SENIOR SECONDARY IMPROVEMENT PROGRAMME 2013

education
Department: Education
GAUTENG PROVINCE

GRADE 12

MATHEMATICS

LEARNER NOTES

The SSIP is supported by

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LEARNER NOTES

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SESSION 15

TOPIC: REVISION OF ANALYTICAL GEOMETRY (GRADE 11)

Learner Note: Analytical Geometry is an important topic that carries a lot of marks in the matric final exam. Make sure that you know the basic formulae and then practise lots of examples involving applications of these formulae. The properties of quadrilaterals are extremely important in Analytical Geometry. Make sure you can prove that a quadrilateral is a parallelogram, rectangle, square, rhombus or trapezium by knowing the properties of these quadrilaterals.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1: **15 minutes**

In the diagram, PQRS is a trapezium with vertices $P(5;2)$, $Q(1;-1)$, $R(9;-5)$ and S, and PS//QR. PT is the perpendicular height of PQRS and W is the midpoint of QR. Point S lies on the *x*-axis and $P\hat{R}Q = \theta$.

QUESTION 3

3(a)	$E\left(\frac{2+5}{2};\frac{4+(-1)}{2}\right)$ $=E\left(\frac{7}{2};\frac{3}{2}\right)$	✓✓ $E\left(\frac{7}{2};\frac{3}{2}\right)$ (2)
3(b)	$E\left(\frac{7}{2};\frac{3}{2}\right)=\left(\frac{x_A+x_C}{2};\frac{y_A+y_C}{2}\right)$ $\therefore E\left(\frac{7}{2};\frac{3}{2}\right)=\left(\frac{1+x_C}{2};\frac{3+y_C}{2}\right)$ $\therefore \frac{7}{2}=\frac{1+x_C}{2}$ or $\frac{3}{2}=\frac{3+y_C}{2}$ $\therefore 7=1+x_C$ or $3=3+y_C$ $\therefore x_C=6$ or $y_C=0$ $\therefore C(6;0)$	✓ $\frac{7}{2}=\frac{1+x_C}{2}$ ✓ $\frac{3}{2}=\frac{3+y_C}{2}$ ✓ $x_C=6$ ✓ $y_C=0$ ✓ $C(6;0)$ (5)

3(c)	$m_{AB}=\frac{4-3}{2-1}=\frac{1}{1}=1$ $m_{CD}=\frac{0-(-1)}{6-5}=\frac{1}{1}=1$ $\therefore m_{AB}=m_{CD}$ $\therefore AB\parallel CD$ $m_{AD}=\frac{3-(-1)}{1-5}=\frac{4}{-4}=-1$ $m_{BC}=\frac{4-0}{2-6}=\frac{4}{-4}=-1$ $\therefore m_{AD}=m_{BC}$ $\therefore AD\parallel BC$ ∴ ABCD is a parallelogram Now $m_{AB}\times m_{AD}=(1)\times(-1)=-1$ $\therefore AB\perp AD$ $\therefore \hat{A}=90°$ ∴ ABCD is a rectangle (since one interior angle of parallelogram ABCD is 90°)	✓ m_{AB} ✓ m_{CD} ✓ $AB\parallel CD$ ✓ m_{AD} ✓ m_{BC} ✓ $AD\parallel BC$ ✓ ∴ ABCD is a parallelogram ✓ $m_{AB}\times m_{AD}=-1$ ✓ $\hat{A}=90°$ ✓ ABCD is a rectangle (10)

QUESTION 2

2(a)	$(-1;3)=\left(\frac{1+(-3)}{2};\frac{2+k}{2}\right)$ $(-1;3)=\left(-1;\frac{2+k}{2}\right)$ $\therefore 3=\frac{2+k}{2}$ $\therefore 6=2+k$ $\therefore k=4$	✓ $(-1;3)=\left(\frac{1+(-3)}{2};\frac{2+k}{2}\right)$ ✓ $\therefore 3=\frac{2+k}{2}$ ✓ $\therefore k=4$ (3)
2(b)	$m_{AB}=m_{CD}$ (lines //) $\therefore \frac{2-1}{1-3}=\frac{k-(-3)}{-3-2}$ $\therefore \frac{1}{-2}=\frac{k+3}{-5}$ $\therefore -5=-2(k+3)$ $\therefore -5=-2k-6$ $\therefore 2k=-1$ $\therefore k=-\frac{1}{2}$	✓✓ $\therefore \frac{2-1}{1-3}=\frac{k-(-3)}{-3-2}$ ✓ $\therefore k=-\frac{1}{2}$ (3)
2(c)	$m_{AB}\times m_{CD}=-1$ (lines $\perp$) $\therefore \frac{1}{-2}\times\frac{k+3}{-5}=-1$ $\therefore \frac{k+3}{10}=-1$ $\therefore k+3=-10$ $\therefore k=-13$	✓✓ $\therefore \frac{1}{-2}\times\frac{k+3}{-5}=-1$ ✓ $\therefore k=-13$ (3)
2(d)	$m_{AB}=m_{BC}$ (lines //) $\therefore \frac{1}{-2}=\frac{1-k}{3-(-3)}$ $\therefore \frac{1}{-2}=\frac{1-k}{6}$ $\therefore 6=-2(1-k)$ $\therefore 6=-2+2k$ $\therefore 8=2k$ $\therefore k=4$	✓✓ $\therefore \frac{1}{-2}=\frac{1-k}{3-(-3)}$ ✓ $\therefore k=4$ (3)
2(e)	$CD=5\sqrt{2}$ And $CD^2=(2-(-3))^2+(-3-k)^2$ $\therefore (5\sqrt{2})^2=5^2+9+6k+k^2$ $\therefore 50=25+9+6k+k^2$ $\therefore 0=k^2+6k-16$ $\therefore 0=(k+8)(k-2)$ $\therefore k=-8$ or $k=2$	✓ $CD^2=(2-(-3))^2+(-3-k)^2$ ✓ $\therefore (5\sqrt{2})^2=5^2+9+6k+k^2$ ✓ $\therefore 0=k^2+6k-16$ ✓ $\therefore 0=(k+8)(k-2)$ ✓ $\therefore k=-8$ or $k=2$ (5) **[17]**

1(e)	$QT^2 = (1-3)^2 + (-1-(-2))^2$ $\therefore QT^2 = 4+1$ $\therefore QT^2 = 5$ $\therefore QT = \sqrt{5}$ $TR^2 = (3-9)^2 + (-2-(-5))^2$ $\therefore TR^2 = 36+9$ $\therefore TR^2 = 45$ $\therefore TR = \sqrt{45} = \sqrt{9\times 5} = 3\sqrt{5}$ $\therefore \frac{1}{3}TR = \sqrt{5} = QT$ $\therefore QT = \frac{1}{3}TR$	✓ correct substitution to get QT ✓ answer for QT ✓ correct substitution to get TR ✓ answer for TR ✓ establishing that $QT = \frac{1}{3}TR$ (5)
1(f)	$\tan\alpha = m_{PR}$ $\therefore \tan\alpha = \frac{2-(-5)}{5-9}$ $\therefore \tan\alpha = -1$ $\therefore \alpha = 180° - 45°$ $\therefore \alpha = 135°$ $\tan\beta = m_{PT}$ $\therefore \tan\beta = 2$ $\therefore \beta = 63,43494882°$ Now $T\hat{P}R + \beta = \alpha$ $\therefore T\hat{P}R = \alpha - \beta$ $\therefore T\hat{P}R = 135° - 63,43494882°$ $\therefore T\hat{P}R = 71,56505118°$ $\theta + 90° + 71,56505118° = 180°$ $\therefore \theta = 18,43°$	✓ $\tan\alpha = -1$ ✓ $\alpha = 135°$ ✓ $\beta = 63,43494882°$ ✓ $T\hat{P}R = 71,56505118°$ ✓ $\theta = 18,43°$ (5)

[24]

1(a)	$W\left(\frac{1+(9)}{2}\,;\,\frac{-1+(-5)}{2}\right)$ $= W(5\,;\,-3)$ The equation of PW is $x=5$	✓ midpoint ✓ $x=5$ (2)
1(b)	$m_{QR}=\frac{-5-(-1)}{9-1}=\frac{-4}{8}=-\frac{1}{2}$ $\therefore m_{PS}=-\frac{1}{2}$ (PS‖QR) $y-2=-\frac{1}{2}(x-5)$ $\therefore y-2=-\frac{1}{2}x+\frac{5}{2}$ $\therefore y=-\frac{1}{2}x+\frac{9}{2}$	✓ m_{QR} ✓ m_{PS} ✓ correct substitution into formula for equation ✓ $y=-\frac{1}{2}x+\frac{9}{2}$ (4)
1(c)	$m_{PT}=2$ $(PT\perp QR)$ $y-2=2(x-5)$ $\therefore y-2=2x-10$ $\therefore y=2x-8$	✓ m_{PT} ✓ correct substitution into formula for equation ✓ $y=2x-8$ (3)
1(d)	$m_{QR}=-\frac{1}{2}$ $y-(-1)=-\frac{1}{2}(x-1)$ $\therefore y+1=-\frac{1}{2}x+\frac{1}{2}$ $\therefore y=-\frac{1}{2}x-\frac{1}{2}$ $\therefore -\frac{1}{2}x-\frac{1}{2}=2x-8$ $\therefore -x-1=4x-16$ $\therefore -5x=-15$ $\therefore x=3$ $\therefore y=2(3)-8=-2$ $\therefore T(3;-2)$	✓ correct substitution into formula for equation ✓ $y=-\frac{1}{2}x-\frac{1}{2}$ ✓ $-\frac{1}{2}x-\frac{1}{2}=2x-8$ ✓ $x=3$ ✓ $T(3;-2)$ (5)

(a) Determine the equation of PW if W is the midpoint of QR. (2)
(b) Determine the equation of PS. (4)
(c) Determine the equation of PT. (3)
(d) Determine the coordinates of T. (5)
(e) Show that $QT = \frac{1}{3}TR$. (5)
(f) Calculate the size of θ rounded off to two decimal places. (5)
[24]

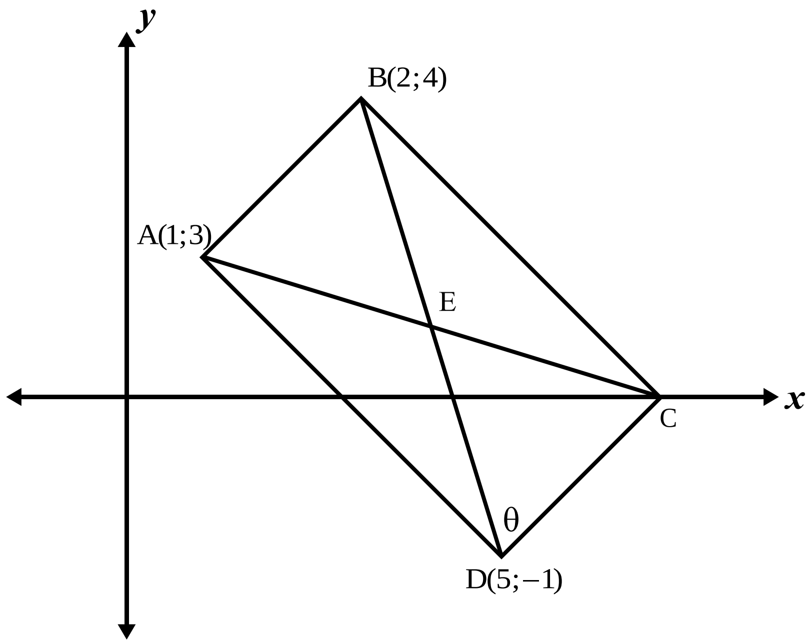
QUESTION 2: 15 minutes

Consider the following points on a Cartesian plane:
A(1;2), B(3;1), C(-3;*k*) and D(2;-3)
Determine the value(s) of k if:

(a) $(-1;3)$ is the midpoint of AC. (3)
(b) AB is parallel to CD. (3)
(c) AB $\perp$ CD. (3)
(d) A, B and C are collinear. (3)
(e) $CD = 5\sqrt{2}$ (5)
[17]

QUESTION 3: 25 minutes

ABCD is a quadrilateral with vertices $A(1;3)$, $B(2;4)$, C and $D(5;-1)$.
The diagonals BD and AC bisect each other at point E.



SESSION 20

TOPIC: REVISION OF ANALYTICAL GEOMETRY (GRADE 11)

Learner Note: Analytical Geometry is an important topic that carries a lot of marks in the matric final exam. Make sure that you know the basic formulae and then practise lots of examples involving applications of these formulae. The properties of quadrilaterals are extremely important in Analytical Geometry. Make sure you can prove that a quadrilateral is a parallelogram, rectangle, square, rhombus or trapezium by knowing the properties of these quadrilaterals.

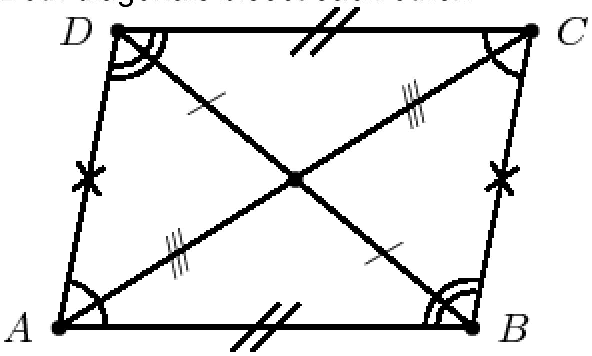
SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1: 15 minutes

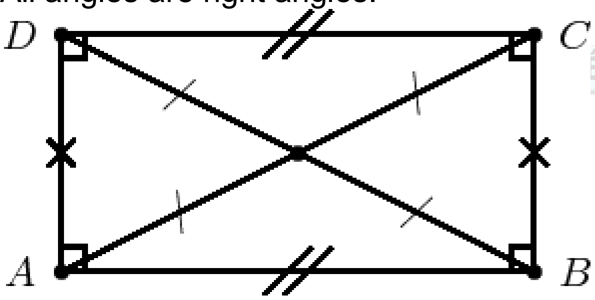
In the diagram, PQRS is a trapezium with vertices $P(5;2)$, $Q(1;-1)$, $R(9;-5)$ and S, and PS//QR. PT is the perpendicular height of PQRS and W is the midpoint of QR. Point S lies on the *x*-axis and $P\hat{R}Q = \theta$.

Properties of Quadrilaterals

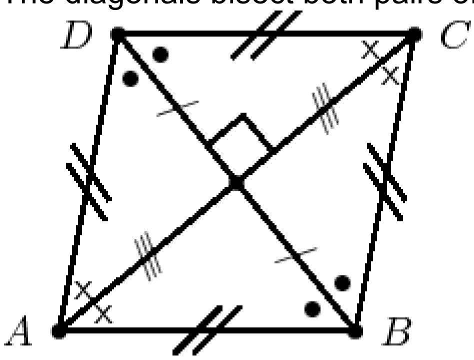
Parallelogram:
Both pairs of opposite sides are parallel.
Both pairs of opposite sides are equal in length.
Both pairs of opposite angles are equal.
Both diagonals bisect each other.



Rectangle:
Both pairs of opposite sides are parallel.
Both pairs of opposite sides are equal in length.
Both pairs of opposite angles are equal.
Both diagonals bisect each other.
Diagonals are equal in length.
All angles are right angles.



Rhombus:
Both pairs of opposite sides are parallel.
Both pairs of opposite sides are equal in length.
Both pairs of opposite angles are equal.
Both diagonals bisect each other.
All sides are equal in length.
The diagonals bisect each other at 90°.
The diagonals bisect both pairs of opposite angles.



SECTION B: ADDITIONAL CONTENT NOTES

REVISION OF ANALYTICAL GEOMETRY (GRADE 11)

If AB is the line segment joining the points $A(x_A\ ;\ y_A)$ and $B(x_B\ ;\ y_B)$, then the following formulas apply to line segment AB.

The Distance Formula

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

or $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

The Midpoint Formula

$M\left(\dfrac{x_A + x_B}{2}\ ;\ \dfrac{y_A + y_B}{2}\right)$ **where M is the midpoint of AB.**

The Gradient of a line segment joining two points

Gradient of $AB = \dfrac{y_B - y_A}{x_B - x_A}$

Parallel lines

Parallel lines have equal gradients. If AB||CD then $m_{AB} = m_{CD}$

Perpendicular lines

The product of the gradients of two perpendicular lines is -1. If AB$\perp$CD, then $m_{AB} \times m_{CD} = -1$

The equation of the line $y = mx + c$ or $y - y_A = m(x - x_A)$

Inclination of a line

$\tan\theta = m_{AB}$

If $m_{AB} > 0$, then θ is acute

If $m_{AB} < 0$, then θ is obtuse

Collinear points (A, B and C)

Using the gradient formula:

$m_{AB} = m_{BC}$

or $m_{AC} = m_{AB}$

or $m_{AC} = m_{BC}$

Using the distance formula:

$d_{AB} + d_{BC} = d_{AC}$

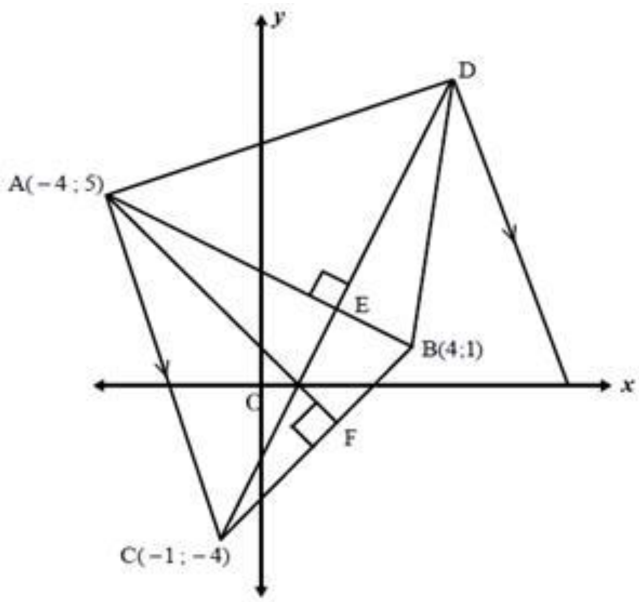
(a) Determine the coordinates of E, the midpoint of BD. (2)
(b) Determine the coordinates of C. (5)
(c) Show that ABCD is a rectangle. (10)
(d) Determine the area of ABCD. (6)
(e) Calculate the size of the angle θ rounded off to the nearest degree. (8)
[31]

QUESTION 4: 5 minutes

(a) Determine the numerical value of p if the straight line defined by the equation $px + 3y = 6$ has an angle of inclination of $135°$ with respect to the positive x-axis. (4)
(b) Calculate the value of k if the points $A(6;5)$, $B(3;2)$ and $C(2k; k+4)$ are collinear. (3)
[7]

QUESTION 5: 15 minutes

In the diagram below, a(-4;5), C(-1;-4) and B94;1) are the vertices of a triangle in a Cartesian plane. CE⊥AB with E on AB. E is the midpoint of line CD. AF⊥BC with F on CB. The equation of AF is $x + y = 1$.



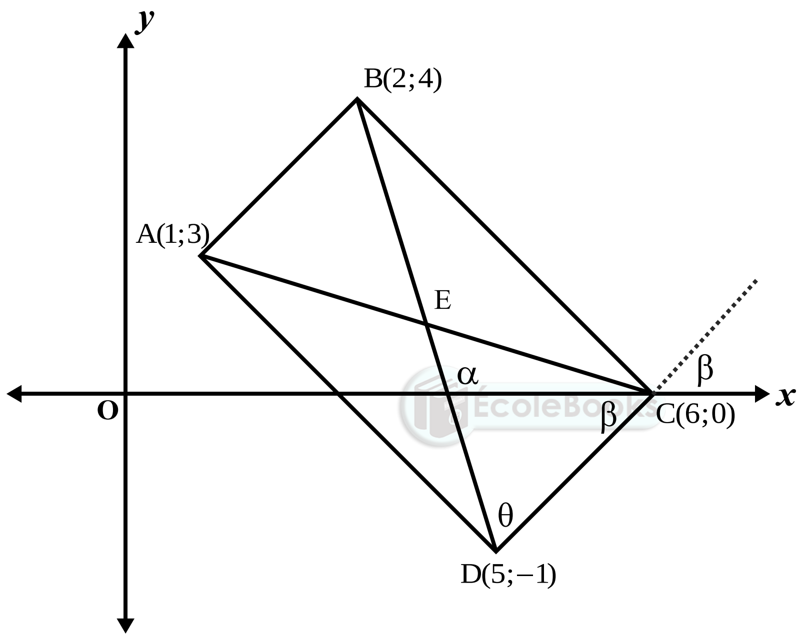
(a) Determine the equation of CD. (4)
(b) Determine the coordinates of E. (5)
(c) Determine the equation of the line through D and parallel to line AC. (6)
(d) Determine, showing all calculations, whether the x-intercept of line CD also lies on the line through AF with the equation $x + y = 1$. (3)
[18]

QUESTION 3

3(a)	$\text{E}\left(\frac{2+5}{2};\frac{4+(-1)}{2}\right)$ $=\text{E}\left(\frac{7}{2};\frac{3}{2}\right)$	✓✓ $\text{E}\left(\frac{7}{2};\frac{3}{2}\right)$ (2)
3(b)	$\text{E}\left(\frac{7}{2};\frac{3}{2}\right)=\left(\frac{x_A+x_C}{2};\frac{y_A+y_C}{2}\right)$ $\therefore \text{E}\left(\frac{7}{2};\frac{3}{2}\right)=\left(\frac{1+x_C}{2};\frac{3+y_C}{2}\right)$ $\therefore \frac{7}{2}=\frac{1+x_C}{2}$ or $\frac{3}{2}=\frac{3+y_C}{2}$ $\therefore 7=1+x_C$ or $3=3+y_C$ $\therefore x_C=6$ or $y_C=0$ $\therefore \text{C}(6;0)$	✓ $\frac{7}{2}=\frac{1+x_C}{2}$ ✓ $\frac{3}{2}=\frac{3+y_C}{2}$ ✓ $x_C=6$ ✓ $y_C=0$ ✓ $\text{C}(6;0)$ (5)

3(c)	$m_{AB}=\frac{4-3}{2-1}=\frac{1}{1}=1$ $m_{CD}=\frac{0-(-1)}{6-5}=\frac{1}{1}=1$ $\therefore m_{AB}=m_{CD}$ $\therefore$ AB||CD $m_{AD}=\frac{3-(-1)}{1-5}=\frac{4}{-4}=-1$ $m_{BC}=\frac{4-0}{2-6}=\frac{4}{-4}=-1$ $\therefore m_{AD}=m_{BC}$ $\therefore$ AD||BC $\therefore$ ABCD is a parallelogram Now $m_{AB}\times m_{AD}=(1)\times(-1)=-1$ $\therefore AB\perp AD$ $\therefore \hat{A}=90^\circ$ $\therefore$ ABCD is a rectangle (since one interior angle of parallelogram ABCD is 90°)	✓ m_{AB} ✓ m_{CD} ✓ AB||CD ✓ m_{AD} ✓ m_{BC} ✓ AD||BC ✓ $\therefore$ ABCD is a parallelogram ✓ $m_{AB}\times m_{AD}=-1$ ✓ $\hat{A}=90^\circ$ ✓ ABCD is a rectangle (10)

3(d)	$AB^2 = (2-1)^2 + (4-3)^2$ $\therefore AB^2 = 1+1$ $\therefore AB^2 = 2$ $\therefore AB = \sqrt{2}$ $AD^2 = (5-1)^2 + (-1-3)^2$ $\therefore AD^2 = 16+16$ $\therefore AD = \sqrt{32} = \sqrt{16\times 2} = 4\sqrt{2}$ $\text{Area ABCD} = AD \times AB$ $\therefore \text{Area ABCD} = (4\sqrt{2})(\sqrt{2}) = 8 \text{ units}^2$	✓ $AB^2 = (2-1)^2 + (4-3)^2$ ✓ $AB = \sqrt{2}$ ✓ $AD^2 = (5-1)^2 + (-1-3)^2$ ✓ $AD = \sqrt{32}$ ✓ $\text{Area ABCD} = (4\sqrt{2})(\sqrt{2})$ ✓ $\text{Area ABCD} = 8 \text{ units}^2$ (6)



3(e)	$\tan\alpha = m_{BD}$ $\therefore \tan\alpha = \dfrac{4-(-1)}{2-5}$ $\therefore \tan\alpha = -\dfrac{5}{3}$ $\therefore \alpha = 180° - 59°$ $\therefore \alpha = 121°$	✓ $\tan\alpha = -\dfrac{5}{3}$ ✓ $\alpha = 121°$ ✓ $\tan\beta = 1$ ✓ $\beta = 45°$ ✓ $\theta = 18{,}43°$ ✓ $O\hat{C}D = \beta = 45°$ ✓ $121° = \theta + 45°$ ✓ $\theta = 76°$ (8)

	$\tan\beta = m_{DC}$ $\therefore \tan\beta = \frac{0-(-1)}{6-5} = \frac{1}{1} = 1$ $\therefore \tan\beta = 1$ $\therefore \beta = 45°$ $\therefore O\hat{C}D = \beta = 45°$ $\alpha = \theta + \beta$ $\therefore 121° = \theta + 45°$ $\therefore \theta = 76°$	

[31]

QUESTION 4

4(a)	$3y = -px + 6$ $\therefore y = -\frac{p}{3}x + 2$ $\therefore \tan 135° = -\frac{p}{3}$ $\therefore -1 = -\frac{p}{3}$ $\therefore p = 3$	✓ $y = -\frac{p}{3}x + 2$ ✓ $\tan 135° = -\frac{p}{3}$ ✓ $-1 = -\frac{p}{3}$ ✓ $p = 3$ (4)
4(b)	$m_{AB} = m_{BC}$ $\therefore \frac{2-5}{3-6} = \frac{k+4-2}{2k-3}$ $\therefore 1 = \frac{k+2}{2k-3}$ $\therefore 2k-3 = k+2$ $\therefore k = 5$	✓ $m_{AB} = m_{BC}$ ✓ working out gradients ✓ $k = 5$ (3)

[7]

QUESTION 5

5(a)	$m_{AB} = \frac{5-1}{-4-4} = -\frac{1}{2}$ $\therefore m_{CD} = 2$ $y - 4 = 2(x - 1)$ $\therefore y + 4 = 2x + 2$ $\therefore y = 2x - 2$	✓ m_{AB} ✓ m_{CD} ✓ correct substitution into formula for equation of line ✓ $y = 2x - 2$ (4)
5(b)	Equation of AB $y - 1 = -\frac{1}{2}(x-4)$ $\therefore y - 1 = -\frac{1}{2}x + 2$ $\therefore y = -\frac{1}{2}x + 3$ Equation of CD $y = 2x - 2$ $\therefore -\frac{1}{2}x + 3 = 2x - 2$ $\therefore -x + 6 = 4x - 4$ $\therefore -5x = -10$ $\therefore x = 2$ $\therefore y = 2$ $\therefore E(2;2)$	✓ $y - 1 = -\frac{1}{2}(x-4)$ ✓ $y = -\frac{1}{2}x + 3$ ✓ $-\frac{1}{2}x + 3 = 2x - 2$ ✓ $x = 2$ ✓ E(2;2) (5)
5(c)	C(-1;-4) E(2;2) D$(x_D; y_D)$ $2 = \frac{-1+x_D}{2}$ and $2 = \frac{-4+y_D}{2}$ $4 = -1 + x_D$ $\therefore x_D = 5$ $4 = -4 + y_D$ $\therefore y_D = 8$ $D(5;8)$ Now $m_{AC} = \frac{5-(-4)}{-4-(-1\_} = \frac{9}{-3} = -3$ Equation of line required: $y - 8 = -3(x-5)$ $\therefore y - 8 = -3x + 15$ $\therefore y = -3x + 23$	✓ $x_D = 5$ ✓ $y_D = 8$ ✓ m_{AB} ✓ m_{CD} ✓ correct substitution into formula for equation of line ✓ $y = 2x - 2$ (6)
5(d)	x-intercept: $0 = 2x - 2$ $\therefore 2 = 2x$ $\therefore x = 1$ $(1;0)$ Substitute (1;0) into $x + y = 1$ $LHS = x + y = 1 + 0 = 1 = RHS$ $\therefore (1;0)$ lies on the line AF	✓ $x = 1$ ✓ Substitute (1;0) ✓ LHS=RHS (3)

[18]