boardworks

KS4 Mathematics

N4 Decimals and rounding

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N4 Decimals and rounding

The decimal number system

The decimal number system allows us to use the digits 0 to 9 and the place value system to represent numbers of any size.

To represent whole numbers we label columns using increasing powers of 10:

×10 ×10 ×10 ×10

	Ten thousands	Thousands	Hundreds	Tens	Units
	10^4	10^3	10^2	10^1	10^0
For example,	5	7	4	0	6

The decimal number system

To represent fractions of whole numbers in our place value system we extend the column headings in the other direction by dividing by 10.

Starting with the tens column we have:

÷10 ÷10 ÷10 ÷10

For example,

Tens	Units	Tenths	Hundredths	Thousandths
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
4	8 •	0	3	5

The decimal point

The decimal number system

Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
2	4	3	5	7	4

WHOLE NUMBER PART — DECIMAL PART

The decimal point separates the whole number part from the fractional part.

We can think of the number in this example as:

$$243.574 = 200 + 40 + 3 + \frac{5}{10} + \frac{7}{100} + \frac{4}{1000}$$

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Terminating decimals

A **terminating decimal** is a decimal that has a fixed number of digits after the decimal point.

For example,

0.6, 0.85 and 3.536

Terminating decimals can be written as vulgar fractions by writing them over the appropriate power of 10 and canceling if possible.

For example,

$$0.85 = \frac{\overset{17}{\cancel{85}}}{\underset{20}{\cancel{100}}} = \frac{17}{20}$$

Recurring decimals

Convert $\frac{8}{11}$ into a decimal using your calculator.

A calculator displays this as:

0.72727273

This decimal has been rounded to eight decimal places.

The last digit is a 3 because it has been rounded up.

The digits 2 and 7 actually repeat infinitely. This is an example of a **recurring decimal**.

We can show this by writing dots above the 7 and the 2.

$$\frac{8}{11} = 0.\dot{7}\dot{2}$$

Recurring decimals

Here are some more examples of recurring decimals:

$\frac{4}{9} = 0.\dot{4}$

This decimal is made up of an infinite number of repeating 4's.

$\frac{5}{6} = 0.8\dot{3}$

This decimal starts with an 8 and is followed by an infinite number of repeating 3's.

$\frac{2}{7} = 0.\dot{2}8571\dot{4}$

In this decimal, the six digits 285714 repeat an infinite number of times in the same order.

$\frac{9}{22} = 0.4\dot{0}\dot{9}$

This decimal starts with a 4. The two digits 09 then repeat an infinite number of times.

Recurring decimals and fractions

It is important to remember that *all* fractions convert to either terminating or recurring decimals.

If the denominator of the fraction divides into a power of 10 then the fraction will convert into a terminating decimal.

If the denominator of the fraction does not divide into a power of 10 then the fraction will convert into a recurring decimal.

The converse is also true. All terminating and recurring decimals can be written as fractions in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Converting recurring decimals to fractions

We can use place value to convert terminating decimals to fractions.

Converting recurring decimals to fractions is more difficult. For example,

Write 0.88888… as a fraction.

Start by letting the recurring decimal be equal to x.

$$x = 0.88888\ldots$$

One digit recurs so multiply both sides by 10.

$$10x = 8.88888\ldots$$

Converting recurring decimals to fractions

Write 0.88888… as a fraction.

Call these equations 1 and 2 .

$$x = 0.88888\ldots \qquad (1)$$

$$10x = 8.88888\ldots \qquad (2)$$

Now, subtracting 1 from 2 we have,

$$9x = 8$$

Dividing both sides of the equation by 9 we have,

$$x = \frac{8}{9}$$

Converting recurring decimals to fractions

Write 0.63636… as a fraction.

Let $x = 0.63636\ldots$ ①

Two digits recur so multiply both sides by 100,

$100x = 63.63636\ldots$ ②

Now, subtracting ① from ② we have,

$$99x = 63$$

Dividing both sides of the equation by 99 we have,

$$x = \frac{\cancel{63}^{\,7}}{\cancel{99}_{\,11}} = \frac{7}{11}$$

Converting recurring decimals to fractions

Write 0.370370… as a fraction.

Let $x = 0.370370\ldots$ ①

Three digits recur so multiply both sides by 1000,

$1000x = 370.370370\ldots$ ②

Now, subtracting ① from ② we have,

$999x = 370$

Dividing both sides of the equation by 999 we have,

$$x = \frac{\cancel{370}^{10}}{\cancel{999}_{27}} = \frac{10}{27}$$

Nought point nine recurring

The number 0.99999… gives us an interesting result when we use this method to convert it into a fraction.

Let $x = 0.99999\ldots$ (1)

Multiply both sides by 10,

$10x = 9.99999\ldots$ (2)

Now, subtracting (1) from (2) we have,

$9x = 9$

Dividing both sides of the equation by 9 we have,

$x = 1$

0.99999… is exactly equal to 1.

Rational numbers

Any number that can be written in the form $\frac{a}{b}$ (where a and b are integers and b ≠ 0) is called a **rational number**.

All of the following are rational:

$\frac{6}{7}$ 7 –12 $0.\dot{3}$ $8\frac{3}{4}$ 43.721

All integers are rational because they can be written as

$$\frac{\text{the integer}}{1}$$

We have seen that all terminating and recurring decimals can be written as fractions in the form $\frac{a}{b}$. This means that they are also rational.

Irrational numbers

Some numbers cannot be written in the form $\frac{a}{b}$.

These numbers are called **irrational numbers**.

If we tried to write an irrational number as a decimal it would neither terminate nor recur. It would be represented by an infinite non-repeating string of digits.

This means that irrational numbers can only be written as approximations to a given number of decimal places.

Examples of irrational numbers include:

π $\sqrt{3}$ and $\sin 50°$

Rational or irrational?

$$-19\frac{2}{3}$$

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Adding and subtracting decimals

6•154 | 4•228 | 85•94

+

Short multiplication

What is 2.28×7?

Start by finding an approximate answer:

$2.28 \times 7 \approx 2 \times 7 = 14$

2.28×7 is equivalent to $228 \times 7 \div 100$.

$$\begin{array}{r} 228 \\ \times \quad 7 \\ \hline 1596 \\ \hline {}_{1\ 5} \end{array}$$

Answer

$2.28 \times 7 = 1596 \div 100 = \mathbf{15.96}$

Short multiplication

What is 392.7 × 0.8?

Again, start by finding an approximate answer:

$392.7 \times 0.8 \approx 400 \times 1 = 400$

392.7×0.8 is equivalent to $3927 \times 8 \div 100$

$$\begin{array}{r} 3927 \\ \times \quad 8 \\ \hline 31416 \\ \hline {}_{7\,2\,5} \end{array}$$

Answer

$392.7 \times 0.8 = 31416 \div 100$

$= \mathbf{314.16}$

Long multiplication

Calculate 57.4×24

Estimate: $60 \times 25 = 1500$

Equivalent calculation: $57.4 \times 10 \times 24 \div 10$

$= 574 \times 24 \div 10$

$$
\begin{array}{rr}
 & 574 \\
 & \times\ 24 \\
\hline
4 \times 574 = & 2296 \\
20 \times 574 = & 11480 \\
\hline
 & 13776 \\
\hline
\end{array}
$$

Answer: $13776 \div 10 =$ **1377.6**

Long multiplication

Calculate 23.2×1.8

Estimate: $23 \times 2 = 46$

Equivalent calculation: $23.2 \times 10 \times 1.8 \times 10 \div 100$

$= 232 \times 18 \div 100$

	232
	× 18
8 × 232 =	1856
10 × 232	2320
	4176

Answer: $4176 \div 100 =$ **41.76**

Long multiplication

Calculate 394×0.47

Estimate: $400 \times 0.5 = 200$

Equivalent calculation: $394 \times 0.47 \times 100 \div 100$

$= 394 \times 47 \div 100$

$$
\begin{array}{lr}
 & 394 \\
 & \times\ 47 \\
\hline
7 \times 394 = & 2758 \\
40 \times 394 & 15760 \\
\hline
 & 18518 \\
\hline
\end{array}
$$

Answer: $18518 \div 100 =$ **185.18**

Long multiplication

A supermarket sells 935 tins of food in a day. How much money does it take if the tins are priced at £0.55?

Short division

Calculate 259.2 ÷ 6

Start by finding an approximate answer:

$259.2 \div 6 \approx 240 \div 6 = 40$

$$\begin{array}{r} 0\;4\;3\;.\;2 \\ 6\,\overline{)\,2\,{}^{2}5\,{}^{1}9\;.\,{}^{1}2} \end{array}$$

$259.2 \div 6 =$ **43.2**

Short division by a decimal

Calculate 57.23 ÷ 0.8 to 2 decimal places.

When we are asked to divide by a decimal we should first write an equivalent calculation with a whole number divider.

$$\frac{57.23}{0.8} \overset{\times 10}{=} \frac{572.3}{8}$$

We can then find an approximate answer:

$$572.3 \div 8 \approx 560 \div 8 = 70$$

Short division by a decimal

Calculate 57.23 ÷ 0.8 to 2 decimal places.

We are dividing by a single digit and so it is most efficient to use short division:

$$\begin{array}{r} 0\ 7\ 1\ .\ 5\ 3\ 7 \\ 8\,\overline{)\,5\,^{5}7\,^{1}2\ .\,^{4}3\,^{3}0\,^{6}0} \end{array}$$

We need to find the value of this digit to see whether we need to round up or down.

57.23 ÷ 0.8 = **71.54 (to 2 d.p.)**

Dividing by two-digit numbers

Calculate 75.4 ÷ 3.1

Estimate: 75 ÷ 3 = 25

Equivalent calculation: 75.4 ÷ 3.1 = 754 ÷ 31

$$
\begin{array}{rrl}
31 \,\big)\, & 754 & \\
- & 620 & \quad 20 \times 31 \\
\hline
 & 134 & \\
- & 124 & \quad 4 \times 31 \\
\hline
 & 10.0 & \\
- & 9.3 & \quad 0.3 \times 31 \\
\hline
 & 0.70 & \\
- & 0.62 & \quad 0.02 \times 31 \\
\hline
 & 0.08 &
\end{array}
$$

Answer: 75.4 ÷ 3.1 = 24.32 R 0.08

= 24.3 to 1 d.p.

Dividing by two-digit numbers

Calculate 8.12 ÷ 0.46

Estimate: 8 ÷ 0.5 = 16

Equivalent calculation: 8.12 ÷ 0.46 = 812 ÷ 46

46) 812		
- 460		**10** × 46
352		
- 322		**7** × 46
30.0		
- 27.6		**0.6** × 46
2.40		
- 2.30		**0.05** × 46
0.10		

Answer: 8.12 ÷ 0.43 = **17.65** R 0.1

= 17.7 to 1 d.p.

Long division

One day a supermarket takes £98.21 selling tins of food.
How many tins did it sell if they were priced £0.23?

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Rounding

We do not always need to know the exact value of a number.

For example,

There are 1432 pupils at Eastpark Secondary School.

There are about one and a half thousand pupils at Eastpark Secondary School.

Rounding

There are four main ways of rounding a number:

1) To the nearest 10, 100, 1000, or other power of ten.

2) To the nearest whole number.

3) To a given number of decimal places.

4) To a given number of significant figures.

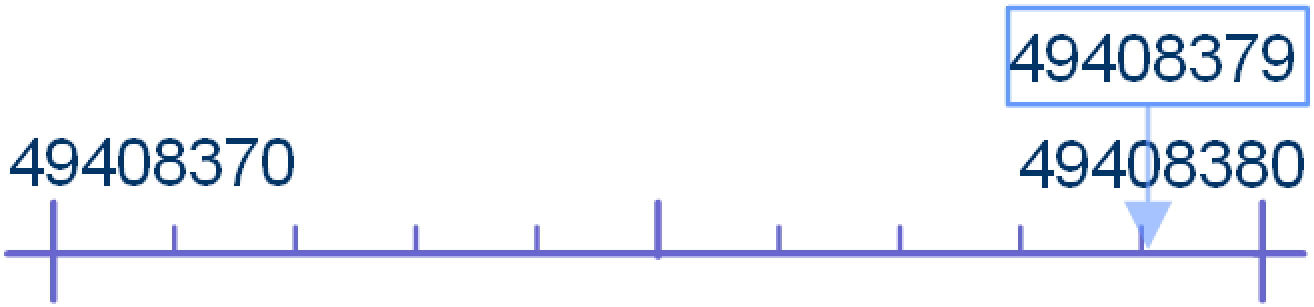
The method of rounding used usually depends on what kind of numbers we are dealing with.

Whole numbers, for example, can be rounded to the nearest power of ten or to a given number of significant figures.

Rounding to powers of ten

49408379 to the nearest

| 10 | 100 | 1000 | 10 000 | 100 000 | 1000 000 |



Rounding to powers of ten

Example

Round 34 871 to the nearest 100.

Look at the digit in the hundreds position.

We need to write down every digit up to this.

Look at the digit in the tens position.

If this digit is 5 or more then we need to round up the digit in the hundreds position.

Solution: 34871 = 34900 (to the nearest 100)

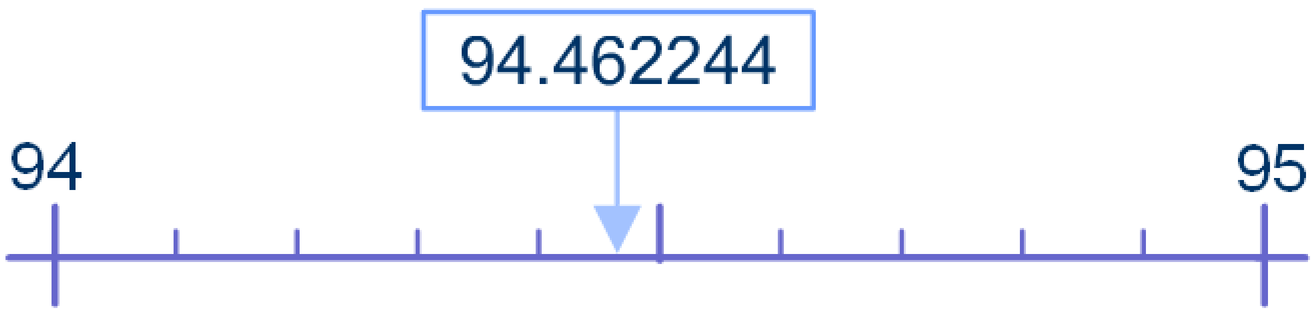
Rounding to powers of ten

Complete this table:

	to the nearest 1000	to the nearest 100	to the nearest 10
37521	38000	37500	37520
274503	275000	274500	274500
7630918	7631000	7630900	7630920
9875	10000	9900	9880
452	0	500	450

Rounding to decimal places

94.462244 to the nearest | 1 | 0.1 | 0.01 | 0.001



Rounding to decimal places

Example

Round 2.75241302 to one decimal place.

Look at the digit in the first decimal place.

We need to write down every digit up to this.

Look at the digit in the second decimal place.

If this digit is 5 or more then we need to round up the digit in the first decimal place.

2.75241302 to 1 decimal place is 2.8.

Rounding to decimal places

Complete this table:

	to the nearest whole number	to 1 d.p.	to 2 d.p.	to 3 d.p.
63.4721	63	63.5	63.47	63.472
87.6564	88	87.7	87.66	87.656
149.9875	150	150.0	149.99	149.988
3.54029	4	3.5	3.54	3.540
0.59999	1	0.6	0.60	0.600

Rounding to significant figures

Numbers can also be rounded to a given number of **significant figures**.

The first significant figure of a number is the first digit which is not a zero.

For example,

4 890 351

This is the first significant figure

and

0.0007506

This is the first significant figure

Rounding to significant figures

The second, third and fourth significant figures are the digits immediately following the first significant figure, including zeros.

For example,

4 890 351

This is the fourth significant figure

and

0.0007506

This is the fourth significant figure

Rounding to significant figures

Complete this table:

	to 3 s. f.	to 2 s. f.	to 1 s. f.
6.3528	6.35	6.4	6
34.026	34.0	34	30
0.005708	0.00571	0.0057	0.006
150.932	151	150	200
0.00007835	0.0000784	0.000078	0.00008

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Discrete and continuous quantities

Numerical data can be **discrete** or **continuous**.

Discrete data can only take certain values.

For example, shoe sizes,
the number of children in a class,
amounts of money

Continuous data comes from measuring and can take any value within a given range.

For example, the weight of a banana,
the time it takes for pupils to get to school,
heights of 15 year-olds.

Upper and lower bounds for discrete data

The population of the United Kingdom is 59 million to the nearest million.

What is the least this number could be?

The *least* this number could be before being rounded *up* is:

58 500 000

What is the most this number could be?

The *most* this number could be before being rounded *down* is:

59 499 999

Upper and lower bounds

We can give the possible range for the population as:

58 500 000 ≤ population ≤ 59 499 999

This value is called the **lower bound** …

… and this value is called the **upper bound**.

This is an inequality. It says that the actual population of the United Kingdom is between 58 500 000 and 59 499 999.

Because we have used ≤ symbols it means that these two numbers are included in the range of possible values. We could also write the range as:

58 500 000 ≤ population < 59 500 000

Upper and lower bounds for discrete data

Last year a shopkeeper made a profit of £43 250, to the nearest £50.

What range of values could this amount be in?

The lower bound is half-way between £43 200 and £43 250:

£43 225

The upper bound is half-way between £43 250 and £43 300, minus 1p:

£43 274.99

The range for this profit is:

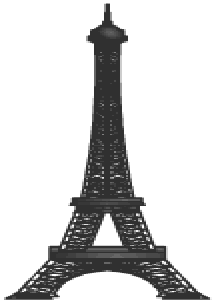
£43 225 ≤ profit ≤ £43 274.99

Upper and lower bounds for discrete data

Number of people	Lower bound	Upper bound
6000 to the nearest 500	5750	6249
75 to the nearest 5	73	77
14500 to the nearest 500	14250	14749
760 to the nearest 10	755	764
5200 to the nearest 100	5150	5249
8000 to the nearest 500	7750	8249
2450 to the nearest 50	2425	2474
49000 to the nearest 1000	48500	49499

Upper and lower bounds for continuous data

The height of the Eiffel Tower is 324 meters to the nearest meter.



What is the least this measurement could be?

The *least* this measurement could be before being rounded *up* is:

323.5 m

What is the most this measurement could be?

The *most* this measurement could be before being rounded *down* is up to but not including:

324.5 m

Upper and lower bounds

We can write the range for this measurement as:

$$323.5 \text{ m} \leq \text{height} < 324.5 \text{ m}$$

This value is called the **lower bound** …

… and this value is called the **upper bound**.

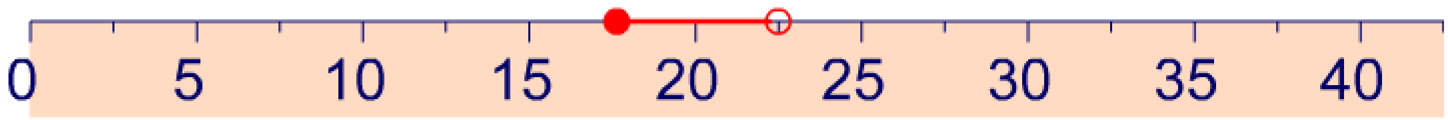
The height could be equal to 323.5 m so we use a greater than or equal to symbol.

If the length was equal to 324.5 m however, it would have been rounded up to 325 m. The length is therefore "*strictly less than*" 324.5 m and so we use the < symbol.

Upper and lower bounds

A length *l* is given as 20 cm to the nearest

5 cm



The range for this measurement is:

≤ / <