Geometric Sequences

Plus a review of arithmetic sequences

Definitions

- **Definitions:** (yes, that's right, this is important, know these!)
- **A sequence is a set of numbers, called terms, arranged in some particular order.**
- **An arithmetic sequence is a sequence with the difference between two consecutive terms constant. The difference is called the *common difference.* (isn't that clever!)**
- **A geometric sequence is a sequence with a common ratio, r. (cleverness two!)**
  - i.e. The ratio of successive terms in a geometric sequence is a constant called the common ratio, denoted r.

Examples: Find the common ratio of the following:

1) 1, 2, 4, 8, 16, ...

   r = 2

2) 27, 9, 3, 1, 1/3, ...

   r = 1/3

3) 3, 6, 12, 24, 48, ...

   r = 2

4) 1/2, -1, 2, -4, 8, ...

   r = -2

Examples: Find the next term in each of the previous sequences.

1) 1, 2, 4, 8, 16, ...

   32

2) 27, 9, 3, 1, 1/3, ...

   1/9

3) 3, 6, 12, 24, 48, ...

   96

4) 1/2, -1, 2, -4, 8, ...

   -16

Let's play guess the sequence!: I give you a sequence and you guess the type.

1. 3, 8, 13, 18, 23, . . .
2. **1, 2, 4, 8, 16, . . .**
3. 24, 12, 6, 3, 3/2, 3/4, . . .
4. **55, 51, 47, 43, 39, 35, . . .**
5. 2, 5, 10, 17, . . .
6. **1, 4, 9, 16, 25, 36, . . .**

Answers!

1) Arithmetic, the common difference d = 5
2) Geometric, the common ratio r = 2
3) Geometric, r = 1/2
4) Arithmetic, d = -4
5) Neither, why? (How about no common difference or ratio!)
6) Neither again! (This looks familiar, could it be from geometry?)

This is important!

Arithmetic formula:

$$a_n = a_1 + (n - 1)d$$

a_n is the nth term, a_1 is the first term, and d is the common difference.

Geometric formula:

$$a_n = a_1 \cdot r^{(n - 1)}$$

a_n is the nth term, a_1 is the first term, and r is the common ratio.

Sample problems:

Find the first four terms and state whether the sequence is arithmetic, geometric, or neither.

1) $a_n = 3n + 2$
2) $a_n = n^2 + 1$
3) $a_n = 3*2^n$

Answers:

1) $a_n = 3n + 2$

To find the first four terms, in a row, replace n with 1, then 2, then 3 and 4

Answer: 5, 8, 11, 14

The sequence is arithmetic! d = 3

2) $a_n = n^2 + 1$

To find the first four terms, do the same as above!

Answer: 2, 5, 10, 17

The sequence is neither. Why?

3) $a_n = 3*2^n$

Ditto for this one (got it by now?)

Answer: 6, 12, 24, 48

The sequence is geometric with r = 2

Find a formula for each sequence.

1) 2, 5, 8, 11, 14, . . .

Work: It is arithmetic! So use the arithmetic formula you learned!

$a_1 = 2$, look at the first number in the sequence!

$d = 3$, look at the common difference!

Therefore, $a_n = 2 + (n - 1)3$ and simplifying yields : $a_n = 3n - 1$ (tada!)

Try putting in 1, then 2, then 3, etc. and you will get the sequence!

2) 4, 8, 16, 32, . . .

Work: It is geometric! So use the geometric formula you learned up yonder!

a_1 = 4, look at the first number in the sequence!

r = 2, look at the common ratio! Therefore,

$a_n = 4 * 2^{(n - 1)}$ and simplifying gives us:

$a_n = 2 * 2n$ (Yikes stripes! Where did this come from. rewrite 2(n - 1) as 2n . 2-1 and cancel with the four!)

Try putting in 1, 2, 3, etc and see if you get the sequence back!

3) 21, 201, 2001, 20001, . . .

Work: Bummer! It's not geometric or arithmetic. What do I do now? Don't panic! Use your head and think!

Think of the sequence as (20 +1), (200+1), (2000 + 1), (20000 + 1), . . .

Then as this sequence:[(2)(10) +1],[(2)(100) +1], [(2)(1000) +1], [(2)(10000) +1]

Wait! Hold on here! I see a pattern! Cool, without a formula! Powers of 10!

How does this grab ya! $a_n = 2*10^n + 1$ Does this work? Try it and see!

Find the indicated term of the sequence.

1) **sequence is arithmetic with $t_1 = 5$ and $t_7 = 29$. Find t_{53}**

Work: Use the formula! $29 = 5 + 6d$

Where oh where did I get that!

Substitution!

$24 = 6d$ **means** $d = 4$

$t_{53} = 5 + 52 \cdot 4 = 213$

2) Find the number of multiples of 9 between 30 and 901.

Work: What's the first multiple of 9 in the range? How about 36.

What's the last multiple of 9 in the range? How about 900.

Use the formula: $900 = 36 + 9(n - 1)$ and solve for n!

$$864 = 9n - 9$$

$$873 = 9n$$

$97 = n$ There are 97 multiples in the range!

How to find the sum of a finite Geometric Series

$$S_n = a_1(1 - r^n)/(1 - r)$$

where r is the common ratio and

(r doesn't = 0)

To find the sum of a finite geometric series, you need to know three things: the first term, how many terms to add and the common ratio!! (piece of cake!)

Definition

geometric series - the expression formed by adding the terms of a geometric sequence.

Finding the Sum of the First *n* Terms of a Geometric Sequence.

- Use $S_n = [a_1(1 - r^n)/(1 - r)]$, S_n is the sum of the first *n* terms.
- Substitute the *n*, *a*, and *r* values into $S_n = [a_1(1 - r^n)/(1 - r)]$.
- Simplify to find the sum.

Example problem:

Find the sum of the first 10 terms of the geometric series:

4, 8, 16, 32, 64, $\cdots$

Answer: $t_1 = 4$

$r = 2$ $t_{10} = 4 \cdot 2^9 = 2048$ **(This is the formula for a geometric sequence!)**

Therefore: $S_n = [a_1(1 - r^n)/(1 - r)]$,

$S_{10} = 4(1-2^{10})/(1 - 2) = 4 \cdot 1023 = 4092$

Example:

Find the sum of the first 10 terms of the geometric series 9 + 36 + 144 + 576 + ...

Answer: $S_n = [(1 - r^n)/(1 - r)]$

$S_n = [9(1- 4^{10})/(1-4)]$

$S_n = [9(-1048575)/(-3)]$

$S_n = 28,311,525$

Example:

Find the sum of the first 10 terms of the geometric series -6 + -30 + -150 + -750 + ...

Answer: $S_n = [(1 - r^n)/(1 - r)]$

$S_n = [-6(1- 5^{10})/(1-5)]$

$S_n = [-6(-9765624)/(-4)]$

$S_n = -234,374,976$

Example:

Find the sum of the first 10 terms of the geometric series 8 + 56 + 392 + 2744 + ...

Answer: $S_n = [(1 - r^n)/(1 - r)]$

$S_n = [8(1- 7^{10})/(1-7)]$

$S_n = [8(-282,475,248)/(-6)]$

$S_n = 13,558,811,900$

Example:

Find the sum of the first 10 terms of the geometric series $4 + 12 + 36 + 108 + \ldots$

Answer: $S_n = [(1 - r^n)/(1 - r)]$

$S_n = [4(1- 3^{10})/(1-3)]$

$S_n = [4(-59{,}048)/(-2)]$

$S_n = 472{,}384$

The End!