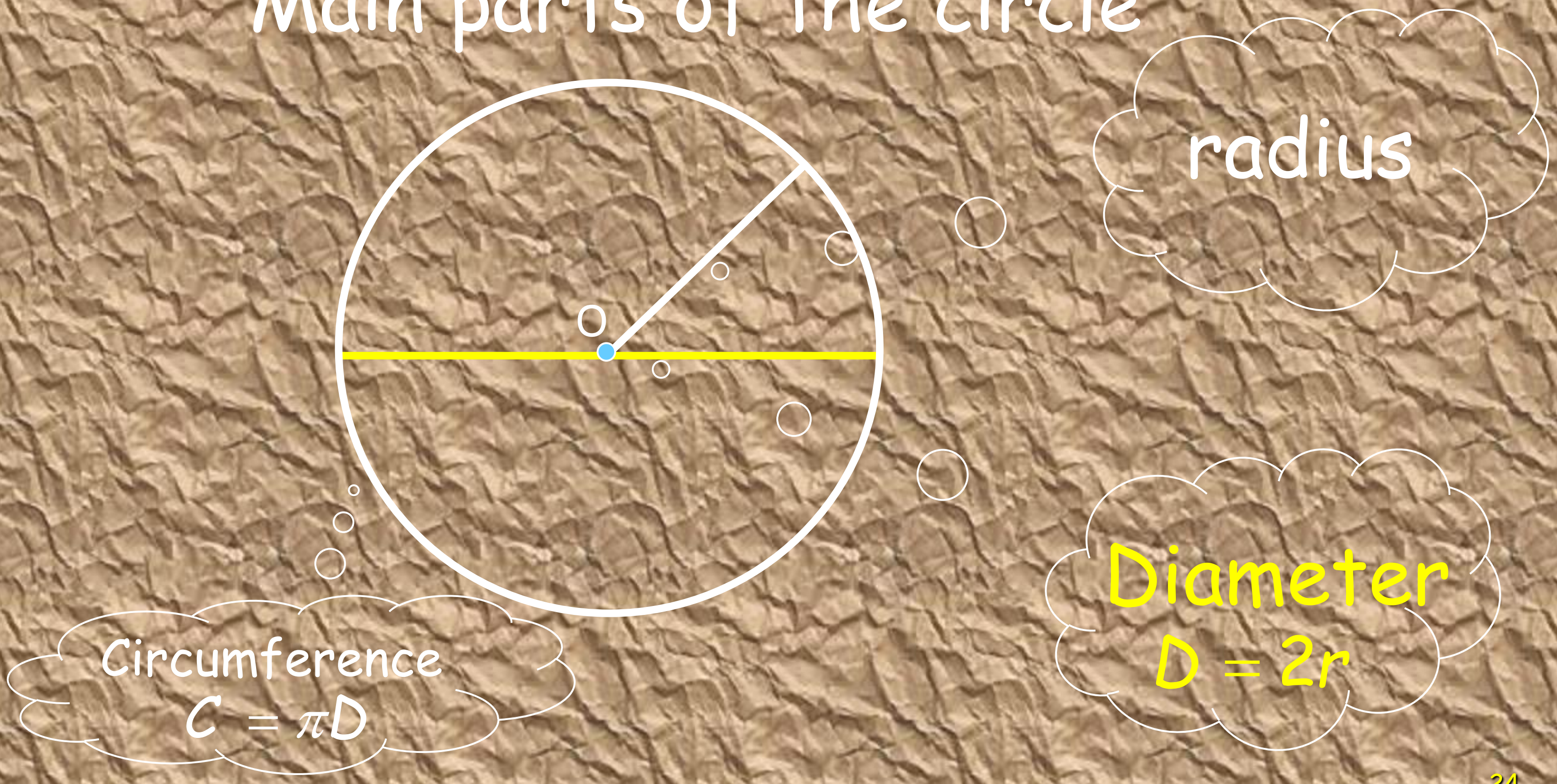
Circumference of a circle

Aim of Today's Lesson

To be able to use the formula for calculating the circumference of a circle

Circumference of a circle

Main parts of the circle



Circumference of a circle

Q. Find the circumference of the circle ?

<u>Solution</u>

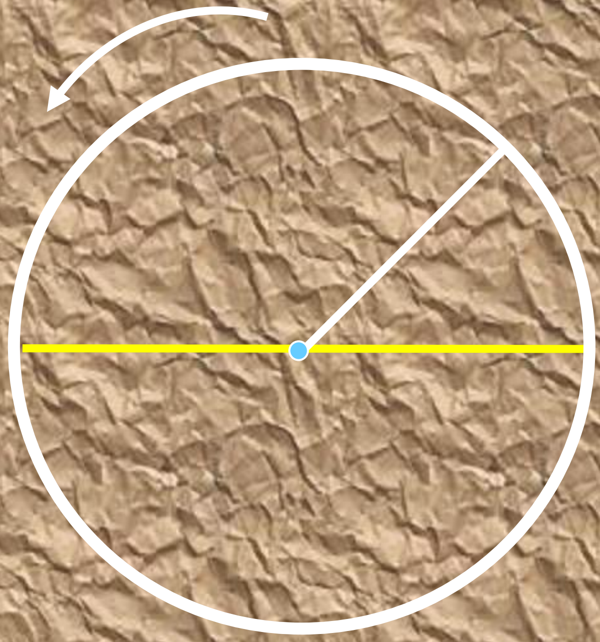
$C = \pi D$

$C = \pi \times 8$

$C = 25.12cm$

Circumference of a circle

Q. The circumference of the circle is 60cm ? Find the length of the diameter and radius.



Solution

$$C = \pi D$$

$$60 = \pi \times D$$

$$D = \frac{60}{\pi} cm$$

$$D \approx 19 \ cm$$

$$D = 2r$$

$$19 = 2r$$

$$r = \frac{19}{2}$$

$$r = 9.5cm$$

Circumference of a circle

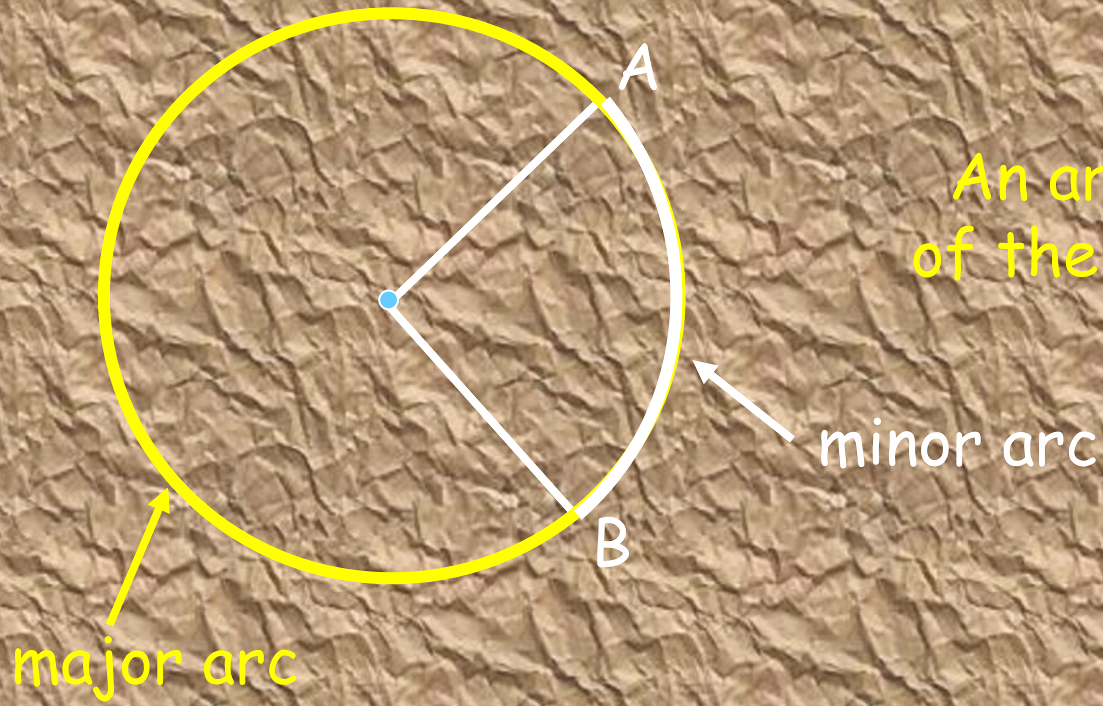
length of the arc of a circle

Aim of Today's Lesson

To find and be able to use the formula for calculating the length of an arc.

Arc length of a circle

Q. What is an arc ?

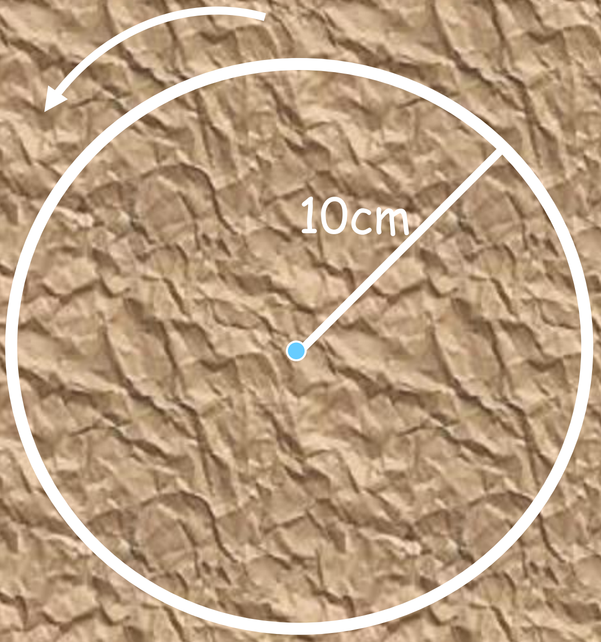


Answer

An arc is a fraction of the circumference.

Arc length of a circle

Q. Find the circumference of the circle ?



Solution

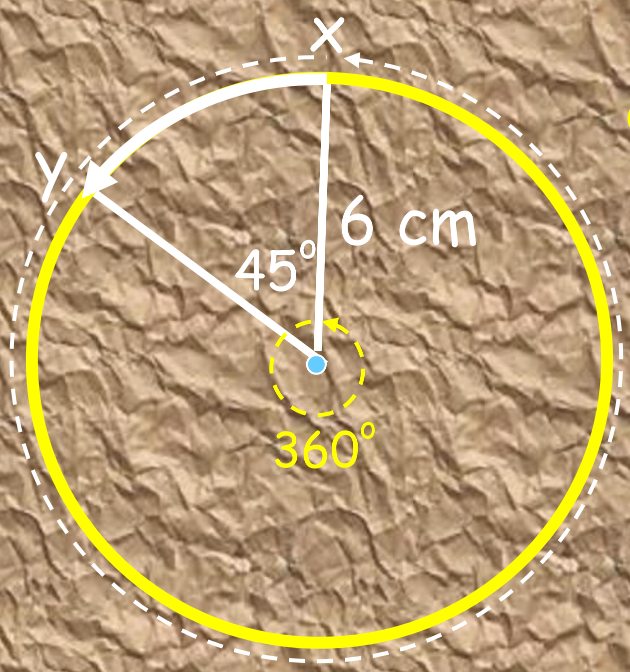
$$C = \pi D$$

$$C = \pi \times 20$$

$$C = 62.8cm$$

Arc length of a circle

Q. Find the length of the minor arc XY below ?



connection

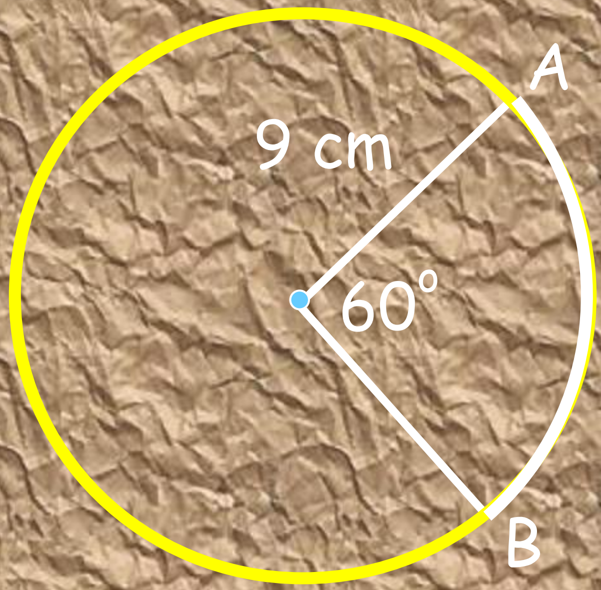
$$\frac{\text{Arc length}}{\pi D} = \frac{\text{Arc angle}}{360^o}$$

$$arc\ length = \frac{45^o}{360^o} \times (\pi \times 12)$$

$$arc\ length = 4.71cm$$

Arc length of a circle

Q. Find the length of the minor arc AB below ?



connection

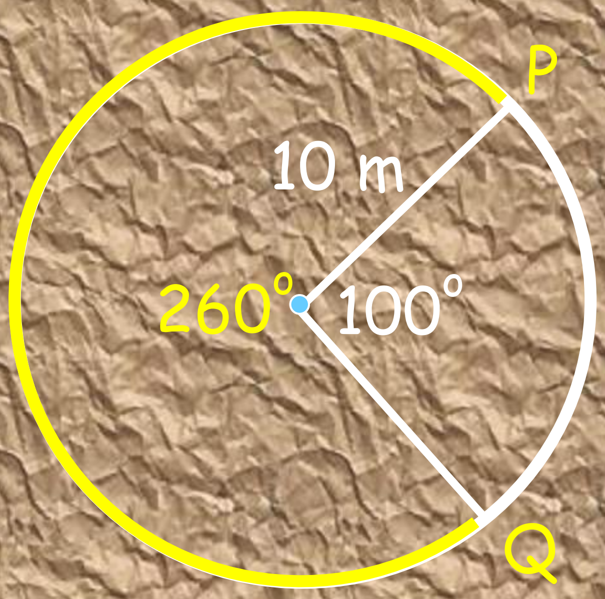
$$\frac{\text{Arc length}}{\pi D} = \frac{\text{Arc angle}}{360^{o}}$$

$$arc\ length = \frac{60^{o}}{360^{o}} \times (\pi \times 18)$$

$$arc\ length = 9.42cm$$

Arc length of a circle

Q. Find the length of the major arc PQ below ?



connection

$$\frac{\text{Arc length}}{\pi D} = \frac{\text{Arc angle}}{360^o}$$

$$\text{arc length} = \frac{260^o}{360^o} \times (\pi \times 20)$$

$$\text{arc length} = 45.38cm$$

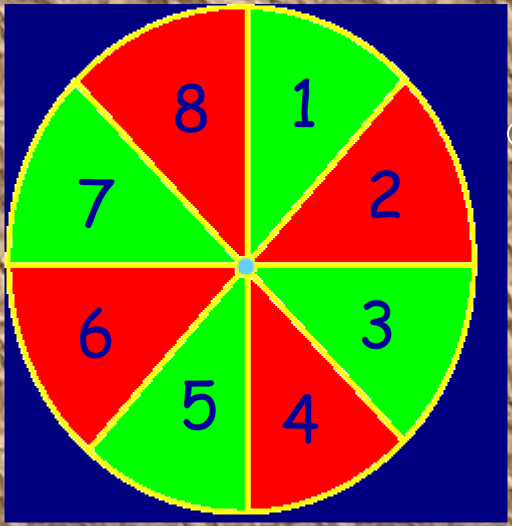
length of the arc of a circle

The Area of a circle

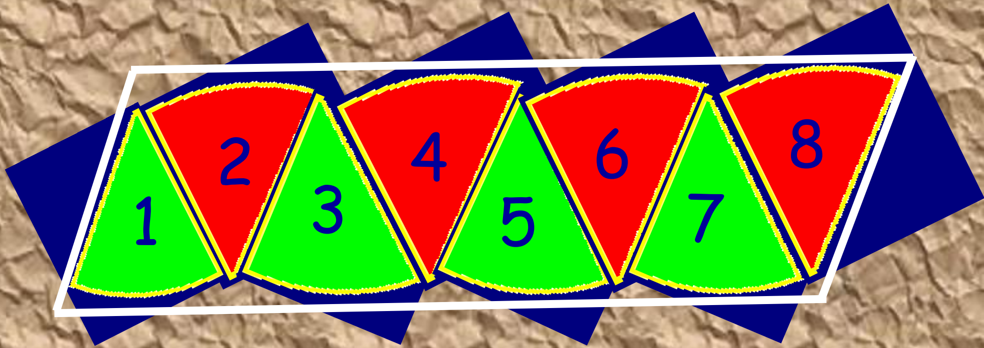
Aim of Today's Lesson

To come up with and be able to use the formula for calculating the area of a circle

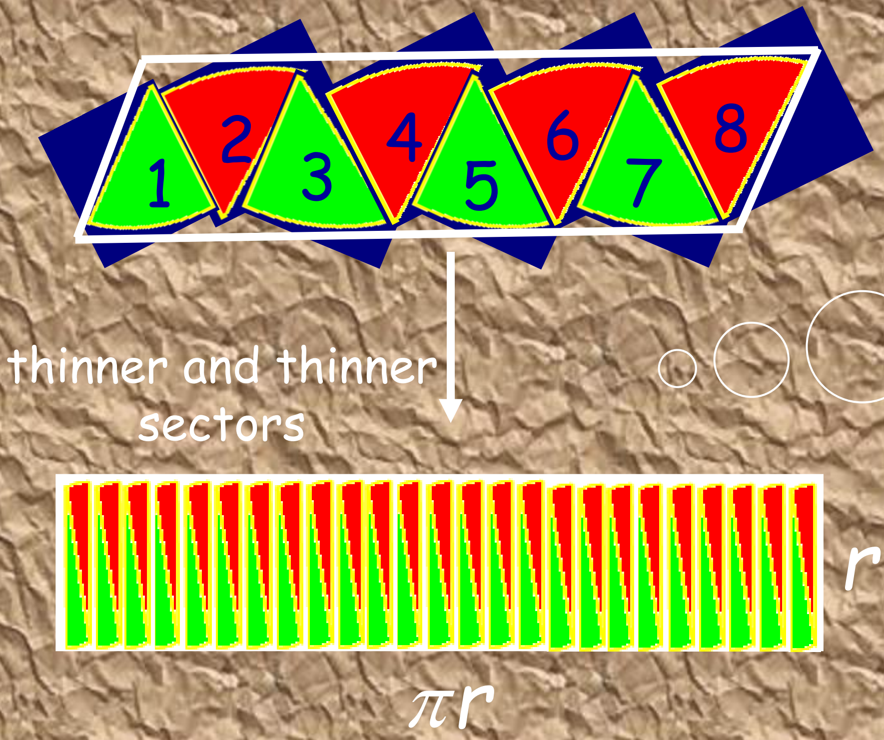
The Area of a circle



If we break the circle
into equal sectors
And lay them out side by side
We get very close
to a rectangle.

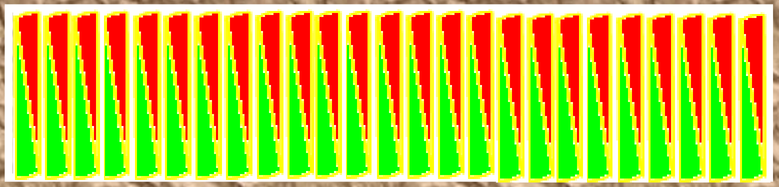


The Area of a circle



If we cut the sectors Thinner and thinner then we get closer and closer to a rectangle. Hence we can represent the area of a circle by a rectangle.

The Area of a circle



$$\text{Area of a rectangle} = l \times b$$

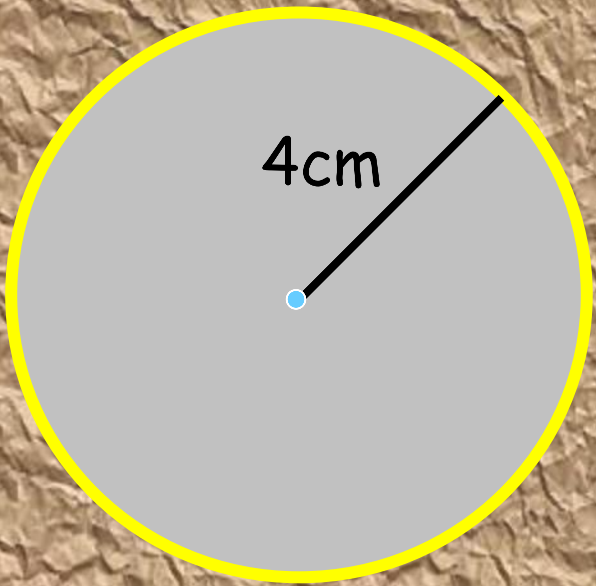
$$\text{Area of a rectangle} = \pi r \times r = \pi r^2$$

But the area inside this rectangle is also the area of the circle

$$\text{Area of a circle} = \pi r^2$$

The Area of a circle

Q. Find the area of the circle ?



Solution

$$A = \pi r^2$$

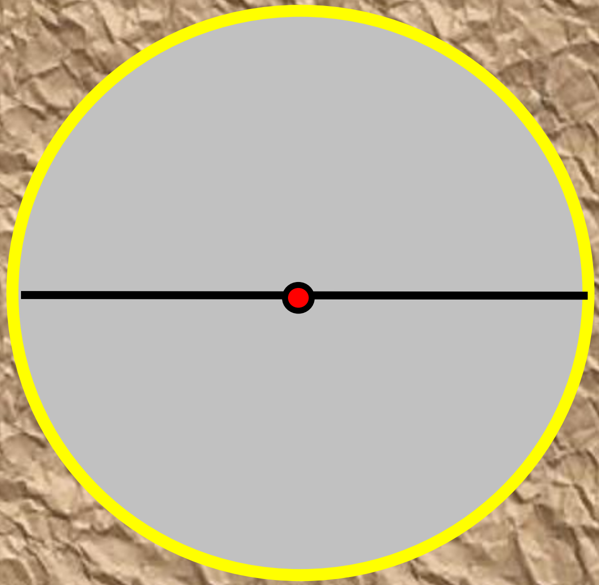
$$A = \pi \times 4^2$$

$$A = 50.26cm^2$$

The Area of a circle

The Area of a circle

Q. The diameter of the circle is 60cm. Find area of the circle?



<u>Solution</u>

$$A = \pi r^2$$

$$r = \frac{D}{2} = \frac{60}{2} = 30cm$$

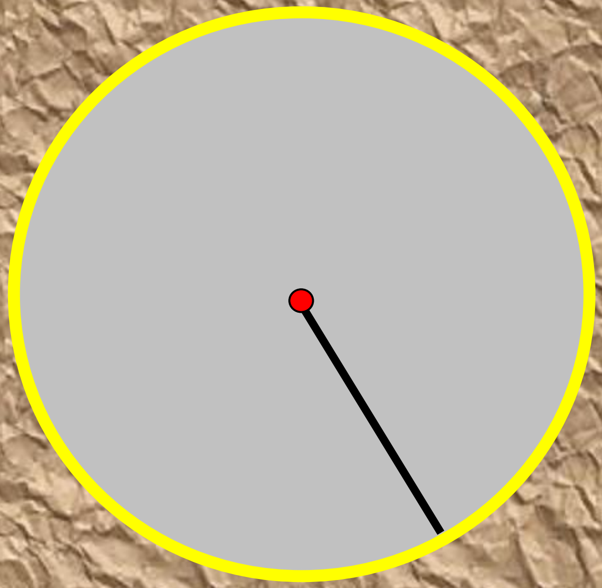
$$A = \pi \times 30^2$$

$$A = 2827.43cm^2$$

The Area of a circle

The Area of a circle

Q. The area of a circle is 12.64 cm^2.
Find its radius?



Solution

$$A = \pi r^2$$

$$12.64 = \pi \times r^2$$

$$r^2 = \frac{12.64}{\pi} = 4cm$$

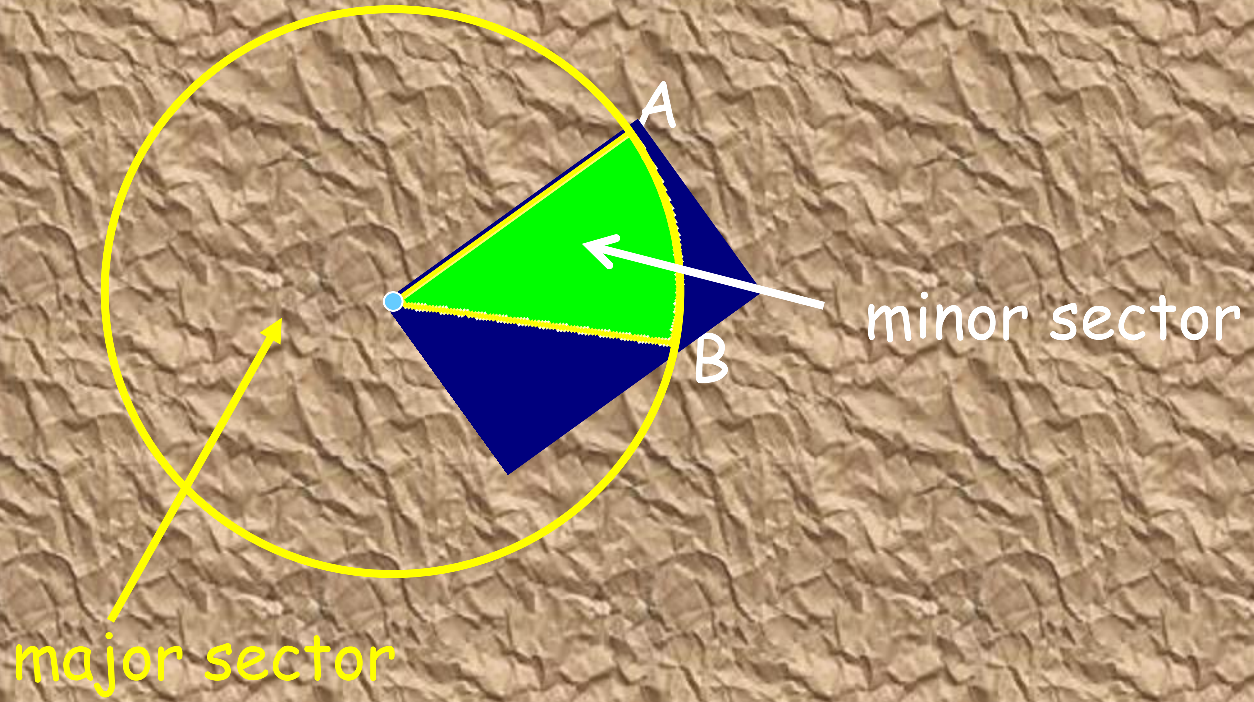
$$r = \sqrt{4} = 2cm$$

Sector area of a circle

Aim of Today's Lesson

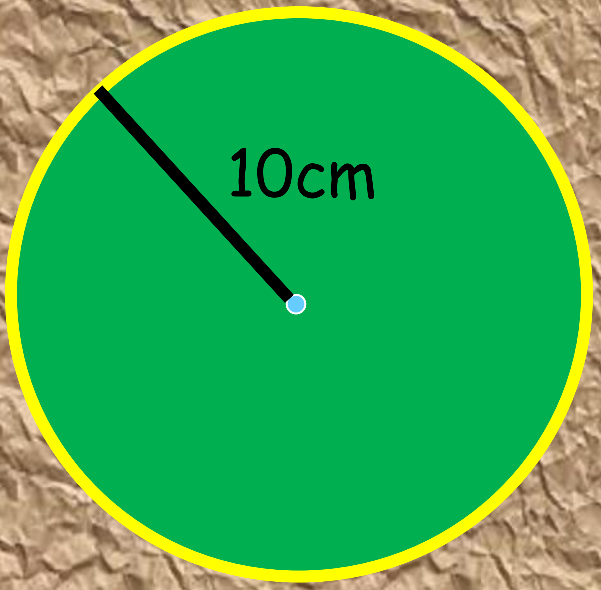
To find and be able to use the formula for calculating the sector of an circle.

Area of Sector in a circle



Area of Sector in a circle

Q. Find the area of the circle ?



Solution

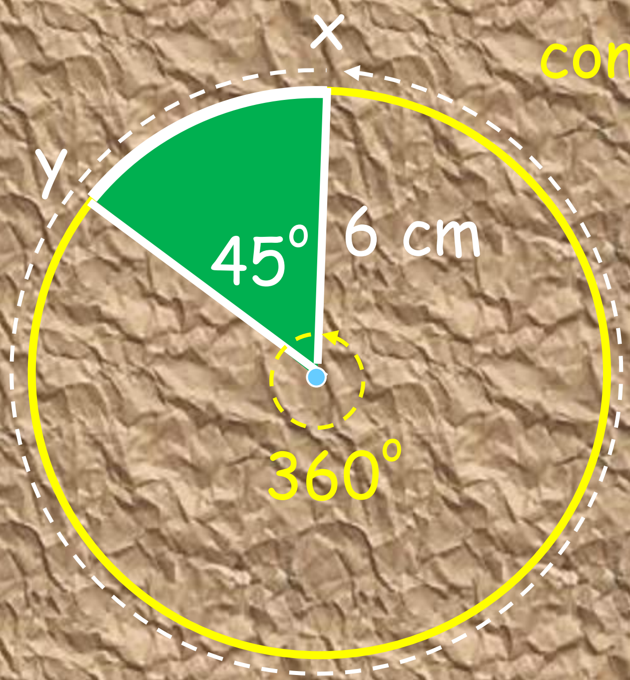
$$A = \pi r^2$$

$$A = \pi \times 10^2$$

$$A = 314cm^2$$

Area of Sector in a circle

Find the area of the minor sector XY below ?



$$\frac{\text{Area Sector}}{\pi r^2} = \frac{\text{Sector angle}}{360^o}$$

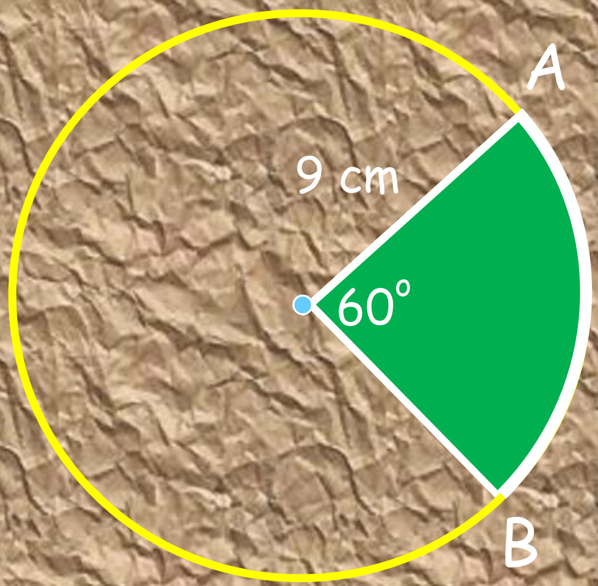
$$\text{Area of Sector} = \frac{45^o}{360^o} \times (\pi \times 6^2)$$

$$\text{Area Sector} = 14.14cm^2$$

Area of Sector in a circle

Q. Find the area of the minor sector AB below ?

connection



$$\frac{\text{Area Sector}}{\pi r^2} = \frac{\text{Sector angle}}{360^\circ}$$

$$\text{Area Sector} = \frac{60^\circ}{360^\circ} \times (\pi \times 9^2)$$

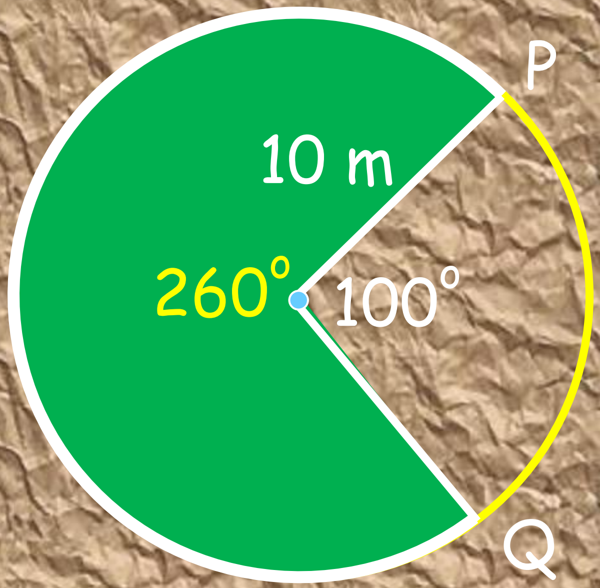
$$\text{Area Sector} = 42.41 cm^2$$

Area of Sector in a circle

Q. Find the area of the major sector PQ below ?

connection

$$\frac{\text{Sector Area}}{\pi r^2} = \frac{\text{Sector angle}}{360^o}$$



$$\text{Sector Area} = \frac{260^o}{360^o} \times (\pi \times 10^2)$$

$$\text{Area Sector} = 226.89cm^2$$

Sector area of a circle

Summary of Circle Topic

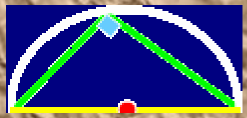
Arc length is

$$\frac{Arc_{length}}{\pi D} = \frac{\text{centre angle}^{o}}{360^{o}}$$

line that bisects a chord

1. Splits the chord into 2 equal halves.
2. Makes right-angle with the chord.
3. Passes through centre of the circle

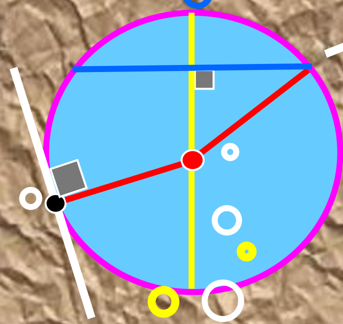
Pythagoras Theorem
SOHCAHTOA



Semi-circle angle is always 90^{o}

Circumference is

$$C = \pi D$$



Tangent touches circle at one point and make angle 90^{o} with point of contact radius

Area is

$$A = \pi r^{2}$$

Diameter

$$D = 2r$$

Radius

$$r = \frac{1}{2}D$$

Sector area

$$\frac{Area_{sector}}{\pi r^{2}} = \frac{\text{centre angle}^{o}}{360^{o}}$$