SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2021

GRADE 12
GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12

SUBJECT: MATHEMATICS

TEACHER /LEARNER SOLUTIONS

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SESSION 5

	SOLUTIONS	21 MARKS
1.1.1	5 24 55 98 …. 874 19 31 43 12 12 $2a = 12$ $3a + b = 19$ $a + b + c = 5$ $a = 6$ $b = 1$ $c = -2$ $\therefore T_n = 6n^2 + n - 2$	✓a = 2 ✓b = 1 ✓c = −2 ✓$T_n = 2n^2 + n - 2$ (4)
1.1.2	$\sum_{k=1}^{12}(6k^2 + k - 2)$	✓$\sum_{k=1}^{n}$ ✓$6k^2 + k - 2$ (2)
1.2	$\sum_{k=-2}^{5} 5\left(\frac{1}{2}\right)^{1-k} = \frac{5}{8} + \frac{5}{4} + \frac{5}{2} + \cdots$ $a = \frac{5}{8}$ $r = \frac{1}{2}$ $n = 5 - (-2) + 1 = 8$ $S_n = \frac{a(1 - r^n)}{1 - r}$ $S_8 = \frac{\frac{5}{8}\left(1 - \left(\frac{1}{2}\right)^8\right)}{1 - \frac{1}{2}} = \frac{1275}{1024}$ $\sum_{k=-2}^{\infty} 5\left(\frac{2}{3}\right)^{k} = \frac{45}{4} + \frac{15}{2} + 5 + \cdots$ $a = \frac{45}{4}$ $r = \frac{2}{3}$ $S_\infty = \frac{a}{1 - r}$ $S_\infty = \frac{\frac{45}{4}}{1 - \frac{2}{3}} = \frac{135}{4}$ $S_8 + S_\infty = \frac{1275}{1024} + \frac{135}{4}$ ≈ 35	✓$a = \frac{5}{8}$ & $r = \frac{1}{2}$ ✓$\frac{\frac{5}{8}\left(1 - \left(\frac{1}{2}\right)^8\right)}{1 - \frac{1}{2}}$ ✓$\frac{1275}{1024}$ ✓$a = \frac{45}{4}$ & $r = \frac{2}{3}$ ✓$\frac{\frac{45}{4}}{1 - \frac{2}{3}}$ ✓$\frac{135}{4}$ ✓$\frac{1275}{1024} + \frac{135}{4}$ ✓ ≈ 35 ✓ handling expressions differently (9)
		[15]

2.1	$108\,;72$	✓108 ✓72 (2)
2.2	$S_n = \frac{a(1-r^n)}{1-r}$ $\frac{108\left(1-\left(\frac{2}{3}\right)^x\right)}{1-\frac{2}{3}} = \frac{25220}{81}$ $\left(\frac{2}{3}\right)^x = \frac{256}{6561}$ $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^5$ $\therefore x = 5$	✓ $\frac{108\left(1-\left(\frac{2}{3}\right)^x\right)}{1-\frac{2}{3}} = \frac{25220}{81}$ ✓ $\left(\frac{2}{3}\right)^x = \frac{256}{6561}$ ✓ $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^5$ ✓$x = 5$ (4)
		[6]

QUESTION 3

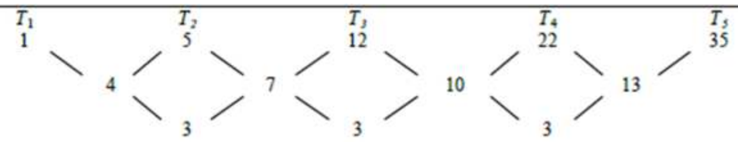
3.1.1	$Second\ constant\ diff = 4$	✓ 4
3.1.2	$2a = 4$ $\quad 3a + b = 5$ $\quad a + b + c = 1$ $a = 2$ $\quad b = -1$ $\quad c = 0$ $\therefore T_n = 2n^2 - n$	✓a = 2 ✓b = −1 ✓c = 0 ✓$T_n = 2n^2 - n$ (4)
3.1.3	$2n^2 - n = 2701$ $2n^2 - n - 2701 = 0$ $(2n + 73)(n - 37) = 0$ $n = 37 \quad OR \quad n \neq -36{,}5$	✓$2n^2 - n = 2701$ ✓standard form ✓factors/quadratic formula ✓$n = 37 \quad OR \quad n \neq -36{,}5$ (4)
3.2.1	$10 + 15 + 20 + 25 + \cdots + 185$ $a = 10 \quad d = 5$ $10 + (n-1)(5) = 185$ $5n = 180$ $\therefore n = 36$	✓$10 + (n-1) \times 5 = 185$ ✓$5n = 180$ ✓$n = 36$ (3)
3.2.2	Natural numbers Divisible by 5 from 10 to 185 $S_n = \frac{n}{2}[a + l]$ $S_{36} = \frac{36}{2}[10 + 185]$ $S_{36} = 3510$ All Natural numbers from 10 to 185	✓ Subt ✓3510

	$n = 185 - 10 + 1$ $n = 176$ $S_{176} = \frac{176}{2}[10 + 185]$ $S_{176} = 17160$ Natural numbers NOT Divisible by 5 from 10 to 185 $S_{140} = 17160 - 3510$ $= 13650$	✓Subt ✓17160 ✓$17160 - 3510$ ✓1350 (6)
QUESTION 4		
4.1.1	$T_n = \frac{1}{2}r^{n-1}$ $\frac{1}{2}r^{5-1} = 40{,}5$ $r^4 = 81$ $\therefore r = 3$	✓$\frac{1}{2}r^{5-1} = 40{,}5$ ✓$r^4 = 81$ ✓$r = 3$ (3)
4.1.2	$\frac{1}{2}(3)^{n-1} = \frac{59049}{2}$ $3^{n-1} = 3^{10}$ $n - 1 = 10$ $\therefore n = 11$	✓$\frac{1}{2}(3)^{n-1} = \frac{59049}{2}$ ✓ $3^{n-1} = 3^{10}$ ✓$n = 11$ (3)
4.2.1	A.S $a = 8$ $d = r$ G.S $a = 8$ $r = d$ $T_5 = ar^4 = 2048$ $ar^4 = 2048$ $8.r^4 = 2048$ $r = 4$ $\Rightarrow d = 4$ $S_5 = 8 + 12 + 16 + 20 + 24$ $= 80$	✓$ar^4 = 2048$ ✓$8.r^4 = 2048$ ✓$r = 4$ ✓summation ✓ 80 (5)
4.2.2	$T_n = 8 + (n - 1)(4)$ $= 4n + 4$ $\sum_{k=1}^{5}(4k + 4)$	✓subt into general formula ✓4k + 4 ✓$\sum_{k=1}^{5}$ (3)
		[14]

QUESTION 5

5.1.4	$T_n = 4n - 7$ **OR** $T_n = -3 + (n-1)(4)$	✓ $4n$ ✓ -7 (2) ✓ -3 ✓ $(n-1)(4)$ (2)
5.1.2	$T_4 = 9$ $T_5 = 13$ $T_6 = 17$ $T_7 = 21$	✓ any TWO consecutive answers correct ✓ last TWO answers correct (2)
5.1.3	0 ; 1 ; 2 ; 0 ; 1 ; 2 ; 0	2 marks for all 7 correct **OR** 1 mark for only **first** / **last** 3 correct **OR** 0 marks if less than 3 correct (2)
5.1.4	Multiples of 3 in the pattern are: $-3 ; 9 ; 21$ $T_n = -3 + 12(n-1)$ $T_n = 12n - 15$ $393 = 12n - 15$ $12n = 408$ $n = 34$ or $T_n = a + (n-1)d$ $393 = -3 + (n-1)(12)$ $393 = 12n - 15$ $12n = 408$ $n = 34$ $S_n = \frac{n}{2}[a + L]$ $S_{34} = \frac{34}{2}[-3 + 393]$ $S_{34} = 6630$ or $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{34} = \frac{34}{2}[2(-3) + 33(12)]$ $S_{34} = 6630$ **NOTE:** • If the candidate does not show the working to get to $n = 34$: no penalty • If a candidate sums the whole sequence: 0/5 marks • Answer only: max 1/5 marks	✓ $12n - 15$ ✓ $393 = 12n - 15$ ✓ $n = 34$ ✓ subs $a = -3$ and $d = 12$ into correct formula ✓ $S_{34} = 6630$ (5)

5.2.1

3.2.1	 $T_5 = 35$	✓✓ answer (2)
5.2.2	**OR** The sequence is 1, 5, 12, 22, 35. Therefore $T_5 = 35$	✓✓ answer (2)
	OR $T_5 = 22 + 13 = 35$	✓✓ answer (2)

3.2.2	$T_{50} = T_1 + \frac{49}{2}[2(4)+48(3)]$ $= 1 + 3724$ $= 3725$ **NOTE:** • Answer only: max 1 mark • If the candidate calculates the general formula in 3.2.1, they can be awarded 5/5 marks in 3.2.2	✓ $a = 4$ ✓ $d = 3$ ✓ $n = 49$ ✓ substitution into correct formula ✓ answer (5)
	OR $2a = 3$ $a = \frac{3}{2}$ $3\left(\frac{3}{2}\right) + b = 4$ $b = -\frac{1}{2}$ $\left(\frac{3}{2}\right) + \left(-\frac{1}{2}\right) + c = 1$ $c = 0$ $T_n = \frac{3}{2}n^2 - \frac{1}{2}n$ $T_{50} = \frac{3}{2}(50)^2 - \frac{1}{2}(50)$ $= 3725$	✓ $a = \frac{3}{2}$ ✓ $b = -\frac{1}{2}$ ✓ $c = 0$ ✓ subs $n = 50$ ✓ answer (5)
	OR $T_1 = 1$ $T_2 - T_1 = 4$ $T_3 - T_2 = 7$ $T_4 - T_3 = 10$ ... $T_{50} - T_{49} = ?$ Add both sides $T_{50} = 1 + 4 + 7 + 10 + \ldots$ to 50 terms $= \frac{50}{2}(2 + 49(3))$ $= 3725$	✓✓ expansion ✓ $T_{50} = 1 + 4 + 7 + 10 + \ldots$ to 50 terms ✓ subs into correct formula ✓ answer (5) **[18]**

QUESTION 6

6.1.1	$w-3;\ 2w-4;\ 23-w$ $(2w-4)-(w-3)=(23-w)-(2w-4)$ $w-1=27-3w$ $4w=28$ $w=7$	✓ $(2w-4)-(w-3)$ $=(23-w)-(2w-4)$ ✓ $w=7$ (2)
6.1.2	Sequence is: 4 ; 10 ; 16 First difference / *Eerste verskil* = 6 **OR** $d=w-1$ $=6$	✓ answer (1) ✓ answer (1)
6.2	$T_{50}=3+(4+10+16+\ldots$ to 49 terms$)$ $T_{50}=3+\frac{49}{2}[2(4)+(49-1)(6)]$ $=3+7252$ $=7255$ **OR** $2a=6$ $a=3$ $3a+b=4$ $3(3)+b=4$ $b=-5$ $a+b+c=3$ $3-5+c=3$ $c=5$ $T_n=3n^2-5n+5$ $T_{50}=3(50)^2-5(50)+5$ $=7255$	✓ $T_{50}=3+$ sum of 49 linear terms ✓ $a=4$ ✓ $n=49$ ✓ 7252(sum of 49 terms) ✓ answer (5) ✓ $a=3$ ✓ $b=-5$ ✓ $c=5$ ✓ substitution 50 ✓ answer (5) **[8]**

QUESTION 7

7.1	$S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ $a = p\left[1-\left(\frac{1}{2}\right)^1\right]$ $= \frac{p}{2}$ $r = \frac{1}{2}$ $\therefore 10 = \frac{\frac{p}{2}}{1-\frac{1}{2}}$ $5 = \frac{p}{2}$ $p = 10$	✓ $a = \frac{p}{2}$ ✓ $r = \frac{1}{2}$ ✓ substitute in correct formula ✓ answer (4)
7.2	$r = \frac{1}{2}$ $\frac{a}{1-\frac{1}{2}} = 10$ $a = 5$ $T_2 = ar = \frac{5}{2}$	✓ $r = \frac{1}{2}$ ✓ substitution ✓ $a = 5$ ✓ answer
	OR $T_2 = S_2 - S_1$ $= p\left(1-\left(\frac{1}{2}\right)^2\right) - p\left(1-\frac{1}{2}\right)$ $= \frac{p}{4}$ $= \frac{10}{4}$ $= \frac{5}{2}$	✓ $T_2 = S_2 - S_1$ ✓ substitution ✓ $\frac{p}{4}$ ✓ answer (4) **[8]**

QUESTION 8

8.1.1	$r=-\frac{32}{64}=-\frac{1}{2}$ $p=256\left(-\frac{1}{2}\right)$ $p=-128$ **OR** $\frac{p}{256}=\frac{64}{p}$ $p^2=16384$ $p=\pm128$ $p=-128$	✓ $-\frac{1}{2}$ ✓ substitution ✓ answer (3) ✓ $\frac{p}{256}=\frac{64}{p}$ ✓ $p=\pm128$ ✓ answer (3)
8.1.2	$S_n=\frac{a\left[1-r^n\right]}{1-r}$ $S_8=\frac{256\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $=\frac{512}{3}\left(\frac{255}{256}\right)$ $=170$	✓ formula ✓ substitution ✓ answer (3)
8.1.3	$-1<r<1$ **OR** The common ratio is $-\frac{1}{2}$ which is between -1 and 1. **OR** $-1<-\frac{1}{2}<1$	✓ answer (1) ✓ answer (1) ✓ answer (1)
8.1.4	$S_\infty=\frac{a}{1-r}$ $=\frac{256}{1-\left(-\frac{1}{2}\right)}$ $=\frac{512}{3}$ $=170{,}67$	✓ formula ✓ substitution ✓ answer (3)

8.2.1	16	✓ answer (1)
8.2.2	$T_n = -8 + 6(n-1)$ $148 = 6n - 14$ $6n = 162$ $n = 27$	✓ substitution into equation ✓ $T_n = 148$ ✓ answer (3)
8.2.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $\frac{n}{2}[2(-8) + (n-1)(6)] > 10\,140$ $3n^2 - 11n > 10\,140$ $3n^2 - 11n - 10\,140 > 0$ $(3n + 169)(n - 60) > 0$ When $n = 60$, $S_n = 10\,140$ Smallest $n = 61$	✓ $\frac{n}{2}[2(-8) + (n-1)(6)]$ ✓ $3n^2 - 11n > 10\,140$ ✓ factors ✓ $n = 60$ ✓ answer (5)
8.3	$\sum_{k=1}^{30}(3k+5)$ $a = 8 \quad n = 30 \quad d = 3$ $\sum_{k=1}^{30}(3k+5) = \frac{30}{2}[2(8) + 29(3)]$ $= 15(103)$ $= 1545$	✓ $n = 30$ ✓ substitution into correct formula ✓ answer (3) **[22]**

QUESTION 9

9.1	Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence is quadratic. **OR** Jacob has multiplied each term by 3 to get the next term. Vusi sees it as a sequence with a constant second difference. **OR** Jacob calculated that the sequence is geometric or exponential. Vusi calculated that the sequence can be seen as a combination of exponential and cubic sequences.	✓Jacob (geometric/exponential) ✓ Vusi (quadratic) (2) ✓ Jacob (multiplied each term by 3) ✓ Vusi (constant second difference) (2) ✓Jacob (geometric/exponential) ✓ Vusi (exponential and cubic combined) (2)

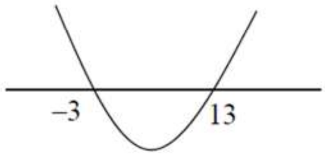
9.2.1	$T_n = 3^n$ **OR** $T_n = 3.3^{n-1}$	✓answer (1) ✓answer (1)
9.2.2	3 9 27 57 6 18 30 12 12 $2a = 12$ $3a + b = 6$ $a + b + c = 3$ $a = 6$ $18 + b = 6$ $6 - 12 + c = 3$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$	✓ $a = 6$ ✓ method ✓ $b = -12$ ✓ $c = 9$ (4)

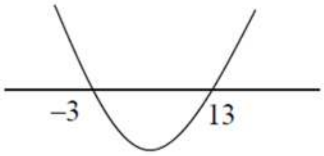
[7]

QUESTION 10

10.1.1	$T_n = ar^{n-1}$ $= 27\left(\frac{1}{3}\right)^{n-1}$ **Note:** The final answer can also be written as 3^{4-n} **or** $\left(\frac{1}{3}\right)^{n-4}$	✓ $a = 27$ and $r = \frac{1}{3}$ ✓ substitute into correct formula (2)
10.1.2	$-1 < r < 1$ or $\lvert r \rvert < 1$ **Note:** If candidate concludes series is not convergent, award 0 marks. **OR** The common ratio (r) is $\frac{1}{3}$ which is between -1 and 1. **OR** $-1 < \frac{1}{3} < 1$	✓ answer (1) ✓ answer (1) ✓ answer (1)
10.1.3	$S_\infty = \frac{a}{1-r}$ $= \frac{27}{1-\frac{1}{3}}$ $= \frac{81}{2}$ or 40,5 or 41 **Note:** If $r > 1$ or $r < -1$ is substituted then 0/2 marks.	✓ substitution ✓ answer (2)

10.2	Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$ $S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$ $=\frac{524287}{524288}V$ $=0,9999980927\ V$ $<V$ Yes, the water will fill the first tank without spilling over. **Note:** If candidate lets the volume of the first tank be a specific value (instead of a variable) and his/her argument follows correctly, award 4/4 marks **Note:** If candidate answers 'Yes' only with no justification: 1/4 marks	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ answer ✓ conclusion (4)
	OR Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$ $S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$ $=V\left[1-\left(\frac{1}{2}\right)^{19}\right]$ $<V\cdot 1$ $=V$ Yes, the water will fill the first tank without spilling over.	✓ $\frac{V}{2}$ ✓ substitute into correct formula ✓ observes that $\left[1-\left(\frac{1}{2}\right)^{19}\right]<1$ ✓ conclusion (4)
	OR If the tanks are emptied one by one, starting from the second, each tank **will fill only half the remaining space**, so the first tank can hold all the water from the other 19 tanks.	✓ Yes (explicit or understood from the argument.) ✓✓✓ argument (4)
10.3.1	$T_n=-2(n-5)^2+18$ Term 1 = – 14 Term 2 = 0 Term 3 = 10	✓ – 14 ✓ 0 ✓ 10 (3)

10.3.2	Term 5 **OR** $n=5$ **OR** T_5	✓ answer (1)
10.3.3	Second difference = $2a$ Second difference = $2(-2)$ Second difference = -4 **OR** -14 0 10 14 10 -4 Second difference = -4	✓ subs -2 into $2a$ ✓ answer (2) ✓ first differences ✓second difference (2)
10.3.4	$-2(n-5)^2+18<-110$ $-2(n-5)^2+128<0$ $-2n^2+20n-50+128<0$ $-2n^2+20n+78<0$ $n^2-10n-39>0$ $(n-13)(n+3)>0$ + 0 − 0 + -3 13 $n<-3$ or $n>13$ $n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$ **Note:** Answer only award 2/6 marks  **OR**	✓ $T_n<-110$ ✓standard form ✓factors ✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$) (6)

	$-2(n-5)^2+18<-110$ $-2(n-5)^2+128<0$ $(n-5)^2-64>0$ $[(n-5)-8][(n-5)+8]>0$ $(n-13)(n+3)>0$ + 0 − 0 + -3 13  $n<-3$ or n > 13 $n\geq 14$; $n\in N$ **OR** $n>13$; $n\in N$	✓ $T_n<-110$ ✓ $(n-5)^2-64>0$ ✓factors ✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR -14 ; 0 ; 10 ; 16 ; 18 ; 16 ; 10 ; 0 ; -14 ; -32 ; -54 ; -80 ; -110 $n\geq 14$; $n\in N$	✓✓✓✓ expansion ✓✓ conclusion of $n\geq 14$ (accept $n>13$) (6) **[21]**

SESSION 6

QUESTION 1

1.1.1	$T_3 = 20$ and $T_4 = 40$ $r = \frac{T_4}{T_3} = 2$	✓ answer (1)
1.1.2	$T_n = ar^{n-1}$ $20 = a.2^{3-1}$ $a = 5$ $T_n = 5.2^{n-1}$ **OR** $40 = a.2^{4-1}$ $a = 5$ $T_n = 5.2^{n-1}$	✓subs into correct formula ✓ $a = 5$ ✓answer (3) ✓subs into correct formula ✓ $a = 5$ ✓answer (3)
1.2.1	$\frac{-7}{125}$	✓ answer (1)
1.2.2	$T_n = \frac{2+(n-1)(-3)}{(1).5^{n-1}}$ $T_n = \frac{5-3n}{5^{n-1}}$	✓ 5 ✓ 5^{n-1} ✓ $-3n$ (3)
1.2.3	$T_n = \frac{5-3n}{5^{n-1}}$ $T_{500} = \frac{5-3(500)}{5^{499}}$ $= \frac{-1495}{5^{499}}$	✓ numerator ✓ denominator (2)
1.2.4	$5-3n < -59$ $-3n < -64$ $n > 21{,}333...$ $n = 22$	✓ $5-3n < -59$ ✓ $n > 21{,}333...$ ✓ $n = 22$ (3) **[13]**

	QUESTION 2	
2.1	Given: Sequence: $3\ ;\ x\ ;\ 25$ 1st diff: $9\ ;\ y$ $x - 3 = 9$ $\therefore x = 12$ $25 - x = y$ $25 - 12 = y$ $13 = y$	✓$x = 12$ ✓$y = 13$ (2)
2.2	Sequence: $3\ ;12\ ;25$ 1st diff: $9\ ;13$ 2nd diff: 4 $2a = 4$ $a = 2$ $3a + b = 9$ $3(2) + b = 9$ $b = 3$ $a + b + c = 3$ $2 + 3 + c = 3$ $c = -2$ $T_n = 2n^2 + 3n - 2$	✓$a = 2$ ✓$b = 3$ ✓$c = -2$ ✓T_n (4)
		[6]
	QUESTION 3	
3.1.1	Sequence: $100\ km; 100(1 + 0{,}1)km\ ;\ \ 100(1 + 0{,}1)^2\ km$ $\therefore 100\ km\ ;\ 110\ km;\ 121\ km$ $T_n = ar^{n-1}$ $T_8 = 100(1{,}1)^7$ $= 194{,}87\ km$	✓$r = 1{,}1$ ✓Subs into correct formula ✓answer (3)
3.1.2	$S_8 = \frac{a(r^n - 1)}{r - 1}$ $= \frac{100(1{,}1^8 - 1)}{1{,}1 - 1}$ $= 1143{,}59\ km$	✓Calculate sum ✓Correct substition ✓answer (3)
3.1.3	$\sum_{n=1}^{8} 100(1{,}1)^{n-1} = 1143{,}596$	✓$T_n = 10(1{,}1)^{n-1}$ ✓correct values (2)
3.2.1	$T_5 = a + 4d = 0$ …………….① $T_{14} = a + 13d = -36$ …………..② ①-②: $-9d = 36$ $d = -4$ $a = -4(-4) = 16$ $T_1 = 16$	✓ $a + 4d = 0$ ✓ $a + 13d = -36$ ✓ $d = -4$ ✓$T_1 = 16$ (4)

3.2.2	$T_{23}+T_{23-p}=-96$ $[16-4(23-1)]+[16-4(23-p-1)]=-96$ $-72+16-88+4p=-96$ $4p=48$ $p=12$	✓T_{23} ✓T_{23-p} ✓simplification ✓$p=12$ (4)
3.3.1	$r=\frac{T_1}{T_2}=\frac{2}{3}$ $-1<\frac{2}{3}<1$ Therefor the series converge	✓$r=\frac{2}{3}$ ✓$\frac{2}{3}<1$ (2)
3.3.2	$\sqrt[3]{16}\times\sqrt[9]{256}\times\sqrt[27]{65536}\times\ldots$ $\sqrt[3]{2^4}\times\sqrt[9]{2^8}\times\sqrt[27]{2^{16}}\times\ldots$ $2^{\frac{4}{3}}\times2^{\frac{8}{9}}\times2^{\frac{16}{27}}\times\ldots$ $2^{\frac{4}{3}+\frac{8}{9}+\frac{16}{27}+\ldots}$ $\therefore\frac{4}{3}+\frac{8}{9}+\frac{16}{27}+\cdots=S_\infty$ $S_\infty=\frac{\frac{4}{3}}{1-\frac{2}{3}}=4$ $\sqrt[3]{16}\times\sqrt[9]{256}\times\sqrt[27]{65536}\times\ldots=2^4=16$ **OR** $\sqrt[3]{16}\times\sqrt[9]{256}\times\sqrt[27]{65536}\times\ldots$ $=16^{\frac{1}{3}}\times16^{\frac{2}{9}}\times16^{\frac{4}{27}}\ldots$ *notice* $a=\frac{1}{3}$ *and* $r=\frac{2}{3}$ $S_\infty=\frac{\frac{1}{3}}{1-\frac{2}{3}}=1$ $\therefore16^1=16$	✓write in exponent form ✓add the exponents ✓Subs into S_∞ ✓$S_\infty=4$ ✓16 (5)
		[23]

OCT/NOV 2017

QUESTION/*VRAAG* 2

2.1.1	5 –4 –19 –40 –9 –15 –21 –6 –6 first differences: –9; –15; –21 second difference = – 6	✓ first differences ✓ – 6 (2)
2.1.2	$T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2} = -3$ $3a + b = -9$ $3(-3) + b = -9$ $b = 0$ $a + b + c = 5$ $-3 + 0 + c = 5$ $c = 8$ $T_n = -3n^2 + 8$ **OR/*OF*** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 5 + (n-1)(-9) + \frac{(n-1)(n-2)(-6)}{2}$ $= 5 - 9n + 9 - 3n^2 + 9n - 6$ $T_n = -3n^2 + 8$	✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ **OR/*OF*** ✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ (4)
2.1.3	$-3n^2 + 8 = -25\ 939$ $-3n^2 = -25947$ $n^2 = 8649$ $n = -93$ *or* $n = 93$ The 93rd term has a value of –25 939	✓ $T_n = -25\ 939$ ✓ $n^2 = 8649$ ✓ answer (3)

2.2.1	$2k-7\,;\,k+8$ and $2k-1$ $k+8-(2k-7)=2k-1-(k+8)$ $-k+15=k-9$ $2k=24$ $k=12$ $2k-7;k+8$ and $2k-1$ $17\,;20\,;23.......$ $d=3$ $T_{15}=17+14(3)$ $=59$	✓ $k+8-(2k-7)=2k-1-(k+8)$ ✓$k=12$ ✓17 ✓ $d=3$ ✓$T_{15}=59$ (5)
2.2.2	Sequence is $17\,;\;20\,;23\,;\;26\,;29\,;32\,.......$ Every alternate term of the sequence will be even / *Elke tweede term van die ry sal ewe wees* $20+26+32+.......$ $S_{30}=\frac{30}{2}[2(20)+(29)(6)]$ $=15[40+174]$ $=3210$ **OR/*OF*** $T_{30}=20+29(6)$ $=94$ $S_{30}=\frac{30}{2}(20+194)$ $=3210$	✓ $20+26+32+.......$ ✓$a=20\;\;d=6$ ✓subst into correct formula ✓ answer (4) ✓ $a=20\;\;d=6$ ✓ $T_{30}=94$ ✓ $S_{30}=\frac{30}{2}(20+194)$ ✓ answer (4) **[18]**

QUESTION/*VRAAG* 3

3.1	$a+ar=2$ $a(1+r)=2$ $a=\frac{2}{1+r}$ **OR/*OF*** $\frac{a}{1-r}-2=\frac{1}{4}$ $4a-8(1-r)=1-r$ $4a-8+8r=1-r$ $4a=9-9r$ $a=\frac{9-9r}{4}$	✓ $a+ar=2$ ✓ $a=\frac{2}{1+r}$ (2) ✓ $\frac{a}{1-r}-2=\frac{1}{4}$ ✓ $a=\frac{9-9r}{4}$ (2)

3.2	$S_\infty=T_1+T_2+\sum_{n=3}^{\infty}T_n$ $S_\infty=2+\frac{1}{4}$ $\frac{a}{1-r}=2+\frac{1}{4}$ $\frac{a}{1-r}=\frac{9}{4}$ $\left(\frac{2}{1+r}\right)\times\left(\frac{1}{1-r}\right)=\frac{9}{4}$ $\frac{2}{1-r^2}=\frac{9}{4}$ $8=9-9r^2$ $9r^2=1$ $r=\frac{1}{3}$ $a=\frac{3}{2}$	✓ $S_\infty=2+\frac{1}{4}$ ✓ $\frac{a}{1-r}=\frac{9}{4}$ ✓ substitution of a into the correct formula ✓ $9r^2=1$ ✓ $r=\frac{1}{3}$ ✓ $a=\frac{3}{2}$ (6)

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QUESTION/*VRAAG* 2

2.1.1	42	✓answer (1)
2.1.2	$2a = 6$ $a = 3$ $3a + b = 1$ $3(3) + b = 1$ $b = -8$ $a + b + c = 2$ $(3) + (-8) + c = 2$ $c = 7$ $T_n = 3n^2 - 8n + 7$	✓ $a = 3$ ✓ $b = -8$ ✓ $c = 7$ ✓ $T_n = an^2 + bn + c$ (4)
	OR/OF $2a = 6$ $a = 3$ $T_n = 3n^2 + bn + c$ $T_1 : 3 + b + c = 2$ $\quad b + c = -1$ ......(1) $T_2 : 12 + 2b + c = 3$ $\quad 2b + c = -9$ ......(2) $T_2 - T_1 : b = -8$ Subst. in (1): $-8 + c = -1$ $c = 7$ $T_n = 3n^2 - 8n + 7$	**OR/OF** ✓ $a = 3$ ✓ $b = -8$ ✓ $c = 7$ ✓ $T_n = an^2 + bn + c$ (4)
2.1.3	$T_{20} = 3(20)^2 - 8(20) + 7$ $= 1047$	✓substitution ✓answer (2)

2.2	$T_n = -7n + 42$ $-7n + 42 = -140$ $-7n = -182$ $n = 26$	✓ $T_n = -7n + 42$ ✓ $-7n + 42 = -140$ ✓ $n = 26$ (3)
2.3	$S_n = \frac{n}{2}(a + l)$ **OR/OF** $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}(35 - 7n + 42)$ $S_n = \frac{n}{2}(70 - 7n + 7)$ $S_n = \frac{n}{2}(-7n + 77)$ $S_n = -\frac{7}{2}n^2 + \frac{77}{2}n$ $-\frac{7}{2}n^2 + \frac{77}{2}n = 3n^2 - 8n + 7$ $13n^2 - 93n + 14 = 0$ $(n-7)(13n-2) = 0$ $n = 7$ or $n = \frac{2}{13}$ NA $\therefore n = 7$	✓ $S_n = \frac{n}{2}(35 - 7n + 42)$ or $S_n = \frac{n}{2}(70 - 7n + 7)$ ✓simplification of S_n ✓equating ✓standard form ✓factors ✓answer with selection (6)
		[16]

QUESTION/*VRAAG* 3

3.1	$r = \frac{1}{2}$ and $S_\infty = 6$ $S_\infty = \frac{a}{1-r}$ $6 = \frac{a}{1-\frac{1}{2}}$ $a = 3$	✓substitution ✓answer (2)
3.2	$T_n = ar^{n-1}$ $T_8 = 3\left(\frac{1}{2}\right)^7$ $T_8 = \frac{3}{128}$	✓✓ $T_8 = 3\left(\frac{1}{2}\right)^7$ (2)

3.3	$\sum_{k=1}^{n} 3(2)^{1-k} = 5,8125$ $3+\frac{3}{2}+\frac{3}{4}+\ldots = 5,8125$ $S_n = \frac{a(1-r^n)}{1-r} = 5,8125$ $\frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = 5,8125$ $6\left[1-\left(\frac{1}{2}\right)^n\right]=5,8125$ $\left(\frac{1}{2}\right)^n = \frac{1}{32} = 0,03125$ $2^{-n} = 2^{-5}$ or $n\log\frac{1}{2} = \log\frac{1}{32}$ $n = 5$ $n = 5$	✓ $r = \frac{1}{2}$ ✓substitution ✓simplification ✓answer (4)
3.4	$\sum_{k=1}^{20} 3(2)^{1-k} = p$ $3+\frac{3}{2}+\frac{3}{4}+\ldots\ldots+3.2^{-19} = p$ $\sum_{k=1}^{20} 24(2)^{-k}$ $=12+6+3+\ldots..+24.2^{-20}$ $=4\left(3+\frac{3}{2}+\frac{3}{4}+\ldots..+3.2^{-19}\right)$ $= 4p$ **OR/OF** $\sum_{k=1}^{20} 3(2)^{1-k} = p$ $\sum_{k=1}^{20} 6(2)^{-k} = p$ $\therefore \sum_{k=1}^{20} 24(2)^{-k} = 4p$	✓expansion ✓ expansion ✓ answer (3) **OR/OF** ✓ $\sum_{k=1}^{20} 6(2)^{-k} = p$ ✓ $\sum_{k=1}^{20} 4\times 6(2)^{-k}$ ✓$4p$ (3)

	OR/OF $S_{20} = \dfrac{3\left(\left(\frac{1}{2}\right)^{20} - 1\right)}{\frac{1}{2} - 1} = 6 = p$ $S_{20} = \dfrac{12\left(\left(\frac{1}{2}\right)^{20} - 1\right)}{\frac{1}{2} - 1} = 24$ $24 = 4 \times 6 = 4p$	**OR/OF** ✓ substitution and answer ✓substitution and answer ✓$4p$ (3)
		[11]

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QUESTION/*VRAAG* 2

2.1.1	209 ; 186	✓209 ✓186 (2)
2.1.2	321 ; 290 ; 261 ; 234 *1st diff* -31 -29 -27 *2nd diff* 2 2 $2a = 2$ $3a + b = -31$ $a + b + c = 321$ $a = 1$ $3(1) + b = -31$ $1 + (-34) + c = 321$ $b = -34$ $c = 354$ $T_n = n^2 - 34n + 354$	✓ 2^{nd} diff = 2 ✓ $a = 1$ ✓ $b = -34$ ✓ $c = 354$ (4)
2.1.3	$n^2 - 34n + 354 = 74$ $n^2 - 34n + 280 = 0$ $(n - 14)(n - 20) = 0$ $n = 14$ or $n = 20$	✓equating T_n to 74 ✓standard form ✓14 ✓ 20 (4)

2.1.4	$f'(n)=0$ $2n-34=0$ $2n=34$ $n=17$ Term 17 will have the smallest value	✓ $2n-34=0$ ✓answer (2)
	OR/OF	**OR/OF**
	$n=\frac{-b}{2a}$ $n=\frac{34}{2}$ $n=17$ Term 17 will have the smallest value	✓substitution ✓answer (2)
	OR/OF	**OR/OF**
	$n=\frac{14+20}{2}=17$ Term 17 will have the smallest value	✓substitution ✓answer (2)

2.2.1	$a=\frac{5}{8}$; $r=\frac{1}{2}$; $n=21$ $S_n=\frac{a(1-r^n)}{1-r}$ $S_{21}=\frac{\frac{5}{8}\left(1-\left(\frac{1}{2}\right)^{21}\right)}{1-\frac{1}{2}}$ $=1,2499...$ $=1,25$	✓ r ✓ substitution into the correct formula ✓ answer (3)

2.2.2	$T_n > \frac{5}{8192}$ $ar^{n-1} > \frac{5}{8192}$ $\frac{5}{8}\left(\frac{1}{2}\right)^{n-1} > \frac{5}{8192}$ $\left(\frac{1}{2}\right)^{n-1} > \frac{1}{1024}$ $\left(\frac{1}{2}\right)^{n-1} > \left(\frac{1}{2}\right)^{10}$ *or* $2^{-n+1} > 2^{-10}$ $\therefore n-1<10$ $\quad -n+1>-10$ $n<11$ $\quad n<11$ $\therefore n=10$ $\quad \therefore n=10$	✓ substitution into the correct formula ✓ method /same base or log ✓calculating n ✓ answer (4)

[19]

QUESTION/*VRAAG* 3

3.1	$\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ $=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)-\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ $=1-\frac{1}{9}$ $=\frac{8}{9}$	✓ $\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)$ ✓ $\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ ✓answer (3)

3.2	$\left(\frac{1}{3}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{2}{3}\right)+\left(1\times\frac{2}{3}\right)+....+\left(4\times\frac{2}{3}\right)$ $=\frac{2}{9}+\frac{4}{9}+\frac{2}{3}+....+\frac{8}{3}$ $a=\frac{2}{9}$ and $d=\frac{2}{3}-\frac{4}{9}=\frac{2}{9}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ OR $S_n=\frac{n}{2}(a+l)$ $S_{12}=\frac{12}{2}\left[2\left(\frac{2}{9}\right)+(12-1)\frac{2}{9}\right]$ $S_{12}=\frac{12}{2}\left(\frac{2}{9}+\frac{8}{3}\right)$ $=\frac{52}{3}\text{m}^2$ $=\frac{52}{3}\text{m}^2$ $\therefore$ for both sides $=2\times\frac{52}{3}=\frac{104}{3}=34{,}67\text{m}^2$	✓✓a ✓d ✓ substitution into the correct formula ✓ answer ✓ answer for both sides
	OR/OF $\frac{2}{9}\times(1+2+3+4+5+6+7+8+9+10+11+12)\times 2$ $=34{,}67\text{ m}^2$ **OR/OF** $T_1=\frac{2}{9}\times 12=\frac{8}{3}$ $l=\frac{2}{9}\times 1=\frac{2}{9}$ $2S_{12}=2\left(\frac{12}{2}\right)\left(\frac{8}{3}+\frac{2}{9}\right)$ $=34{,}67\text{ m}^2$	**OR/OF** ✓✓ a ✓✓ (1 + …. + 12) ✓ ×2 ✓ answer (6) **OR/OF** ✓✓ a ✓ $T_1=\frac{8}{3}$ ✓ $l=\frac{2}{9}$ ✓ substitution into correct formula ✓ answer (6)
		[9]

OCT/NOV 2020

2.1	$7;\ x\ ;\ y\ ;-11$ $x-7=y-x=-11-y$ $x-7=y-x$ $y=2x-7$ Also $y-x=-11-y$ $2y=x-11$ $\Rightarrow 2(2x-7)=x-11$ $4x-14=x-11$ $3x-14=x-11$ $3x=3$ $\therefore x=1$ $\therefore y=-5$	✓ making y subject of a formula ✓ substituting for y ✓$x=1$ ✓$y=-5$ (4)
2.2.1	$-3\ ;6\ ;37\ ;60$ $9\quad 21\quad 33$ $12\quad 12$ $2a=12$ $\quad 3a+b=9$ $\quad a+b+c=-3$ $a=6$ $\quad b=-9$ $\quad c=0$ $\therefore T_n=6n^2-9n$	✓$a=6$ ✓$b=-9$ ✓$c=0$ ✓$6n^2-9n$ (4)
2.2.2	$T_{50}=6(50)^2-9(50)$ $=14\ 550$	✓substituting 50 ✓ 14 550 (2)
2.2.3	$a=9\qquad d=12$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $=\frac{n}{2}[2(9)+(n-1)(1)2]$ $=\frac{n}{2}[12n+6]$ $S_n=6n^2+3n$	✓ formula ✓ subt into the formula ✓$\frac{n}{2}[12n+6]$ (3)
2.2.4	$6n^2+3n=21060$ $6n^2+3n-21060=0$ $(n-60)(2n+117)=0$ $n=60\ \ OR\ \ n\neq-\frac{117}{2}$ $\therefore n=59$ first differences	✓Equating to 21 060 ✓ standard form ✓ factors /quadratic formula ✓$n=59$ (4)
		[17]

QUESTION 3		
3.1	$\sum_{k=1}^{\infty} 4.3^{2-k} = 12 + 4 + \frac{4}{3} + \cdots$ $r = \frac{1}{3}$ $-1 < r < 1$ $\therefore$ it is a convergent series.	✓$12 + 4 + \frac{4}{3} + \cdots$ ✓ constant ratio ✓ condition $-1 < r < 1$ (3)
3.2	$a = 4.3^{2-p}$ $r = \frac{1}{3}$ $S_\infty = \frac{a}{1-r}$ $\frac{4.3^{2-p}}{1-\frac{1}{3}} = \frac{2}{9}$ $3^{2-p} = \frac{1}{27}$ $3^{2-p} = 3^{-3}$ $2 - p = -3$ $p = 5$	✓$a = 4.3^{2-p}$ ✓ substituting & equating to $\frac{2}{9}$ ✓$3^{2-p} = \frac{1}{27}$ ✓$2 - p = -3$ ✓$\text{p} = 5$ (5)
		[8]

SESSION 7

QUESTION 1

1.1	$\hat{B}_1 = 40°$ Tan-chord	✓ $\hat{B}_1 = 40°$ ✓ reason (2)
1.2	$\hat{D}_2 = 40°$ Angles opp equal sides	✓ $\hat{D}_2 = 40°$ ✓ reason (2)
1.3	$\hat{C} = 100°$ Sum of the $\angle$'s of a Δ	✓ $\hat{C} = 100°$ ✓ reason (2)
1.4	$\hat{O}_2 = 200°$ $\angle$ at centre $= 2\times\angle$ at circle	✓ $\hat{O}_2 = 200°$ ✓ reason (2)
1.5	$\hat{O}_1 = 160°$ $\angle$'s round a point	✓ $\hat{O}_1 = 160°$ ✓ reason (2)
1.6	$\hat{D}_3 + \hat{B}_2 + \hat{O}_1 = 180°$ Sum of the $\angle$'s of a Δ $\therefore \hat{D}_3 + \hat{B}_2 + 160° = 180°$ $\therefore \hat{D}_3 + \hat{B}_2 = 20°$ But $\hat{D}_3 = \hat{B}_2$ Angles opp equal radii $\therefore \hat{D}_3 + \hat{D}_3 = 20°$ $\therefore 2\hat{D}_3 = 20°$ $\therefore \hat{D}_3 = 10°$	✓ $\hat{D}_3 + \hat{B}_2 + \hat{O}_1 = 180°$ ✓ $\hat{D}_3 = \hat{B}_2$ ✓ $\hat{D}_3 = 10°$ (3)
1.7	$\hat{A} = 80°$ Opp $\angle$'s cyclic quad or $\angle$ at centre $= 2\times\angle$ at circle	✓ $\hat{A} = 80°$ ✓ reason (2)

QUESTION 2

2.1	$\hat{C}_1 = 20°$ Alt angles equal	✓ $\hat{C}_1 = 20°$ ✓ reason (2)
2.2	$\hat{O}_1 = 40°$ $\angle$ at centre $= 2 \times \angle$ at circle	✓ $\hat{O}_1 = 40°$ ✓ reason (2)
2.3	$\hat{D} = 20°$ $\angle$ at centre $= 2 \times \angle$ at circle or Angles in same segment	✓ $\hat{D} = 20°$ ✓ reason (2)
2.4	$\hat{E}_1 = 40°$ Ext $\angle$ of triangle	✓ $\hat{E}_1 = 40°$ ✓ reason (2)
2.5	$\hat{E}_1 = \hat{O}_1 = 40°$	✓ answer (1)

QUESTION 3

3.1	$\hat{N}_1 + \hat{N}_2 = 90°$ Given $\hat{T}_3 = 90°$ $\angle$ in semi-circle $\therefore \hat{N}_1 + \hat{N}_2 = \hat{T}_3$ $\therefore$ MNPT is a cyclic quad Ext $\angle$ = int opp $\angle$	✓ $\hat{N}_1 + \hat{N}_2 = 90°$ ✓ $\hat{T}_3 = 90°$ ✓ $\hat{N}_1 + \hat{N}_2 = \hat{T}_3$ ✓ reasons (4)
3.2	$\hat{T}_1 = \hat{T}_4$ Vertically opp angles $\hat{T}_4 = \hat{M}_1$ Tan chord $\hat{M}_1 = \hat{P}$ Ext $\angle$ of cyclic quad $\therefore \hat{T}_1 = \hat{P}$ $\therefore$ NP = NT Sides opp equal $\angle$ s	✓ $\hat{T}_1 = \hat{T}_4$ ✓ $\hat{T}_4 = \hat{M}_1$ ✓ $\hat{M}_1 = \hat{P}$ ✓ $\hat{T}_1 = \hat{P}$ ✓ reasons (5)

QUESTION 4

4.1	$\hat{A}_1 + \hat{A}_2 = 90°$ Tan $\perp$ radius $\hat{A}_2 = x$ Tan-chord $\therefore \hat{A}_1 + x = 90°$ $\therefore \hat{A}_1 = 90° - x$ $\hat{A}_1 + \hat{A}_2 + \hat{B}_1 + \hat{E} = 180°$ Sum of the $\angle$'s of a Δ $\therefore 90° + x + \hat{E} = 180°$ $\therefore \hat{E} = 90° - x$ $\therefore \hat{A}_1 = \hat{E}$ $\therefore$ AB is a tangent to circle ADE since $\angle$ between line and chord equals $\angle$ in alt segment.	✓ $\hat{A}_1 + \hat{A}_2 = 90°$ ✓ $\hat{A}_2 = x$ ✓ $\hat{A}_1 = 90° - x$ ✓ $\hat{A}_1 + \hat{A}_2 + \hat{B}_1 + \hat{E} = 180°$ ✓ $\hat{E} = 90° - x$ ✓ $\hat{A}_1 = \hat{E}$ ✓ reasons (7)
4.2	$\hat{C}_1 = \hat{A}_1$ Ext $\angle$ of cyclic quad $\hat{A}_1 = \hat{E} = 90° - x$ Proved $\therefore \hat{C}_1 = \hat{E}$	✓ $\hat{C}_1 = \hat{A}_1$ ✓ $\hat{A}_1 = \hat{E} = 90° - x$ (2)

HOMEWORK SOLUTIONS

QUESTION 1

1.1	$\hat{D}_1 + \hat{D}_2 = 90°$ $\angle$ in a semi-circle But $\hat{D}_2 = 50°$ given $\hat{D}_1 = 40°$	✓ $\hat{D}_1 + \hat{D}_2 = 90°$ ✓ $\hat{D}_1 = 40°$ (2)
1.2	$\hat{M}_1 = 2\hat{D}_1$ $\angle$ at centre $= 2 \times \angle$ at circumference $\hat{M}_1 = 2(40°)$ $\therefore \hat{M}_1 = 80°$	✓ $\hat{M}_1 = 2\hat{D}_1$ ✓ $\hat{M}_1 = 80°$ (2)
1.3	$\hat{E}_2 = 50°$ $\angle$'s in same segment $\hat{F}_2 = 50°$ $\angle$'s opp equal sides ($ME = FE$, equal radii)	✓ $\hat{M}_1 = 2\hat{D}_1$ ✓ $\hat{F}_2 = 50°$ (2)
1.4	$\hat{G} = \hat{F}_1 + \hat{F}_2$ $\angle$'s in same segment $\therefore \hat{G} = 10° + 50°$ $\therefore \hat{G} = 60°$	✓ $\hat{G} = \hat{F}_1 + \hat{F}_2$ ✓ $\hat{G} = 60°$ (2)
1.5	$\hat{D}_1 + \hat{D}_2 + \hat{G} + \hat{E}_1 = 180°$ sum of the $\angle$'s of a triangle $\therefore 40° + 50° + 60° + \hat{E}_1 = 180°$ $\therefore \hat{E}_1 = 30°$	✓ $\hat{D}_1 + \hat{D}_2 + \hat{G} + \hat{E}_1 = 180°$ ✓ $\hat{E}_1 = 30°$ (2)

QUESTION 2

2.1	$\hat{P}_2 = \hat{P}_1$ given But $\hat{P}_1 = 22°$ given $\therefore \hat{P}_2 = 22°$	✓ $\hat{P}_2 = \hat{P}_1$ ✓ $\hat{P}_2 = 22°$ (2)
2.2	$\hat{R}_2 = 22°$ tan-chord	✓ $\hat{R}_2 = 22°$

		✓ reason (2)
2.3	$\hat{P}_2 + \hat{P}_3 + \hat{P}_4 = 90°$ $\angle$ in a semi-circle But $\hat{P}_2 = 22°$ $\therefore \hat{P}_3 + \hat{P}_4 = 90° - 22° = 68°$	✓ $\hat{P}_2 + \hat{P}_3 + \hat{P}_4 = 90°$ ✓ $\hat{P}_3 = 68°$ ✓ reason (3)
2.4	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ tan-chord But $\hat{P}_1 + \hat{P}_2 = 44°$ given $\therefore \hat{R}_1 + \hat{R}_2 = 44°$ But $\hat{R}_2 = 22°$ tan-chord $\therefore \hat{R}_1 + 22° = 44°$ $\therefore \hat{R}_1 = 22°$	✓ $\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ ✓ $\hat{R}_1 + \hat{R}_2 = 44°$ ✓ $\hat{R}_1 = 22°$ ✓ reasons (4)
2.5	$\hat{R}_1 = 22°$ proved $\therefore \hat{T}_1 = 22°$ equal radii, $\angle$'s opp equal sides $\therefore \hat{O}_1 = 44°$ ext $\angle$ of triangle	✓ $\hat{T}_1 = 22°$ ✓ $\hat{O}_1 = 44°$ ✓ reasons (3)
2.6	$\hat{R}_2 + \hat{P} + \hat{Q}_2 = 180°$ Sum of the $\angle$'s of a triangle $\therefore 22° + (90° + 22°) + \hat{Q}_2 = 180°$ $\therefore \hat{Q}_2 = 46°$	✓ $\hat{R}_2 + \hat{P} + \hat{Q}_2 = 180°$ ✓ $\hat{Q}_2 = 46°$ ✓ reason (3)

QUESTION 3

3.1	$\hat{L}_3 = \hat{M}_1$	alt $\angle$'s equal	✓ $\hat{L}_3 = \hat{M}_1$
	$\hat{L}_3 = \hat{P}_1$	tan-chord	✓ $\hat{L}_3 = \hat{P}_1$
	$\therefore \hat{M}_1 = \hat{P}_1$		✓ LM = LP
	$\therefore$ LM = LP	sides opp equal $\angle$'s	✓ reasons (4)
3.2	$\hat{N}_1 = \hat{P}_1$	ML subtends equal $\angle$'s	✓ $\hat{N}_1 = \hat{P}_1$
	$\hat{P}_1 = \hat{M}_1$	proved	✓ $\hat{M}_1 = \hat{N}_2$
	$\hat{M}_1 = \hat{N}_2$	PL subtends equal $\angle$'s	✓ $\hat{M}_1 = \hat{N}_2$
	$\therefore \hat{N}_1 = \hat{N}_2$		✓ reasons
3.3	$\hat{M}_1 = \hat{P}_1$	proved	✓ $\hat{M}_1 = \hat{P}_1$
	$\hat{N}_1 = \hat{P}_1$	ML subtends equal $\angle$'s	✓ $\hat{N}_1 = \hat{P}_1$
	$\therefore \hat{M}_1 = \hat{N}_1$		✓ $\hat{M}_1 = \hat{N}_1$
	$\therefore$ LM is a tangent to circle MNQ.	$\angle$ between line and chord	✓ reasons

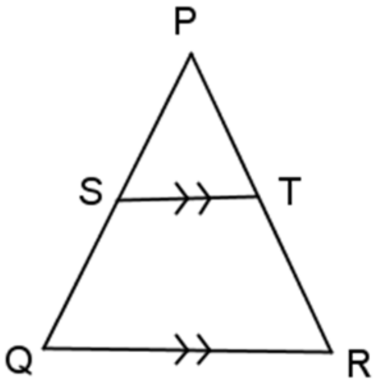
QUESTION 4

4.1	$\hat{D}_3 = 90°$	$\angle$ in semi-circle	✓ $\hat{D}_3 = 90°$
	$\hat{B}_1 + \hat{B}_2 = 90°$	given	✓ $\hat{B}_1 + \hat{B}_2 = 90°$
	$\therefore \hat{D}_3 = \hat{B}_1 + \hat{B}_2$		✓ $\hat{D}_3 = \hat{B}_1 + \hat{B}_2$
	$\therefore$ ABCD is a cyclic quad	ext $\angle$ of quad equals int opp $\angle$	✓ reasons
4.2	$\hat{A}_1 = \hat{D}_1$	BC subtends equal angles	✓ $\hat{A}_1 = \hat{D}_1$
	$\hat{D}_1 = \hat{E}$	tan-chord	✓ $\hat{D}_1 = \hat{E}$
	$\therefore \hat{A}_1 = \hat{E}$		✓ reasons (3)

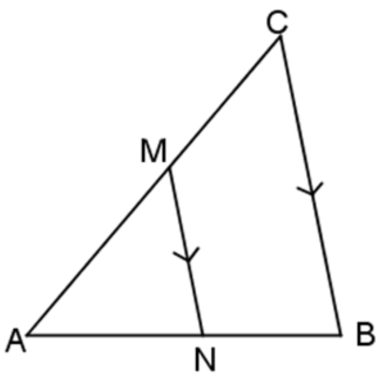
4.3	$\hat{A}_1 + \hat{A}_2 = \hat{C}_3$	ext $\angle$ = int opp $\angle$	✓ $\hat{A}_1 + \hat{A}_2 = \hat{C}_3$
	$\hat{C}_3 = \hat{D}_4$	tan-chord	✓ $\hat{C}_3 = \hat{D}_4$
	$\hat{D}_4 = \hat{D}_2$	vertically opp $\angle$'s	✓ $\hat{D}_4 = \hat{D}_2$
	$\therefore \hat{A}_1 + \hat{A}_2 = \hat{D}_2$		✓ $\hat{A}_1 + \hat{A}_2 = \hat{D}_2$
	$\therefore$ BD = BA	sides opp equal $\angle$'s	✓ reasons (5)
4.4	$\hat{C}_2 = \hat{D}_2$	AB subtends equal $\angle$'s	✓ $\hat{C}_2 = \hat{D}_2$
	$\hat{D}_2 = \hat{D}_4$	vertically opp $\angle$'s	✓ $\hat{D}_2 = \hat{D}_4$
	$\hat{D}_4 = \hat{C}_3$	tan-chord	✓ $\hat{D}_4 = \hat{C}_3$
	$\therefore \hat{C}_2 = \hat{C}_3$		✓ reasons (4)

SESSION 8

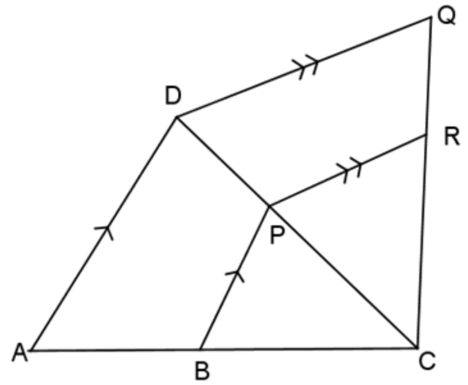
QUESTION 1

		
	$\frac{TR}{PT} = \frac{SQ}{PS}$ line ‖ to one side △ $\frac{TR}{10} = \frac{2}{5}$ $TR = 4cm$	

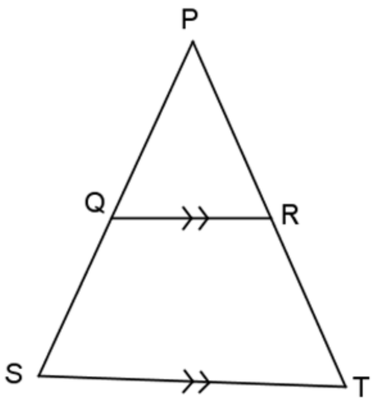
QUESTION 2

		
	$\frac{CM}{AC} = \frac{NB}{AB}$ line ‖ to one side △ $\frac{CM}{35} = \frac{18}{42}$ $CM = 15cm$	

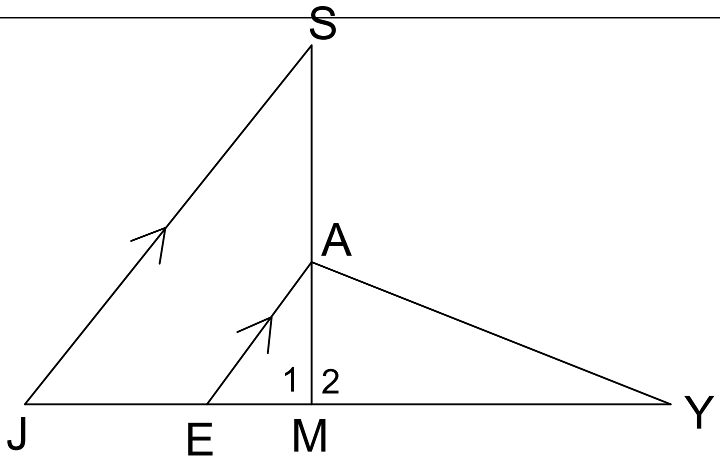
QUESTION 3

		
	$\frac{AB}{BC}=\frac{DP}{PC}$ line ∥ to one side △ $\frac{DP}{PC}=\frac{22}{33}$ $\frac{DP}{PC}=\frac{QR}{RC}$ line ∥ to one side △ $\frac{2}{3}=\frac{QR}{15}$ $QR=10cm$	

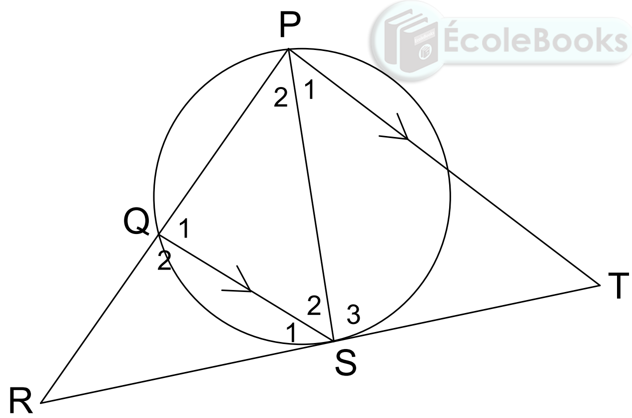
QUESTION 4

		
4.1	$PQ=\frac{5}{8}\times 32mm=20mm$	
4.2	$QS=\frac{3}{8}\times 32mm=12mm$	
4.3	$\frac{PR}{RT}=\frac{5}{3}$ $PR=\frac{5}{8}\times 24mm=15mm$	
4.4	$RT=\frac{3}{8}\times 24mm=9mm$	

QUESTION 5

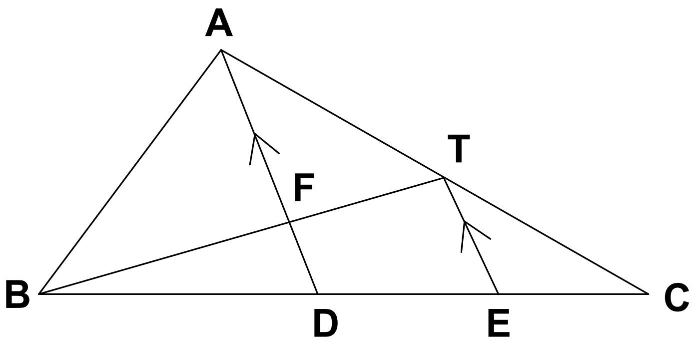
		
	$\frac{EM}{JE} = \frac{AM}{AS}$ line ∥ to one side △ $\frac{EM}{9} = \frac{6}{12}$ $EM = 4{,}5cm$	

QUESTION 6

		
6.1	$\Delta SPQ \mid\mid\mid \Delta PTS$ In ΔSPQ and ΔPTS $\hat{S}_2 = \hat{P}_1$ alt angles $QS \parallel PT$ $\hat{Q}_1 = \hat{S}_3$ tan chord theorem $\hat{P}_2 = \hat{T}$ sum of angles of △ $\therefore \Delta SPQ \mid\mid\mid \Delta PTS$ A,A,A	
6.2	$\frac{\boldsymbol{SP}}{\boldsymbol{PT}} = \frac{\boldsymbol{PQ}}{\boldsymbol{TS}} = \frac{\boldsymbol{SQ}}{\boldsymbol{PS}}$ ∣∣∣ △ s $\frac{SP}{PT} = \frac{SQ}{PS}$ $SP^2 = PT.SQ$	

6.3	In $\triangle RPT$ $\frac{RQ}{RP} = \frac{RS}{RT}$ line ∥ to one side △ $RQ.RT = RS.RP$	

QUESTION 7

		
7.1	$\frac{TC}{TA} = \frac{CE}{ED}$ line ∥ to one side △ $\frac{CE}{ED} = \frac{1}{2}$	
7.2	$DE = \frac{2}{3} \times 9cm = 6cm$ $BD = DE = 6cm$ $\therefore D$ is the midpoint	
7.3	$\frac{FD}{TE} = \frac{BD}{BE}$ line ∥ to one side △ $\frac{2}{TE} = \frac{6}{12}$ $\therefore TE = 4cm$	
7.4.1	$\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ $= \frac{\frac{1}{2} \times DC \times Height}{\frac{1}{2} \times BD \times Height}$ $= \frac{9}{6}$ $= \frac{3}{2}$	
7.4.2	$\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ $= \frac{\frac{1}{2} \times TC \times CE \times \sin\hat{C}}{\frac{1}{2} \times AC \times BC \times \sin\hat{C}}$ $= \frac{1}{3} \times \frac{1}{5}$ $= \frac{1}{15}$	

SESSION 9

QUESTION 1

1.1	$\hat{C} = \theta$ $D\hat{A}C = \theta$ $\therefore A\hat{D}C = 180^0 - 2\theta$	
1.2	$A\hat{D}C = 180^0 - 2\theta$ $a^2 = b^2 + b^2 - 2.b.b\cos 2\theta$ $a^2 = 2b^2 + 2b^{2.}\cos 2\theta$ $a^2 = 2b^2(1 + \cos 2\theta)$ $1 + \cos 2\theta = \frac{a^2}{2b^2}$ $\cos 2\theta = \frac{a^2}{2b^2} - 1$	
1.3	$\cos 2\theta = \frac{(3)^2}{2(2)^2} - 1$ $\cos 2\theta = \frac{1}{8}$ $2\theta = 82{,}819^0$ $\theta = 41{,}41^0$	

QUESTION 2

2.1	$\frac{\sin 2x}{CB} = \frac{\sin(90^0 - x)}{k}$ $\frac{2\sin x\cos x}{CB} = \frac{\cos x}{k}$ $CB = \frac{2k\sin x\cos x}{\cos x}$ $CB = 2k\sin x$	
2.2	$\cos x = \frac{CB}{HC}$ $\cos x = \frac{2k\sin x}{HC}$ $HC = \frac{2k\sin x}{\cos x}$ $HC = 2k\tan x$	

2.3	$k^2 = HC^2 + HD^2 - HC.HDcos\theta$ $40^2 = 33{,}9579853^2 + 31{,}8^2 - 2(31{,}8)(33{,}957)cos\theta$ $0{,}2613221669 \ldots \ldots = cos\theta$ $\theta = 74{,}85°$	

QUESTION 3

	$\hat{A} + \hat{B} + \hat{C} = 180^0$ $\sin A = \sin[180^0 - (B + C)]$ $\sin A = \sin(B + C)$ $\sin A = 0{,}8$ $\frac{\sin B}{AC} = \frac{\sin A}{BC}$ $\frac{AC}{\sin 30^0} = \frac{10}{0{,}8}$ $AC = 10.\frac{1}{2}.\frac{10}{8}$ $AC = 6\frac{1}{4} units$	

QUESTION 4

4.1	$AC^2 = 10^2 + 6^2 - 2(10)(6)\cos 100^0$ $AC^2 = 156{,}83777813$ $AC = 12{,}5 units$ $\hat{B} = 80^0$ Opp. <'s of a cyclic quad $\frac{\sin 40^0}{BC} = \frac{\sin B}{AC}$ In ΔABC: $\frac{\sin 40^0}{BC} = \frac{\sin 80^0}{12.5}$ $BC = \frac{12{,}5 \times \sin 40^0}{\sin 80^0}$ $BC = 8{,}2 units$	

4.2	Area of $\Delta ABC = \frac{1}{2}.AC.\sin 60^0$ $= \frac{1}{2}(12,5)(82)\sin 60^0$ $= 44.4 units^2$	

QUESTION 5

5.1	In $\Delta\, ABD$: $\tan x = \frac{p}{DB}$ $P = DB \tan x$	
5.2	$\frac{\sin[180^0 - (y+\theta)]}{k} = \frac{\sin\theta}{DB}$ $\frac{\sin(y+\theta)}{k} = \frac{\sin\theta}{\frac{p}{\tan x}}$ $\frac{\sin(y+\theta)}{k} = \frac{\sin\theta \tan x}{p}$ $p = \frac{k\sin\theta\tan x}{\sin(y+\theta)}$ $p = \frac{k\sin\theta\tan x}{\sin y.\cos\theta + \cos y.\sin\theta}$	
5.3	$\tan x = \frac{p}{DB}$ $\tan 51,7^0 = \frac{80}{DB}$ $DB = \frac{80}{\tan 51,7^0}$ $DB = 63,18m$ $(BC)^2 = (DB)^2 + (k)^2 - 2(DB)(k)\cos y$ $(BC)^2 = (63.,18)^2 + (95)^2 - 2(63,18)(95)\cos 62,5^0$ $BC^2 = 7473,789697.....$ $BC = 86,45$ $BC \approx 86m$	

QUESTION 6

6.1	$\tan x = \frac{h}{BD}$ $BD = \frac{h}{\tan x}$	
6.2	$CD^2 = \left(\frac{h}{\tan x}\right)\left(\frac{h}{\tan x}\right) - 2\left(\frac{h}{\tan x}\right)\left(\frac{h}{\tan x}\right)\cos y$ c $CD^2 = \frac{2h^2}{\tan^2 x} - 2\left(\frac{h^2}{\tan^2 x}\right)\cos y$ $CD^2 = \frac{2h^2(1-\cos y)}{\tan^2 x}$	

QUESTION 7

7.1	$\cos\theta = \frac{DC}{p}$ $DC = p\cos\theta$ $\frac{\sin(90^9 - \theta)}{BD} = \frac{\sin 2\theta}{DC}$ $\frac{\cos\theta}{BD} = \frac{2\sin\theta\cos\theta}{DC}$ $\frac{\cos\theta}{BD} = \frac{2\sin\theta\cos\theta}{DC}$ $BD = \frac{p\cos^2\theta}{2\sin\theta\cos\theta}$ $BD = \frac{p\cos\theta}{2\sin\theta}$	
7.2	$\sin 30^0 = \frac{AC}{p}$ $AC = p\sin 30^0$ $AC = 3.\sin 30^0$ $AC = 3\left(\frac{1}{2}\right)$ $AC = \frac{3}{2}m$	

7.3	$BD=\frac{3\cos 30^0}{2\sin 30^0}$ $BD=\frac{3\left(\frac{\sqrt{3}}{2}\right)}{2\left(\frac{1}{2}\right)}$ $BD=\frac{3\sqrt{3}}{2}$ $AB^2=AD^2+BD^2-2AD.BD\cos A\hat{D}B$ $AB^2=(3)^2+\left(\frac{3\sqrt{3}}{2}\right)^2-2(3)\left(\frac{3\sqrt{3}}{2}\right)\cos 70^0$ $AB^2=9+\frac{27}{4}-9\sqrt{3}\cos 70^0$ $AB^2=10,418...$ $AB=3,23m$	

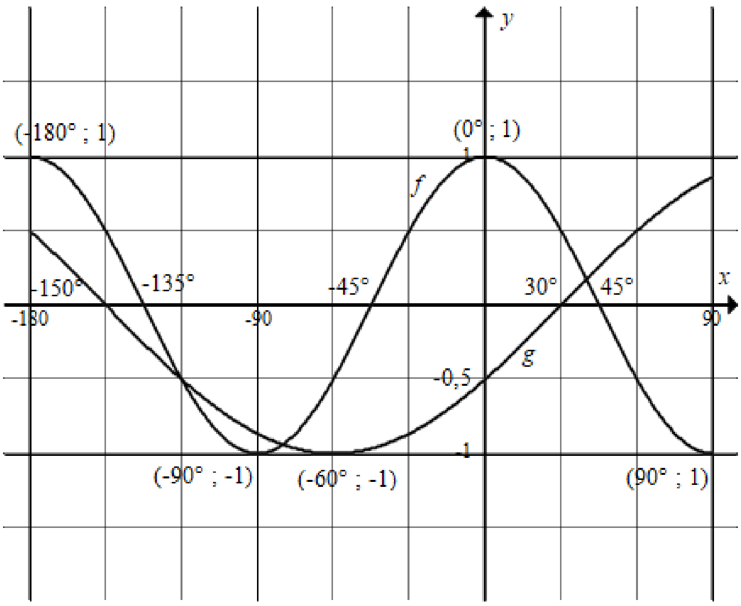
QUESTION 8

8.1	$(QP)^2=(PQ)^2+(RP)^2-2(PQ)(RP)\cos\hat{P}$ $\left(\sqrt{3}x\right)^2=x^2+x^2-2..x..x.\cos\hat{P}$ $3x^2=2x^2-2x^2\cos\hat{P}$ $2x^2\cos\hat{P}=-x^2$ $\cos\hat{p}=\frac{-x^2}{2x^2}$ $\cos\hat{P}=\frac{-1}{2}$ $\hat{P}=120^0$	

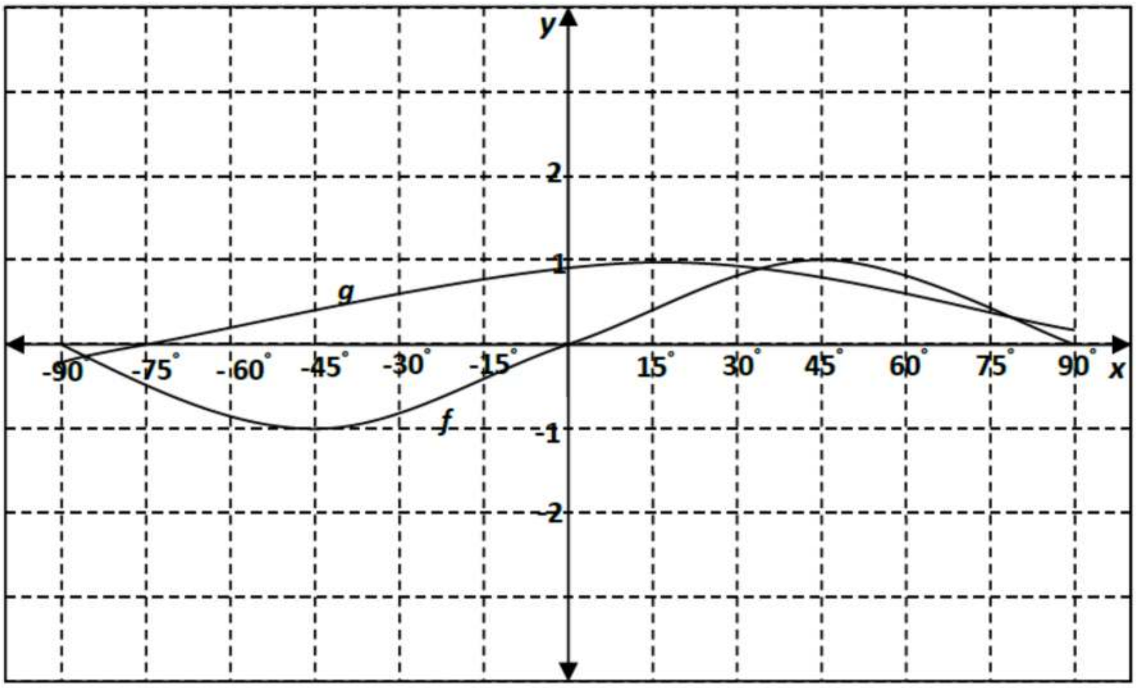
8.2	$P\hat{R}Q = P\hat{Q}R = 30^0$ $\hat{R} = \hat{Q}$ $S\hat{R}Q = 150^0$ Area of $\Delta QRS = \frac{1}{2}.QR.RS\sin Q\hat{R}S$ $= \frac{1}{2}\left(\sqrt{3}x\right)\left(\frac{3}{2}x\right)\sin 150^0$ $= \left(\left(\frac{3\sqrt{3}}{4}x^2\right)\right)\left(\frac{1}{2}\right)$ $= 3\frac{\sqrt{3}}{8}x^2$ $= 0{,}65x^2$	

SESSION 10

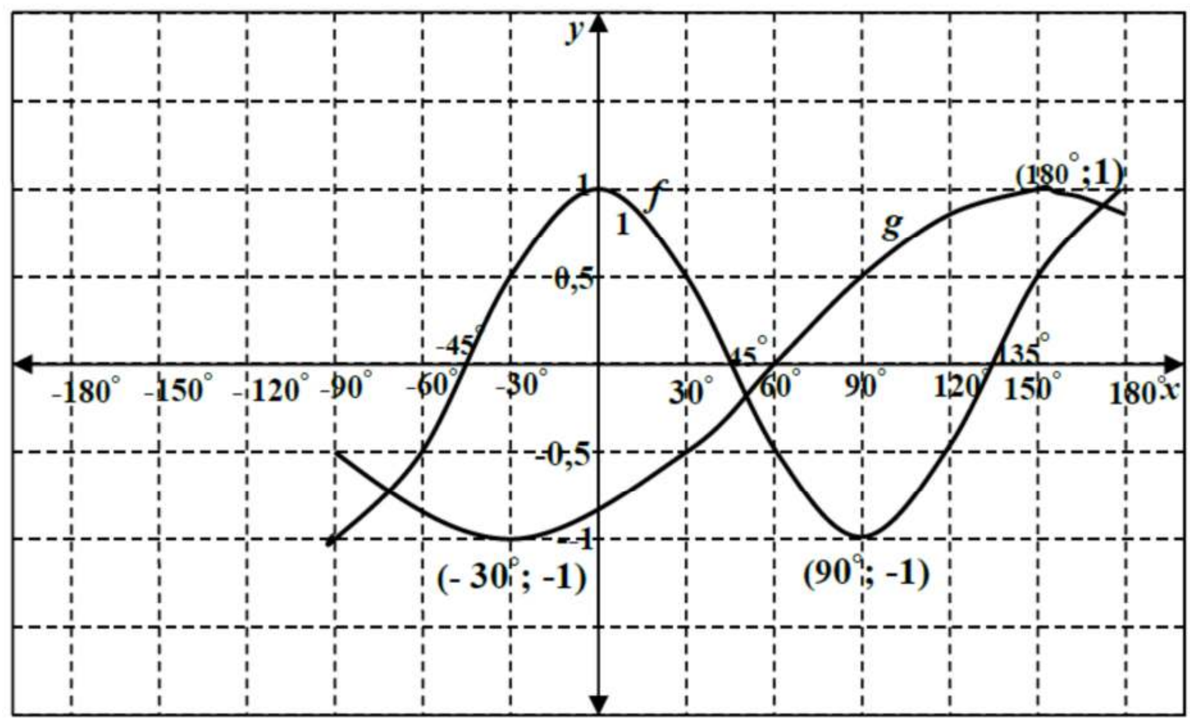
QUESTION 1

1.1	$\cos 2x = \sin(x - 30°)$ $= \cos[90° - (x - 30°)]$ $= \cos(120° - x)$ key angle $= 120° - x$ $2x = 120° - x + n.360°;\ n \in Z$ $3x = 120° + n.360°; n \in Z$ $x = 40° + n.120°; n \in Z$ or $2x = 360° - (120° - x) + n.360°;\ n \in Z$ $2x = 240° + x + n.360°;\ n \in Z$ $\qquad x = 120° + n.180°;\ n \in Z$	
1.2		
(a)	$-120° < x < -80°$ or $40° < x \le 90°$	

QUESTION 2

2.1	Period of $f = 180^0$	
2.2		
2.3	$\sin 2x = \cos(x - 15^0)$ $\cos(90^0 - 2x) = \cos(x - 15^0)$ $90^0 - 2x = x - 15^0 + k.360^0$ or $90^0 - 2x = -(x - 15^0) + k.360^0$ $-3x = 105^0 + k.360^0$ $-x = 75^0 + k.360^0 ; k \in Z$ $x = 35^0 + k.120^0$ $x = 75^0 + k.360^0$ $x = -85^0 ; 35^0 ; 75^0$ $-85^0 < x < 35^0$ or $75^0 < x < 90^0$	

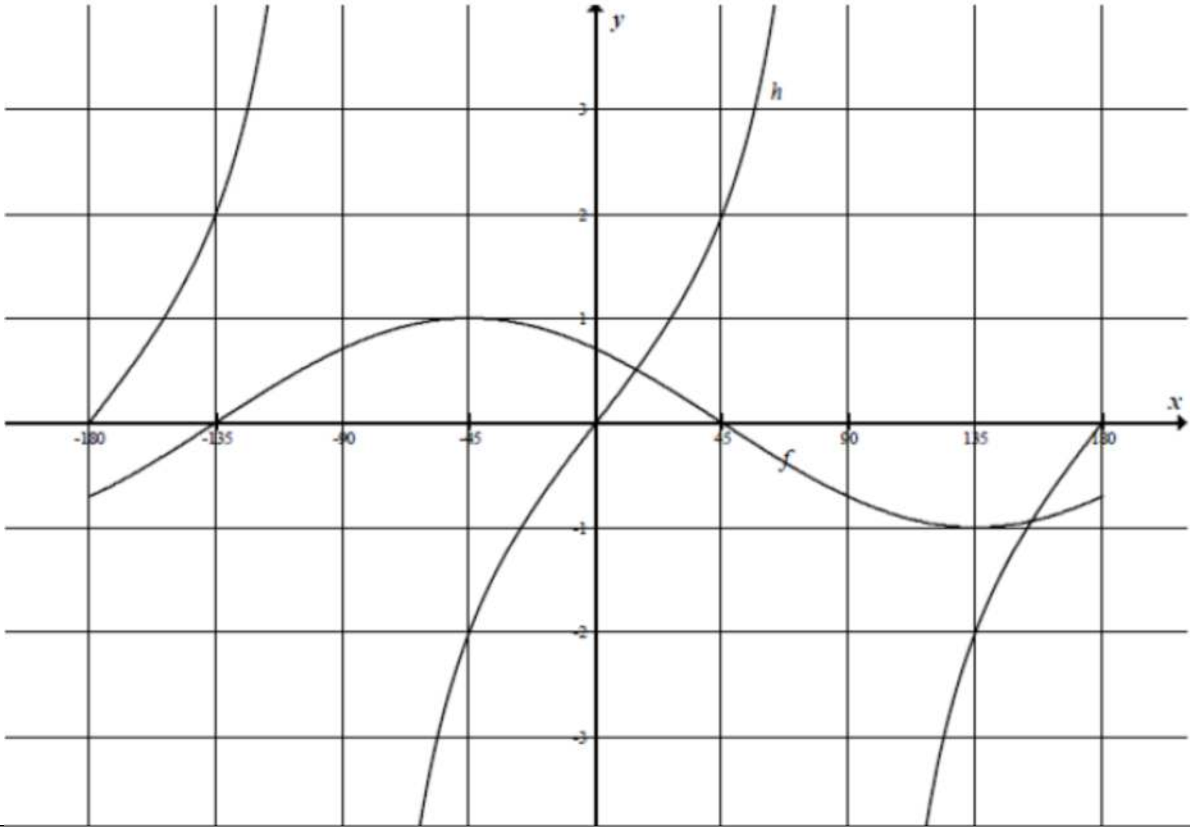
QUESTION 3

3.1		
3.2	$(60^0;180^0]$ or $60^0 < x \leq 180^0$	

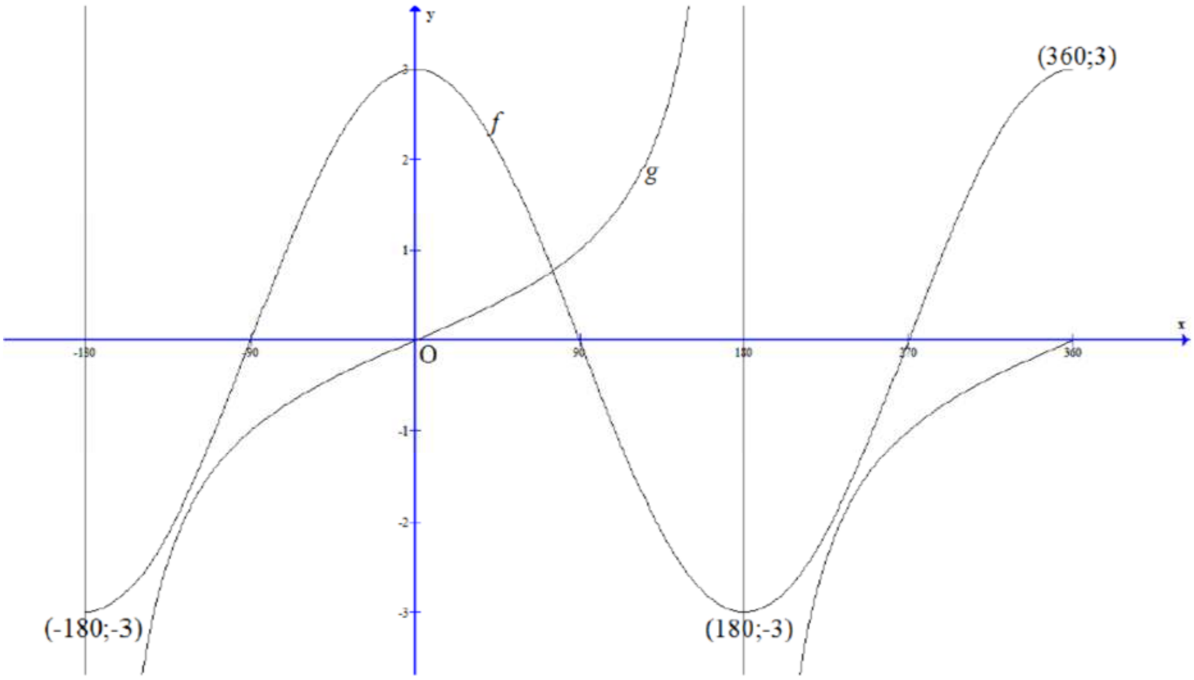
QUESTION 4

4.1	$\sin(x+30^0)=\cos 3x$ $\sin(x+30^0)=\sin(90^0-3x)$ $x+30^0=90^0-3x+k.360^0$ $4x=60^0+k.360^0$ $x=15^0+k.90^o$	
4.2.1		
4.2.2	120°	
4.2.3	$15° \leq x \leq 105°$	

QUESTION 5

5.1	$a\tan 45^0 = 2$ $a = 2$	
5.2		
5.3	2 solutions.	

QUESTION 6

6.1		
6.2.1	360^0	
6.2.2	$(0;3)$ and $(180^0;-3)$ $[180^0;-3]$ and $(360^0;3)$	
6.2.3	$-180^0 < x < 0^0 \cup 180^0 < x < 360^0$	
6.2.4	$y = 3\cos(x - 45^0)$	

QUESTION 7

7.1.1	$x = 90^0$	
7.1.2	$x \in [90^0;180^0]$	
7.1.3	$x \in [0^0;90^0] \cup x = 180^0$	
7.2.1	$g(x) = 2\sin x$ $c = 2$ d=1	
7.2.2	$h(x) = 2\cos(x - 90^0)$ $a = 2$ $b = 90°$	

SESSION 11

QUESTION 10

	Using sin(A+B) + sin(A – B) = 2.sinA.cosB With A = 32° and B = 28° $\sin 60° + \sin(4°) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4° = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$ **OR** Using $\sin 4° = 2\sin 2°.\cos 2°$ and $\sin 2° = \sin(30° - 28°) = \frac{1}{2}\left(\sqrt{1-a^2} - \sqrt{3}a\right)$ and $\sin 2° = \sin(32° - 30°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-b^2} - b\right)$ and $\cos 2° = \cos(30° - 28°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-a^2} + a\right)$ and $\cos 2° = \cos(32° - 30°) = \frac{1}{2}\left(\sqrt{3}b + \sqrt{1-b^2}\right)$ then $\sin 4° = \frac{1}{2}\left\{\sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2}\right\}$ **OR** $\sin 4° = \frac{1}{2}\left\{3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab\right\}$	
10.2	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32°.\sqrt{1-\sin^2 28°} - \sin 28°\sqrt{1-\cos^2 32°}$ $= \cos 32°.\cos 28° - \sin 28°.\sin 32°$ $= \cos(32° + 28°)$ $= \cos 60°$ $= \frac{1}{2}$	✓ substitution ✓ cos 28° ✓ sin 32° ✓ compound angle formula (4)
10.3.1	$\frac{\sin 130°.\tan 60°}{\cos 540°.\tan 230°.\sin 400°}$ $= \frac{\sin 50° \times \tan 60°}{\cos 180° \times \tan 50° \times \sin 40°}$ $= \frac{\sin 50° \times \sqrt{3}}{-1 \times \frac{\sin 50°}{\cos 50°} \times \cos 50°}$ $= -\frac{\sqrt{3}\cos 50°}{\cos 50°}$ $= -\sqrt{3}$	✓ sin 50° ✓ tan 50° ✓sin40° ✓cos50° ✓ $\frac{\sin 50°}{\cos 50°}$ ✓ –1 ✓answer (7)

10.3.2	$(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=1-2\sin^2 75°$ $=\cos 150°$ $=\frac{-\sqrt{3}}{2}$ **OR** $\sin 75°$ $=\sin(45°+30°)$ $=\sin 45°.\cos 30°+\cos 45°.\sin 30°$ $=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}.\frac{1}{2}$ $\sqrt{2}\sin 75°=\frac{\sqrt{3}}{2}+\frac{1}{2}=a$ $(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=(1-a)(1+a)$ $=1-a^2$ $=1-\left(\frac{3}{4}+\frac{1}{4}+2.\frac{\sqrt{3}}{2}.\frac{1}{2}\right)$ $=-\frac{\sqrt{3}}{2}$	✓simplification ✓$1-2\sin^2 75°$ ✓$\cos 150°$ ✓answer (4) ✓simplification ✓$1-2\sin^2 75°$ ✓$\cos 150°$ ✓answer (4)
10.4	$\sin^2 x+\cos 2x-\cos x=0$ $\sin^2 x+(\cos^2 x-\sin^2 x)-\cos x=0$ $\cos^2 x-\cos x=0$ $\cos x(\cos x-1)=0$ $\cos x=0$ *or* $\cos x=1$ $x=\pm 90°+k.360°$ *or* $x=0°+k.360°$ $k\in Z$ $=k.360°$ (i.e. $x=90°+k.180°$ or $x=k.360°\pm 90°, k\in Z$)	✓$(\cos^2 x-\sin^2 x)$ ✓$\cos^2 x-\cos x=0$ ✓factors ✓$\cos x=0$ *or* $\cos x=1$ ✓$90°+k.360°$ ✓$k.360°$ ✓ $x=-90°+k.360°$ (7)
10.5.1	$x=0°;90°;180°$	✓✓✓ each value (3)

10.5.2	$\frac{\cos 2x.\tan x}{\sin^2 x}=\frac{(\cos^2 x-\sin^2 x).\frac{\sin x}{\cos x}}{\sin^2 x}$ $=\frac{\cos^2 x-\sin^2 x}{\cos x.\sin x}$ $=\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$ $=\frac{\cos x}{\sin x}-\tan x$	✓ $(\cos^2 x-\sin^2 x)$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer ✓ $\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$ ✓answer (5) **[39]**

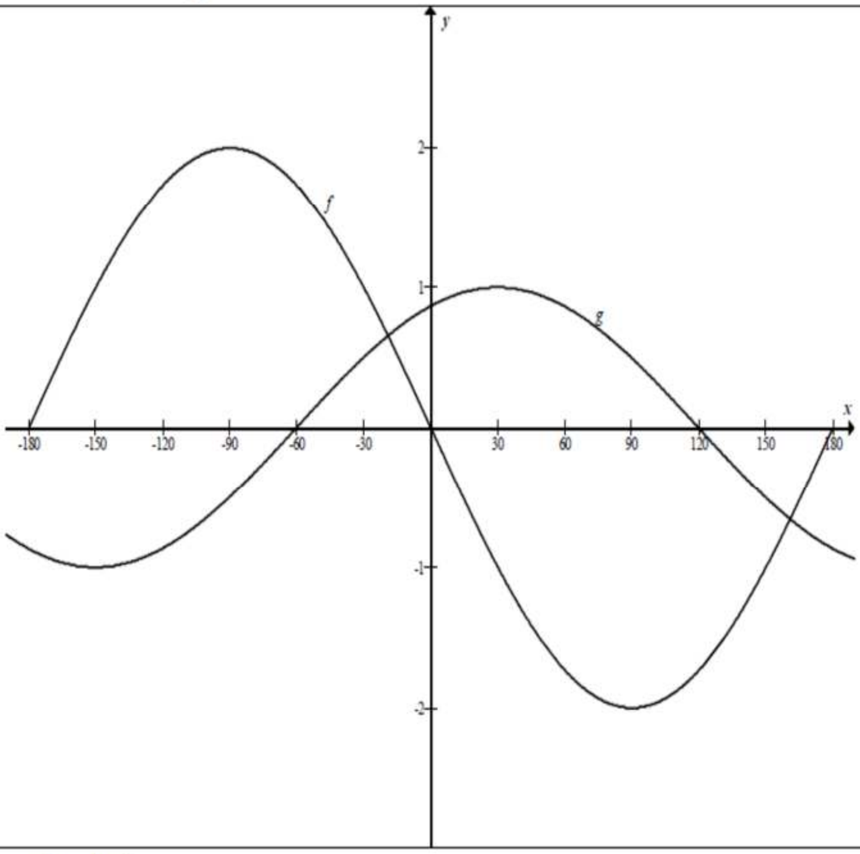
QUESTION 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$ $= (7,5)^2 + (9,4)^2 - 2.(7,5)(9,4)\cos 32°$ $= 25,03521844...$ $EC = 5,0$ metres	✓substitution into cosine rule ✓25,03521844... ✓answer (3)
11.2	$\frac{\sin D\hat{C}E}{7,5} = \frac{\sin 32°}{5,0}$ $\sin D\hat{C}E = \frac{7,5.\sin 32°}{5,0}$ $= 0,7948788963$ $D\hat{C}E = 52,6°$	✓ sin rule ✓0,7948788963 ✓answer (3)
11.3	Area of ΔDEC $= \frac{1}{2}DE.DC\sin\hat{D}$ $= \frac{1}{2}(7,5)(9,4)\sin 32°$ $= 18,7m^2$ OR Area of ΔDEC $= \frac{1}{2}CE.DC\sin 52,6°$ $= \frac{1}{2}(5,0)(9,4)\sin 52,6°$ $= 18,7m^2$	✓ substitution ✓answer (2)

11.4	$\sin 32° = \frac{EG}{7,5}$ $EG = 7,5.\sin 32°$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5$ metres **OR** $EG = EC.\sin 52,6°$ $= (5,0).\sin 52,6°$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ **OR**	✓ratio ✓substitution ✓answer (3) **[11]**

	$\frac{1}{2}.DC.EG = \text{area } \Delta DEC$ $\frac{1}{2}(9,4)EG = 18,7$ $\therefore EG = \frac{18,7 \times 2}{9,4}$	

QUESTION 12

12.1	Period = 360°	✓answer (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓answer (2)
12.3		✓shape ✓x intercepts ✓ amplitude (3)
12.4	2 solutions	✓answer (1)
12.5	$-60° \leq x \leq 120°$ or $x \in [-60°;120°]$	✓ $-60°;120°$ ✓ notation (2)
12.6	$-90° < x < 30°$ or $x \in (-90°;30°)$	✓✓ $-90°;30°$ ✓ notation (3) **[12]**

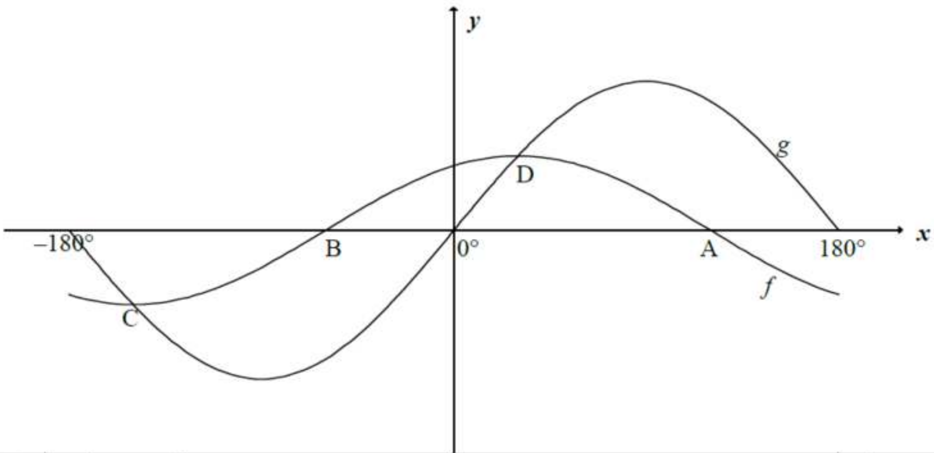
JUNE 2019

QUESTION/*VRAAG* 5

5.1.1	$\sin 191°$ $= -\sin 11°$	✓ $-\sin 11°$ (1)
5.1.2	$\cos 22°$ $= \cos(2 \times 11°)$ $= 1 - 2\sin^2 11°$	✓ answer (1)
5.2	$\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ $= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$ $= -\cos x + \sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x\left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ **OR** $\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ $= -\cos x + \sqrt{2}(\sin x \cos 45° + \cos x \sin 45°)$ $= -\cos x + \sqrt{2}\left(\sin x\left(\frac{\sqrt{2}}{2}\right) + \cos x\left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓simplification of last 2 terms ✓answer (5) ✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓simplification of last 2 terms ✓answer (5)
5.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$ $\sin^2 P + 2\sin P \cos P + \cos^2 P = \frac{49}{25}$ $2\sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$ $= \frac{24}{25}$	✓ $\sin Q = \cos P$ ✓ squaring ✓ expansion ✓ $\sin^2 P + \cos^2 P = 1$ ✓answer (5)
		[12]

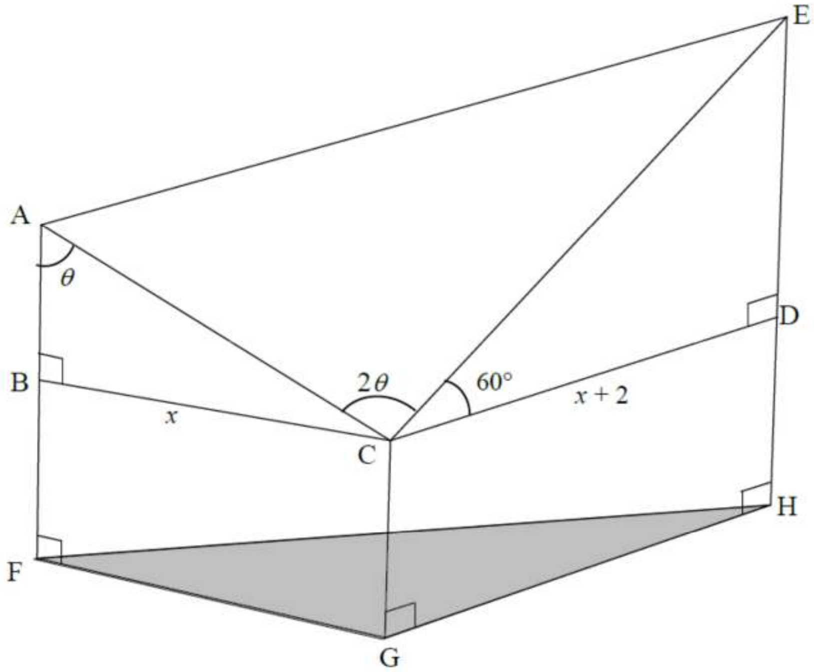
QUESTION/*VRAAG* 6

6.1	$\cos(x-30°) = 2\sin x$ $\cos x\cos 30° + \sin x \sin 30° = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x = \frac{3}{2}\sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30° + k.180°;\quad k \in Z$ **OR** $x = 30° + k.360°$ or $x = 210° + k.360°;\quad k \in Z$	✓ expansion ✓ special ∠s ✓ simplification ✓ equation in tan ✓ 30° ✓ $k.180°; k \in Z$ **OR** ✓ 30° and 210° ✓ $k.360°;\quad k \in Z$ (6)



6.2.1(a)	$A(120°\ ;\ 0)$	✓ answer (1)
6.2.1(b)	$C(-150°\ ;\ -1)$	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90°\ ;\ 30°)$ OR $-90° < x < 30°$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160°\ ;20°)$ OR $-160° < x < 20°$	✓endpoints ✓correct interval (2)
6.2.3	$y = 2^{2\sin x+3}$ Range of $y = 2\sin x$: $y \in [-2\ ;\ 2]$ **OR** $-2 \le y \le 2$ Range of $y = 2\sin x + 3$: $y \in [1\ ;\ 5]$ **OR** $1 \le y \le 5$ Range: $y = 2^{2\sin x+3}$: $y \in [2\ ;\ 32]$ **OR** $2 \le y \le 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
		[18]

QUESTION/*VRAAG* 7

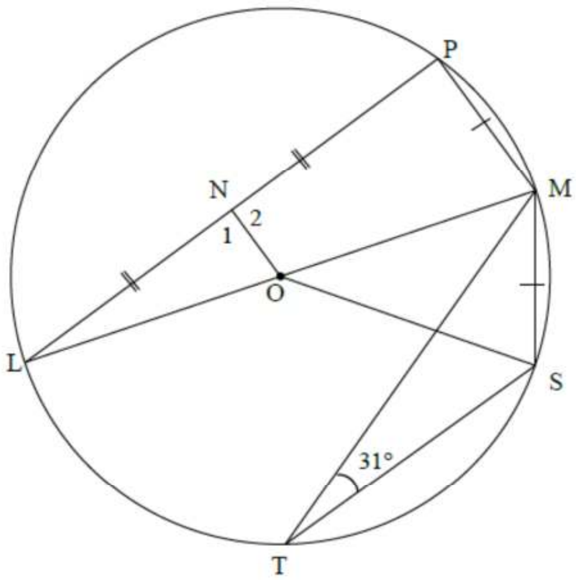


7.1.1	$\sin\theta = \frac{x}{AC}$ **OR** $\frac{\sin\theta}{x} = \frac{\sin 90°}{AC}$ $AC = \frac{x}{\sin\theta}$ $AC = \frac{x}{\sin\theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60° = \frac{x+2}{CE}$ **OR** $\frac{\sin 30}{x+2} = \frac{\sin 90°}{CE}$ $CE = \frac{x+2}{\cos 60°}$ $CE = \frac{x+2}{\sin 30°}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$ $= 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	$\text{Area } \Delta ACE = \frac{1}{2} AC.EC.\sin A\hat{C}E$ $= \frac{1}{2}\left(\frac{x}{\sin\theta}\right)(2(x+2))\sin 2\theta$ $= \frac{x(x+2)\times 2\sin\theta\cos\theta}{\sin\theta}$ $= 2x(x+2)\cos\theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin\theta}(2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

7.3	$EC = 2(12+2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos A\hat{C}E$ $= \left(\frac{12}{\sin 55°}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55°}\right)(28)\cos 110°$ $AE = 35{,}77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]

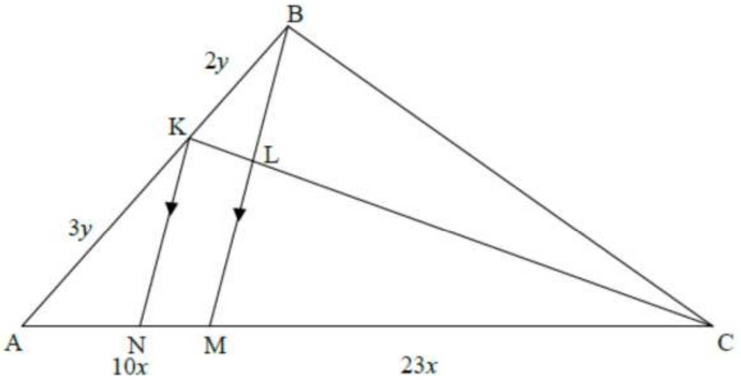
QUESTION/*VRAAG* 8

8.1



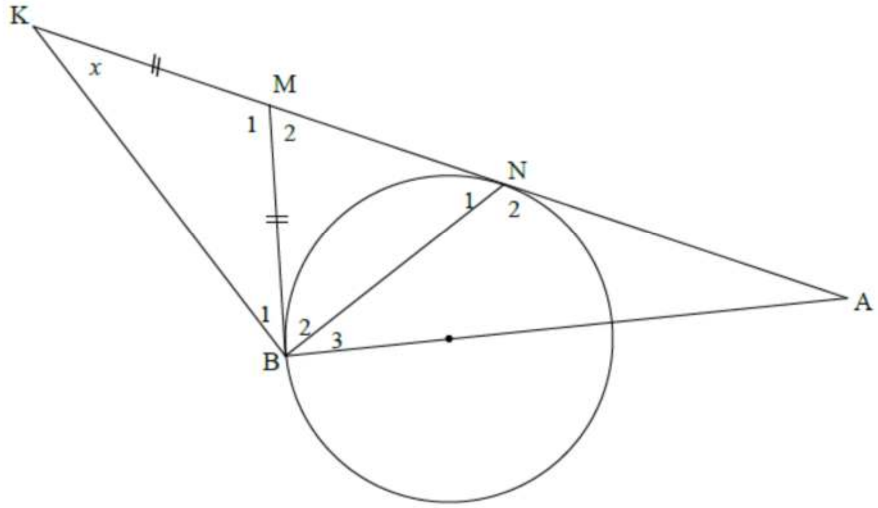
8.1.1(a)	$M\hat{O}S = 62°$ [∠ at centre = 2× ∠ at circumf/*middelpts∠ =2omtreks∠*]	✓ S ✓ R (2)
8.1.1(b)	$\hat{L} = 31°$ [equal chords; equal ∠s /= *koorde; = ∠e*]	✓ S ✓ R (2)
8.1.2	LN = NP and LO = OM $\therefore ON = \frac{1}{2} PM$ [midpoint theorem/*middelpuntstelling*] $\therefore ON = \frac{1}{2} MS$ [PM = MS] **OR** $\hat{N}_1 = 90°$ [line from centre to midpt chord/*lyn v midpt na midpt kd*] $\hat{P} = 90°$ [∠ in semi-circle/*∠ in halfsirkel*] $\hat{L}$ is common/*gemeen* $\therefore \Delta NLO \parallel\mid \Delta PLM$ (∠∠∠) $\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}$ $\therefore ON = \frac{1}{2} PM$ $\therefore ON = \frac{1}{2} MS$ [PM = MS]	✓ LO = OM ✓ S ✓ R ✓ S (4) ✓ S R ✓ S/R ✓ S ✓ S (4)

8.2



8.2.1	$\frac{AN}{AM}=\frac{AK}{AB}$ [line ‖ one side of Δ **OR** prop theorem; KN ‖BM/ *lyn ‖ sy van Δ **OR** eweredigheidst; KN‖BM*] $\frac{AN}{AM}=\frac{3y}{5y}=\frac{3}{5}$	✓ R ✓ S (2)
8.2.2	$\frac{AM}{MC}=\frac{10x}{23x}$ [given] $AM = 5y = 10x \quad \therefore \quad y = 2x$ $\frac{LC}{KL}=\frac{MC}{NM}$ [line ‖ one side of Δ **OR** prop theorem; KN ‖LM/ *lyn ‖ sy van Δ **OR** eweredigheidst; KN‖BM*] $=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$ **OR** $\frac{AM}{MC}=\frac{10x}{23x}$ [given] $\frac{AN}{MN}=\frac{3y}{2y}=\frac{6x}{4x}$ $\frac{LC}{KL}=\frac{MC}{NM}$ [line ‖ one side of Δ **OR** prop theorem; KN ‖LM/ *lyn ‖ sy van Δ **OR** eweredigheidst; KN‖BM*] $=\frac{23x}{2y}=\frac{23x}{4x}=\frac{23}{4}$	✓ S ✓ R ✓ S (3) ✓ S ✓ R ✓ S (3)
		[13]

QUESTION/*VRAAG* 9

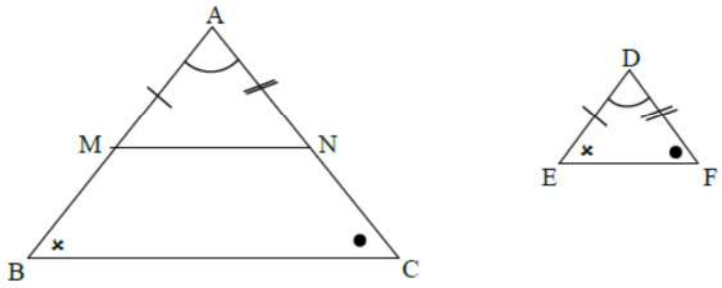


9.1	$\hat{B}_1 = x$ [∠'s opp = sides/*∠e teenoor = sye*] $\hat{M}_2 = 2x$ [ext ∠ of Δ] **OR** $\hat{M}_1 = 180° - 2x$ [∠s of Δ] BM = MN [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{N}_1 = \frac{180° - 2x}{2} = 90° - x$ [∠'s opp = sides/*∠e teenoor = sye*] **OR** NM = BM [2 tans from a common point/*raaklyne vanuit dieselfde punt*] $\hat{B}_2 = \hat{N}_1$ [∠'s opp = sides/*∠e teenoor = sye*] $\hat{B}_1 = x$ [∠'s opp = sides/*∠e teenoor = sye*] In Δ KBN: $x + x + \hat{B}_2 + \hat{N}_1 = 180°$ [sum of ∠'s of Δ] $2x + 2\hat{N}_1 = 180°$ $x + \hat{N}_1 = 90°$ $\hat{N}_1 = 90° - x$	✓S ✓S ✓R ✓S ✓R ✓answer (6) ✓S ✓R ✓S ✓R ✓S ✓answer (6)
9.2	$M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent⊥diameter/*raaklyn⊥middellyn*] $\hat{B}_3 = 90° - \hat{B}_2$ $= 90° - (90° - x) = x$ $\hat{B}_3 = \hat{K} = x$ ∴ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ S ✓ R (5)

	OR $\hat{B}_2 = \hat{N}_1$ $\hat{B}_1 + \hat{B}_2 = x + (90° - x) = 90°$ $\therefore$ KN is diameter/*middellyn* [converse $\angle$ in semi-circle/ *omgekeerde $\angle$ in halfsirkel*] $M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90°$ [tangent $\perp$ diameter] $\therefore$ AB is a tangent/*raaklyn* converse tan-chord theorem/ *omgekeerde raakl koordst*]]	✓S ✓ R ✓ S ✓ R ✓ R (5)
		[11]

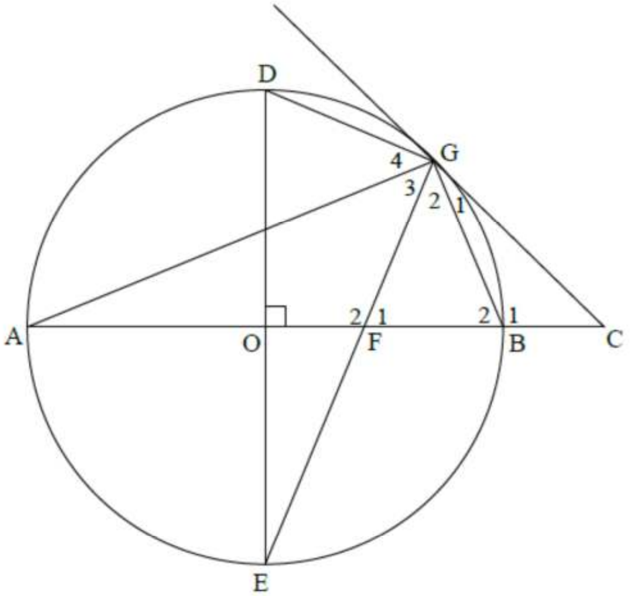
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. *Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.* Proof: In Δ AMN and Δ DEF AM = DE [Constr] AN = DF [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \Delta AMN \equiv \Delta DEF$ (SAS) $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ∥ BC [corresp ∠'s are equal/*ooreenkomstige ∠e* =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ∥ one side of Δ **OR** prop theorem; MN ∥BC] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]	✓Constr / *Konstr* ✓ $\Delta AMN \equiv \Delta DEF$ ✓SAS ✓MN ∥ BC and R ✓ $\frac{AB}{AM} = \frac{AC}{AN}$ ✓R (6)

10.2

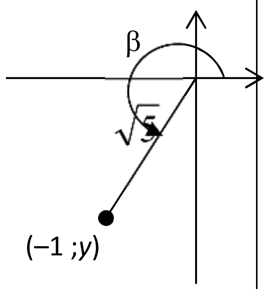


10.2.1(a)	$D\hat{O}B = 90°$ $D\hat{G}F = \hat{G}_3 + \hat{G}_4 = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $D\hat{O}B + D\hat{G}F = 180°$ ∴ DGFO is a cyclic quad. [converse: opp ∠s of cyclic quad/ *omgekeerde teenoorst ∠e v koordevh*] OR ∠s of quad = 180°/*∠e van koordevh = 180°*] **OR** $E\hat{O}B = 90°$ $D\hat{G}F = \hat{G}_3 + \hat{G}_4 = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $E\hat{O}B = D\hat{G}F$ ∴ DGFO is a cyclic quad. . [converse: ext ∠ = opp int ∠/ *omgekeerde buite∠ = teenoorst ∠*] OR ext∠ of quad = opp int ∠/*buite∠ v vh = teenoorst ∠*]	✓S ✓R ✓R (3) ✓S ✓R ✓R (3)
10.2.1(b)	$\hat{F}_1 = \hat{D}$ [ext ∠of cyclic quad/buite∠ v koordevh] $\hat{G}_1 + \hat{G}_2 = \hat{D}$ [tan-chord theorem/raakl koordst] $\therefore \hat{F}_1 = \hat{G}_1 + \hat{G}_2$ ∴ GC = CF [sides opp equal ∠s/sye teenoor = ∠e]	✓S ✓R ✓S ✓R ✓R (5)

10.2.2(a)	AB = DE = 14 [diameters/*middellyne*] $\therefore$ OB = 7 units $\therefore$ BC = OC – OB = 11 – 7 = 4 units Answer only: full marks	✓ S ✓ S ✓ S (3)
10.2.2(b)	In ΔCGB and ΔCAG $\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/*raakl koordst*] $\hat{C} = \hat{C}$ [common] $\Delta CGB \parallel\!\mid \Delta CAG$ [$\angle, \angle, \angle$] $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ *or* 8,49 units	✓ S/R ✓ S ✓ S ✓ CA = 18 ✓answer (5)
10.2.2(c)	OF = OC – FC $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11-\sqrt{72}}{7} = 0{,}36$ $\hat{E} = 19{,}76°$ **OR** OF = OC – FC $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11-\sqrt{72})^2$ FE = 7,437.. = 7,44 $\cos E = \frac{OE}{FE}$ *OR* $\sin E = \frac{OF}{FE}$ $= \frac{7}{7{,}44} = 0{,}94$ $= \frac{11-\sqrt{72}}{7{,}44} = 0{,}338$ $\hat{E} = 19{,}76°$ $\hat{E} = 19{,}76°$	✓ OF ✓ trig ratio ✓ substitution ✓ answer (4) ✓ OF ✓ trig ratio ✓ substitution ✓ answer (4)
		[26]

MAY /JUNE 2015

QUESTION/*VRAAG* 5

5.1	cos $\beta = -\frac{1}{\sqrt{5}}$ and/*en* 180° < β < 360°  $(-1)^2 + y^2 = (\sqrt{5})^2$ $1 + y^2 = 5$ $y^2 = 4$ $y = -2$ $\therefore \sin\beta = -\frac{2}{\sqrt{5}}$	<u>sketch/*skets*:</u> ✓ correct quad/ *korrekte kwadr* ✓ $x = -1$ ✓subst into Pyth/ *subst in Pyth* ✓ value of/*waarde van y* ✓ value of/*waarde van* sin β (5)
5.2	$\frac{(-\tan x).(-\sin(90° - x))}{4\sin x}$ $\frac{(-\tan x).(-\cos x)}{4\sin x}$ $= \frac{(-\frac{\sin x}{\cos x}).(-\cos x)}{4\sin x}$ $= \frac{1}{4}$	✓– tan *x* ✓ –sin(90° – *x*) ✓– cos*x* ✓ sin *x* ✓ $\frac{\sin x}{\cos x}$ ✓ answer/*antw*
5.3.1	tan A = $\frac{\sin A}{\cos A} = \frac{p}{q}$	✓ answer/*antw* (1)

5.3.2	$p^4 - q^4 = (p^2 + q^2)(p^2 - q^2)$ $= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$ $= (1)(\sin^2 A - \cos^2 A)$ $= -1(\cos^2 A - \sin^2 A)$ $= -\cos 2A$	✓ factors/*faktore* ✓identity/*identiteit* ✓ –1 as CF/*GF* ✓ answer/*antw* (4)
5.4.1	LHS/*LK* $= \dfrac{\cos^2\theta - \cos 2\theta}{\sin\theta.\cos\theta}$ $= \dfrac{\cos^2\theta - (2\cos^2\theta - 1)}{\sin\theta.\cos\theta}$ $= \dfrac{1 - \cos^2\theta}{\sin\theta.\cos\theta}$ $= \dfrac{\sin^2\theta}{\sin\theta.\cos\theta}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta =$ RHS/*RK*	✓ writing as single term/*skryf as enkelterm* ✓ expansion/ *uitbreiding* ✓ simplify/*vereenv* ✓ identity/*identiteit* ✓ simplify/*vereenv* (5)
5.4.2	Undefined when/*Ongedefinieerd as:* cos θ = 0, sin θ = 0 $\therefore$ θ = 90°	✓✓ answer/*antw* (2)
5.5	2(2sin *x*. cos *x*) + 3 sin *x* = 0 4sin *x*. cos *x* + 3 sin *x* = 0 sin *x* (4cos *x* + 3) = 0 sin *x* = 0 *x* = 0° + *k*.360° or 180° + *k*.360° **OR/*OF*** *x* = *k*.180° ; *k*∈Z	✓ expansion/ *uitbreiding* ✓ factorise/ *faktoriseer* ✓ both equations/ *beide vgls* ✓ *x* = *k*.180° **OR/*OF*** *x* = 0° + *k*.360°

	or/*of* cos $x = -\frac{3}{4}$ x = 138,59° + k.360° or/*of* 221,41° + k.360° ; $k \in Z$ **OR/*OF*** x = ±138,59° + k.360° ; $k \in Z$	or 180° + k.360° ✓138,59°; 221,41° **OR/*OF*** ±138,59° ✓k.360°, $k \in Z$

QUESTION/*VRAAG* 6

6.1	Period of/*Periode van* f = 120°	✓ 120°
6.2	$b = 3$	✓b = 3
6.3	x = –45° or/*of* x = –22,5° or/*of* x = 67,5°	✓x = –45° ✓x = –22,5° ✓x = 67,5°
6.4	$x \in$ (–45° ; –22,5°) $\cup$ (67,5° ; 90°] **OR/*OF*** $-45° < x < -22,5°$ or/*of* $67,5° < x \leq 90°$	✓ critical values ✓ notation ✓ critical values ✓ notation OR ✓*kritieke waardes* ✓*notasie* ✓*kritieke waardes* ✓*notasie* (4) **[9]**

QUESTION/*VRAAG* 7

7.1	$QR^2 = PQ^2 + RP^2 - 2.PQ.RP.\cos\hat{P}$ $(\sqrt{3}x)^2 = x^2 + x^2 - 2.x.x.\cos\hat{P}$ $\cos\hat{P} = \dfrac{x^2 + x^2 - (\sqrt{3}x)^2}{2x.x}$ $\cos\hat{P} = \dfrac{-x^2}{2x^2}$ $\cos\hat{P} = -\dfrac{1}{2}$ $\hat{P} = 120°$	✓ correct subst into cosine rule/*korrek subst in cos-reël* ✓ $\cos\hat{P}$ as subj/ *onderw* ✓simplify/*vereenv* ✓ answer/*antw* (4)
7.2	$P\hat{R}Q = P\hat{Q}R = 30°$ (∠s opp equal sides/∠*e teenoor gelyke sye*) $Q\hat{R}S = 150°$ (∠s on a str line/∠*e op reguitlyn*) Area of/*Opp van* Δ QRS = $\frac{1}{2}(QR)(RS)(\sin Q\hat{R}S)$ $= \frac{1}{2}(\sqrt{3}x)(\frac{3}{2}x)(\sin 150°)$ $= (\frac{3\sqrt{3}}{4}x^2)(\frac{1}{2})$ $= \frac{3\sqrt{3}}{8}x^2$	✓S ✓ S ✓ correct subst into area rule/*korrek subst in opp-reël* ✓simplify/*vereenv* ✓ answer/*antw* (5) **[9]**

QUESTION/*VRAAG* 8

8.1.1	$\hat{P}_2 = 65°$ (∠s opp equal sides/∠*e teenoor gelyke sye*)	✓S ✓ R (2)
8.1.2	$\hat{D} = 40°$ (ext∠ of ΔCDP/*buite*∠ *v ΔCDP*) **OR/*OF*** (∠s on a str line; sum of ∠s in Δ/ ∠*e op regt lyn; som v* ∠*e in Δ*)	✓ S ✓R (2)
8.1.3	$\hat{A}_1 = 40°$ (ext∠ of ΔCDP/*buite*∠ *v ΔCDP*) **OR/*OF*** (∠s on a str line; sum of ∠s in Δ/ ∠*e op regt lyn; som v* ∠*e in Δ*)	✓S✓ R (2)
8.2	$\hat{A}_1 = \hat{D} = 40°$ ∴CA is a tangent to the circle (converse tan chord theorem)/ *CA is 'n raaklyn aan die sirkel (omgek rkl-kd stelling)*	✓S ✓R (2) **[8]**

QUESTION/*VRAAG* 9

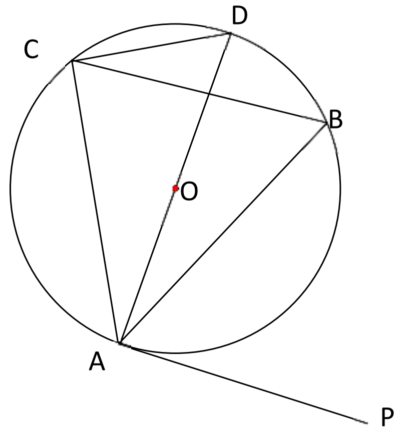
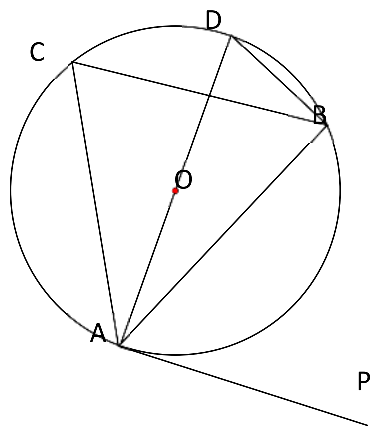
9.1.1	ext∠ of cyclic quad/*buite* ∠ *van koordevh*	✓R (1)
9.1.2	∠ at centre = 2 ×∠ at circumference / *midpts*∠ = *2 × omtreks*∠	✓ R (1)
9.2.1	$C\hat{D}A = x$ (corresp∠s/*ooreenk* ∠*e*; EB || DC) ∴ AC = AD (sides opp equal ∠s/*sye teenoor gelyke* ∠*e*)	✓ S ✓R ✓ S ✓R (4)

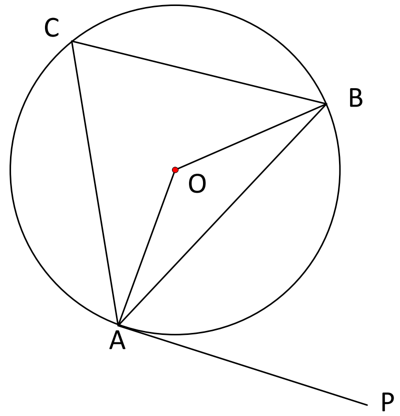
9.2.2	$\hat{A} = 180° - 2x$ (sum of ∠s in Δ/*som van ∠e in Δ*) $\therefore \hat{A} + \hat{O}_1 = 180° - 2x + 2x = 180°$ ∴ ABOD = cyclic quad/*koordevh* (opp∠s quad supp/ *teenoorst ∠e kdvh*)	✓ S ✓ S ✓ R (3) **[9]**

QUESTION/*VRAAG* 10

10.1	then the line is parallel to the third side/*is die lyn ewewydig aan die derde sy.*	✓ S (1)
10.2.1	$\frac{AE}{AC} = \frac{12}{20} = \frac{3}{5}$ $\frac{AD}{AF} = \frac{3}{5}$ $\therefore \frac{AE}{AC} = \frac{AD}{AF}$ $\therefore$ DE || FC (line divides two sides of Δ in prop/ *lyn verdeel twee sye v Δ in dieselfde verh*)	✓ S ✓ S ✓ S (3)
10.2.2	$\frac{BF}{BA} = \frac{8}{20}$ (prop theorem/*eweredigh st*; BC || FE) $\therefore BF = \frac{8}{20}(14)$ $\therefore BF = \frac{28}{5}$ **OR/*OF*** $FB = 5\frac{3}{5}$ **OR/*OF*** $FB = 5{,}6$	✓ S/R ✓ substitution/ *substitusie* ✓ answer/*antw* (3) **[7]**

QUESTION/*VRAAG 11*

11.1	Draw diameter AD and join DC..	✓ construction/ *konstruksie*
	 	
	Proof/*Bewys*:	
	$B\hat{A}P + B\hat{A}D = 90°$ (tangent/*raaklyn* ⊥ radius)	✓ S ✓ R
	$D\hat{C}B + A\hat{C}B = 90°$ (∠ in semi circle/*halfsirkel*)	✓ S ✓ R
	but	
	$B\hat{A}D = D\hat{C}B$ (∠s in same segment/∠*e in dies segm*)	
	$\therefore B\hat{A}P = A\hat{C}B$	✓ S/R
		(6)
	OR/*OF*	
	Draw diameter AD and join DB. *Trek middellyn AD en verbind DB.*	✓ construction/ *konstruksie*
	 	
	Proof/*Bewys*:	✓ S ✓ R ✓ S ✓ R
	$P\hat{A}B + B\hat{A}D = 90°$ (tangent/*raaklyn* ⊥ radius)	
	$D\hat{B}A = 90°$ (∠ in semi circle/*halfsirkel*)	
	$B\hat{A}D + A\hat{D}B = 90°$ (sum of ∠s in Δ/*som van ∠e in Δ*)	✓ S/R
	$A\hat{D}B = A\hat{C}B$ (∠s in same segment/∠*e in dies segm*)	
	$\therefore B\hat{A}P = A\hat{C}B$	(6)

	OR/*OF* Draw radii OA and OB. *Trek radii OA en OB.*  Proof/*Bewys*: $O\hat{A}B + B\hat{A}P = 90°$ (tangent/*raaklyn* ⊥ radius) $\therefore B\hat{A}P = 90° - O\hat{A}B$ $O\hat{A}B = O\hat{B}A$ (∠s opp equal sides/*∠e to gelyke sye*) $A\hat{O}B = 180° - 2O\hat{A}B$ (sum of ∠s in Δ/*som van ∠e in Δ*) $\therefore A\hat{C}B = 90° - O\hat{A}B$ (∠ at centre = 2 ×∠ at circumference/ *midpts∠ = 2 × omtreks∠*) $\therefore B\hat{A}P = A\hat{C}B$	✓ construction/ *konstruksie* ✓ S ✓ R ✓ S ✓ S/R ✓ S (6)
11.2.1	$\hat{A}_1 = \hat{P} = x$ (tangent-chord theorem/*rkl-kd st*) $D\hat{C}A = 2x$ (∠ at centre = 2 ×∠ at circumference/ *midpts∠ = 2 × omtreks∠*) In ΔBAD and ΔBCE: $\hat{B} = \hat{B}$ (common/*gemeen*) $\hat{A}_1 = \hat{C}_1 = x$ $(\hat{C}_1 = \hat{C}_2)$ ∴ΔBAD ||| ΔBCE (∠∠∠) **OR/*OF*** $\hat{A}_1 = \hat{P} = x$ (tangent-chord theorem/*rkl-kd st*) $D\hat{C}A = 2x$ (∠ at centre = 2 ×∠ at circumference/ *midpts∠ = 2 × omtreks∠*) In ΔBAD and ΔBCE: $\hat{B} = \hat{B}$ (common/*gemeen*) $\hat{A}_1 = \hat{C}_1 = x$ $(\hat{C}_1 = \hat{C}_2)$ $\hat{D}_1 = \hat{E}_1$ ∴ΔBAD ||| ΔBCE	✓ S ✓ R ✓ S ✓ R ✓ S ✓ S ✓ R (7) ✓ S ✓ R ✓ S ✓ R ✓ S ✓ S ✓ S (7)

11.2.2(a)	$B\hat{A}C = 90°$ (tangent/*raakl* ⊥ radius) $\therefore BC^2 = 8^2 + 6^2 = 100$ (Pythagoras theorem/*stelling*) BC = 10 AC = DC = 6 (radii) ∴ BD = 10 – 6 = 4 units/*eenhede*	✓ S ✓ R ✓ BC = 10 ✓ DC = 6 ✓ BD = 4 (5)
11.2.2(b)	$\frac{BA}{BC} = \frac{BD}{BE}$ (ΔBAD ||| ΔBCE) $\therefore \frac{8}{10} = \frac{4}{BE}$ ∴ BE = 5 units/*eenhede*	✓ S ✓ substitution/ *substitusie* ✓ BE = 5 (3)
11.2.3(c)	AE = 3 In ΔACE: $\tan x = \frac{3}{6}$ $\therefore x = 26{,}57°$ **OR/*OF*** $\sin 2x = \frac{8}{10}$ $\therefore 2x = 53{,}1301...$ $(2x < 90°)$ $\therefore x = 26{,}57°$	✓ correct trig ratio/ *korrekte trigvh* ✓ correct trig eq/ *korrekte trigvgl* ✓ answer/*antw* (3) ✓ correct trig ratio/ *korrekte trigvh* ✓ correct trig eq/ *korrekte trigvgl* ✓ answer/*antw* (3) **[24]**