Quadratic Equations

A quadratic is any expression of the form $ax^2 + bx + c,\ a \neq 0$.

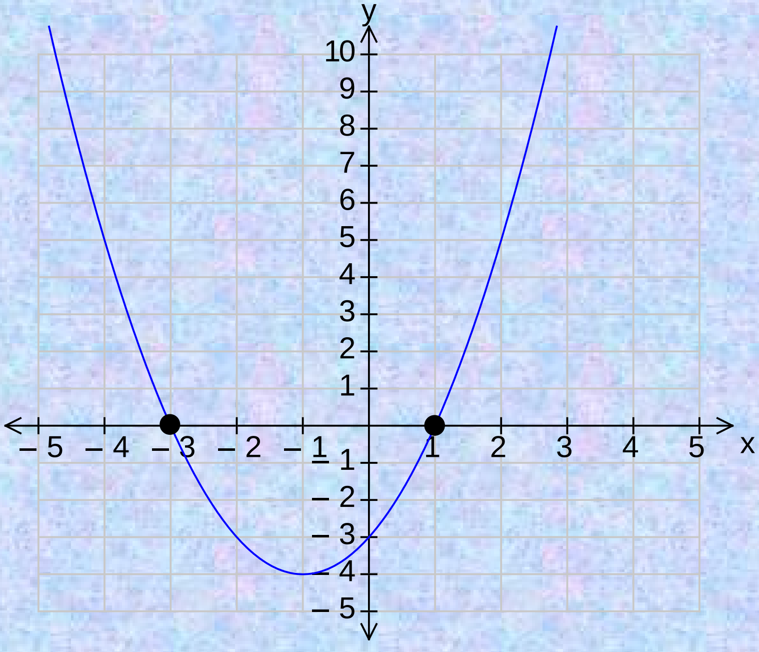
$$(x+1)(x+3) = x^2 + 4x + 3$$

You have already multiplied out pairs of brackets and factorised quadratic expressions.

Quadratic equations can be solved by factorising or by using a graph of the function.

Solving quadratic equations – using graphs

1. Use the graph below to find where $x^2 + 2x - 3 = 0$.



$x^2 + 2x - 3 = 0$ when the graph crosses the x-axis.

$x^2 + 2x - 3 = 0$ when $x = -3$, and $x = 1$.

Solving quadratic equations – using factors

Consider $(x - a)(x - b) = 0$. How do we solve this?

We know that if $c \times d = 0$ then either $c = 0$ or $d = 0$ or $c = d = 0$

$$(x-a)(x-b)=0$$

$x - a = 0$ or $x - b = 0$

$x = a$ $\quad$ $x = b$

1. Solve $3t - t^2 = 0$

$$3t - t^2 = 0$$
$$t(3-t) = 0$$

$t = 0$ $\quad$ $3 - t = 0$

$t = 3$

$$\underline{\underline{t = 0 \text{ or } t = 3}}$$

2. Solve $(x+6)(2x-3)=0$

$x+6=0 \qquad 2x-3=0$

$x=-6 \qquad 2x=3$

$x=3/2$

$x=-6$ or $x=3/2$

Reminder about factorising

1. Common factor. $6x^2 - 18 = 6(x^2 - 3)$

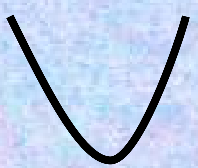
2. Difference of two squares. $4x^2 - 9 = (2x+3)(2x-3)$

3. Factorise. $x^2 - x - 2 = (x-2)(x+1)$

Sketching quadratic functions

To sketch a quadratic function we need to identify where possible:

The shape: $ax^2 + bc + c$

If $a > 0$ then  If $a < 0$ then 

The y intercept (0, c)

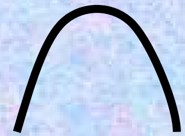
The roots by solving $ax^2 + bx + c = 0$

The axis of symmetry (mid way between the roots)

The coordinates of the turning point.

1. Sketch the graph of $y = 5 - 4x - x^2$

The shape

The coefficient of x^2 is -1 so the shape is 

The Y intercept

(0 , 5)

The roots

$$5 - 4x - x^2 = 0$$
$$(5 + x)(x - 1) = 0$$

(-5 , 0) (1 , 0)

The axis of symmetry

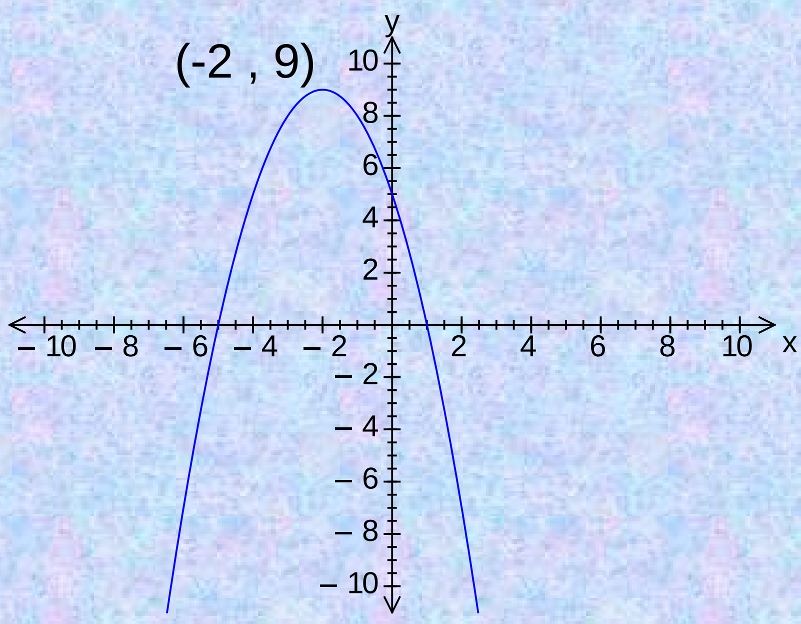
Mid way between -5 and 1 is -2

x = -2

The coordinates of the turning point

When $x = -2, \ y = 9$

(-2 , 9)



Standard form of a quadratic equation

Before solving a quadratic equation make sure it is in its standard form.

$$ax^2 + bx + c = 0$$

1. Solve $4x^2 + 1 = 5x$

$$4x^2 - 5x + 1 = 0$$

$$(4x - 1)(x - 1) = 0$$

$$4x - 1 = 0 \qquad x - 1 = 0$$

$$4x = 1 \qquad x = 1$$

$$x = \frac{1}{4}$$

$$x = 1/4 \text{ or } x = 1$$

Solving quadratic equations using a formula

What happens if you cannot factorise the quadratic equation?

You've guessed it. We use a formula.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Solve the equation $2x^2 - 5x - 1 = 0.$

compare with $ax^2 + bx + c = 0$

$$a = 2,\ b = -5,\ c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

WATCH YOUR NEGATIVES !!!

$$= \frac{5 \pm \sqrt{25 + 8}}{4}$$

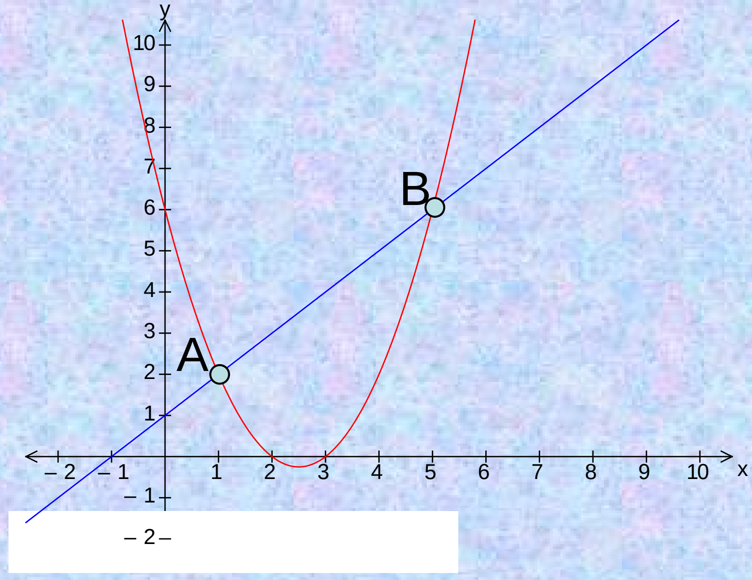
$$= \frac{5 + \sqrt{33}}{4} \text{ and } \frac{5 - \sqrt{33}}{4}$$

$$= \underline{\underline{2.69 \text{ and } -0.19}} \text{ correct to 2 d.p.}$$

Straight lines and parabolas

In this chapter we will find the points where a straight line intersects a parabola.

1. Find the coordinates of the points where the line $y = x + 1$ cuts the parabola with equation $y = x^2 - 5x + 6$.



At the points of intersection A and B, the equations are equal.

$$x^2 - 5x + 6 = x + 1$$

$$x^2 - 6x + 5 = 0$$

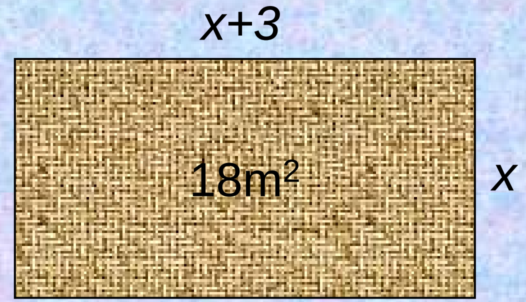
$$(x-1)(x-5) = 0$$

$$x = 1 \text{ and } x = 5 \qquad y = x + 1$$

$$\underline{\underline{A(1,2) \text{ and } B(5,6)}}$$

Quadratic equations as mathematical models

1. The length of a rectangular tile is 3m more than its breadth. It's area is 18m^2. Find the length and breadth of the carpet.



$$A = l \times b$$

$$18 = x(x+3)$$

$$x^2 + 3x = 18$$

$$x^2 + 3x - 18 = 0$$

$$(x+6)(x-3) = 0$$

$$x + 6 = 0 \qquad x - 3 = 0$$

$$x = -6 \qquad x = 3$$

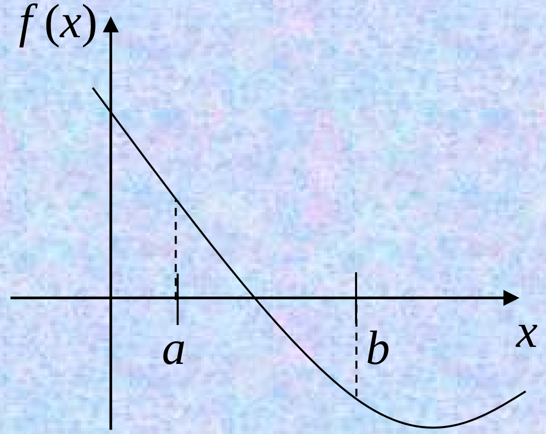
Not a possible solution

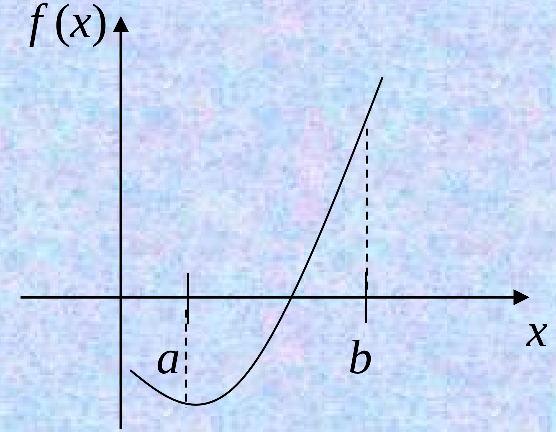
Breadth of the carpet is 3m and the length is 6m.

Trial and Improvement

The point at which a graph crosses the x-axis is known as a root of the function.

When a graph crosses the x-axis the y value changes from negative to positive or positive to negative.





$f(x) > 0 \; at \; x = a$

$f(x) < 0 \; at \; x = b$

A root exists between *a* and *b*.

$f(x) < 0 \; at \; x = a$

$f(x) > 0 \; at \; x = b$

A root exists between *a* and *b*.

The process for finding the root is known as iteration.

If f is the function defined by $f(x) = x^2 + x - 4$, show that a root exists between 1 and 2 and find this root to 2 decimal places.

$f(1) = -2$

$f(2) = 2$

Hence the graph crosses the x - axis between 1 and 2.

x	$f(x)$	Root lies between
1	-2	
2	2	1 and 2
1.5	-0.25	1.5 and 2
1.6	0.16	1.5 and 1.6
1.55	-0.048	1.55 and 1.6
1.56	-0.006	1.56 and 1.6
1.57	0.035	1.56 and 1.57
1.565	0.014	1.56 and 1.565

Hence the root is 1.56 to 2 d.p.

Solving Quadratic Equations

Graphically

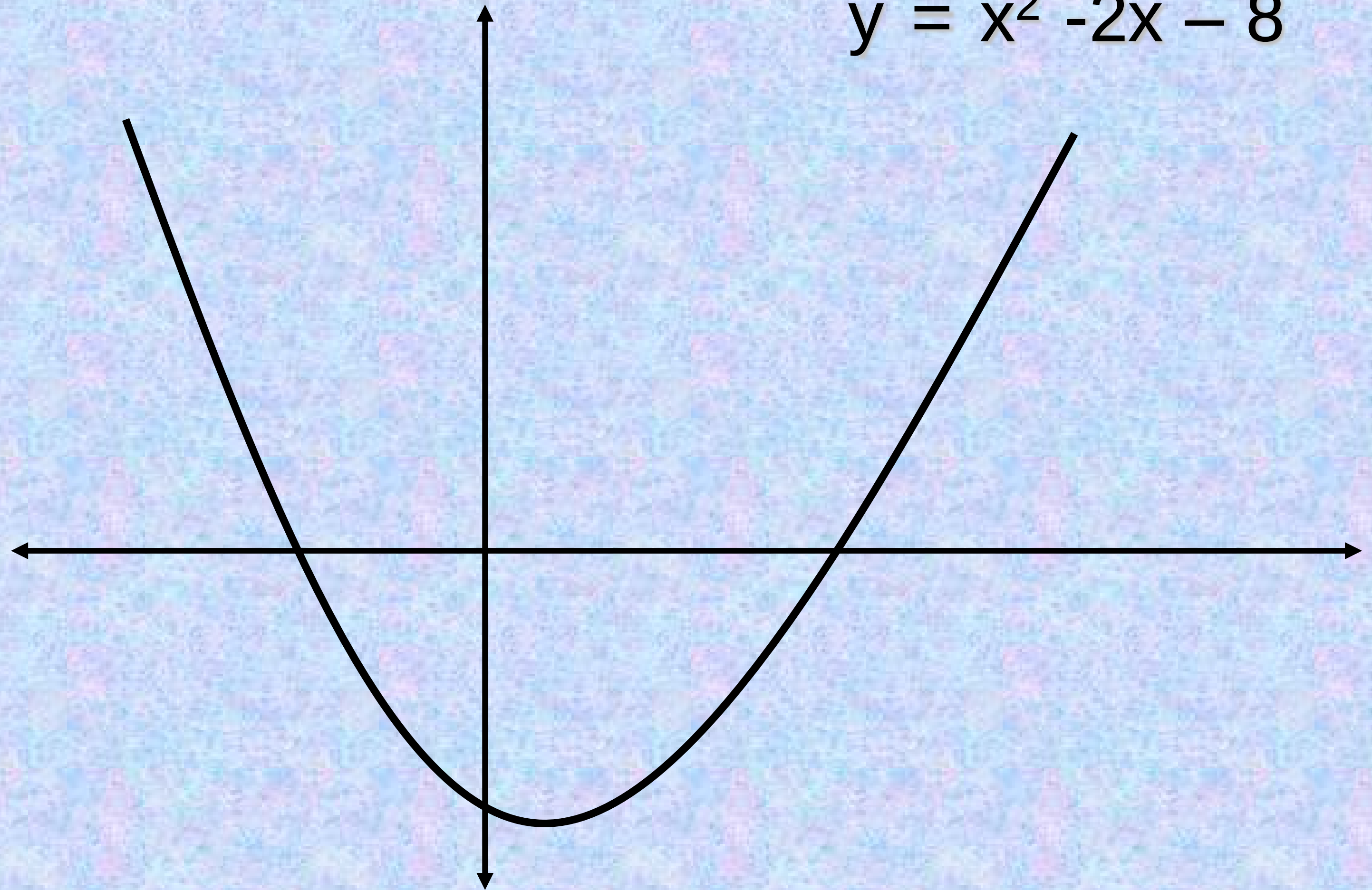
What is to be learned?

- How to solve quadratic equations by looking at a graph.

Laughably Easy
(sometimes)

Solve $x^2 - 2x - 8 = 0$

$y = x^2 - 2x - 8$



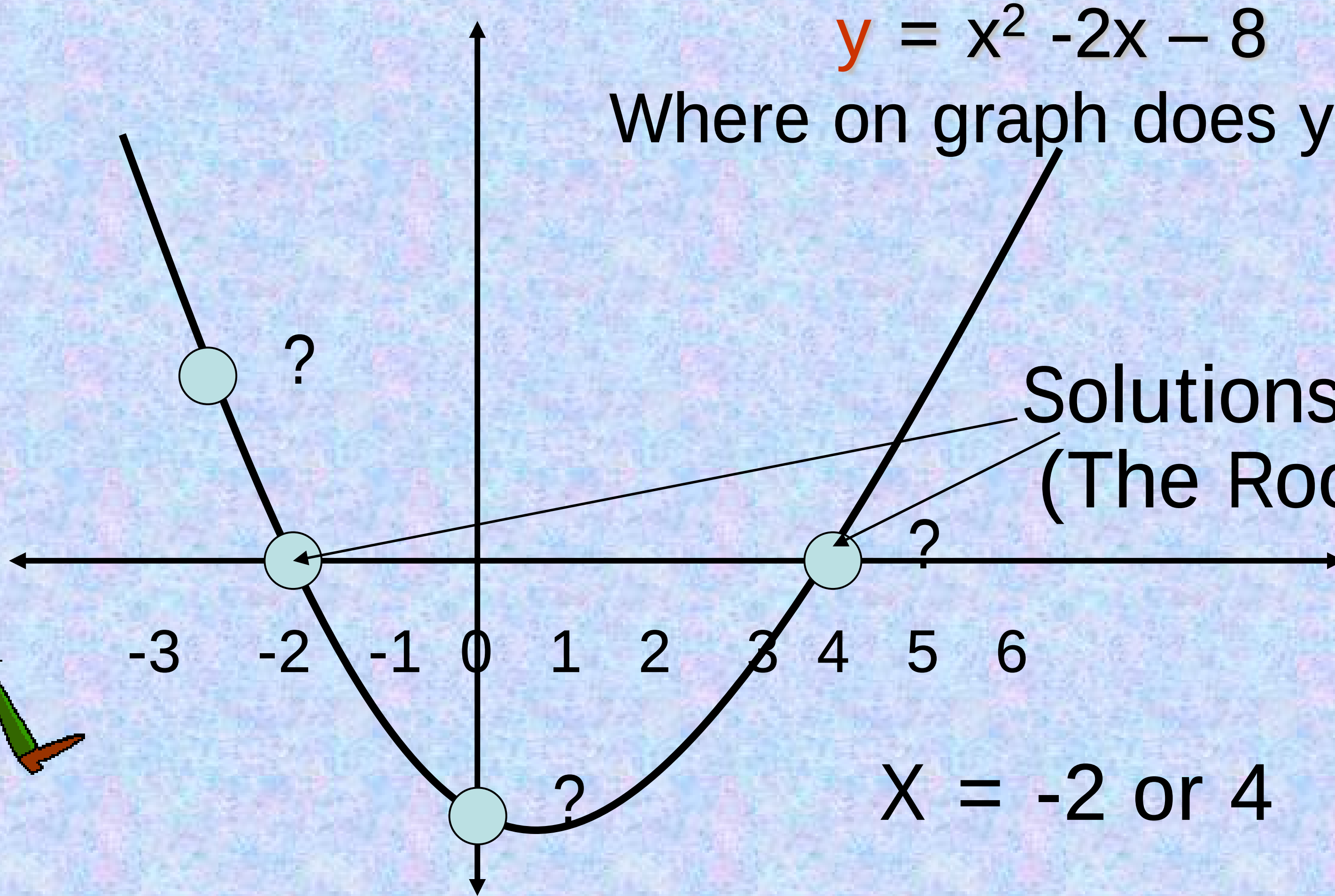
Laughably Easy

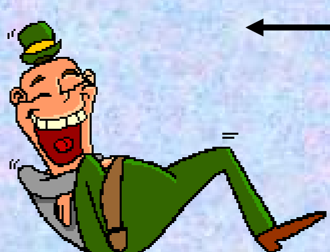
(sometimes)

Solve $x^2 - 2x - 8 = 0$

$y = x^2 - 2x - 8$

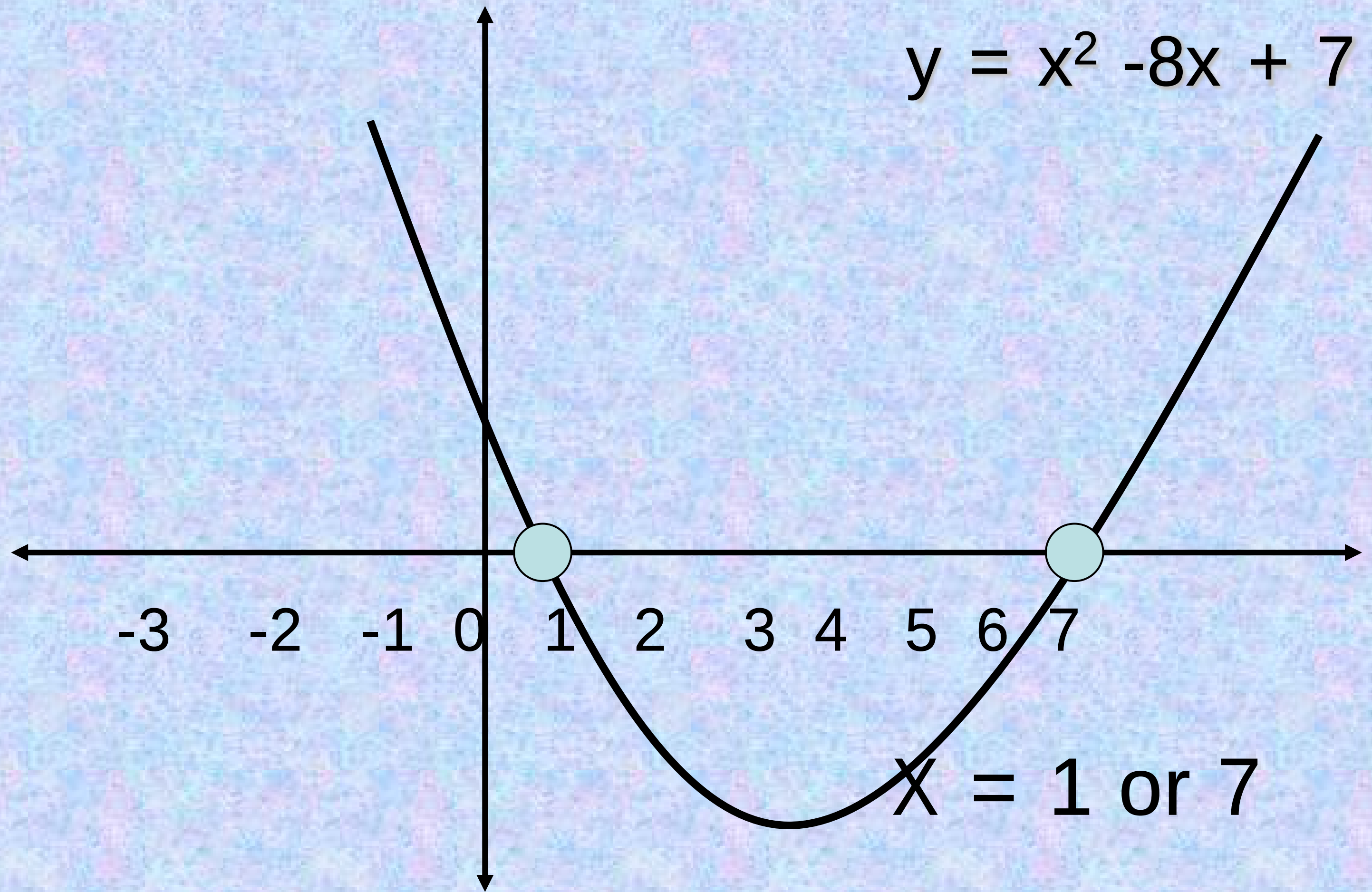
Where on graph does y = 0?





X = -2 or 4

Solve $x^2 - 8x + 7 = 0$



But....

Exam Type Question

$Y = x^2 + 6x + 8$ Find A and B

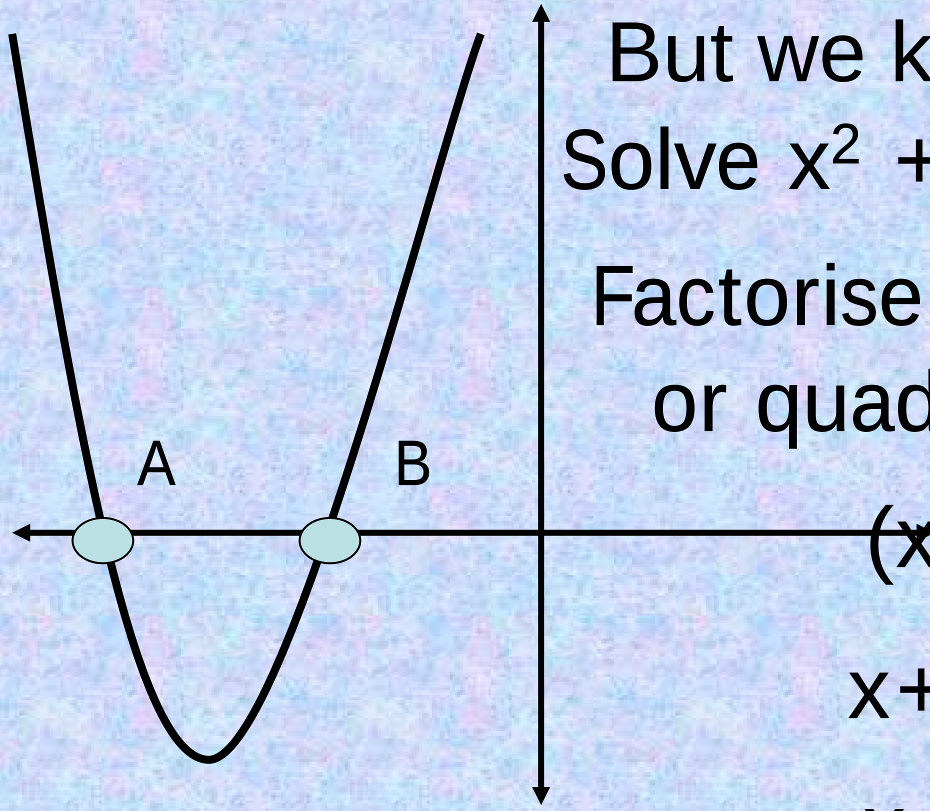
Not given x values

But we know y = 0

Solve $x^2 + 6x + 8 = 0$

Factorise

or quadratic formula

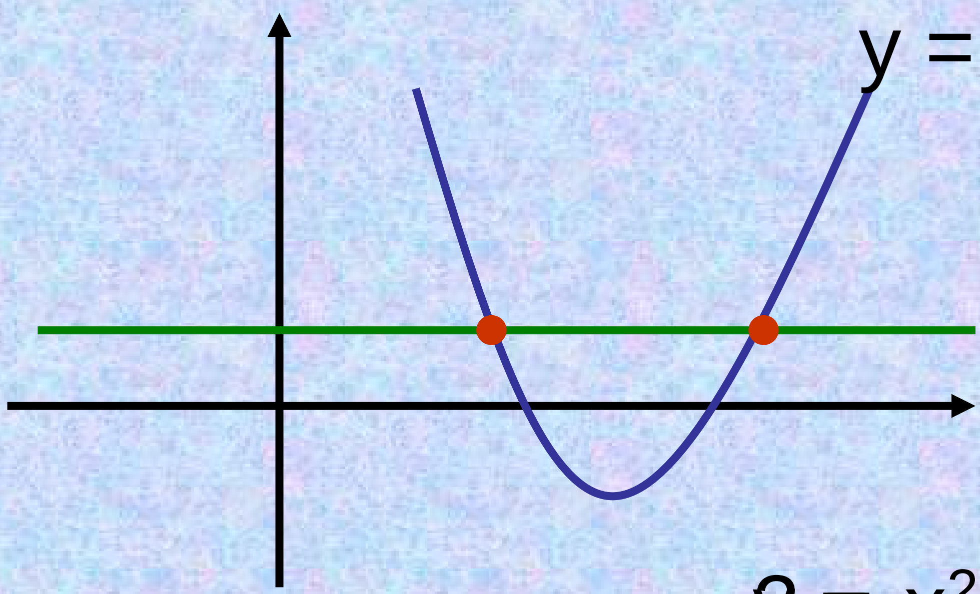


$(x + 2)(x + 4) = 0$

$x+2 = 0$ or $x+4 = 0$

$x = -2$ or $x = -4$

A (-4 , 0)   B (-2 , 0)

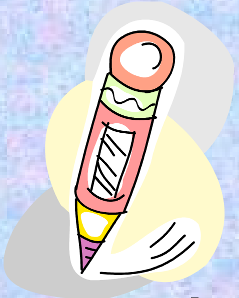


$$2 = x^2 - 7x + 10$$

$$x^2 - 7x + 10 = 2$$

$$x^2 - 7x + 8 = 0$$

Factorise or quadratic formula



Solving Quadratic Equations Graphically

Solutions occur where y = 0

Where graph cuts **X** axis

Known as roots.