SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2021

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12

SUBJECT: MATHEMATICS

SOLUTIONS TERM 2

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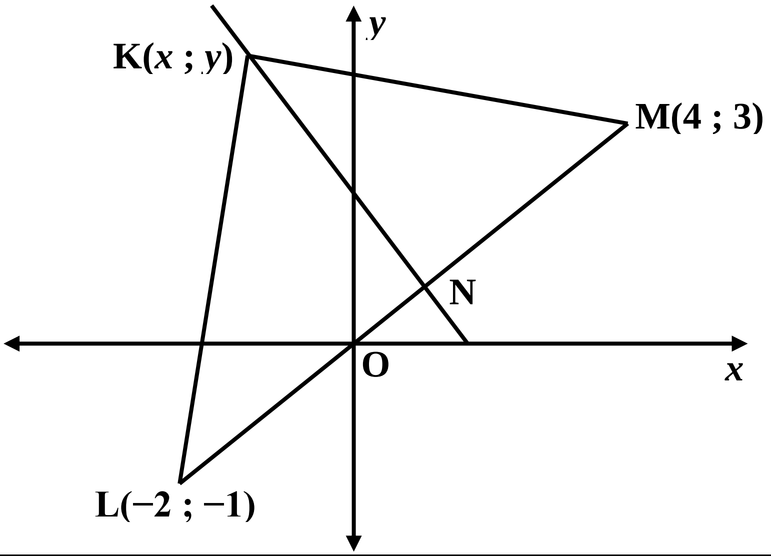
SESSION 1

ACTIVITY 1 SOLUTIONS:

QUESTION 1

1.1	$M=\left(\frac{-4+9}{2};\frac{2-2}{2}\right)=\left(\frac{5}{2};0\right)$	✓ x-coord . ✓ y-coord. (2)
1.2	$\left(\frac{x+8}{2};\frac{6+y}{2}\right)=M=\left(\frac{5}{2};0\right)$ parallelogram ✓ $\frac{x+8}{2}=\frac{5}{2}$ $\therefore x=-3$ ✓ $\frac{6+y}{2}=0$ $\therefore y=-6$ ✓	(3)
1.3	$m_{BC}=\frac{y_B-y_C}{x_B-x_C}=\frac{-2+6}{9-8}=4$	✓for subst. ✓ answer (2)
1.4	$m_{AB}=\frac{y_A-y_B}{x_A-x_B}=\frac{6+2}{-3-9}=-\frac{2}{3}$ $y-6=-\frac{2}{3}(x+3)$ $y=-\frac{2}{3}x+4$	✓for subst. ✓ answer ✓for subst. ✓ answer (4)
1.5	$m_{AD}=m_{BC}$ parallelogram ✓ ✓ ✓ $\theta=\left[180+\tan^{-1}\left(-\frac{2}{3}\right)\right]-\tan^{-1}(4)=70,35^{\circ}$ ✓	(4)
1.6	$m_{BC}.m_{BD}=4\times\frac{-2-2}{9+4}$ $=4\times\frac{-4}{13}$ $=\frac{-16}{13}\neq-1$ $\therefore \Delta$DBC is not right angled triangle	✓ m_{BD} ✓ $\frac{-4}{13}$ ✓statement (3)
		[18]

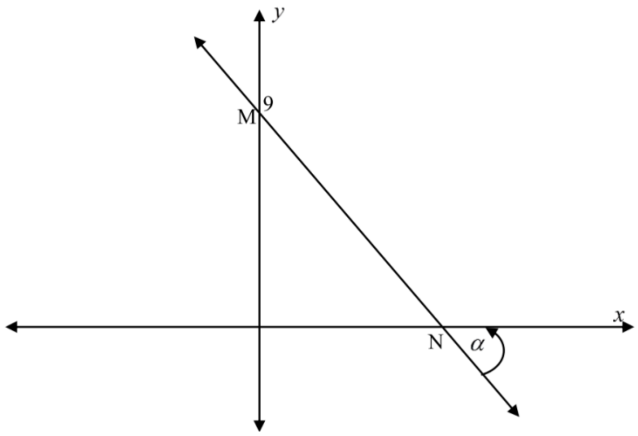
QUESTION 2

		
2.1	$N(\frac{-2+4}{2};\frac{-1+3}{2})$ $N(1;1)$	✓ substitution ✓ answer (2) **Answer Only: Full Marks**
2.2	At K, $y-5x=9$ - - - (1) & $5y+x=19$ - - - (2) (1) ×5: $5y-25x=45$ - - - (3) (2) – (3): $26x=-26$ $\therefore x=-1$ Substitute into (1): $y=5x+9$ $=5(-1)+9$ $y=4$ $\therefore$ K(− 1; 4) ***OR***: substitute $y=5x+9$ into $5y=-x+19$ $5(5x+9)=-x+19$ $26x=19-45$ $x=-1$ $y=4$ K(−1; 4)	✓ (1) ×5 ✓ (2) − (3) ✓ $x=-1$ ✓ $y=5(-1)+9$ (4)
2.3	$m_{KN}=\frac{4-1}{-1-1}=-\frac{3}{2}$ K(−1; 4) N(1; 1) $y-y_1=m(x-x_1)$ $y-1=-\frac{3}{2}(x-1)$ $y=-\frac{3}{2}x+\frac{3}{2}+1$ $y=-\frac{3}{2}x+\frac{5}{2}$	✓ $m_{KN}=-\frac{3}{2}$ ✓ subst N(1; 1) ✓ answer (3)
2.4	$m_{LM}=\frac{3-(-1)}{4-(-2)}=\frac{4}{6}=\frac{2}{3}$ But $m_{LM}\times m_{KN}=(\frac{2}{3})(-\frac{3}{2})=-1$ $\therefore$ KN ⊥ LM and N is the midpoint of LM $\therefore$ KN is the ⊥ bisector of LM	✓ $m_{LM}=\frac{2}{3}$ ✓ $m_{LM}\times m_{KN}=-1$ ✓ KN ⊥ LM (3)

2.5	L, M and J are collinear L(−2; −1) M(4; 3) J(7; a) $\Rightarrow m_{LJ} = m_{LM}$ $\therefore \frac{a+1}{7+2} = \frac{2}{3}$ $\therefore$ 3a + 3 = 18 3a = 15 $\therefore$ a = 5	✓ $m_{LJ} = m_{LM}$ ✓ **substitution** ✓ **answer (3)**
2.6	Let the $\angle$ be θ K(−1; 4) N(−2; −1) $m_{KL} = \frac{-5}{-1} = 5$ $m_{KL} = 5$ $\therefore \tan\theta = 5$ $\therefore \theta = 78,69°$	✓ $m_{KL} = 5$ ✓ $\therefore \theta = 78,69°$ **(2)**
		[17]

QUESTION 3

3.1.1	$3p - 4(p+2) + 5 = 0$ $3p - 4p - 8 + 5 = 0$ $-p = 3$ $p = -3$	✓substitute $x = p$ ✓ substitute $y = p + 2$ ✓ -3 (3)
3.1.2	$3x - 4y + 5 = 0$ $-4y = -3x - 5$ $y = \frac{3}{4}x + \frac{5}{4}$ $m_{AB} = \frac{3}{4}$	✓ $\frac{3}{4}$ (1)
3.1.3	$m_{AC} = \frac{k-2}{-5-1}$ $\frac{k-2}{-6} \times \frac{3}{4} = -1$ $\frac{k-2}{-6} = -\frac{4}{3}$ $k - 2 = 8$ $k = 10$ **OR** $m_{\perp} = -\frac{4}{3}$ $\frac{k-2}{-5-1} = -\frac{4}{3}$ $3k - 6 = 24$ $3k = 30$ $k = 10$	✓ $\frac{k-2}{-6}$ ✓product = –1 **OR** $m_{\perp} = -\frac{4}{3}$ ✓10 (3)



3.2	$\tan\alpha = 3$ $\alpha = 71{,}57°$ $M\hat{N}X = 108{,}43°$ $\tan 108{,}43° = -3{,}0009$ $m_{MN} = -3$ $y = -3x + 9 \quad M(0;9)$ $3x + y - 9 = 0$ $a = 3 \;;\; b = 1 \;;\; c = -9$	✓ $\alpha = 71{,}57°$ ✓ $M\hat{N}X = 108{,}43°$ ✓ $y = -3x + 9$ ✓ $a = 3$ ✓ $b = 1$ ✓ $c = -9$ (6) **[13]**

SESSION NO 2

TOPIC : ANALYTICAL GEOMETRY

ACTIVITY 2 SOLUTIONS:

1.1	$r^2=\left(\frac{5}{2}-5\right)^2+(2-0)^2=\frac{41}{4}$ $\therefore\left(x-\frac{5}{2}\right)^2+(y-2)^2=\frac{41}{4}$ $x^2-5x+\frac{25}{4}+y^2-4y+4=\frac{41}{4}$ $x^2-5x+y^2-4y=0$	✓ subst. ✓ r^2 ✓ for subst. ✓ for expand. (4)
1.2	$m_{BC}=\frac{y_B-y_C}{x_B-x_C}=\frac{0-2}{5-\frac{5}{2}}=-\frac{4}{5}$ $\therefore m_{\tan}=\frac{5}{4}$ $y=\frac{5}{4}(x-5)$ $y=\frac{5}{4}x+\frac{25}{4}$	✓for subst. ✓ answer ✓for *m* ✓for subst. ✓ answer (5)
1.3	$\angle AOB=\theta$ tan-chord theorem $\therefore m_{OA}=\tan\theta=\tan(78.69)=4.999969358...=5$ $\therefore y=5x$	✓ ✓ ✓ answer (3)
1.4	Subst. $y=5x$ into $\left(x-\frac{5}{2}\right)^2+(y-2)^2=\frac{41}{4}$: $\therefore\left(x-\frac{5}{2}\right)^2+(5x-2)^2=\frac{41}{4}$ $x^2-5x+\frac{25}{4}+25x^2-20x+4=\frac{41}{4}$ $26x^2-25x=0$ $x(26x-25)=0$ $\therefore x=0$ or $x=\frac{25}{26}$ $\therefore y=\frac{125}{26}$ $\therefore A=\left(\frac{25}{26};\frac{125}{26}\right)$	✓ **for subst.** ✓ expand ✓ expand ✓ *x* ✓ *y* (5)

1.5.1	Let d be the distance between the centres of the circles. $d^2=\left(\frac{5}{2}+\frac{5}{2}\right)^2+(2-3)^2=26$ $r_1^2+r_2^2=\frac{41}{4}+\frac{61}{4}=\frac{102}{4}=25,5<26$ $\therefore$ circles intersect at 2 distinct points.	✓ d^2 ✓ $r_1^2+r_2^2$ ✓ for <26 (3)
1.5.2	Subst. O and A into $\left(x+\frac{5}{2}\right)^2+(y-3)^2=\frac{61}{4}$: $\left(\frac{5}{2}\right)^2+(-3)^2=\frac{25}{4}+9=\frac{61}{4}=RHS$ $\left(\frac{25}{26}+\frac{5}{2}\right)^2+\left(\frac{125}{26}-3\right)^2=\left(\frac{45}{13}\right)^2+\left(\frac{47}{26}\right)^2=\frac{61}{4}=RHS$	✓ ✓ (2)

[22]

QUESTION 2

2.1	$x^2+y^2=r^2$ $(3)^2+(-4)^2=r^2$ $25=r^2$ $\therefore\ x^2+y^2=25$	✓ substitution ✓ $25=r^2$ ✓ answer (3)
2.2	$r^2=25\Rightarrow r=5$ $T(5;0)$ Q(3;-4) $TQ=\sqrt{(3-5)^2+(-4-0)^2}$ $=\sqrt{20}$ OR $TQ=2\sqrt{5}$	✓ T (5; 0) ✓ substitution ✓ answer (3)
2.3	$m_{OQ}=\frac{-4-0}{3-0}=-\frac{4}{3}$ $y-y_1=m(x-x_1)$ $y+4=-\frac{4}{3}(x-3)$ $y=-\frac{4}{3}x+4-4$ $y=-\frac{4}{3}x$	✓ $m_{OQ}=-\frac{4}{3}$ ✓ substitution ✓ answer (3)

2.4	P(−3; 4) from symmetry **OR** $y = -\frac{4}{3}x$ - - - (1) $x^2 + y^2 = 25$ - - - (2) Subst (1) into (2): $x^2 + (-\frac{4}{3}x)^2 = 25$ $x^2 + \frac{16}{9}x^2 = 25$ $9x^2 + 16x^2 = 225$ $25x^2 = 225$ $x^2 = 9$ $\therefore x = -3$ $y = -\frac{4}{3}(-3)$ $= 4$ $\therefore P(-3;4)$ **Midpoint:** $x_0 = \frac{x_P + x_R}{2}; y_o = \frac{y_P + y_R}{2}$ $0 = \frac{x_P + 3}{2}; 0 = \frac{y_P - 4}{2}$ $0 = x_P + 3; 0 = y_P - 4$ $-3 = x_P$; $4 = y_P$ $\therefore$ P(−3;4)	✓ $x = -3$ ✓ $y = 4$ (2)
2.5	$r = 5 \Rightarrow r^2 = 25$ $\therefore (x+3)^2 + (y-4)^2 = 25$ $\therefore x^2 + y^2 + 6x - 8y = 0$	✓ $r^2 = 25$ ✓ substitution ✓ equation (3)
2.6	$m_{OQ} = -\frac{4}{3}$ (from Q. 4.3) $\therefore m_{QR} = \frac{3}{4}$ (OQ ⊥ QR ⇔ rad ⊥ tan) $y - y_1 = m(x - x_1)$ $y + 4 = \frac{3}{4}(x - 3)$ $y = \frac{3}{4}x - \frac{9}{4} - 4$ $y = \frac{3}{4}x - \frac{25}{4}$ **OR** $4y = 3x - 25$	✓ $m_{QR} = \frac{3}{4}$ ✓ substitutions ✓ simplification ✓ answer (4)
2.7	$y = \frac{3}{4}x - \frac{25}{4}$ $1 = \frac{3}{4}(k) - \frac{25}{4}$ $4 = 3k - 25$ $3k = 29$ $\therefore k = \frac{29}{3}$	✓✓ subst (k; 1) ✓ answer (3)
		[21]

QUESTION 3 [15 Marks]

3.1	EC $= \sqrt{(-3-1)^2 + (2-3)^2}$ $= \sqrt{16+1}$ $= \sqrt{17}$	✓substitution into formula ✓simplification ✓answer
	$m_{EC} = \frac{3-2}{1-(-3)}$ $= \frac{1}{4}$ $m_{tan} = -4$ $y - 3 = -4(x-1)$ $m = -4$ and (1; 3) into equation $y = -4x + 7$	✓gradient of radius ✓gradient of tangent ✓substitution ✓ equation in correct form
	$(x+3)^2 + (x-2)^2 = 17$	✓mark for centre and ✓Mark for $(\text{radius})^2$
	$+x = 4$ and $(x+3)^2 + (x-2)^2 = 17$ Substitute $y = 4 - x$ into $(x+3)^2 + (y-2)^2 = 17$ $(x+3)^2 + (4-x-2)^2 = 17$ $(x+3)^2 + (2-x)^2 = 17$ $x^2 + 6x + 9 + 4 - 4x + x^2 = 17$ $2x^2 + 2x - 4 = 0$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $x = -2$ or $x = 1$ At F $x = -2$ F$(-2\,;6)$	✓substitution into formula ✓substitution into formula ✓simplification ✓solutions ✓ co-ordinates of F

HOME WORK SOLUTIONS

QUESTION 1

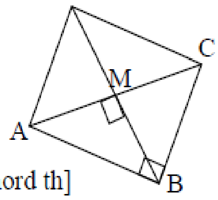
1.1	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $AB = \sqrt{(8 - -2)^2 + (1 - -4)^2}$ $AB = 5\sqrt{5}$	✓ substitution ✓ answer
1.2	$m = \frac{y_1 - y_2}{x_1 - x_2}$ $m_{BC} = \frac{1-(-4)}{8-(-2)}$ $m_{BC} = \frac{1}{2}$	✓ answer
1.3	$m_{AF} \times m_{BC} = -1$ $m_{AF} = -2$ $y = -2x + 1$	✓$m_{AF} = -2$ ✓$y = -2x + 1$

1.4	$M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $E\left(\frac{8+0}{2};\frac{1+1}{2}\right)$ $E(4;1)$	✓ substitution ✓ answer E(4;1)
1.5	$\frac{k+\ -2}{2}=4$ $k=10$	✓ method ✓$k=10$
1.6	$m_{AF}=m_{CG}=-2$ $\frac{0-1}{p-8}=-2$ $p=\frac{17}{2}\ OR\ 16{,}5$	✓$m_{CG}=-2$ ✓$p=\frac{17}{2}\ OR\ 16{,}5$
1.7	$m_{BD}=\frac{6-(-4)}{10-(-2)}=\frac{5}{6}$ $m_{DC}=\frac{6-1}{10-8}=\frac{5}{2}$ $\tan\theta=m$ $\tan\alpha=\frac{5}{6}$ $\alpha=\tan^{-1}\left(\frac{5}{6}\right)=39{,}81°$ $\tan\beta=\frac{5}{2}$ $\beta=\tan^{-1}\left(\frac{5}{2}\right)=68{,}20°$ $\theta=\beta-\alpha$ $\theta=68{,}20°-39{,}81°$ $\therefore\theta=28{,}39°$	✓$m_{BD}=\frac{5}{6}$ ✓$m_{DC}=\frac{5}{2}$ ✓$\tan\theta=m$ ✓39,81° ✓68,20° ∴ ✓$\theta=28{,}39°$

QUESTION 2

2.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $=M\left(\frac{-7+6}{2};\frac{2+3}{2}\right)$ $=M\left(-\frac{1}{2};\frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
2.2	$m_{BC}=\frac{3-0}{6-p}=\frac{3}{6-p}$ **OR/*OF*** $m_{BC}=\frac{0-3}{p-6}=\frac{-3}{p-6}$	✓answer (1) ✓answer (1)

2.3	$m_{AD} = m_{BC}$ [AD || BC] $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF**	✓ $m_{BC} = 2$ ✓ equating ✓answer (3) ✓ $m_{BC} = 2$
2.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
2.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
2.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst* ∠*e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som* ∠*e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3)
2.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r = \frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{5}{2}\right)^2 = \left(\frac{\sqrt{170}}{2}\right)^2 = \frac{85}{2} = 42{,}5$	✓ M is centre ✓ $r = \frac{\sqrt{170}}{2}$ ✓ equation (3)

2.8	$C\hat{B}M = B\hat{A}M = 45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] $\therefore$ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF***  $A\hat{M}B = 90°$ [diag of square bisect ⊥] $\therefore$ AB is diameter $BC \perp AB$ $\therefore$ BC is tangent [line ⊥ radius *or* converse tan-chord th]	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

QUESTION 3

3.1	∠ in semi circle/ ∠ at centre = 2∠ on circle	✓ R
3.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
3.3	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)
3.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2) ✓ substitution ✓ answer (2)

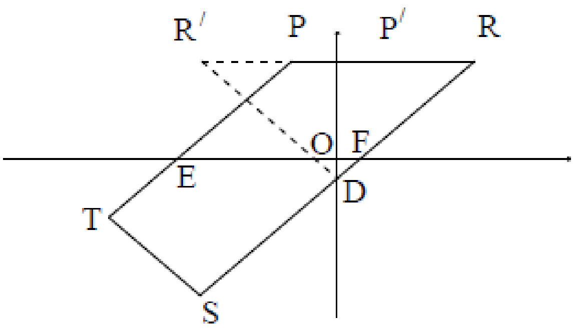
3.4.2	$M\left(9\,;\,6\frac{1}{2}\right)$ $\therefore \frac{x_R+3}{2}=9$ and $\frac{y_R+7}{2}=6\frac{1}{2}$ $\therefore R(15\,;\,6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9\,;\,6\frac{1}{2}\right)$ $\therefore R\left(9+6\,;\,6\frac{1}{2}-\frac{1}{2}\right)=R(15\,;\,6)$	✓ M ✓ x coordinate ✓ y coordinate (3) ✓ M ✓ x coordinate ✓ y coordinate (3)
3.4.3	$ST=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $ST=\sqrt{(5-3)^2+(2-7)^2}$ $ST=\sqrt{4+25}=\sqrt{29}$ $\sin R=\frac{TS}{TR}=\frac{\sqrt{29}}{\sqrt{145}}$ *or* $\frac{\sqrt{5}}{5}$ *or* $\frac{1}{\sqrt{5}}$ *or* 0,45	✓substitution ✓ answer ✓ ratio (3)
3.4.4	$m_{TR}=\frac{7-6\frac{1}{2}}{3-9}=-\frac{1}{12}$ **OR/*OF*** $m_{TR}=\frac{7-6}{3-15}=-\frac{1}{12}$ $m_{TR}\times m_{KTL}=-1$ [$r\perp$ tangent] $m_{KTL}=12$ $y-y_1=12(x-x_1)$ $y-7=12(x-3)$ $y=12x-29$ substitute $K(a;b)$: $b=12a-29$	✓ $m_{TR}=-\frac{1}{12}$ ✓ $m_{KTL}=12$ ✓ $y=12x-29$ (3)

3.4.5	TK = TR $\sqrt{(a-3)^2+(b-7)^2}=\sqrt{145}$ $(a-3)^2+(b-7)^2=145$ Substitute $b=12a-29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2=145$ $(a-3)^2+(12a-36)^2=145$ $a^2-6a+9+144a^2-864a+1296-145=0$ $145a^2-870a+1160=0$ $a=\dfrac{870\pm\sqrt{(870)^2-4(145)(1160)}}{290}$ $a=2$ or $a=4$ $\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $\quad$ $=19$ $\therefore$ K(2 ; –5)	✓ substitution into distance formula ✓ substitution of $b=12a-29$ ✓standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

PAST EXAMINATION QUESTIONS SOLUTIONS

1.1	Equation of PR: $y=5$	✓ answer (1)
1.2.1	$m_{RS}=\dfrac{y_2-y_1}{x_2-x_1}$ $m_{RS}=\dfrac{5-(-7)}{3-(-3)}=\dfrac{12}{6}$ $=2$ Answer only: Full marks	✓ substitution of R & S into gradient formula ✓ answer (2)
1.2.2	$m_{RS}=m_{PT}$ [PT ‖ RS] $\tan\theta=2$ $\theta=63{,}43°$	✓ $m_{RS}=m_{PT}$ ✓ $\tan\theta=2$ ✓ $\theta=63{,}43°$ (3)

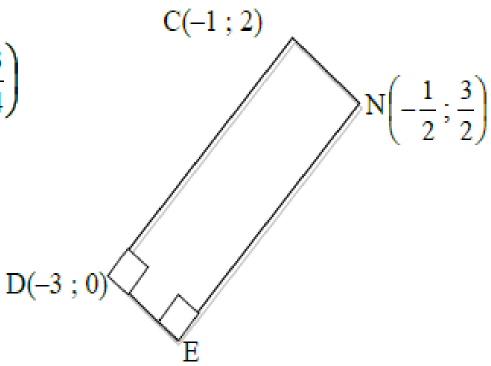
1.2.3	Equation of RS: $y-5=2(x-3)$ or $y-(-7)=2(x-(-3))$ or $5=2(3)+c$ $y-5=2x-6$ $y+7=2x+6$ $c=-1$ $y=2x-1$ $y=2x-1$ $y=2x-1$ $\therefore$ D$(0\,;-1)$ **OR/OF** $m_{RS}=m_{RD}=m_{DS}$ $2=\frac{5-y}{3-0}=\frac{y+7}{0-(-3)}$ $\therefore y=-1$ $\therefore$ D$(0\,;-1)$ Answer only: Full marks	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)
1.3	$ST=2\sqrt{5}=\sqrt{[-5-(-3)]^2+(k-(-7))^2}$ $20=4+(k+7)^2$ $(k+7)^2=16$ $k+7=\pm4$ $k=-11$ or $k=-3$ $\therefore k=-3$ **OR** $ST=2\sqrt{5}=\sqrt{[-5-(-3)]^2+(k-(-7))^2}$ $20=4+k^2+14k+49$ $k^2+14k+33=0$ $(k+11)(k+3)=0$ $k=-11$ or $k=-3$ $\therefore k=-3$	✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer (4) ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer (4)

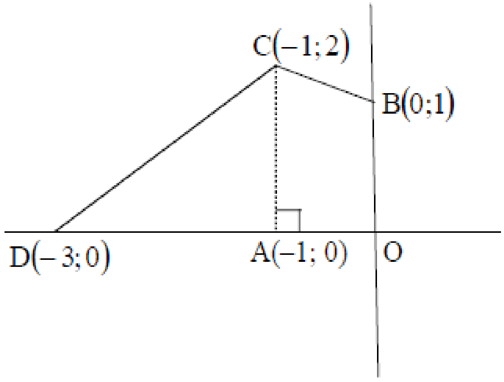
1.4	Method: translation T→S: $(x\,;y) \rightarrow (x+2\,;y-4)$ $\therefore$ by symmetry: D→N: D(0 ; –1) → N(0 + 2 ; –1 – 4) $\therefore$ N(2 ; –5) Answer only: Full marks **OR** Midpoint of TN = Midpoint of SD $\frac{x+(-5)}{2} = \frac{-3+0}{2}$ and $\frac{y+(-3)}{2} = \frac{-7+(-1)}{2}$ $x = 2$ and $y = -5$ $\therefore$ N(2 ; – 5) Answer only: Full marks	✓ method ✓ x-coordinate ✓ y-coordinate (3) ✓ method: midpoint of diagonals ✓ x-coordinate ✓ y-coordinate (3)
1.5	 β is the inclination of RS $\therefore\ \beta = 63{,}434...°$ $O\hat{F}D = 63{,}434...°$ [vert opp ∠s] $O\hat{D}F = 90° - 63{,}434...° = 26{,}565...°$ $R\hat{D}R' = 2(26{,}565...°) = 53{,}13°$	✓ $\beta = 63{,}43°$ ✓ $O\hat{D}F = 26{,}57°$ ✓ answer (3)

QUESTION 2

2.1	M(–1;1) $(x+1)^2 + (y-1)^2 = 1$ Answer only: Full marks	✓M(–1;1) ✓LHS ✓ RHS (3)

2.2	Midpoint of CB, N: (– 0,5 ; 1,5) $\therefore \frac{x_C + 0}{2} = -\frac{1}{2}$ and $\frac{y_C + 1}{2} = \frac{3}{2}$ Answer only: Full marks $\therefore$ C(– 1 ; 2) **OR** B→N: $(x ; y) \rightarrow (x - 0,5 ; y + 0,5)$ N→C: Answer only: Full marks $(x ; y) \rightarrow (x - 0,5 ; y + 0,5)$ $\therefore$ C(– 0,5 – 0,5 ; 1,5 + 0,5) $\therefore$ C(– 1 ; 2)	✓x value ✓ y value (2) ✓x value ✓ y value (2)
2.3	$m_{radius} = \frac{2-1}{-1-0}$ OR $\frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}}$ OR $\frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{tangent} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$	✓ m_{radius} ✓ $m_{tangent}$ ✓ substitute (– 1 ; 2) and m ✓ simplification (4)
2.4	Tangents to circle: $y = x + 3$ and $y = x + 1$ $\therefore t > 3$ or $t < 1$ Answers only: Full marks	✓ $y = x + 1$ ✓ $t > 3$ ✓ $t < 1$ (3)
2.5	Draw rectangle CNED: Midpt of DN $\left(-\frac{7}{4} ; \frac{3}{4}\right)$ $\therefore$ E$(-\frac{5}{2} ; -\frac{1}{2})$ **OR/*OF*** D(– 3 ; 0) C→N: $(x ; y) \rightarrow (x + 0,5 ; y - 0,5)$ D→E: D$(x ; y) \rightarrow$ E$(x + 0,5 ; y - 0,5)$ Answer only: Full marks $\therefore$ E(– 3 + 0,5 ; 0 – 0,5) $\therefore$ E(– 2,5 ; – 0,5)	✓ midpt of DN ✓x value ✓ y value (3) ✓ coordinates of D ✓x value ✓ y value (3)



2.6	 area of trapezium AOBC $= \frac{1}{2}(1+2)(1)$ $= 1\frac{1}{2}$ square units area of ΔACD $= \frac{1}{2}(2)(2)$ $= 2$ square units area of quadrilateral OBCD $= 3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$	✓ substitution into area of trapezium form ✓ area of trapezium ✓ area of triangle ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)
		[20]

QUESTION 3

3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
3.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$ Answer only: Full marks	✓ $\tan\theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)

3.2	Inclination KL = 123,69° – 78,69° = 45° [ext ∠Δ] $\tan 45° = m_{KL} = 1$	✓ S ✓ $\tan 45° = m_{KL} = 1$ (2)
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$	✓ substitute (– 1 ; 2) and m ✓ equation (2) ✓ substitute (– 1 ; 2) and m ✓ equation (2)
3.4	$KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13}$ or 3,61 Answer only: Full marks	✓ substitute K and N into distance formula ✓ answer (2)
3.5.1	$(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3$ ...(2) (2) in (1): $(x+3)^2 + (x+3+5)^2 = 13$ $x^2 + 6x + 9 + x^2 + 16x + 64 = 13$ $2x^2 + 22x + 60 = 0$ $x^2 + 11x + 30 = 0$ $(x+5)(x+6) = 0$ $x = -5$ or $x = -6$ $y = -2$ or $y = -3$ L(–5 ; –2) or (–6 ; –3)	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ x-values ✓ y-values (5)
3.5.2	Midpoint of KM: (– 2 ; – 1,5) $\therefore \frac{x_L + 1}{2} = -2$ and $\frac{y_L - 1}{2} = -\frac{3}{2}$ $\therefore$ L(– 5 ; – 2)	✓ midpoint of KM ✓ x value ✓ y value (3)
	OR/*OF* $m_{KN} = m_{LM}$ $\frac{y - (-5)}{x - (-3)} = -\frac{3}{2}$ $2(x + 3 + 5) = -3(x + 3)$ $2x + 16 = -3x - 9$ $5x = -25$ $x = -5$ $\therefore$ L(– 5 ; – 2) Answer only: Full marks	✓ $m_{LM} = m_{KN}$ ✓ x value ✓ y value (3)

3.6	T(–6 ; –3) (from Question 3.5.1) $KT = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$ $= \sqrt{50}$ $KN = \sqrt{13}$ (CA from 3.4) $\text{Area of } \Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$ $= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78,69°$ = 12,50 square units	✓ coordinates of T ✓ length of KT ✓ substitution into area rule ✓ answer (4)
	OR/*OF* In ΔKLM: $\frac{TL}{\sin 22,62°} = \frac{\sqrt{13}}{\sin 78,69°}$ $TL = 1,414..$ $KL = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$ $= \sqrt{32}$ $\therefore$ KT = 7,0708… $\text{Area of } \Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$ $= \frac{1}{2}(7,0708).\sqrt{13}\sin 78,69°$ = 12,50 square units	✓ length of TL ✓ length of KT ✓ substitution into area rule ✓ answer (4)
		[22]

QUESTION 4

4.1	F(3 ; 1)	✓ x value ✓ y value (2)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ FS = 5	✓ substitution of F & S ✓ answer (2)
4.3	FH(FS) : HG = 1 : 2 $\therefore$ HG = 2 FH = 10	✓ HG = 10 (1)

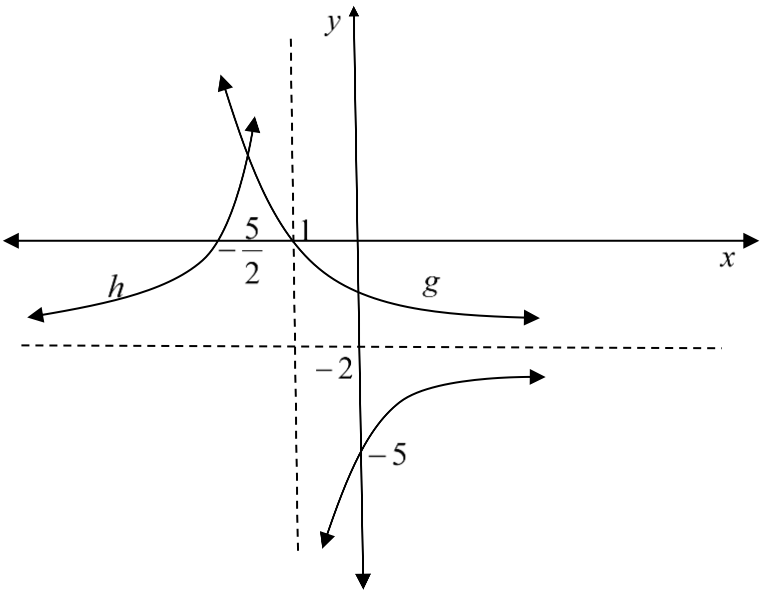
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*	✓ answer (1)
4.5.1	$F\hat{H}J = 90°$ [tan ⊥ radius / *rkl ⊥ radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] $FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x-m)^2 + (y-n)^2 = 100$	✓ answer (1)
4.5.3	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig [HJ is 'n gemeemskaplike raaklyn]* $FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$ $(n-1)^2 = 144$ $n - 1 = \pm 12$ $n \neq 13$ or $n = -11$ $\therefore G(12; -11)$ **OR/*OF*** $n^2 - 2n - 143 = 0$ $(n+11)(n-13) = 0$ $n = -11$ or $n \neq 13$	✓ K(22; n) ✓ value of m ✓ subst. of F and G in distance formula ✓ FG = 15 ✓ simplification/ standard form ✓ value of n ✓ coordinates of G (7)
	OR/*OF* K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ Let J(22 ; y): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$ $\therefore y = 9$ or/of $y \neq -7$ $\therefore n = 9 - 20 = -11$ $\therefore G(12; -11)$	✓ K(22; n) ✓ value of m ✓ subst. of F and J in distance formula ✓ $FJ = \sqrt{425}$ ✓ standard form ✓ value of n ✓ coordinates of G (7)
		[18]

SESSION NO : 3

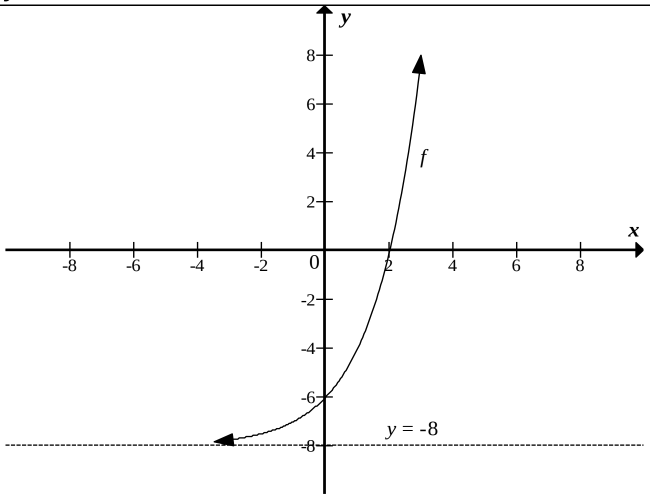
TOPIC : FUNCTIONS AND GRAPHS

QUESTION 1		
1.1	$m_g = 1$	
1.2	A and C are roots of f Therefore, solve for *x*: $2x^2 - x - 3 = 0$ $(2x-3)(x+1) = 0$ $x = \frac{3}{2}$ or $x = -1$ $A(-1;0)$ and $C\left(\frac{3}{2};0\right)$	
1.3	D is the point of intersection of f and g. Solve for *x*: $2x^2 - x - 3 = x + 1$ D (2; 3)	
1.4	$y = -3$	
1.5	$x = \dfrac{-1+\frac{3}{2}}{2} = \frac{1}{4}$	
1.6	Substituting $x = \frac{1}{4}$ into $y = 2x^2 - x - 3$, $y = -\frac{25}{8}$. Turning point is $\left(\frac{1}{4}; -\frac{25}{8}\right)$	
1.7	Domain is $x \in \mathbb{R}$ and range is $\geq -\frac{25}{8}$; $y \in \mathbb{R}$	

QUESTION 2		
2.1	$p = -1$; $q = -8$ and $a = 4$	
2.2	$d = -2$; $t = -\frac{3}{2}$ and $k = 3$	
2.3	$0 \leq x \leq 1$	
2.4	Domain is $x \in \mathbb{R}$ and range is $y \geq -8$; $y \in \mathbb{R}$	
2.5	$y = \pm\left(x - \frac{3}{2}\right) - 2 \Rightarrow y = x - \frac{7}{2}$ or $y = -x - \frac{1}{2}$	

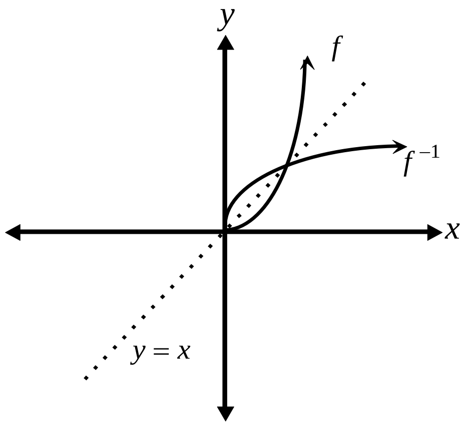
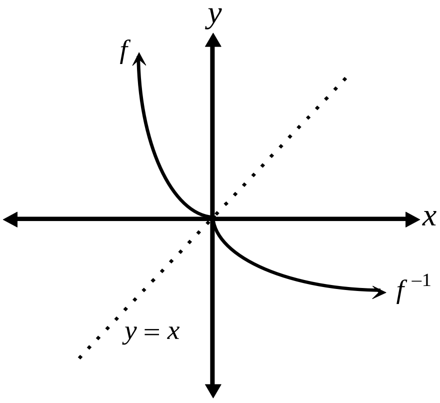
QUESTION 3	
	

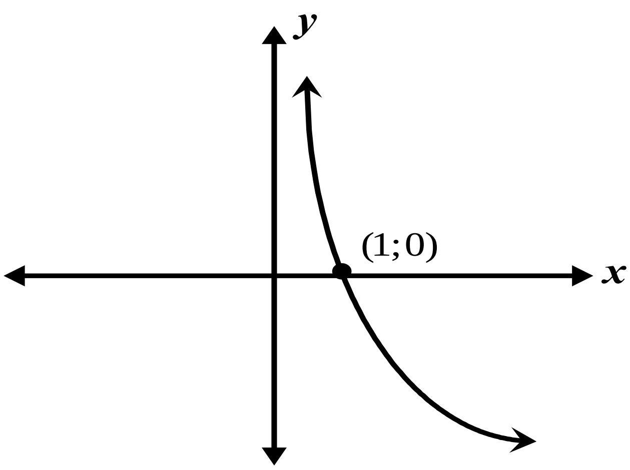
QUESTION 4		
4.1	$B(-1;-4)$	
4.2	$x=-3$ and $y=-4$	
4.3	$A(0;-3)$	
4.4	$g(x)=\frac{1}{x+1}-4$	
4.5	$y=\pm(x+1)-4$ that is $y=x-3$ *or* $y=-x-5$	
4.6	Range of f is $y\geq-4$; $y\in\mathbb{R}$ Range of - f^{-1} is $y\leq-4$; $y\in\mathbb{R}$	

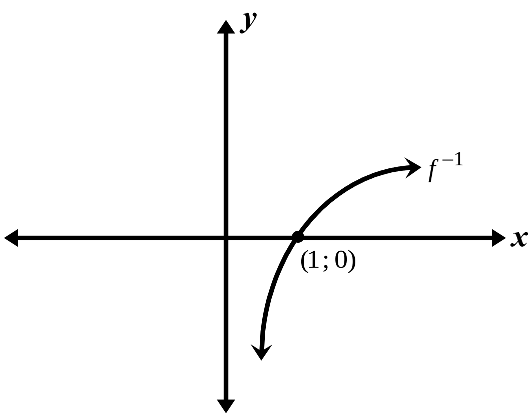
QUESTION 5		
5.1	$y = -8$	
5.2		
5.3	$g(x)=2^{-x+1}-8$ **OR** $g(x)=\left(\frac{1}{2}\right)^{x-1}-8$	

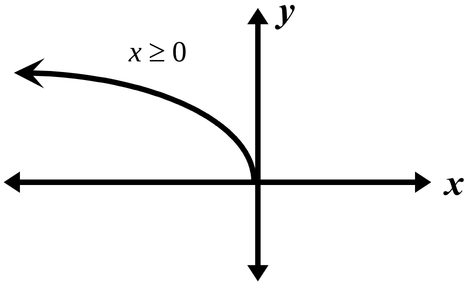
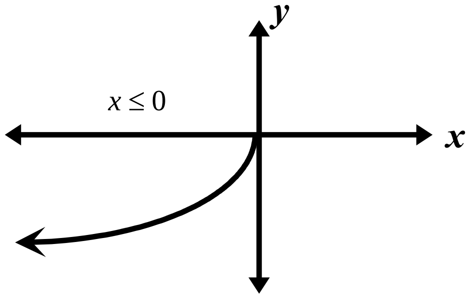
SESSION NO 4

TOPIC : INVERSE FUNCTIONS

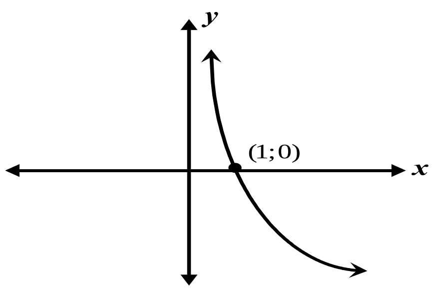
QUESTION 1		
1.1	$f(x)=2x^2$ where $x \geq 0$ OR $f(x)=2x^2$ where $x \leq 0$	
1.2	 	

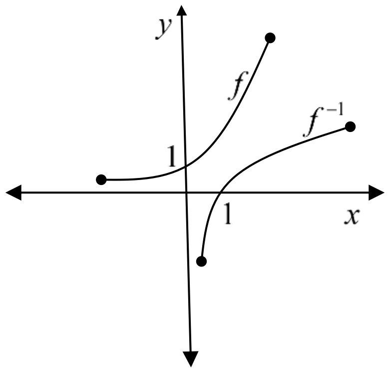
1.3	$y=\left(\frac{1}{2}\right)^x$ $x=\left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x)=\log_{\frac{1}{2}} x$	
1.4		
1.5	$\log_{\frac{1}{2}} x<0$ for $x>1$	

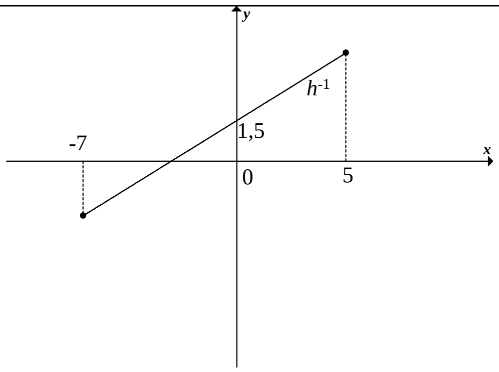
QUESTION 2		
2.1	$y=3^x$ $\therefore x=3^y$ $\therefore \log_3 x = y$ $\therefore f^{-1}(x)=\log_3 x$	
2.2		
2.3	Domain: $x \in (0\,;\infty)$	
2.4	The inverse is a one-to-many relation, which is not a function.	

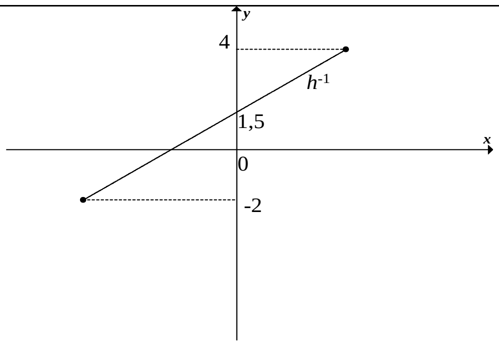
2.5.1	$x \geq 0$ OR $x \leq 0$	
2.5.2	 	

QUESTION 3		
3.1	$m_g = 2$	
3.2	Turning point is $(1;-9)$	
3.3	Range is $y \geq -9$; $y \in \mathbb{R}$	
3.4	$h(x) = (x+4)(x-2)$	
3.5	$k(x) = -f(x) = -(x-4)(x+2) = -x^2 + 2x + 8$	
3.6	$g^{-1}(x) = y = \frac{1}{2}x + 6$	

QUESTION 4		
4.1	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x) = \log_{\frac{1}{2}} x$	
4.2		
4.3	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	

QUESTION 5		
5.1	f is increasing. $\therefore$ range is $2^{-3} \leq y \leq 2^{3}$ that is $\frac{1}{8} \leq y \leq 8$	
5.2	$y = \log_2 x$	
5.3	Domain is $\frac{1}{8} \leq x \leq 8$ and range is $-3 \leq y \leq 3$	
5.4		
5.5	It is a function because there is only one *y* value for each value of x	

QUESTION 6		
6.1	For *x*-intercepts, *y* = 0 $2x - 3 = 0$ $x = 1{,}5$ $Q(1{,}5\ ;\ 0)$	
6.2	$h:$ $x = -2: \quad y = 2(-2) - 3 = -7$ $x = 4: \quad y = 2(4) - 3 = 5$ Domain of $h^{-1}: -7 \leq x \leq 5$ **OR/*OF*** $[-7;\ 5]$	
6.3		

		
6.4	$h(x)=2x-3$ For the inverse of *h*, $x=2y-3$ $y=\frac{x+3}{2}$ $h(x)=h^{-1}(x)$ $2x-3=\frac{x+3}{2}$ $4x-6=x+3$ $3x=9$ $x=3$ OR $h(x)=2x-3$ *h* and h^{-1} intersect when $y=x$ $h(x)=x$ $2x-3=x$ $x=3$ OR $h(x)=2x-3$ For the inverse of *h*, $x=2y-3$ $y=\frac{x+3}{2}$	

	$h^{-1}(x)=x$ $\frac{x+3}{2}=x$ $x+3=2x$ $x=3$	
6.5	$OP^2=(x-0)^2+(y-0)^2$ $=x^2+(2x-3)^2$ $=x^2+4x^2-12x+9$ $=5x^2-12x+9$ For OP to be at its minimum, OP^2 has to be a minimum $x=-\frac{b}{2a}$ $=-\frac{-12}{2(5)}$ $\therefore x=\frac{6}{5}$ Minimum length of OP $=\sqrt{5\left(\frac{6}{5}\right)^2-12\left(\frac{6}{5}\right)+9}=\sqrt{\frac{9}{5}}$ or $\frac{3}{\sqrt{5}}$ or 1,34 units **OR** $m_h=2$ (given) $m_{OP}=\frac{-1}{2}$ $\therefore$ OP has equation $y=\frac{-1}{2}x$ $\frac{-1}{2}x=2x-3$ $-x=4x-6$ $5x=6$ $x_P=1{,}2$ $y_P=-\frac{1}{2}(1{,}2)=-0{,}6$ $OP=\sqrt{(1{,}2-0)^2+(-0{,}6-0)^2}$ $=1{,}34$ or $\sqrt{1{,}8}$ units OR	

	$O(0;0) \quad P(x;\, 2x-3) \quad Q\left(\frac{3}{2};0\right)$ $$OP^2 + PQ^2 = OQ^2 \quad \text{(pythag)}$$ $$(x-0)^2 + (2x-3-0)^2 + \left(x-\frac{3}{2}\right)^2 + (2x-3-0)^2 = \left(\frac{3}{2}\right)^2$$ $$x^2 + 4x^2 - 12x + 9 + x^2 - 3x + \frac{9}{4} + 4x^2 - 12x + 9 = \frac{9}{4}$$ $$10x^2 - 27x + 18 = 0$$ $$(5x-6)(2x-3) = 0$$ $$x = \frac{6}{5} \;\; or \;\; \frac{3}{2}$$ Hence, $x = \frac{6}{5}$ at P $$OP^2 = x^2 + (2x-3)^2$$ $$= \left(\frac{6}{5}\right)^2 + \left(2\left(\frac{6}{5}\right) - 3\right)^2$$ $$= \frac{36}{25} + \frac{9}{25}$$ $$= \frac{9}{5}$$ $$OP = 1{,}34$$ **OR** $$\tan \hat{Q} = 2$$ $$\hat{Q} = 63{,}43^\circ$$ $$\sin 63{,}43^\circ = \frac{OP}{1{,}5}$$ $$OP = 1{,}34$$	