SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2021

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12

SUBJECT: MATHEMATICS

LEARNER NOTES

TABLE OF CONTENTS

SESSION NO: 5

TOPIC: SEQUENCE AND SERIES

Teaching Tips

Introduction

- The teacher introduces the topic and bring to the learners attention the weighting of the topic in the exam at the end of a year.

Sequence and Series

- The teacher reminds learners about the formulae .
- Refer to the Notes in the manual below.
- Explain the importance of understanding the question before answering.
- Before answering the question ascertain the type of sequence if not told.

Learners to work more on exercises and then feedback can be done.

NOTES ON CONTENT

ARITHMETIC Sequence	Arithmetic Series	GEOMETRIC Sequence	GEOMETRIC Series
$T_n = a + (n-1)d$ $T_{23} = a + 22d$	$S_n = \frac{n}{2}[2a + (n-1)d]$ **OR** $S_n = \frac{n}{2}[a + l]$	$T_n = a.r^{n-1}$ $T_{23} = a.r^{22}$	$S_n = \frac{a(1-r^n)}{1-r}$ **OR** $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ Sum to Infinity $S_\infty = \frac{a}{1-r}$; $if\ -1 < r < 1$
Which term equals 210? $210 = a + (n-1)d$ (solve n)		Which term equals 8? $8 = a.r^{n-1}$	
$T_1 = a$		$T_1 = a$	
$T_2 - T_1 = d$ common difference		$\frac{T_2}{T_1} = r$ common ratio	
Test for AS: $T_2 - T_1$ has to be equal to $T_3 - T_2$		Test for GS: $\frac{T_2}{T_1}$ *has to be equal to* $\frac{T_3}{T_2}$	
To solve for x, the equation is: $T_2 - T_1 = T_3 - T_2$		To solve for x the equation is: $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ *and apply cross products* or multiply by the LCD.	
These two equations are often used in simultaneous equations in this section: $T_2 - T_1 = T_3 - T_2$ (1) $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ (2)			
QUADRATICS NUMBER PATTERN $T_n = an^2 + bn + c$			

QUESTION 1

1.1 Given that $5\,;24\,;55\,;98\,;\ldots;874$, is a quadratic sequence.

1.1.1 Determine the general term of the above sequence. (4)

1.1.2 Express $5+24+55+98+\cdots+874$ in sigma notation. (2)

1.2 Evaluate: $\sum_{k=-2}^{5} 5\left(\frac{1}{2}\right)^{1-k} + \sum_{k=-2}^{\infty} 5\left(\frac{2}{3}\right)^{k}$. (Give your answer to a whole number) (9)

[15]

QUESTION 2

Given : $\sum_{r=0}^{x} 108\left(\frac{2}{3}\right)^{r}$

2.1 Determine the first TWO terms. (2)

2.2 If $\sum_{r=0}^{x} 108\left(\frac{2}{3}\right)^{r} = \frac{25220}{81}$, determine the value of x. (4)

[6]

QUESTION 3

3.1 Given the quadratic sequence 1; 6; 15; 28; …

3.1.1 Write down the second difference. (1)

3.1.2 Determine the *n*th term. (4)

3.1.3 Calculate which term of the sequence equals 2701. (3)

3.2 Given the arithmetic series: 10 + 15 + 20 + 25 + … + 185

3.2.1 How many terms are there in the series? (3)

3.2.2 Calculate the sum of all the natural numbers from 10 to 185 that are NOT divisible by 5. (6)

[17]

QUESTION 4

4.1 $T_n = \frac{1}{2}(r)^{n-1}$ is the general term of a geometric sequence.

4.1.1 Calculate the value of the common ratio if the fifth term is 40,5. (3)

4.1.2 Determine the position of the term in the sequence that has a value of $\frac{59049}{2}$. (3)

4.2 Both the arithmetic and geometric sequences have the same first term equal to 8. The common difference of an arithmetic sequence is equal to the common ratio of the geometric sequence. The fifth term of the geometric sequence is 2048.

4.2.1 Calculate the sum of the first five terms of the arithmetic sequence. (5)

4.2.2 Hence, express the sum of the first five terms of the arithmetic sequence in **4.2.1** above in sigma notation. (3)

[14]

QUESTION 5

5.1 Given the arithmetic sequence: $-3; 1; 5; \ldots; 393$.

5.1.1 Determine a formula for the n^{th} term of the sequence. (2)

5.1.2 Write down the 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of the sequence. (2)

5.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)

5.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)

5.2 Consider the following pattern of dots:

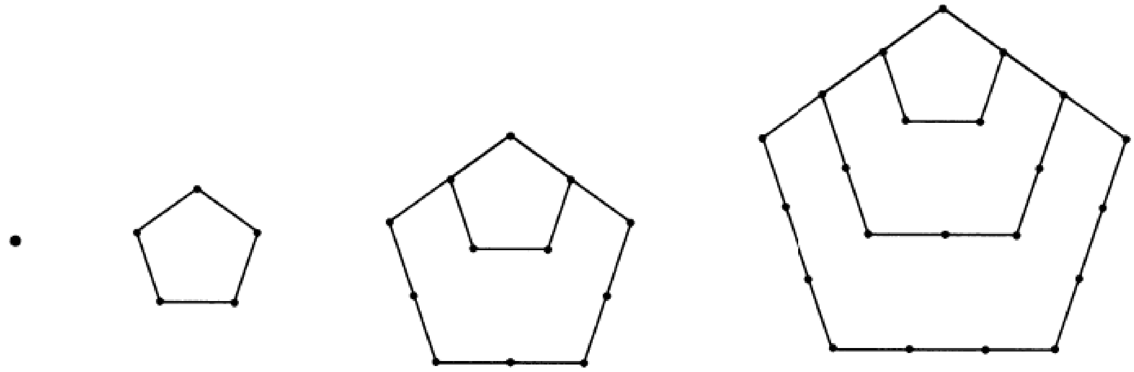


FIGURE 1 FIGURE 2 FIGURE 3 FIGURE 4

If T_n represents the total number of dots in FIGURE n, then $T_1 = 1$ and $T_2 = 5$.

If the pattern continues in the same manner, determine:

5.2.1 T_5 (2)

5.2.2 T_{50}. (5)

[18]

QUESTION 6

6.1 Given the arithmetic sequence: $w-3\,;2w-4\,;23-w$.

6.1.1 Determine the value of w. (2)

6.1.2 Write down the common difference of this sequence. (1)

6.2 The arithmetic sequence $4\,;10\,;16\,;\ldots$ is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50^{th} term of the quadratic sequence. (5)

[8]

QUESTION 7

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

7.1 Calculate the value of p. (4)

7.2 Calculate the second term of the series. (4)

[8]

QUESTION 8

8.1 Given the geometric series: $265 + p + 64 - 32 + \cdots$

8.1.1. Determine the value of p. (3)

8.1.2. Calculate the sum of the first 8 terms of the series. (3)

8.1.3. Why does the sum to infinity for this series exist? (1)

8.1.4. Calculate S_∞. (3)

8.2. Consider the arithmetic sequence: $-8\,;\,-2\,;4\,;10\,;\ldots$

8.2.1 Write down the next term of the sequence. (1)

8.2.2 If the nth term of the sequence is 148, determine the value of n. (3)

8.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

8.3 Calculate $\sum_{k=1}^{30}(3k+5)$. (3)

[22]

QUESTION 9

Consider the sequence: $3\,;9\,;27\,;\ldots$

Jacob says that the fourth term of the sequence is 81.

Vusi disagrees and says that the fourth term of the sequence is 57.

9.1 Explain why Jacob and Vusi could both be correct. (2)

9.2 Jacob and Vusi continue with their number patterns.

Determine a formula for the nth term of:

9.2.1 Jacob's sequence. (1)

9.2.2 Vusi's sequence. (4)

[7]

QUESTION 10

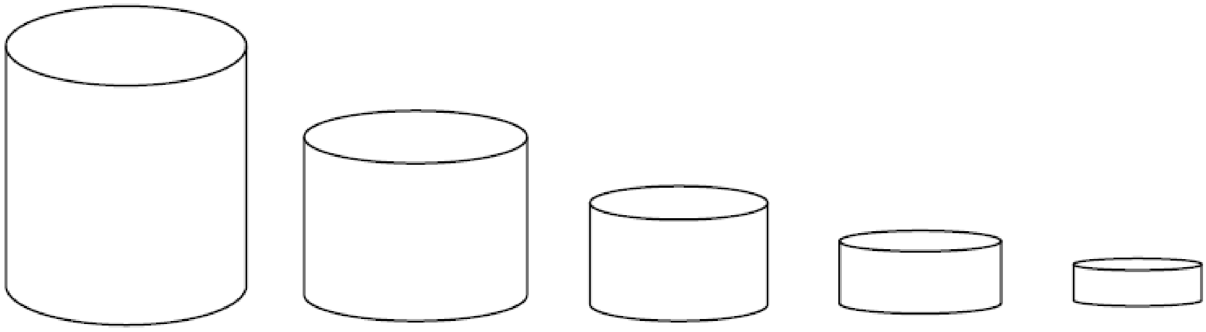
10.1 Given the geometric sequence: $27\,;9\,;3\,;\ldots$

10.1.1 Determine a formula for T_n, the nth term of the sequence. (2)

10.1.2 Why does the sum to infinity for this sequence exist? (1)

10.1.3 Determine S_∞. (2)

10.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is the $\frac{1}{2}$ volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would if be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

10.3 The n[th] term of a sequence is given by $T_n = -2(n-5)^2 + 18$.

10.3.1 Write down the first **THREE** terms of the sequence. (3)

10.3.2 Which term of the sequence will have the greatest value? (1)

10.3.3 What is the second difference of this quadratic sequence? (2)

10.3.4 Determine ALL values of n for which the terms of the sequence will be less than -110. (6)

[21]

SESSION NO: 6

TOPIC: SEQUENCE AND SERIES
Teaching Tips

Introduction

- The teacher introduces the topic and bring to the learners attention the weighting of the topic in the exam at the end of a year.

Sequence and Series

- The teacher reminds learners about the formulae .
- Refer to the Notes in the manual below.
- Explain the importance of understanding the question before answering.
- Before answering the question ascertain the type of sequence if not told.

Learners to work more on exercises the feedback will be discussed .
Time management is again critical

QUESTION 1

1.1 A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

1.1.1 The common ratio. (1)

1.1.2 A formula for T_n. (3)

1.2 The following sequence has the property that the sequence of numerators are arithmetic and the sequence of denominators is geometric:

$$\frac{2}{1} ; \frac{-1}{5} ; \frac{-4}{25} ; \ldots$$

1.2.1 Write down the FOURTH term of the sequence. (1)

1.2.2 Determine the formula for the nth term. (3)

1.2.3 Determine the 500th term of the sequence. (2)

1.2.4 Which term of the sequence will have a numerator which is less than -59? (3)

[13]

QUESTION 2

The sequence $3;\ x;\ 25$ is a quadratic sequence. The sequence of first differences is $9;\ y;\ \ldots$

2.1 Calculate x and y. (2)

2.2 Determine the nth term of the quadratic sequence. (4)

[6]

QUESTION 3

3.1 A cyclist training for the Argus cycle tour does 100 km during the first week. Thereafter, the distance he covers each week is 10% more than that of the previous week.

3.1.1 Determine the distance cycled by the cyclist in the eighth week. (3)

3.1.2 Determine the total distance cycled by the cyclist in the first eight weeks. (3)

3.1.3 Rewrite question **3.1.2** in sigma notation. (2)

3.2 In an arithmetic sequence the fifth term has a value of 0 and the fourteenth term has a value of -36.

3.2.1 Calculate T_1. (4)

3.2.2 Find the value of p if $T_{23} + T_{23-p} = -96$. (4)

3.3 $\frac{4}{3}+\frac{8}{9}+\frac{16}{27}+\cdots$ is an infinite geometric series. ...

3.3.1 Explain why the series converges. (2)

3.3.2 Hence evaluate the sequence below if it continues indefinitely (5)

$$\sqrt[3]{16} \times \sqrt[9]{256} \times \sqrt[27]{65536} \times \ldots$$

OCT/NOV 2017

QUESTION 2			
2.1	Given the following number pattern: $5\,;\,-4\,;\,-19\,;\,-40;\ldots\ldots$		
	2.1.1	Determine the constant second difference of the sequence	(2)
	2.1.2	Determine the nth term (T_n) of the pattern	(4)
	2.1.3	Which term of the pattern will be equal to -25939?	(3)
2.2	The first three terms of an arithmetic sequence are $2k-7; k+8$ and $2k-1$		
	2.2.1	Calculate the value of the 15th term of the sequence	(5)
	2.2.2	Calculate the sum of the first 30 even terms of the sequence	(4)
			[18]

QUESTION 3		
A convergent geometric series consisting of only positive terms has first term a, constant ratio r and nth term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.		
3.1	If $T_1 + T_2 = 2$, write down the expression of a in terms of r	(2)
3.2	Calculate the values of a and r	(6)

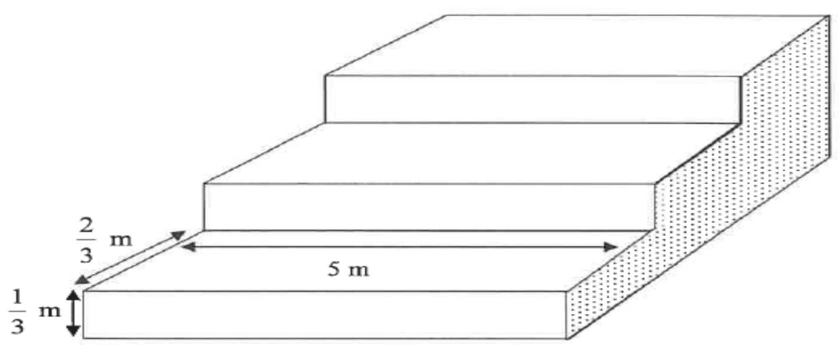
OCT/NOV 2018

QUESTION 2			
2.1	Given the quadratic sequence : $2; 3; 10; 23; \ldots \ldots \ldots$		
	2.1.1	Write down the next term of the sequence	(1)
	2.1.2	Determine the nth term of the sequence	(4)
	2.1.3	Calculate the 20th term of the sequence	(2)
2.2	Given the arithmetic sequence : $35 ; 28 ; 21 ; \ldots \ldots \ldots$ Calculate which term of the sequence will have a value of -140		(3)
2.3	For which values of n will the sum of the first n terms of the arithmetic sequence in QUESTION 2.2 be equal to the nth term of the quadratic sequence in QUESTION 2.1?		(6)
			[16]

QUESTION 3		
A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6		
3.1	Calculate the first term of the series	(2)
3.2	Calculate the 8th term of the series	(2)
3.3	Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$. Calculate the value of n.	(4)
3.4	If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{-k}$, in terms of p	(3)
		[11]

OCT/NOV 2019

QUESTION 2			
2.1	Given the quadratic sequence : $321\,;290\,;261\,;234;\ldots\ldots\ldots$		
	2.1.1	Write down the values of the next TWO terms of the sequence.	(2)
	2.1.2	Determine the general term of the sequence in the form $T_n = an^2 + bn + c$.	(4)
	2.1.3	Which term(s) of the sequence will have a value of 74?	(4)
	2.1.4	Which term in the sequence has the least value?	(2)
2.2	Given the geometric series: $\frac{5}{8}+\frac{5}{16}+\frac{5}{32}+\cdots = K$		
	2.2.1	Determine the value of K if the series has 21 terms.	(3)
	2.2.2	Determine the largest value of n for which $T_n > \frac{5}{8192}$	(4)
			[19]

QUESTION 3		
3.1	Without using a calculator , determine the value of :$\sum_{y=3}^{10}\frac{1}{y-2} - \sum_{y=3}^{10}\frac{1}{y-1}$	(3)
3.2	A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first three are shown on the diagram below. Each step is 5m wide. Each step has a rise of $\frac{1}{3}$m and has a tread of $\frac{2}{3}m$, as shown in the diagram below	
		
	The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area in (m^2) of metal sheeting needed to cover both open sides.	(6)
		[9]

OCT/NOV 2020

QUESTION 2			
2.1	$7\,;x\,;y\,;\ -11\,;...$ is an arithmetic sequence . Determine the values of x and y		(4)
2.2	Given the quadratic number pattern : $-3\,;6\,;27\,;60\,;...$		
	2.2.1	Determine the general term of the pattern in the form $T_n = an^2 + bn + c$.	(4)
	2.2.2	Calculate the value of the 50th term of the pattern.	(2)
	2.2.3	Show that the sum of the first n first – differences of this pattern can be given by $S_n = 6n^2 + 3n$.	(3)
	2.2.4	How many consecutive first difference were added to the first term of the quadratic number pattern to obtain a term in the quadratic number pattern that has a value of 21060?	(4)
			[17]

QUESTION 3		
3.1	Prove that $\sum_{k=1}^{\infty} 4.3^{2-k}$ is a convergent series.Show all your calculatios	(3)
3.2	If $\sum_{k=1}^{\infty} 4.3^{2-k} = \frac{2}{9}$, determine the value of p	(5)
		[8]

SESSION NO: 7

TOPIC: EUCLIDEAN GEOMETRY (REVISION OF GR 11 CIRCLE GEOMETRY)

Teaching Tips

Introduction

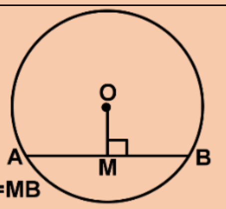
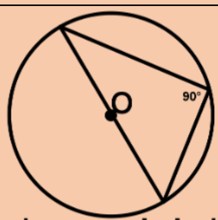
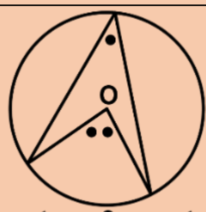
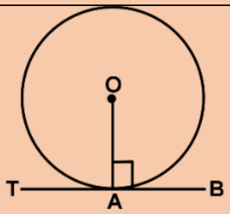
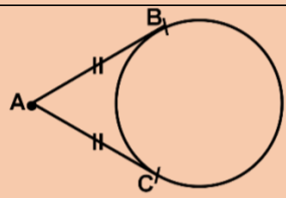
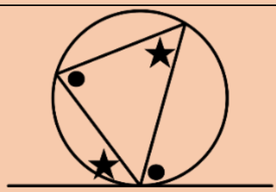
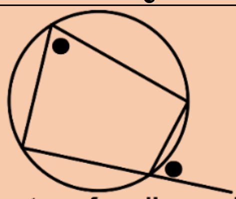
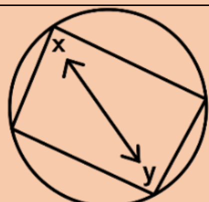
- The teacher introduces the topic and bring to the learners attention the weighting of the topic in the exam at the end of a year.

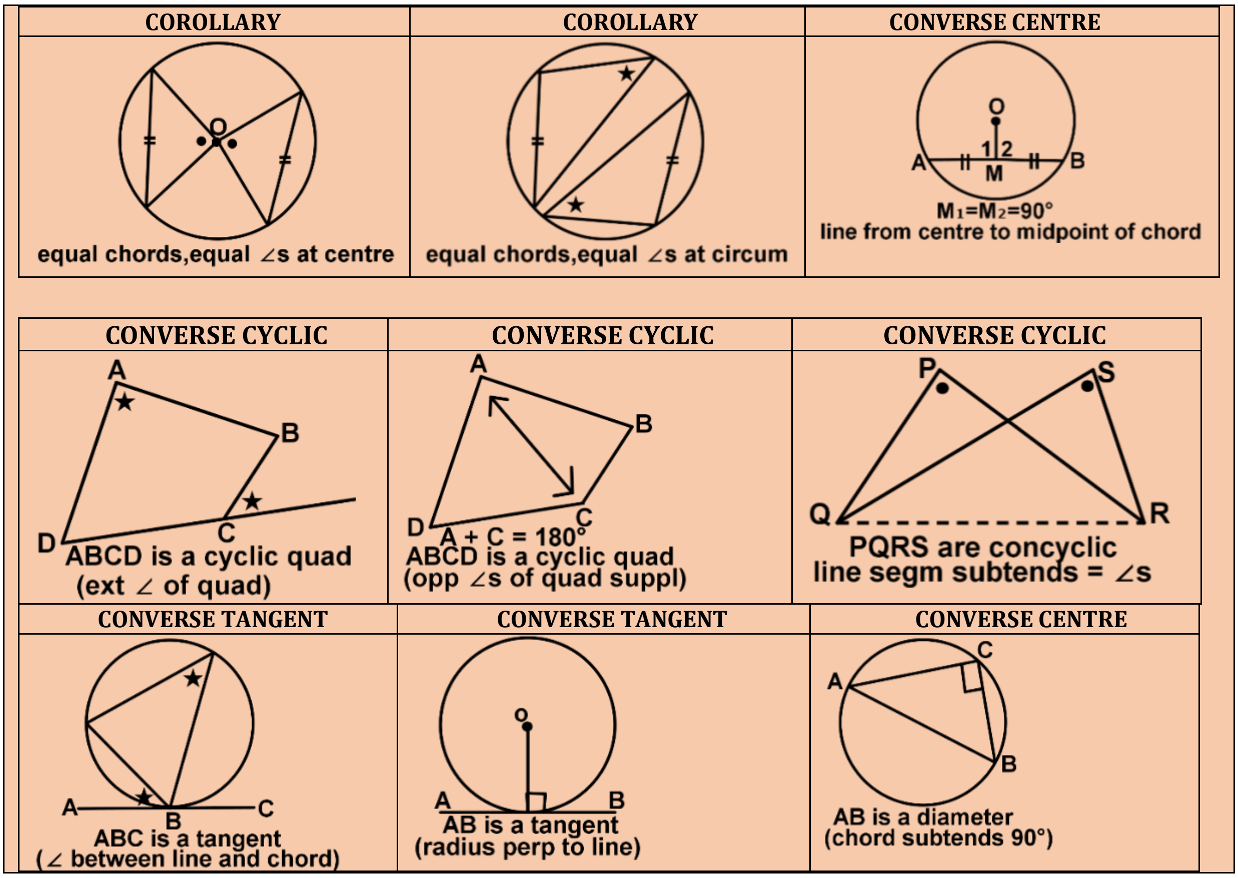
Euclidean Geometry

- The teacher reminds learners about the theorems and the converses.
- Refer to the Notes in the manual.
- Explain the diagram analysis on page before learners start with the activities.
- On the examples on diagram analysis learners to identify possible questions that could be asked.

Learners to work more on exercises the feedback will be discussed .
Time management is again critical

SUMMARY OF GRADE 11 THEOREMS

CENTRE THEOREMS		
 AM =MB line from centre perp to chord	 ∠ in a semi circle	 ∠ at centre = 2 x ∠ at circum
TANGENT THEOREMS		
 tangent ⊥ radius	 AB = AC ..tangents from same point	 tan chord theorem
CYCLIC QUADS		
 ∠s in the same segment	 ext ∠ of cyclic quad	 $x + y = 180°$ opp ∠s of cyclic quad

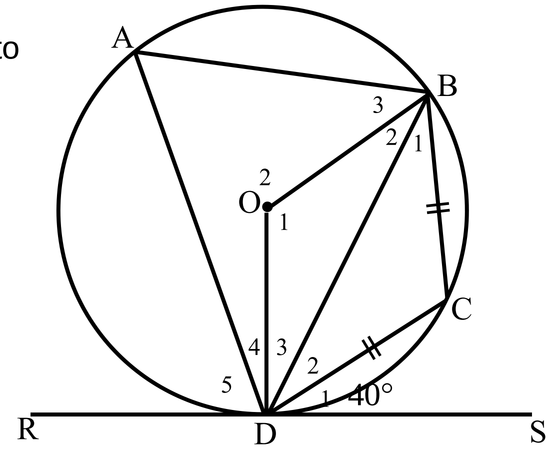


ACTIVITY

QUESTION 1

In the figure below, RDS is a tangent to the circle centre O at D.

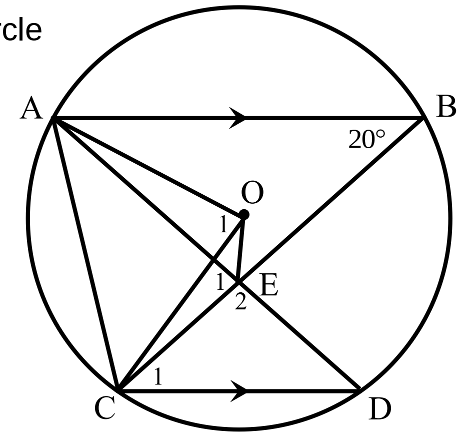
BC = DC and $C\hat{D}S = 40°$



1.1	Write down the size of $\hat{B}_1$. State a reason.	(2)
1.2	Write down the size of $\hat{D}_2$. State a reason.	(2)
1.3	Write down the size of $\hat{C}$. State a reason.	(2)
1.4	Calculate the size of $\hat{O}_2$ State a reason.	(2)
1.5	Calculate the size of $\hat{O}_1$. State a reason.	(2)
1.6	Calculate the size of $\hat{D}_3$ State reasons.	(3)
1.7	Calculate the size of $\hat{A}$ State a reason.	(2)

QUESTION 2

In the diagram, O is the centre of the circle passing through A, B, C and D.
AB||CD and $\hat{B} = 20°$



2.1 Calculate the size of $\hat{C}_1$? State a reason. (2)

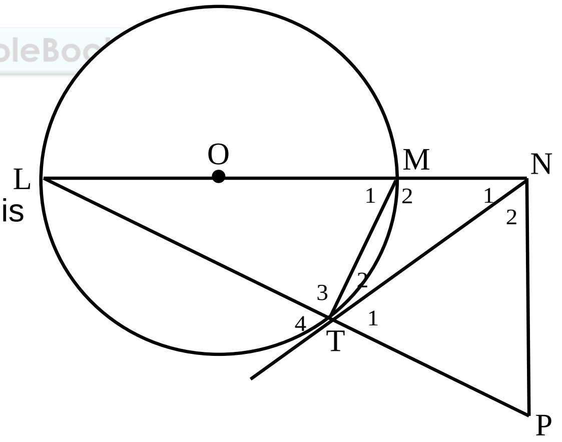
2.2 Calculate the size of $\hat{O}_1$? State a reason. (2)

2.3 Calculate the size of $\hat{D}$? State a reason. (2)

2.4 Calculate the size of $\hat{E}_1$? State a reason. (2)

2.5 Why is AOEC a cyclic quadrilateral? (1)

QUESTION 3



LOM is a diameter of circle LMT. The centre is O. TN is a tangent at T. $LN \perp NP$.
MT is a chord. LT is a chord produced to P.

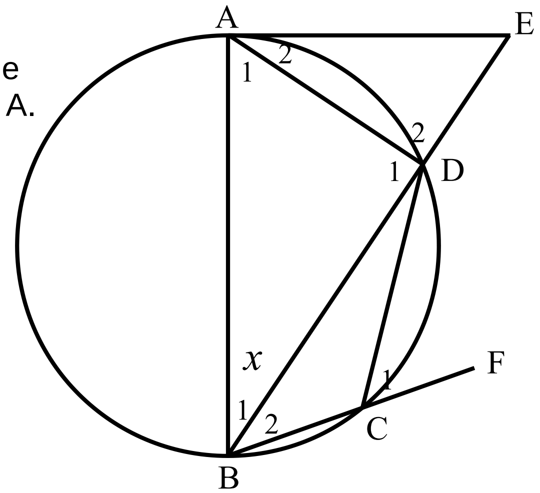
Prove that:

3.1 MNPT is a cyclic quadrilateral (4)

3.2 NP = NT (5)

QUESTION 4

In the diagram below, AB is a diameter of the circle ABCD. AE is a tangent to the circle at A. $\hat{B}_1 = x$.



4.1 Prove that AB is a tangent to the circle through A, D and E. (7)

4.2 Prove that $\hat{C}_1 = \hat{E}$ (2)

HOMEWORK QUESTIONS

QUESTION 1

In the diagram below, M is the centre of the circle. D, E, F and G are points on the circle. If $\hat{F}_1 = 10°$ and $\hat{D}_2 = 50°$, calculate, with reasons, the size of:

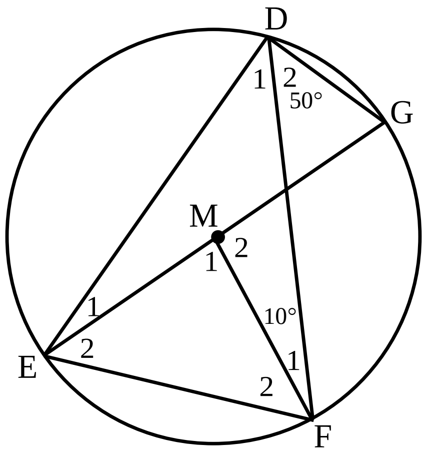
1.1 $\hat{D}_1$ (2)

1.2 $\hat{M}_1$ (2)

1.3 $\hat{F}_2$ (2)

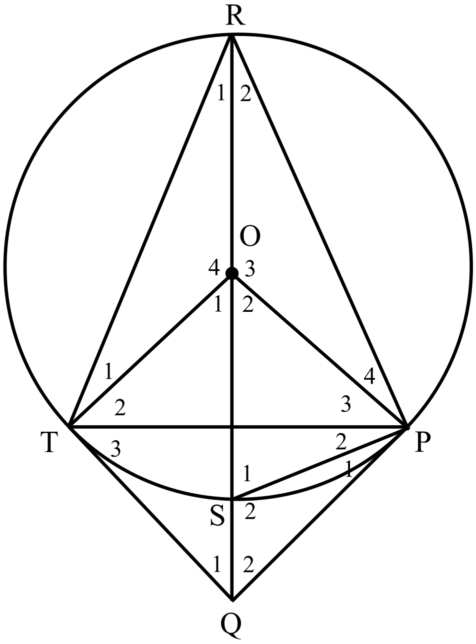
1.4 $\hat{G}$ (2)

1.5 $\hat{E}_1$ (2)



QUESTION 2

In the diagram below, QP is a tangent to a circle with centre O. RS is a diameter of the circle and RQ is a straight line. T is a point on the circle. PS bisects $T\hat{P}Q$ and $S\hat{P}Q = 22°$. Calculate the following, giving reasons:



2.1 $\hat{P}_2$ (2)

2.2 $\hat{R}_2$ (2)

2.3 $\hat{P}_3 + \hat{P}_4$ (3)

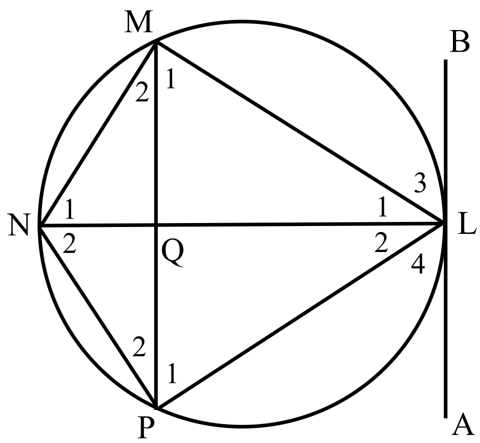
2.4 $\hat{R}_1$ (4)

2.5 $\hat{O}_1$ (3)

2.6 $\hat{Q}_2$ (3)

QUESTION 3

ALB is a tangent to circle LMNP. ALB||MP.
Prove that:



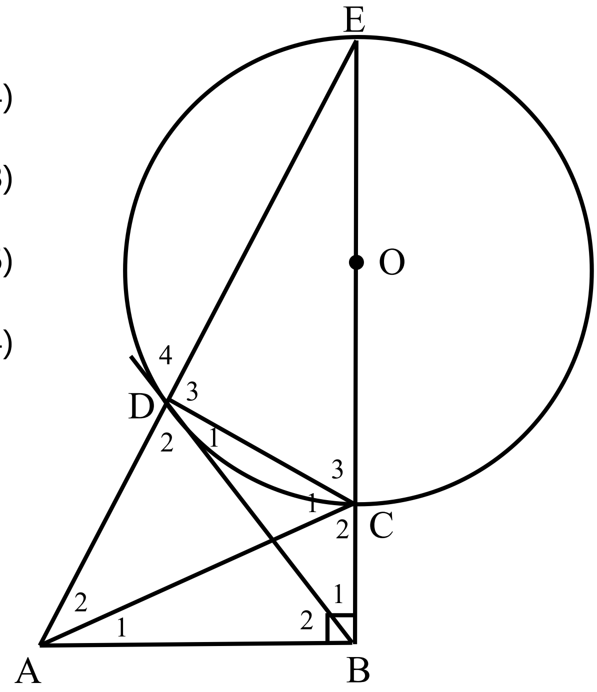
3.1 LM = LP (4)

3.2 LN bisects $M\hat{N}P$ (4)

3.3 LM is a tangent to circle MNQ (4)

QUESTION 4

EC is a diameter of circle DEC. EC is produced to B. BD is a tangent at D. ED is produced to A and $AB \perp BE$.
Prove that:



4.1 ABCD is a cyclic quadrilateral. (4)

4.2 $\hat{A}_1 = \hat{E}$ (3)

4.3 $BD = BA$ (5)

4.4 $\hat{C}_2 = \hat{C}_3$ (4)

SESSION NO: 8

TOPIC: EUCLIDEAN GEOMETRY (GR 12 PROPORTIONALITY AND SIMILARITY)
Teaching Tips

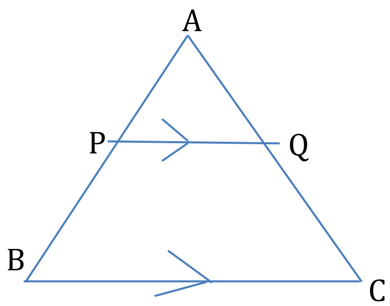
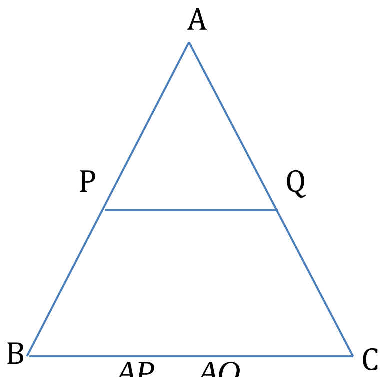
Introduction

- The teacher introduces the topic and bring to the learners attention the weighting of the topic in the exam at the end of a year.

Euclidean Geometry

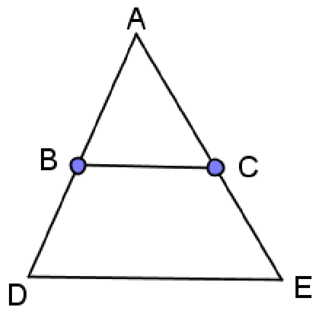
- The teacher reminds learners about the theorems and the converses.
- Refer to the Notes in the manual.
- Explain the diagram analysis before learners start with the activities.
- On the examples on diagram analysis learners to identify possible questions that could be asked.

Learners to work more on exercises the feedback will be discussed .
Time management is again critical

NOTES ON CONTENT	
PROPORTIONALITY	
Theorem A line drawn parallel to one side of a triangle divides the other two sides in the same proportion.  Given: Triangle ΔABC with PQ|| AB RTP: $\frac{AP}{PB}=\frac{AQ}{QC}$	**Converse Theorem** A line dividing two sides of a triangle proportionally is parallel to the third side.  Given: ΔABC with $\frac{AP}{PB}=\frac{AQ}{QC}$ RTP: PQ||BC

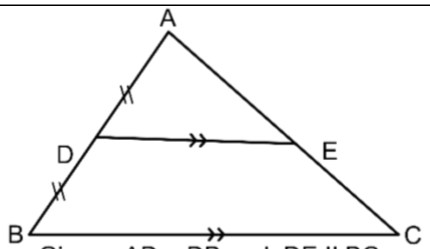
SPECIAL CASE ON PROPORTIONALITY

A line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the third side.(Mid-point Theorem



Given : B and C are the midpoints of AD and AE respectively , then $AB = BD \text{ and } AC = CE$

$\therefore BC \parallel DE$ and $BC = \frac{1}{2}DE$

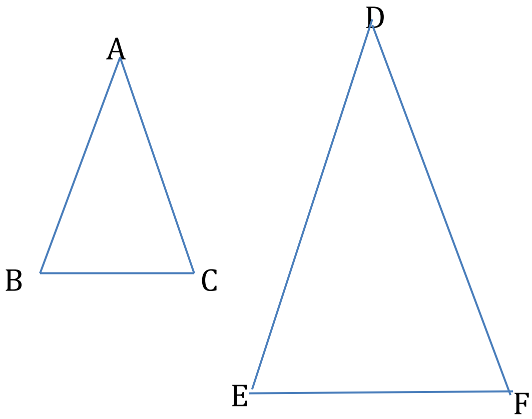


Given : AD = DB and DE II BC

Then $AE = EC$ and $DE = \frac{1}{2}BC$

SIMILARITY

If two triangles are equiangular to one another the lengths of their corresponding sides are proportional.

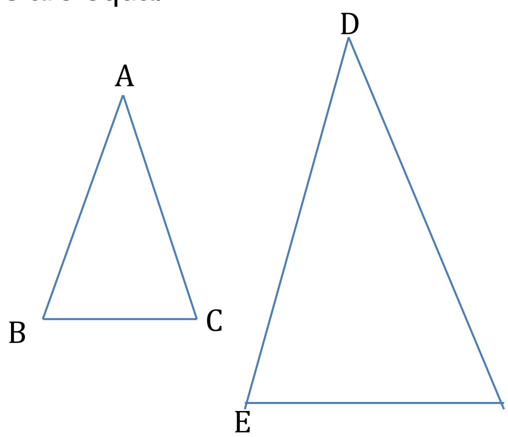


Given: ΔABC and ΔDEF with $\hat{A} = \hat{D}, \hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$

RTP: $\frac{AD}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Converse Theorem:

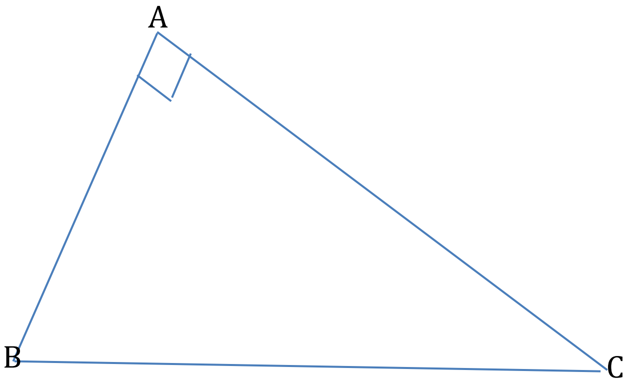
If the corresponding sides of two triangles are proportional, then their corresponding angles are equal

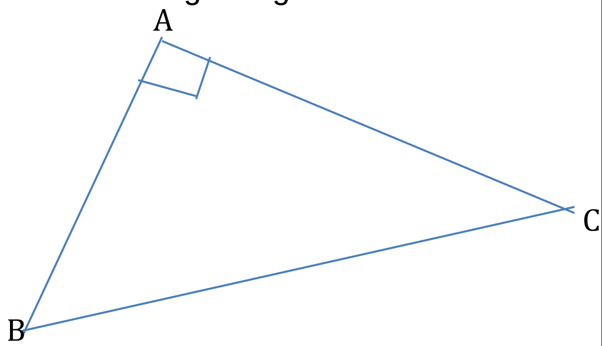


Given: ΔABC and ΔDEF with $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

RTP: $\hat{A} = \hat{D}, \hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$

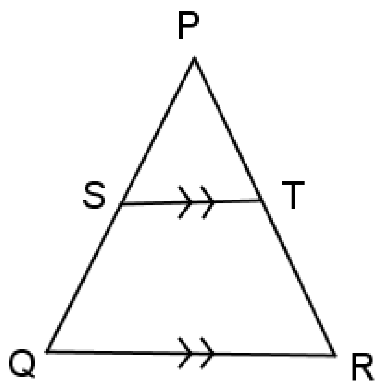
<table>
<tr><th colspan="2">PYTHAGORAS</th></tr>
<tr><td>Theorem

In a right-angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (Pyth)

Given: ΔABC with $\hat{A} = 90^\circ$
RTP: $BC^2 = AB^2 + AC^2$</td><td>Converse Theorem

If the square of one side of triangle equals the sum of the squares of the other two sides, then the angle contained by these two sides is a right angle.

Given: If in ΔABC , $BC^2 = AB^2 + AC^2$
RTP: $\hat{A} = 90^\circ$</td></tr>
</table>

QUESTION 1

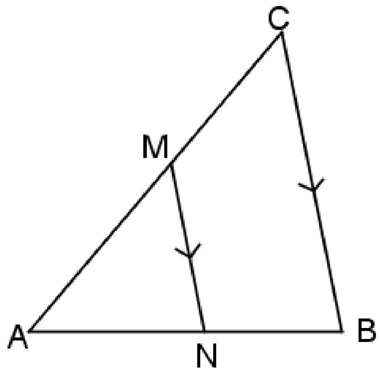
In the diagram below $ST \parallel QR. PS = 5cm, SQ = 2cm \text{ and } PT = 10cm.$



Calculate the length of TR. (3)

QUESTION 2

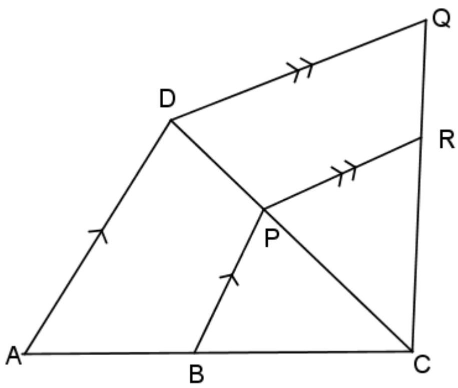
$\triangle ABC$ is given below. $AC = 35cm$, AN = 24 cm and NB = 18 cm



Calculate the length of CM (3)

QUESTION 3

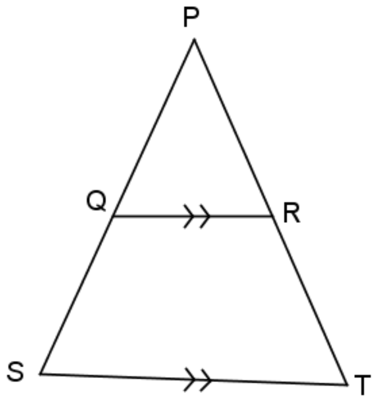
In $\triangle ACD$ $PB \parallel DA$ and in $\triangle CDQ$ $PR \parallel DQ$. AB = 22 cm ,BC = 33 cm and RC = 15 cm



Calculate the length of QR (5)

QUESTION 4

In $\triangle PST$, $QR \parallel ST . PQ : QS = 5 : 3$ and PS =32cm and PT = 24 cm



Calculate the lengths of the following:

4.1 PQ (3)

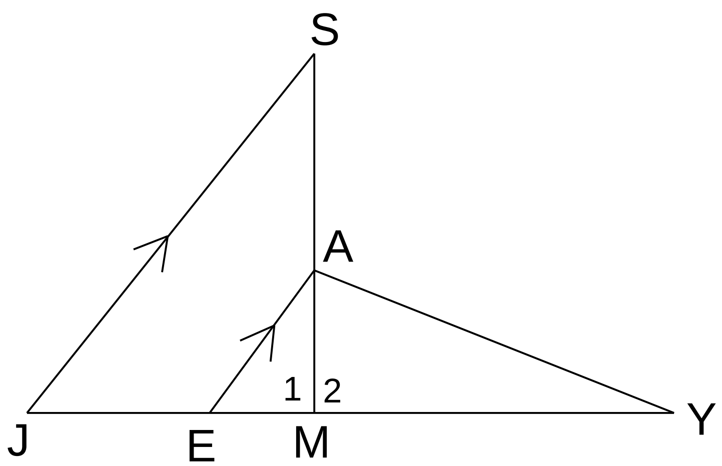
4.2 QS (2)

4.3 PR (2)

4.4 RT (1)

QUESTION 5

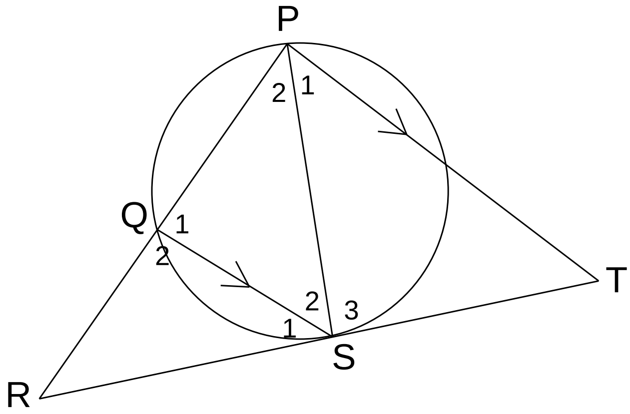
In the diagram *M* is a point on *JY* and *E* is a point on *JM*. $JS \parallel EA$ with *S* on *MA* produced. The diagram is not drawn to scale. *AM* = 6 cm , *JE* = 9 cm , *AS* = 12 cm and *JS* = 22,5 cm.



Calculate the length of EM (3)

QUESTION 6

In the diagram, RST is a tangent. Chord PQ produced meets the tangent at R. $QS \parallel PT$



Prove, giving reasons that:

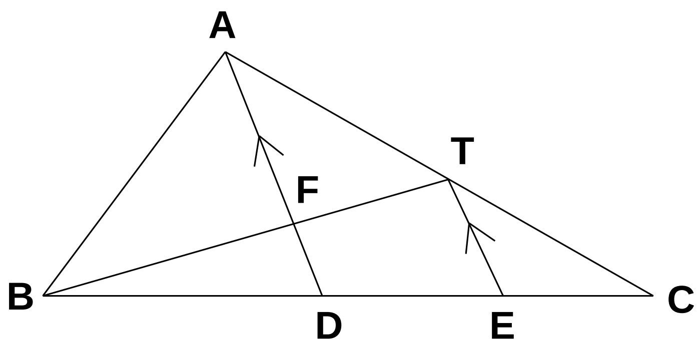
6.1 $\Delta SPQ \,|||\, \Delta PTS$ (5)

6.2 $SP^2 = SQ \,.\, PT$ (1)

6.3 $RS \,.\, RP = RQ \,.\, RT$ (4)

QUESTION 7

In the figure, ΔABC has D and E on BC. $BD = 6$ cm and $DC = 9$ cm. $AT : TC = 2 : 1$ and $AD \parallel TE$.



7.1 Write down the numerical value of $\frac{CE}{ED}$. (1)

7.2 Show that D is the midpoint of BE. (2)

7.3 If $FD = 2$ cm, calculate the length of TE. (2)

7.4 Calculate the numerical value of:

7.4.1 $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (1)

7.4.2 $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ (3)

SESSION NO: 9

TOPIC: TRIGONOMETRY (2D AND 3D)

	SESSION 2
Content	**Duration**
Introduction	15 minutes
trigonometry	30 minutes
Activity	35 minutes
Conclusion	10 minutes
	90 minutes

Learners to work more on exercises the feedback will be discussed .
Time management is again critical.

NOTES ON CONTENT

1. SOLVING PROBLEMS IN TWO AND THREE DIMENSIONS

Any triangle can be solved, if THREE properties of the triangle are given/known, by using:

- ✓ The trig ratios 10) in RIGHT-ANGLED triangles
- ✓ The sine or cosine rule

When to use what?

If the triangle is RIGHT-ANGLED

Use the trig RATIOS

Remember:
3 properties of a triangle must be given in a triangle in order to work in that triangle
*(**NOT** angle, angle, angle)*

If the triangle is NOT right-angled

When to use the rules:

COSINE RULE if ……
- *3 sides of the triangle are given*
- *2 sides and an included angle of the triangle is given*

SINE RULE if ……
- *Any condition that does NOT satisfy the cosine rule*

AREA RULE if ……
- *Only if "area" is mentioned*

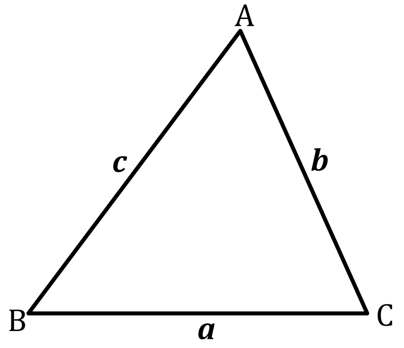
2. The Sine, Cosine and Area Rules

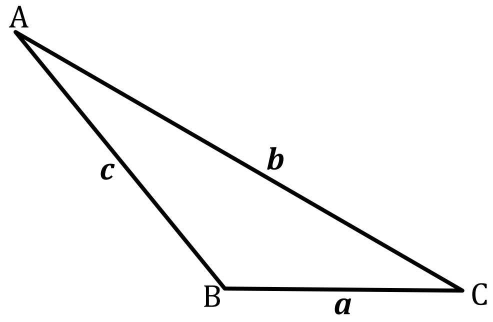
TYPES OF QUESTIONS:

- Numeric (calculations) problems
  Always start with these types of questions (2D and 3D) and make sure that learners master it before moving on to the next level, which is
- Non-numeric (prove type) problems

According to the CAPS document, learners must be able to:
- Establish (prove) the rules
- Apply the rules in solving 2D and 3D problems.

In any ΔABC the rules are applied as follow:





Sine

If an ANGLE is asked

$$\frac{sinA}{a}=\frac{sinB}{b}=\frac{sinC}{c}$$

If a SIDE is asked

$$\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}$$

Cosine

If an ANGLE is asked

- $cosA=\frac{b^2+c^2-a^2}{2bc}$
- $cosB=\frac{a^2+c^2-b^2}{2ac}$
- $cosC=\frac{a^2+b^2-c^2}{2ab}$

If a SIDE is asked

- $a^2=b^2+c^2-2bc\,.\,cosA$
- $b^2=a^2+c^2-2ac\,.\,cosB$
- $c^2=a^2+b^2-2ab\,.\,cosC$

Area

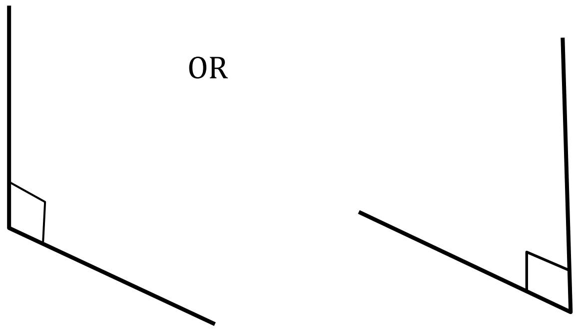
→ **Area of ΔABC** $=\frac{1}{2}ab.sinC$ or

$=\frac{1}{2}bc.sinA$ or

$=\frac{1}{2}ac.sinB$

TIPS FOR SOLVING 2D & 3D PROBLEMS
1. The diagram usually consists of 2 or more triangles with COMMON sides. 2. One of the triangles is often right-angled, so use the trig ratios to solve it. (In triangles without right angles, the sine, Cosine and Area rules must be applied.) 3. Make use of basic Geometry to obtain additional information, such as vertical opposite angles, interior angles of a triangle, etc. 4. In Grade 12, be on the lookout for Compound and Double angles when simplifying a problem. 5. Start in the triangle that contains the most information, then move along to the triangle in which the required line/angle is.

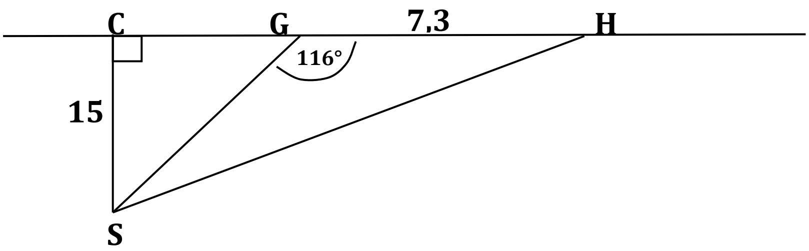
6. When solving problems in three dimensions:
   - It may help to shade the horizontal area
   - In the diagram, right angles may not look like right angles, e.g.



7. In applications, we often use angles of DEPRESSION and ELEVATION.
   Both are measured from the horizontal.

Worked Example 1

A soccer player (S) is 15 m from the back line of a soccer field (CH). She aims towards the goal (GH). The angle from the left goal post (G) to the soccer player is 116°. The goal posts are 7,32 metres apart. The diagram below represents the situation.



a) Calculate the size of $C\hat{G}S$.
b) Calculate SG, the distance between the soccer player and the left goal post FG.
c) Calculate the size of $G\hat{S}H$, the angle within which the soccer player could possibly score a goal.

SOLUTION:

a) $C\hat{G}S = 64°$ (angles on straight line CGH)

b) In ΔCGS , right-angled at C

ΔCGS is right-angled
$\therefore$ use the trig ratios

$$\sin C\hat{G}S = \frac{CS}{GS}$$
$$\therefore \sin 64° = \frac{15}{GS}$$
$$\therefore GS = \frac{15}{\sin 64°}$$
GS = 16,689 02911 … metres
$\therefore GS \approx$ 16,69 metres

c) In ΔSGH we know the lengths of two sides (SG and GH and the size of the included angle $S\hat{G}H$) so we use the cosine rule

$$SH^2 = GH^2 + SG^2 - 2.GH.SG\cos G$$
$$\therefore SH^2 = (7{,}32)^2 + (16{,}69)^2 - 2(7{,}32)(16{,}69)\cos 116°$$
$$\therefore SH = \sqrt{(7{,}32)^2 + (16{,}689\ 02911)^2 - 2(7{,}32)^2(16{,}689\ 02911)^2 \cos 116°}$$
$\therefore SH$ = 20,957 389 36
$\therefore SH \approx$ 20,96 metres

Enough information in ΔSGH is known to use either the sine or cosine rule to calculate $G\hat{S}H$.

Using the sine rule:

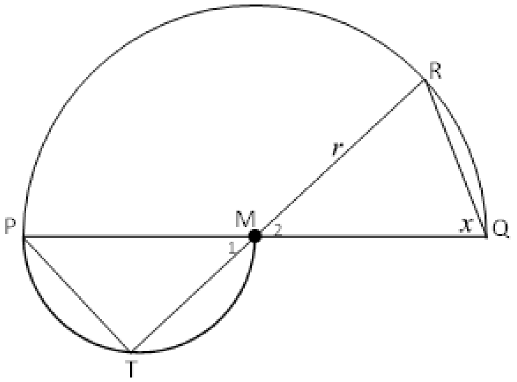
$$\frac{\sin G\hat{S}H}{GH} = \frac{\sin S\hat{G}H}{SH}$$
$$\therefore \frac{\sin G\hat{S}H}{7{,}32} = \frac{\sin 116°}{20{,}96}$$
$$\therefore \sin G\hat{S}H = \frac{7{,}32\sin 116°}{20{,}96}$$
$$\therefore G\hat{S}H = \sin^{-1}\left(\frac{7{,}32\sin 116°}{20{,}96}\right)$$
$\therefore G\hat{S}H$ = 18,293 926 57°
$\therefore G\hat{S}H \approx$ 18,3°

Using the cosine rule:

$$\cos G\hat{S}H = \frac{GS^2 + SH^2 - GH^2}{2.GS.SH}$$
$$\cos G\hat{S}H = \frac{(16{,}689\ 029\ 11)^2 + (20{,}957\ 389\ 36)^2 - (7{,}32)^2}{2\ (16{,}689\ 029\ 11)(20{,}957\ 389\ 36)}$$
$$\therefore G\hat{S}H =$$
$$\cos^{-1}\left(\frac{(16{,}689\ 029\ 11)^2 + (20{,}957\ 389\ 36)^2 - (7{,}32)^2}{2.(16{,}689\ 029\ 11)(20{,}957\ 389\ 36}\right)$$
$\therefore G\hat{S}H$ = 18,296 286 19°
$\therefore G\hat{S}H \approx$ 18,3°

Worked Example 2

In the diagram below, M is the centre of the semicircle PRQ and r is the radius. PM is the diameter of semicircle PTM. $\hat{Q} = x$.



$$RQ = \frac{r\sin 2x}{\sin x}$$
$$= \frac{r.2\sin x\cos x}{\sin x}$$
$$\therefore \underline{\boldsymbol{RQ = 2r\cos x}}$$

In ΔMRQ there is enough information to use the sine or cosine rule. The sine rule is an easier calculation than the cosine

a) $\hat{T} = 90°$ (angle in semi-circle OR diameter subtends a right angle)

Since ΔPMT is a right-angled triangle, we use the formula Area of triangle $\frac{1}{2} \times$ base × height or Area of $\Delta PMT = \frac{1}{2} \times PT \times MT$

$PM = r$ (radii)

$\hat{M}_2 = \hat{M}_1 = 180° - 2x$ (vertically opposite angles

$\sin \hat{M}_1 = \frac{PT}{PM}$

$\therefore \sin(180° - 2x) = \frac{PT}{r}$

$\therefore PT = r\sin(180° - 2x)$

$\therefore \boldsymbol{PT = r\sin 2x}$

$\cos \hat{M}_1 = \frac{MT}{PM}$

$\therefore \cos(180° - 2x) = \frac{MT}{r}$

$\therefore MT = r\cos(180° - 2x)$

$\therefore \boldsymbol{MT = -r\cos 2x}$

ΔPTM is right-angled
∴ use the trig ratios

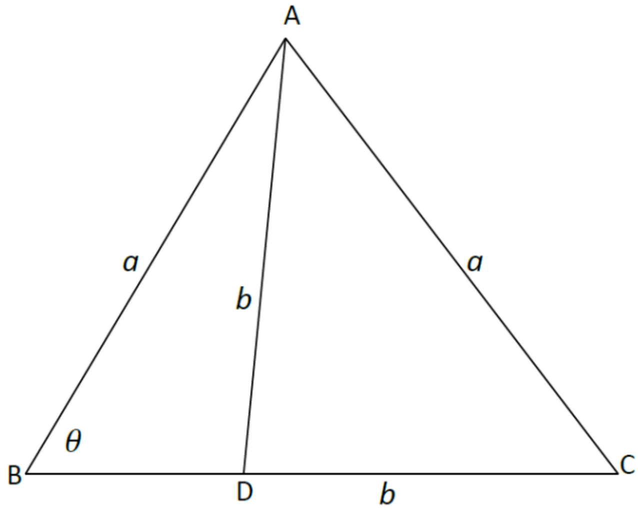
$$Area\ \Delta PMT = \frac{1}{2} \times PT \times MT$$
$$= \frac{1}{2} \times r\sin 2x \times (-r\cos 2x)$$
$$= -\frac{1}{2}r^2 \sin 2x \cos 2x$$
$$= -\frac{1}{2}r^2\left(\frac{1}{2}\sin 4x\right)$$
$$= -\frac{1}{4}r^2 \sin 4x$$

ACTIVITY: TRIGONOMETRY

QUESTION 1

1. In the diagram below, ABC is an isosceles triangle. D lies on BC. AB= $AC = a\ units$

   AD= $DC = b\ units\ \hat{B} = \theta$



1.1 Determine, without reasons, the size of $A\hat{D}C$ in terms of θ.

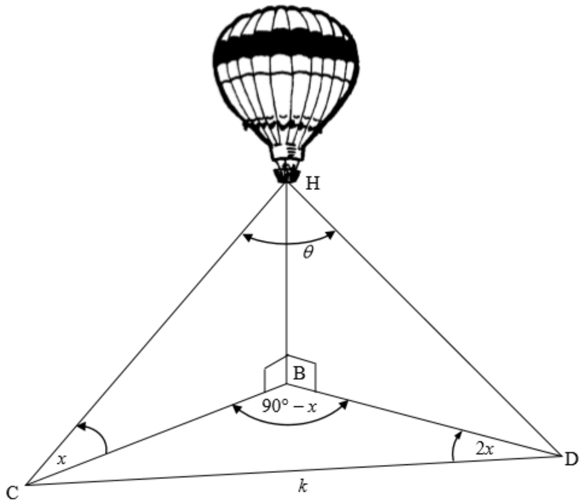
1.2 Prove that: $\cos 2\theta = \frac{a^2}{2b^2} - 1$

1.3 Hence, determine the value of θ it $a = 3$ and $b = 2$ (round off to two decimal digits)

QUESTION 2

A hot-air balloon H is directly above B on the ground. Two ropes are used keep the hot-air balloon in position. The ropes are held by two people on the ground at point C and point D. B, C and D are in the same horizontal plane.

The angle of elevation from C to H is *x*. $C\hat{D}B = 2x$ and $C\hat{B}D = 90° - x$.
The distance between C and D is *k* metres.



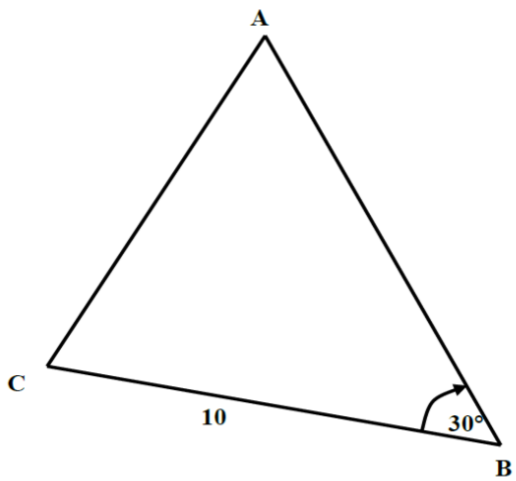
2.1 Show that $CB = 2k\sin x$.

2.2 Hence, show that the length of rope HC is $2k\tan x$.

2.3 If $k = 40\ m$, $x = 23°$ and $HD = 31{,}8\ m$, calculate θ, the angle between the two ropes

QUESTION 3

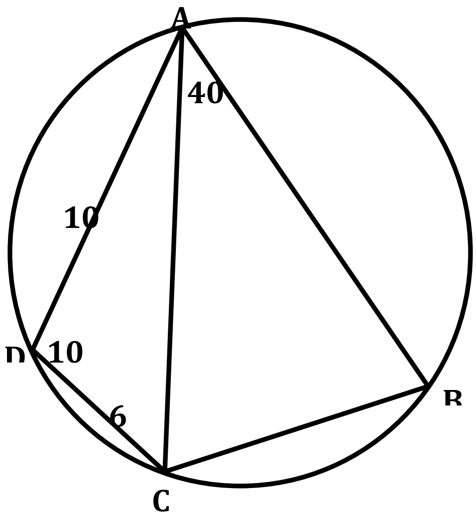
In the diagram, ΔABC is given with BC = 10 units, $\hat{B} = 30^0$ and $\sin(B + C) = 0.8$



Determine the length of AC, WITHOUT USING A CALCULATOR

QUESTION 4

In the diagram below, ABCD is a cyclic quadrilateral with DC = 6 units, AD = 10 units, $A\hat{D}C = 100°$ and $C\hat{A}B = 40°$.



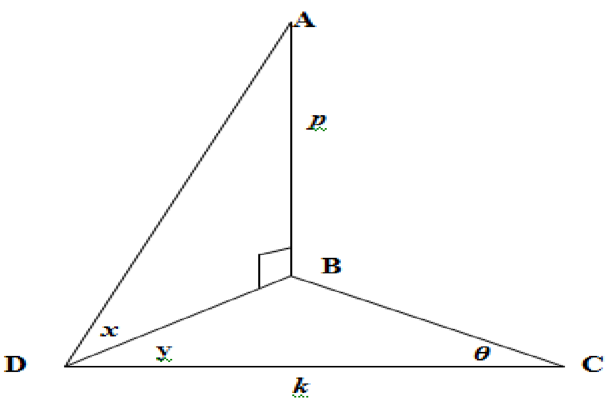
Calculate the following, correct to ONE decimal place:

4.1 The length of BC

4.2 The area of ΔABC

QUESTION 5

AB is a vertical tower *p* units high. D and C are in the same horizontal plane as B, the foot of the tower. The angle of elevation of A form D is *x*. $B\hat{D}C = y$ and $D\hat{C}B = \theta$.The distance between D and C is *k* units.



5.1 Express *p* in terms of DB and *x*.

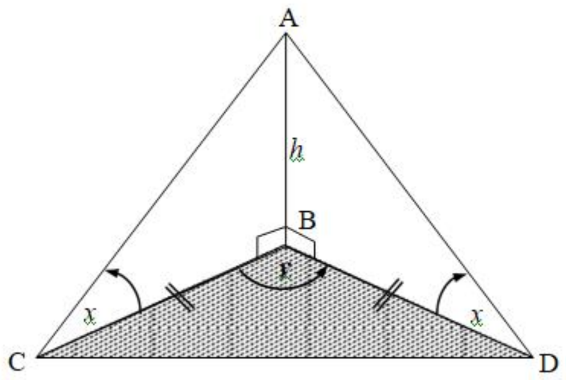
5.2 Hence prove that: $p = \dfrac{k \sin\theta \tan x}{\sin y \cos\theta + \cos y \sin\theta}$

5.3 Calculate BC to the nearest metre if $x = 51{,}7^0$, $y = 62{,}5^0$, $p = 80m$ and . $k = 95m$

QUESTION 6

In the diagram below, C and D represent two ships horizontal plane as B, the bases of a lighthouse AB which is h metres high. Each ship is the same distance away from the base of the lighthouse.

The angle of elevation from C and D to A are both x. $C\hat{B}D = y$

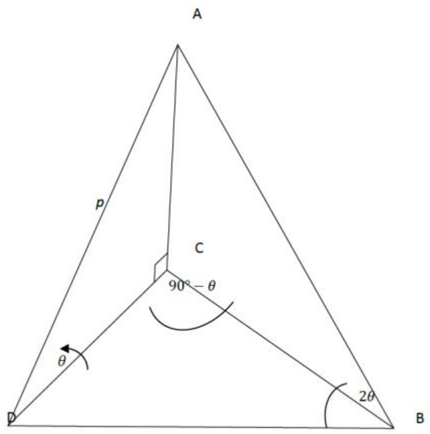


6.1 Write BD in terms of h and a trigonometric ratio of x.

6.2 Hence or otherwise prove $CD^2 = \dfrac{2h^2(1-\cos y)}{\tan^2 x}$

QUESTION 7

In the diagram below, D, B and C are points in the same horizontal plane. AC is a vertical pole and the length of the cable from D to the top of the pole, A, is p meters. AC ⊥ CD. $A\hat{D}C = \theta$; $D\hat{C}B = (90° - \theta)$ and $C\hat{B}D = 2\theta$.



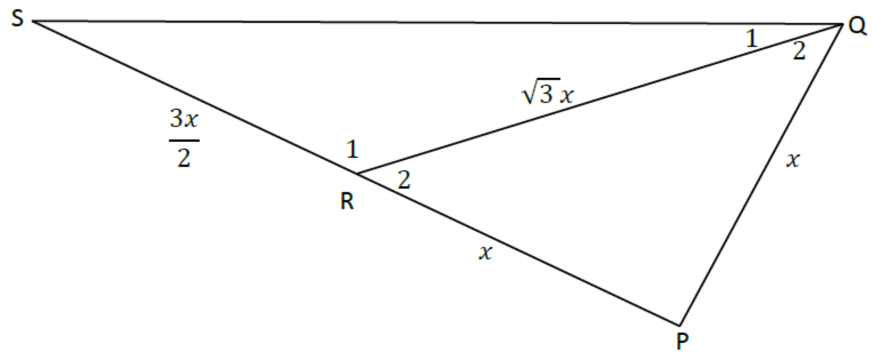
7.1 Prove that: $BD = \dfrac{p\cos\theta}{2\sin\theta}$

7.2.Calculate the height of the flagpole AC if θ = 30° and p = 3 meters.

7.3 Calculate the length of the cable AB if it is further given that $A\hat{D}B = 70^0$.

QUESTION 8

Triangle PQS represents a certain area of a park. R is a point on line PS such that QR divides the area of the park into two triangular parts, as shown below. PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = √3x units



8.1 Calculate the size of P.

8.2 Determine the area of triangle QRS in terms of x.

SESSION NO: 10

TOPIC: TRIGONOMETRY

	SESSION 2
Content	**Duration**
Introduction	15 minutes
Trigonometry: Functions	30 minutes
Activity	35 minutes
Conclusion	10 minutes
	90 minutes

Learners to work more on exercises the feedback will be discussed .
Time management is again critical

NOTES ON CONTENT

1. TRIGONOMETRIC FUNCTIONS

GRADE 10 AND GRADE 11 FUNCTIONS

*In **Grade** 10, the learners plot the basic graphs of* $y = \sin x$; $y = \cos x$; $y = \tan x$ *where* $x \in [0°; 360°]$. *In **Grade 11**, the learners plot graphs within the interval* $[-360°; 360°]$

- Show the learners how to **use the calculator** to draw the functions as it will help them to save time during the exams
- Revision of trigonometric functions should be done in grade 12.

TRANSFORMATIONS:

$$y = a\sin k(x - p) + q$$

$$y = a\cos k(x - p) + q$$

$$y = a\tan k(x - p) + q$$

- In **Grade 10** the learners investigate the effect of ***a*** and ***q***
- In **Grade 11** the learners investigate the effect of ***k*** and ***p*** but the Grade 10 content is needed and can be assessed in Grade 11 and 12.
- The parameters ***a***; ***p***; ***q*** and ***k*** affect **cos *x*** and **sin *x*** in the same way. The **tan *x*** graph behaves differently to both **sin *x*** and **cos *x*,**
- The function for **sin *x*** and **cos *x*** are **wave- like** shapes whereas **tan *x*** is a repeated curve shape.
- Because of the wave-shape of the graphs of ***sin x*** and ***cos x***, these two graphs have an **amplitude (*a*).** The **amplitude** is the height from the rest value ***q*** to the maximum or the minimum.
- All the three functions have a **period** which depends on the value of ***k.*** The **period** is the **length** required for the graph to make **one complete shape.**
- Knowing the features and the characteristics of the function will help in finding the equation and interpreting the graph.
- $y = \tan x$ has **asymptotes**, and they should not be part of the domain i.e. $x \neq 90^\circ + 180^\circ k$

Worked Example 1

Draw the following sets of graphs on the same set of axes and investigate the effect of parameter ***a****;* ***p****;* ***q*** and ***k*** on the graphs.

a) $f(x) = \sin x$, $g(x) = \sin 2x$, $l(x) = \sin 3x$; for $x \in [0°; 360°]$

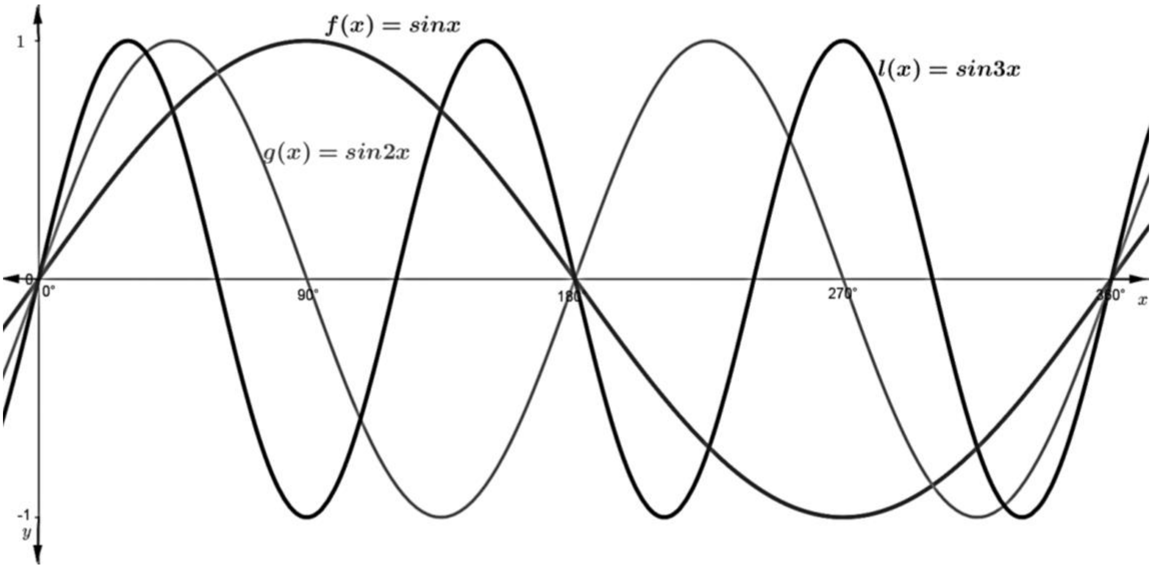
b) $f(x) = \sin x$; $h(x) = 2\sin x$; $k(x) = \sin x + 2$; for $x \in [0°; 360°]$

c) $f(x) = \sin x$; $m(x) = \sin(x + 45°)$; for $x \in [-90°; 360°]$

d) $f(x) = \sin x$; $j(x) = 2\sin(x - 45°)$; for $x \in [-90°; 360°]$

SOLUTIONS

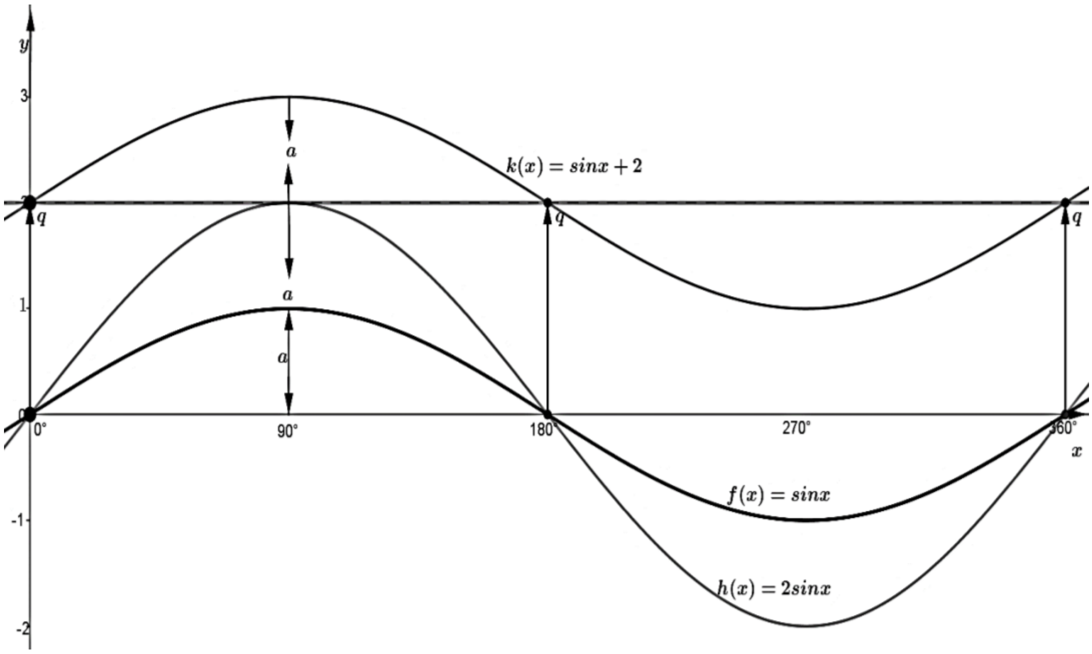
a)



The effect of ***k*** on the graph:

	Value of *k*	**Period**
$f(x) = \sin x$	$k = 1$, so 1 complete shape within 360°	$period = \frac{360°}{1} = 360°$
$g(x) = \sin 2x$	$k = 2$, so 2 complete shapes within 360°	$period = \frac{360°}{2} = 180°$
$l(x) = \sin 3x$	$k = 3$, so 3 complete shapes within 360°	$period = \frac{360°}{3} = 120°$

b)



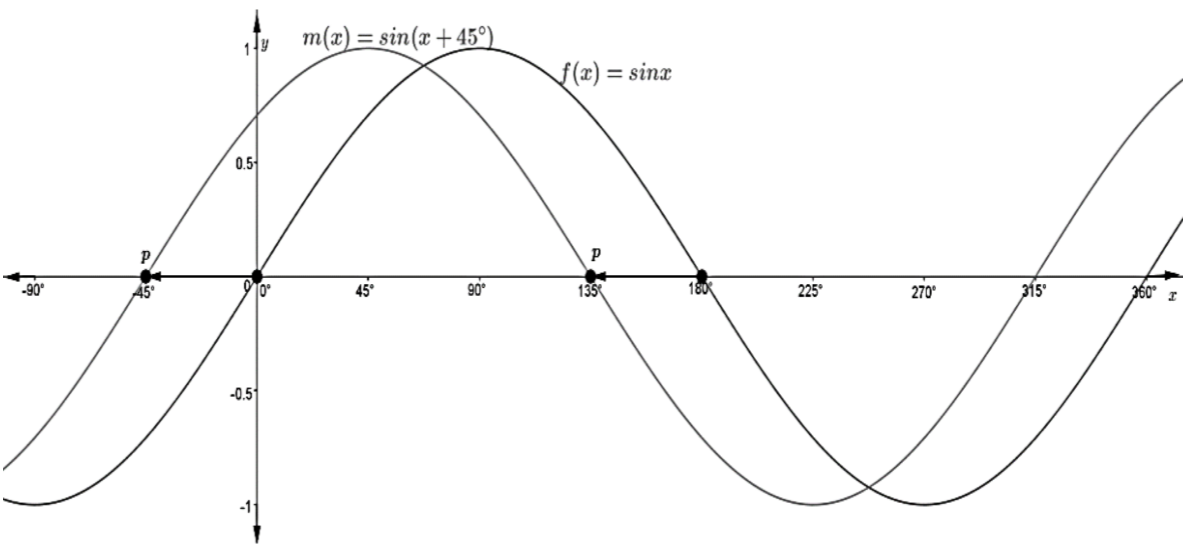
<u>The effect of ***a*** on the graphs:</u>

- $f(x) = \sin x : a = 1 \rightarrow amplitude\ is\ 1$
- $h(x) = 2\sin x : a = 2 \rightarrow amplitude\ is\ 2$
- $k(x) = \sin x + 2 : a = 1 \rightarrow amplitude\ is\ 1$

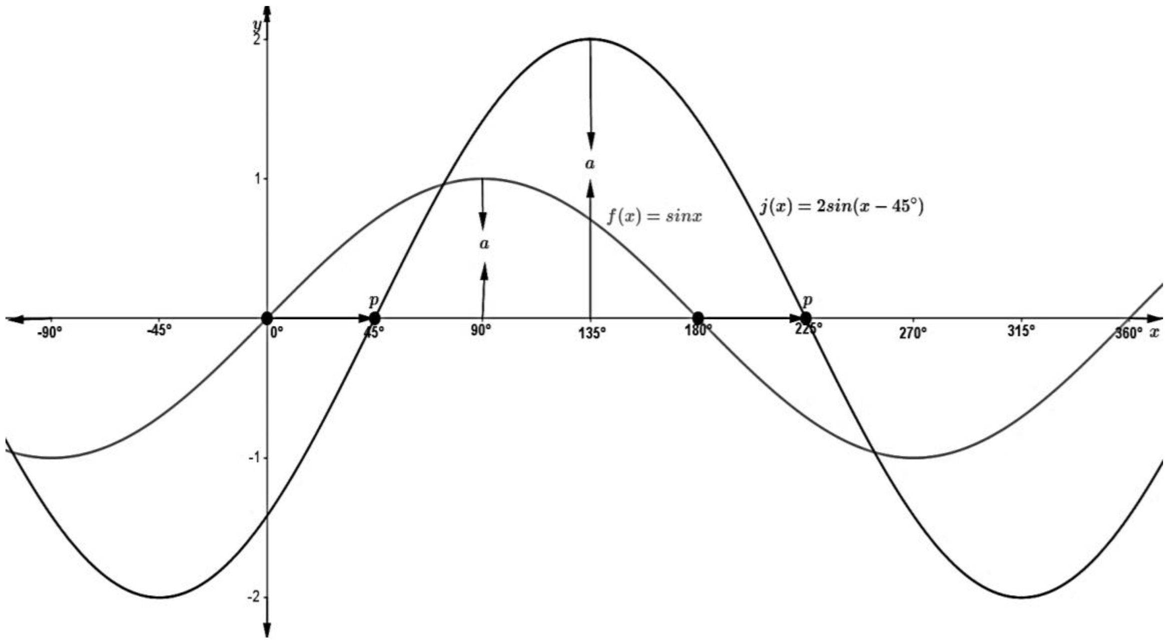
<u>The effect of ***q*** on the graphs: Vertical shift</u>

- The graph of $k(x) = \sin x + 2$ is the shift of *f(x)* by 2 units up,
- If ***q*** is **+** the basic graph will move upward
- If ***q*** is - the basic graph will move downward

c)



d)


y
2
1
-1
-2
x
-90°
-45°
0°
45°
90°
135°
180°
225°
270°
315°
360°
p
p
a
a
$f(x) = sinx$
$j(x) = 2sin(x - 45°)$

ACTIVITY: Trigonometry

QUESTION 1

Answer the following questions:

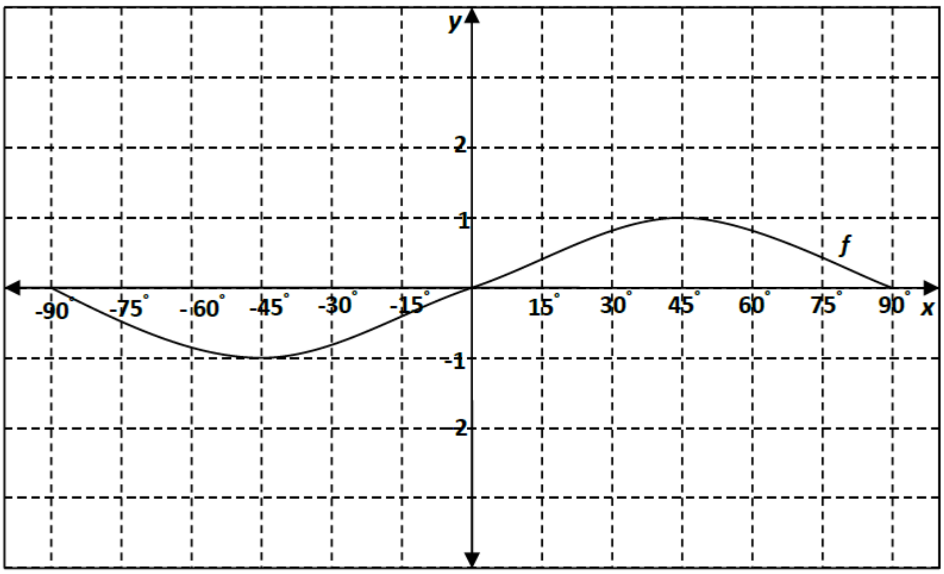
1.1 Determine the general solution of: $cos2x = \sin(x - 30°)$

1.2 Draw the sketch graphs of $f(x) = cos2x$ and $g(x) = \sin(x - 30°)$ for $x \in [-180; 90]$.
Clearly indicate the coordinates of the turning point and intercepts with the axes

a. Write down the value of x for which $g(x) > f(x)$ in the given interval

QUESTION 2

Consider the function f(x) = sin2x for $x \in [-90^0; 90^0]$



2.1 Write down the period of *f*.

2.2 Sketch the graph of $g(x) = \cos(x - 15^0)$ *for* $x \in [-90^0; 90^0]$

2.3 Determine the values of x for which $f(x) < g(x)$

QUESTION 3

Consider: f(x) = cos 2x and g(x) = sin (x – 60°)

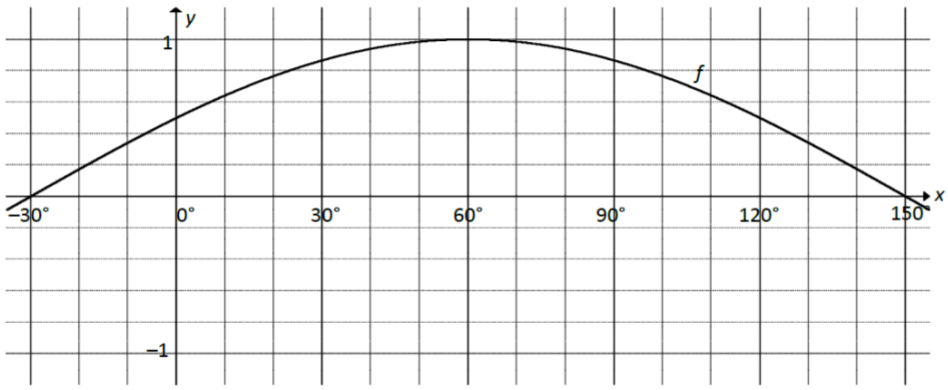
3.1 Sketch the graphs of *f* and g $x \in [-90^0; 90^0]$ on the same set of axes. Show clearly all the intercepts on the axes and the coordinates of the turning points.

3.2 Use your graphs to determine the value(s) of x for which g(x) > 0.

QUESTION 4

4.1 Determine the general solution of: $\sin(x + 30°) = \cos 3x$.

4.2 In the diagram below, the graph of $f(x) = \sin(x + 30°)$ is drawn for the interval $x \in [-30^0; 150^0]$



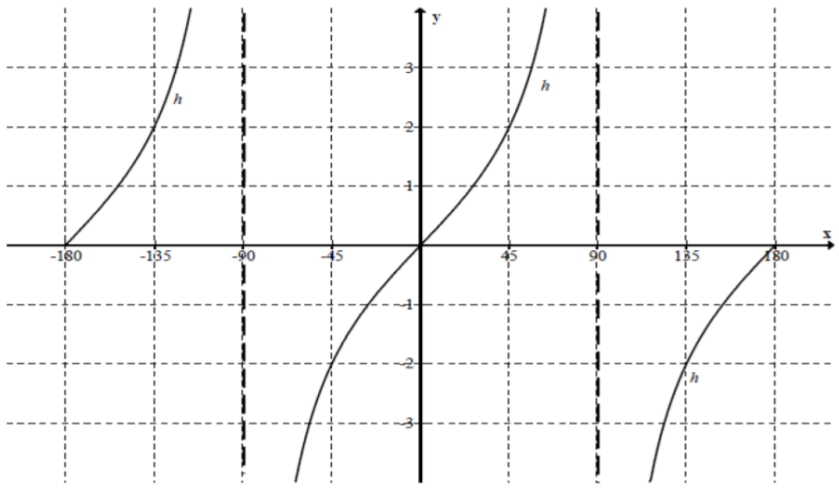
4.2.1 On the same system of axes sketch the graph of g, where $g(x) = \cos 3x$, for the interval $x \in [-30^0; 150^0]$

4.2.2 Write down the period of g.

4.2.3 For which values of x will $f(x) \geq g(x)$ in the interval $x \in [-30^0; 150^0]$

QUESTION 5

5.1 The graph of $h(x) = a\tan x$; for $x \in [-180°; 180°]$, $x \neq -90°$, is sketched below.



5.1 Determine the value of a.

5.2 If $f(x) = \cos(x + 45^0)$, sketch the graph of f for $x \in [-180°; 180°]$, on the diagram

5.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180°; 180°]$?

QUESTION 6

6. 1 On the same system of axes, sketch the graphs of $f(x) = 3\cos x$ and $g(x) = \tan\frac{1}{2}x$ for $-180^0 \leq x \leq 360^0$. Clearly show the intercepts with the axes and all turning points.

6.2 Use the graphs in **6.1** to answer the following questions.

6.2.1 Determine the period of g.

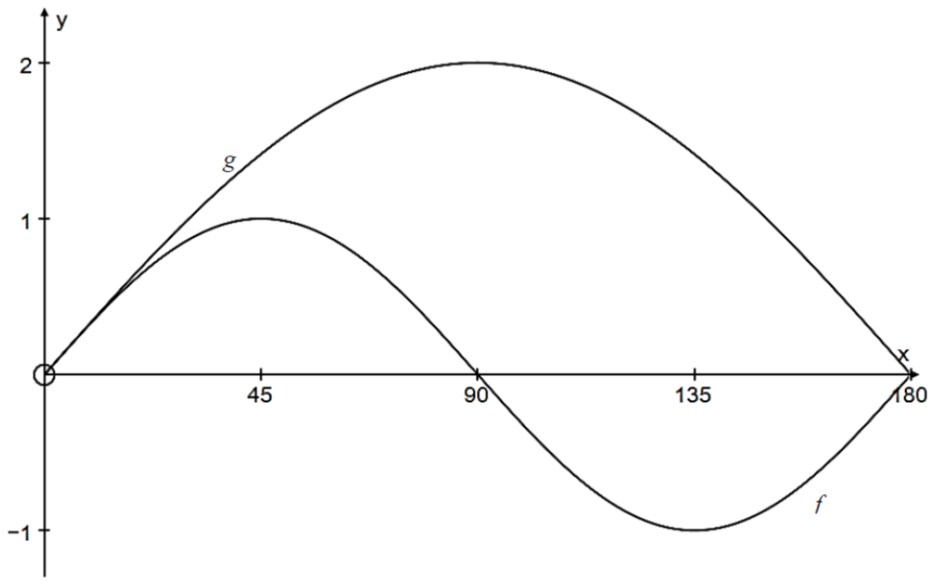
6.2.2 Determine the co-ordinates of the turning points of *f* on the given interval.

6.2.3 For which values of x will both functions increase as x increases for $-180° \leq x \leq 360°$?

6.2.4 If the y–axis is moved 45^0 to the left, then write down the new equation of f in the form $y = ...$

QUESTION 7

The graphs below represent the functions of f and g. f(x) = sin2x and $g(x) = c\sin dx$, $x \in [0°;180°]$



7.1 Determine the value(s) of x, for $x \in [0°;180°]$ where:

7.1.1 $g(x) - f(x) = 2$

7.1.2 $f(x) \leq 0$

7.1.3 $g(x).f(x) \geq 0$

7.2 f in the graph drawn above undergoes transformations to result in *g* and *h* as given below. Determine the values of a, b, c and d if

7.2.1 $g(x) = c\sin dx$

7.2.2 $h(x) = a\cos(x - b)$

SESSION NO: 11

Teaching Tips

Introduction

- The teacher introduces the topic and bring to the learners attention the weighting of the topic in the exam at the end of a year.

Euclidean Geometry

- The teacher reminds learners about the theorems and the converses.
- Refer to the Notes in the manual.
- Explain the diagram analysis on page before learners start with the activities.
- On the examples on diagram analysis learners analysis the diagram before answering questions ..

Trigonometry

Teacher to guide learners to :

- If there is a diagram analyse it before attempting questions.

TOPIC: TRIGONOMETRY AND EUCLIDEAN GEOMETRY

TOPIC	MARK
Trigonometry	50 ± 3
Euclidean Geometry	40 ± 3

A maximum of 12 Marks on Theory in Paper 2

THEORY ON TRIGONOMETRY

- Proofs of Compound Angles

  Accepting $\cos(A - B) = cosAcosB + sinAsinB$ prove the following

  - $\cos(A + B) = cosAcosB - sinAsinB$
  - $\sin(A - B) = sinAcosB - cosAsinB$
  - $\sin(A + B) = sinAcosB + cosAsinB$

- Proofs of Area Rule, Sine Rule and Cosine Rule

THEORY ON EUCLIDEAN GEOMETRY

The following proofs of theorems are examinable (NB. know them by heart)

- ✓ The line drawn from the centre of a circle perpendicular to a chord bisects the chord; **(From Gr.11)**
- ✓ The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); **(From Gr.11)**
- ✓ The opposite angles of a cyclic quadrilateral are supplementary; **(From Gr.11)**
- ✓ The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment; **(From Gr.11)**
- ✓ A line drawn parallel to one side of a triangle divides the other two sides proportionally; **(From Gr.12)**
- ✓ Equiangular triangles are similar. **(From Gr.12)**

ACTIVITY

FEB/MARCH 2011

QUESTION 10

10.1 If sin 28° = *a* and cos 32°= *b*, determine the following in terms of *a* and/or *b* :

10.1.1 $\cos 28°$ (2)

10.1.2 $\cos 64°$ (3)

10.1.3 sin 4° (4)

10.2 Prove without the use of a calculator, that if sin 28° = *a* and cos 32° = *b*, then $b\sqrt{1-a^2} - a\sqrt{1-b^2} = \frac{1}{2}$. (4)

10.3 Evaluate each of the following without using a calculator. Show ALL working.

10.3.1 $\dfrac{\sin 130°.\tan 60°}{\cos 540°.\tan 230°.\sin 400°}$ (7)

10.3.2 $(1-\sqrt{2}\sin 75°)(\sqrt{2}\sin 75°+1)$ (4)

10.4 Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$ (7)

10.5 Consider: $\dfrac{\cos 2x.\tan x}{\sin^2 x}$

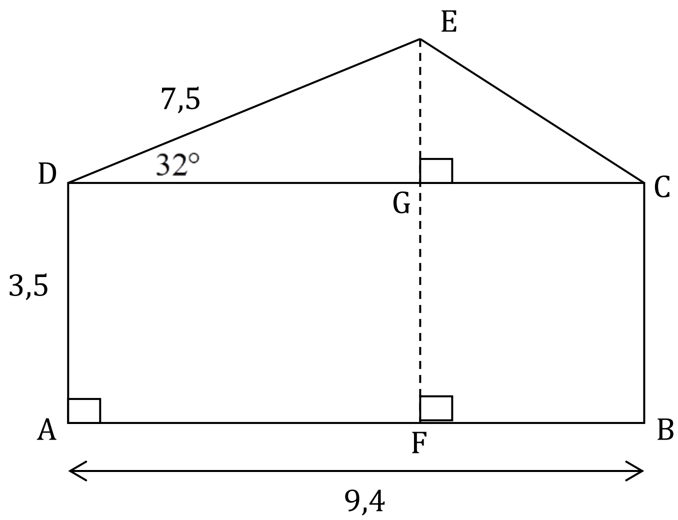
10.5.1 For which values of *x*, $x \in [0°\,;180°]$, will this expression be undefined? (3)

10.5.2 Prove that $\dfrac{\cos 2x.\tan x}{\sin^2 x} = \dfrac{\cos x}{\sin x} - \tan x$ for all other values of *x*. (5)

[39]

QUESTION 11

The sketch below shows one side of the elevation of a house. Some dimensions (in metres) are indicated on the figure.



Calculate, rounded off to ONE decimal place:

11.1 EC (3)

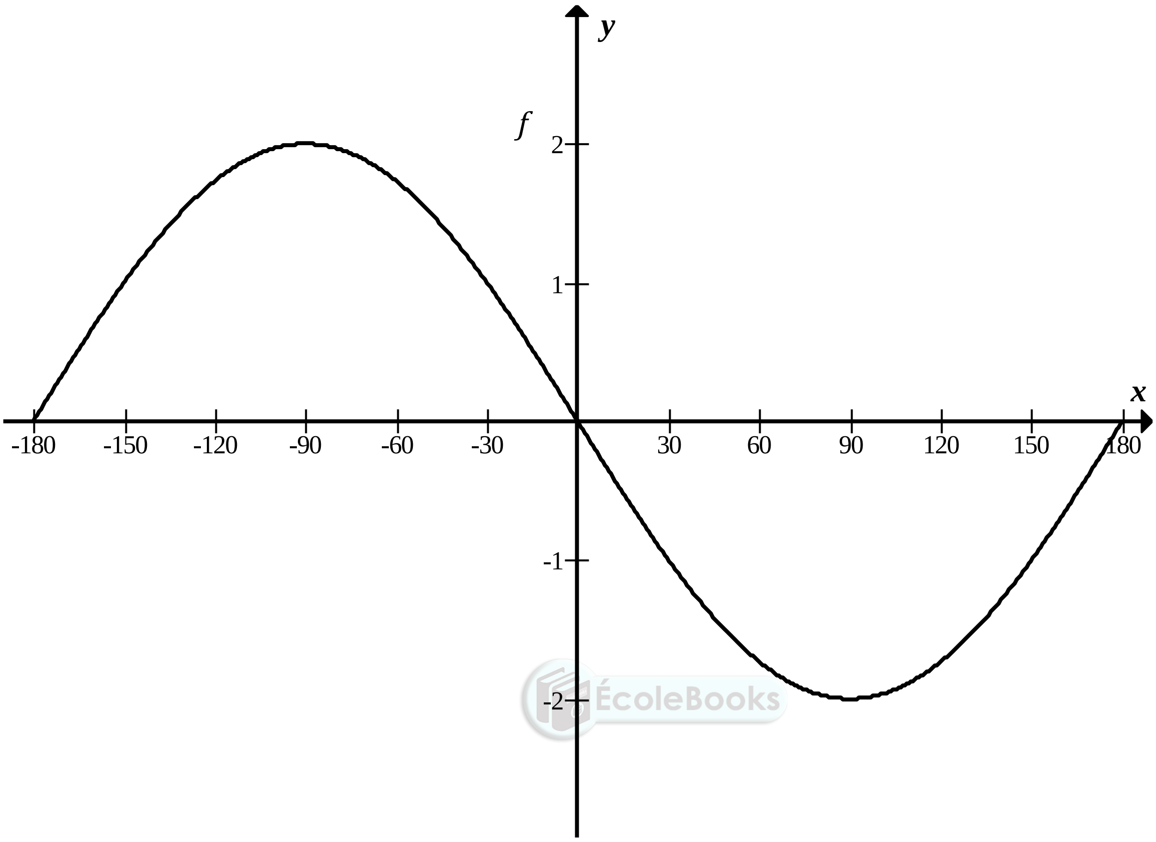
11.2 $D\hat{C}E$ (3)

11.3 Area of ΔDEC (2)

11.4 The height EF (3)
[11]

QUESTION 12

The graph of $f(x) = -2\sin x$ is drawn below.



12.1 Write down the period of *f*. (1)

12.2 Write down the amplitude of *h* if $h(x) = \frac{f(x)}{4}$. (2)

12.3 Draw the graph of $g(x) = \cos(x - 30°)$ for $x \in [-180° ; 180°]$ on the grid provided on DIAGRAM SHEET 5. (3)

12.4 Use the graph to determine the number of solutions for $-2\sin x = \cos(x - 30°)$, $x \in [-180° ; 180°]$. (1)

12.5 For which values of *x* is $g(x) \geq 0$? (2)

12.6 For which values of *x* is $f'(x) < 0$ and $g'(x) > 0$? (3)

[12]

JUNE 2019

QUESTION 5

5.1 **Without using a calculator,** write the following expressions in terms of $\sin 11°$:

5.1.1 $\sin 191°$ (1)

5.1.2 $\cos 22°$ (1)

5.2 Simplify $\cos(x-180°)+\sqrt{2}\sin(x+45°)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin P+\sin Q=\frac{7}{5}$ and $\hat{P}+\hat{Q}=90°$

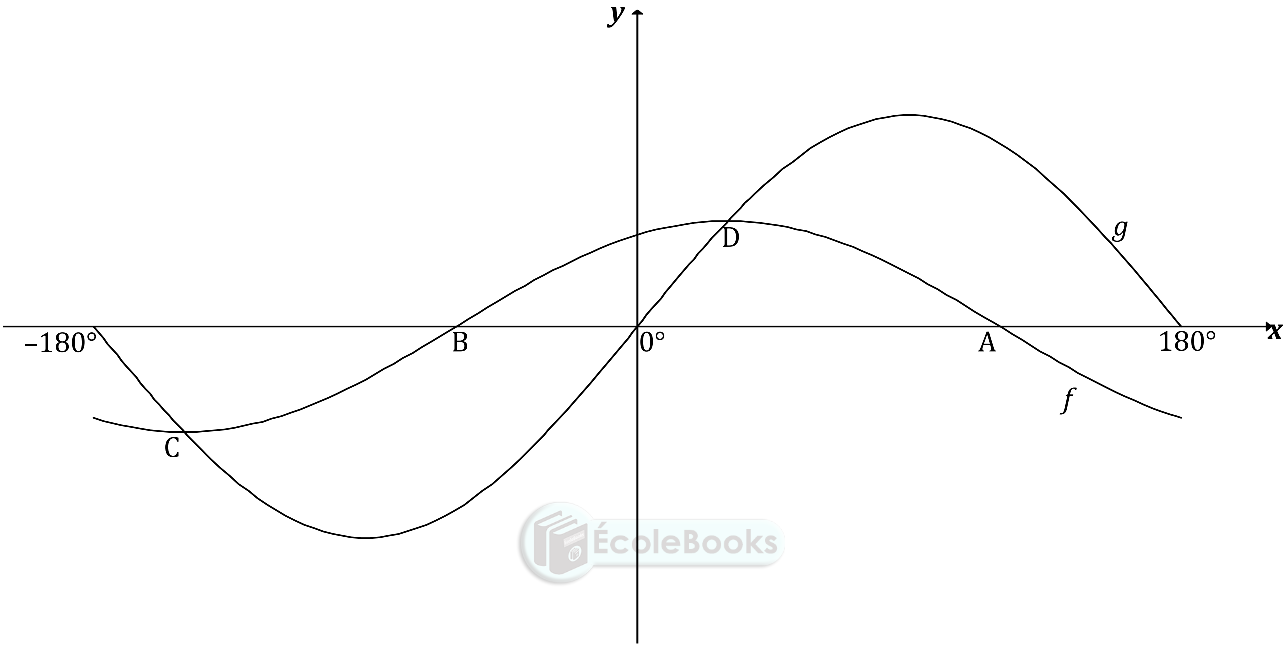
Without using a calculator, determine the value of $\sin 2P$. (5)

[12]

QUESTION 6

6.1 Determine the general solution of $\cos(x - 30°) = 2\sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x - 30°)$ and $g(x) = 2\sin x$ are drawn for the interval $x \in [-180°; 180°]$. A and B are the *x*–intercepts of *f*. The two graphs intersect at C and D, the minimum and maximum turning points respectively of *f*.



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180°; 180°]$, for which:

(a) Both graphs are increasing (2)

(b) $f(x + 10°) > g(x + 10°)$ (2)

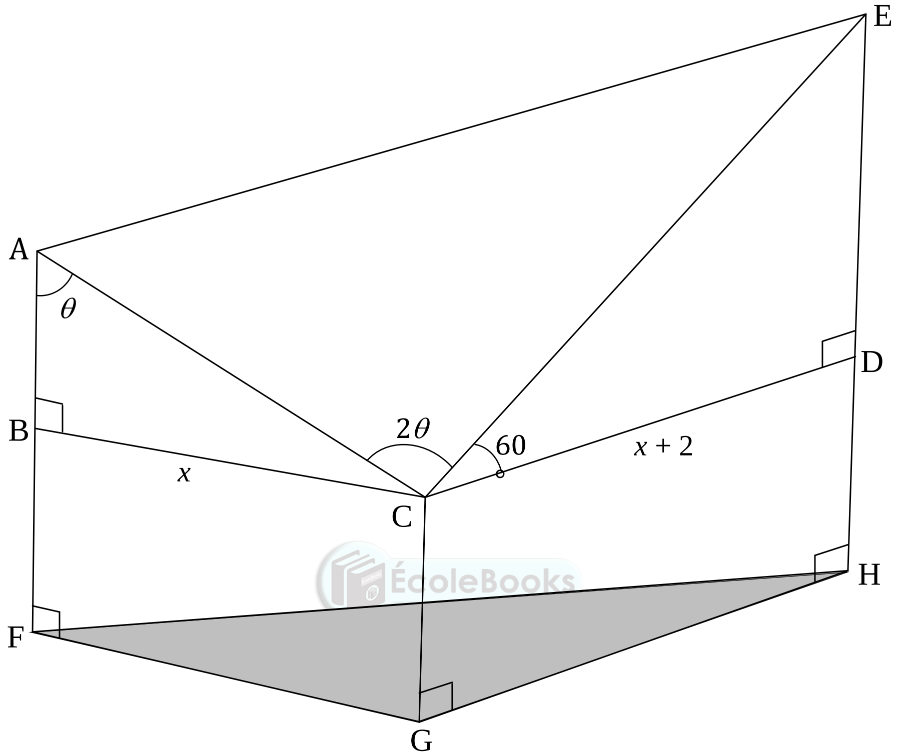
6.2.3 Determine the range of $y = 2^{2\sin x + 3}$ (5)

[18]

QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. ΔACE forms the roof of an entertainment centre.

BC = x, CD = x + 2, $B\hat{A}C = \theta$, $A\hat{C}E = 2\theta$ and $E\hat{C}D = 60°$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

7.1.2 CE in terms of x (2)

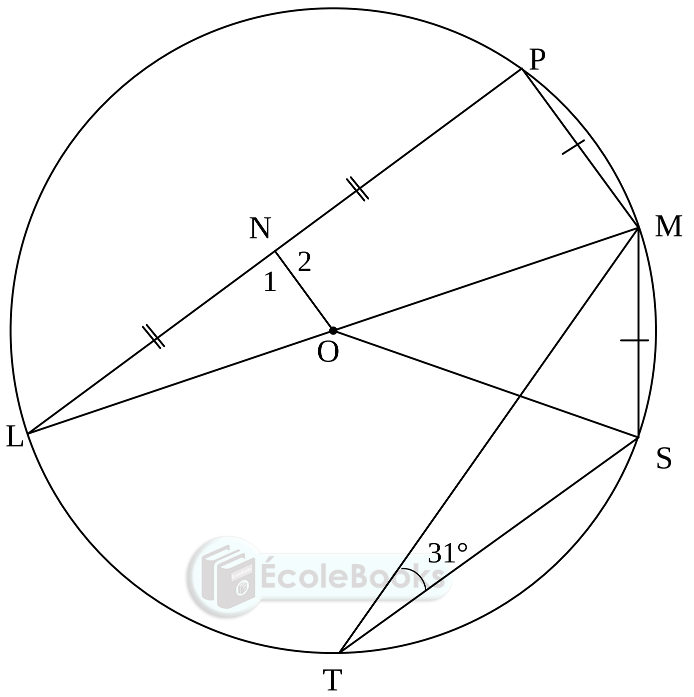
7.2 Show that the area of the roof ΔACE is given by $2x(x+2)\cos\theta$. (3)

7.3 If $\theta = 55°$ and BC = 12 metres, calculate the length of AE. (4)

[11]

QUESTION 8

8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. PM = MS and $M\hat{T}S = 31°$



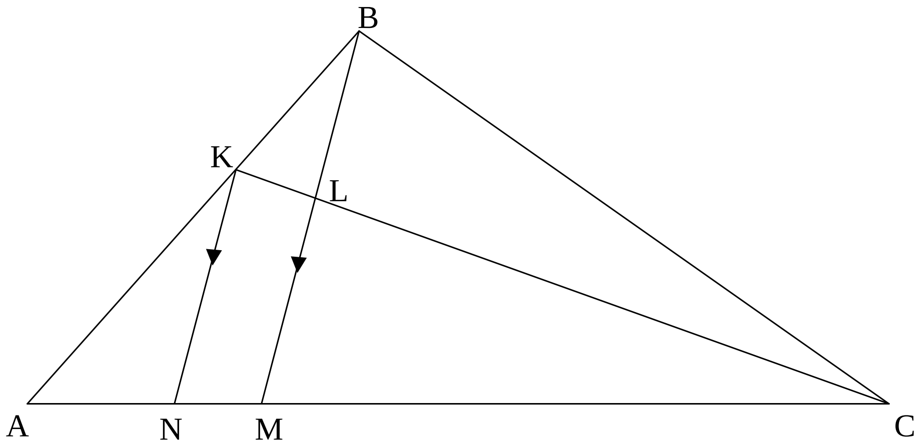
8.1.1 Determine, with reasons, the size of each of the following angles:

(a) $M\hat{O}S$ (2)

(b) $\hat{L}$ (2)

8.1.2 Prove that ON = $\frac{1}{2}$ MS. (4)

8.2 In ΔABC in the diagram, K is a point on AB such that AK : KB = 3 : 2. N and M are points on AC such that KN || BM. BM intersects KC at L. AM : MC = 10 : 23.



Determine, with reasons, the ratio of:

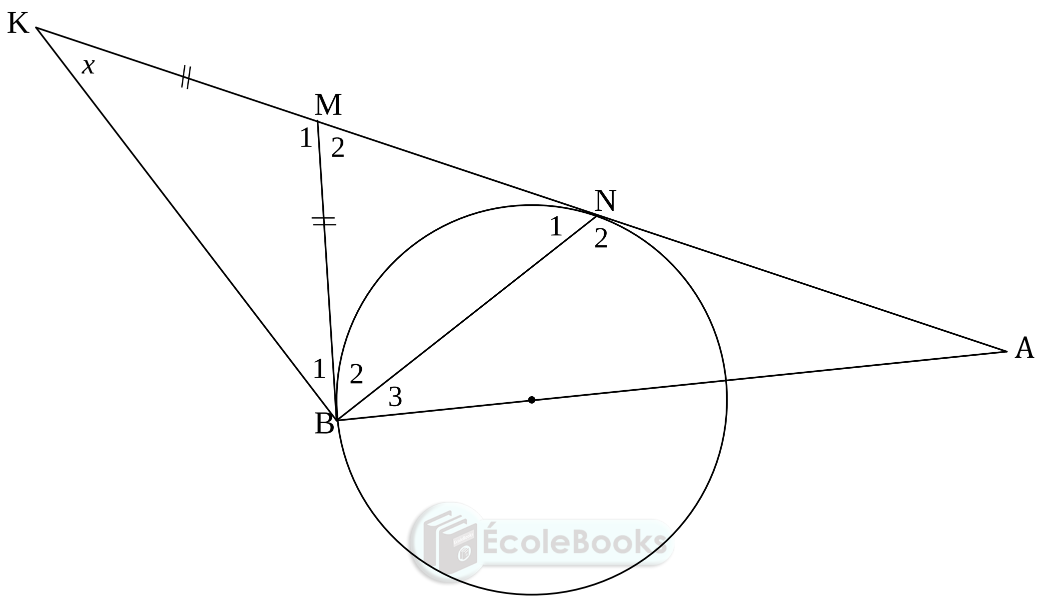
8.2.1 $\frac{AN}{AM}$ (2)

8.2.2 $\frac{CL}{LK}$ (3)

[13]

QUESTION 9

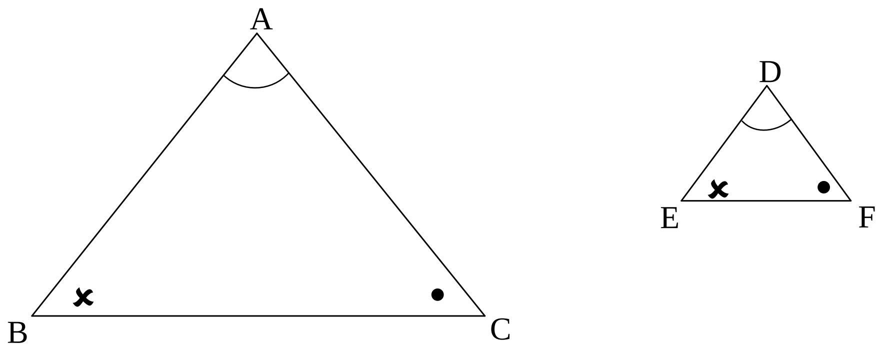
In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N. The straight line from B passing through the centre of the circle meets MN produced in A. NM is produced to K such that BM = MK. BK and BN are drawn.
Let $\hat{K} = x$.



9.1	Determine, with reasons, the size of $\hat{N}_1$ in terms of *x*.	(6)
9.2	Prove that BA is a tangent to the circle passing through K, B and N.	(5) **[11]**

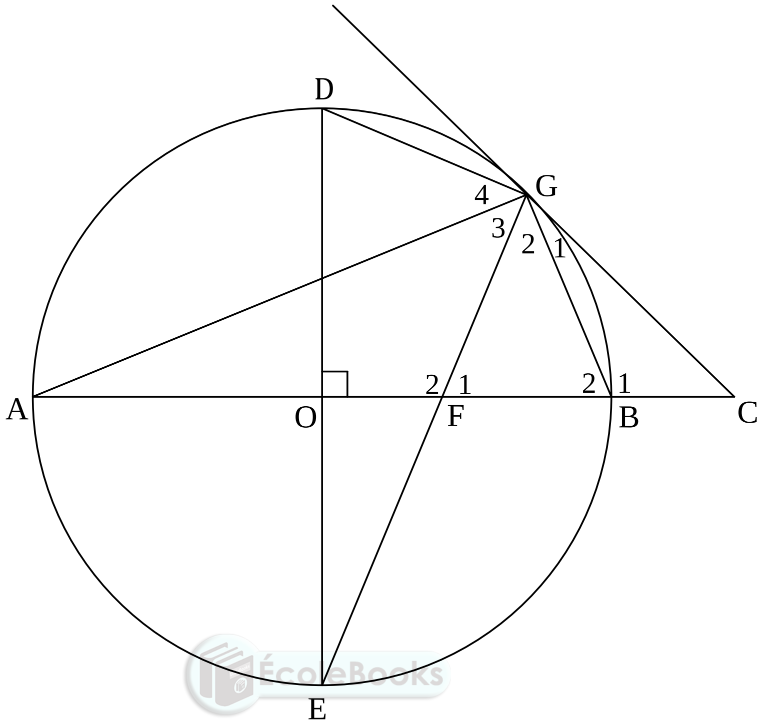
QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G. The straight line from C passing through O cuts the circle at A and B. Diameter DOE is perpendicular to CA. GE and CA intersect at F. Chords DG, BG and AG are drawn.



10.2.1 Prove that:

(a) DGFO is a cyclic quadrilateral (3)

(b) GC = CF (5)

10.2.2 If it is further given that CO = 11 units and DE = 14 units, calculate:

(a) The length of BC (3)

(b) The length of CG (5)

(c) The size of $\hat{E}$. (4)

[26]

MAY /JUNE 2015

QUESTION 5

5.1 Given that $\cos\beta = -\frac{1}{\sqrt{5}}$, where $180° < \beta < 360°$.

Determine, with the aid of a sketch and without using a calculator, the value of $\sin\beta$. (5)

5.2 Determine the value of the following expression:

$$\frac{\tan(180° - x).\sin(x - 90°)}{4\sin(360° + x)}$$ (6)

5.3 If $\sin A = p$ and $\cos A = q$:

5.3.1 Write $\tan A$ in terms of p and q (1)

5.3.2 Simplify $p^4 - q^4$ to a single trigonometric ratio (4)

5.4 Consider the identity: $\frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin\theta.\cos\theta} = \tan\theta$

5.4.1 Prove the identity. (5)

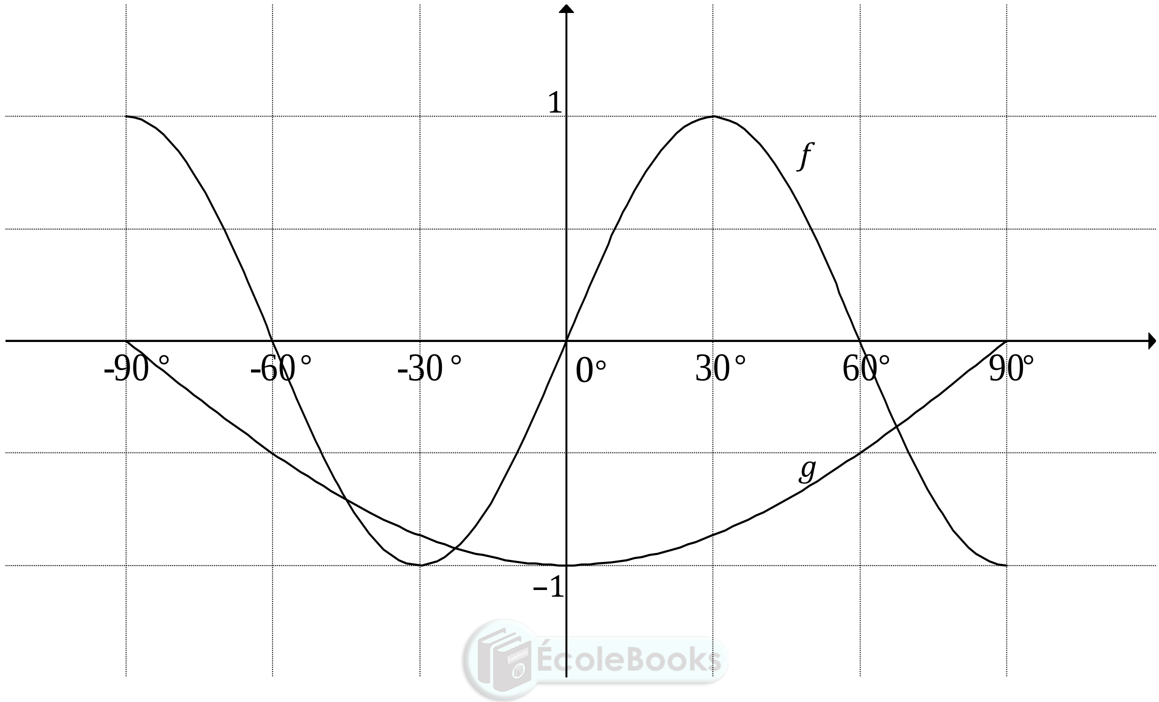
5.4.2 For which value(s) of θ in the interval $0° < \theta < 180°$ will the identity be undefined? (2)

5.5 Determine the general solution of $2\sin 2x + 3\sin x = 0$ (6)

[29]

QUESTION 6

In the diagram below the graphs of $f(x) = \sin bx$ and $g(x) = -\cos x$ are drawn for $-90° \leq x \leq 90°$. Use the diagram to answer the following questions.



6.1 Write down the period of f. (1)

6.2 Determine the value of b. (1)

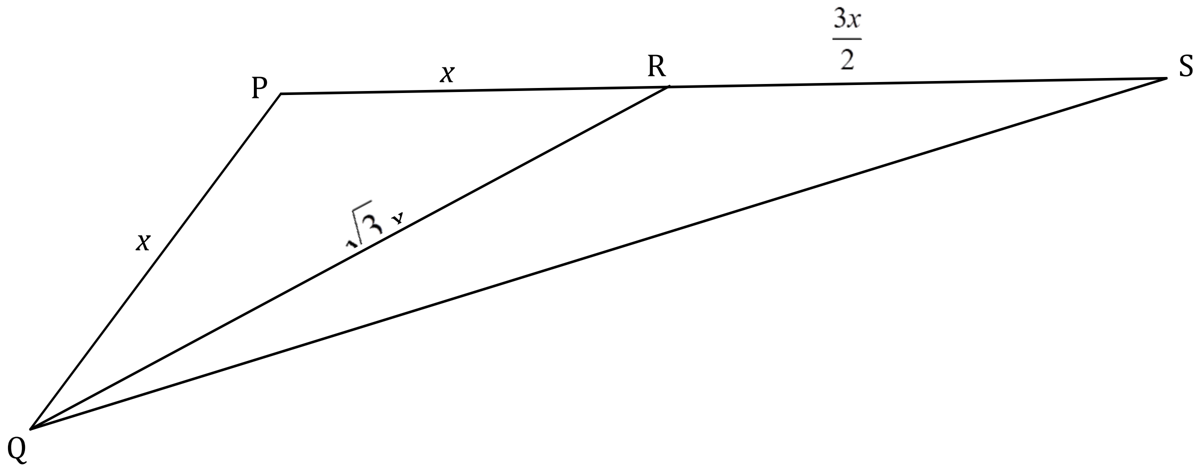
6.3 The general solutions of the equation $\sin bx = -\cos x$ are $x = 67{,}5° + k.90°$ or $x = 135° + k.180°$ where $k \in Z$.
Determine the x-values of the points of intersection of f and g for the given domain. (3)

6.4 Write down the values of x for which $\sin bx + \cos x < 0$ for the given domain. (4)
[9]

QUESTION 7

Triangle PQS forms a certain area of a park. R is a point on PS and QR divides the area of the park into two triangular parts, as shown below, for a festive event.

PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = $\sqrt{3}\,x$ units.



7.1 Calculate the size of $\hat{P}$. (4)

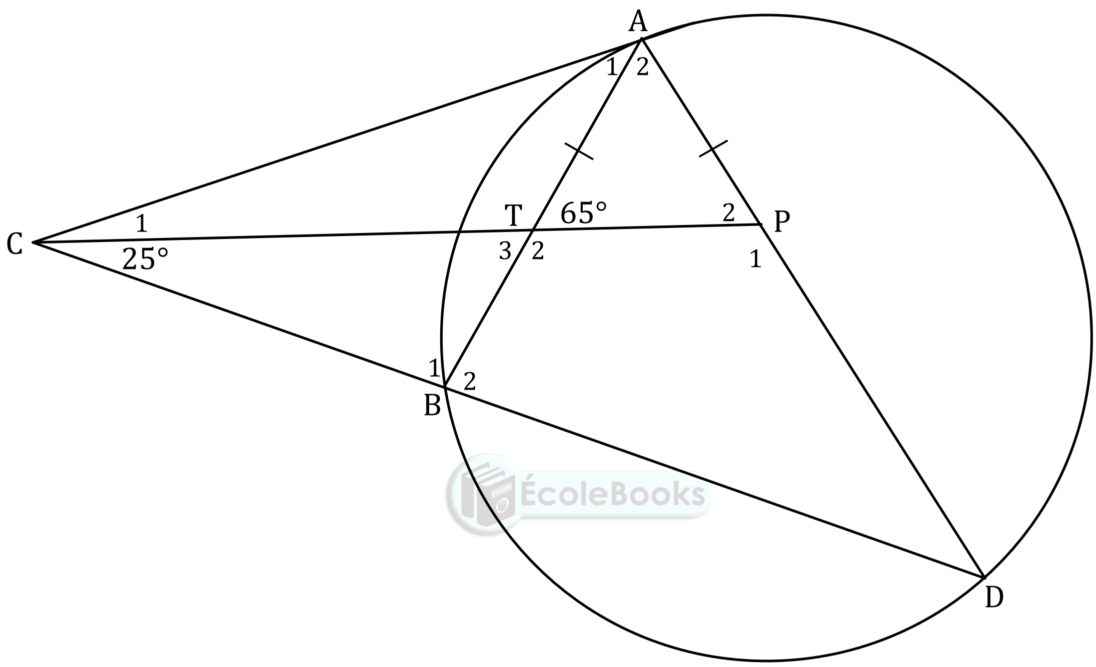
7.2 Hence, calculate the area of triangle QRS in terms of x in its simplest form. (5)

[9]

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram ΔACD is drawn with points A and D on the circumference of a circle. CD cuts the circle at B. P is a point on AD with CP the bisector of $A\hat{C}D$. CP cuts the chord AB at T. AT = AP, $A\hat{T}P = 65°$ and $P\hat{C}D = 25°$.



8.1 Determine the size of each of the following:

8.1.1 $\hat{P}_2$ (2)

8.1.2 $\hat{D}$ (2)

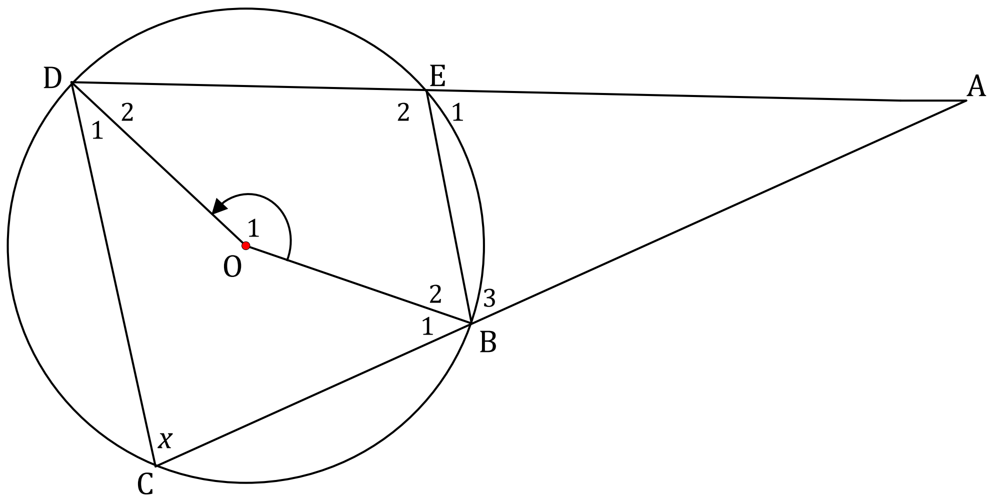
8.1.3 $\hat{A}_1$ (2)

8.2 Is CA a tangent to the circle ABD? Motivate your answer. (2)

[8]

QUESTION 9

In the diagram O is the centre of the circle and BO and OD are drawn. Chords CB and DE are produced to meet in A. Chords BE and CD are drawn. $B\hat{C}D = x$.



9.1 Give the reason for each of the statements in the table. Complete the table provided in the ANSWER BOOK by writing down the reason for each statement. (2)

Statement		Reason
9.1.1	$\hat{E}_1 = x$	
9.1.2	$\hat{O}_1 = 2x$	

9.2 If it is given that BE | | CD, prove that:

9.2.1 AC = AD (4)

9.2.2 ABOD is a cyclic quadrilateral (3)

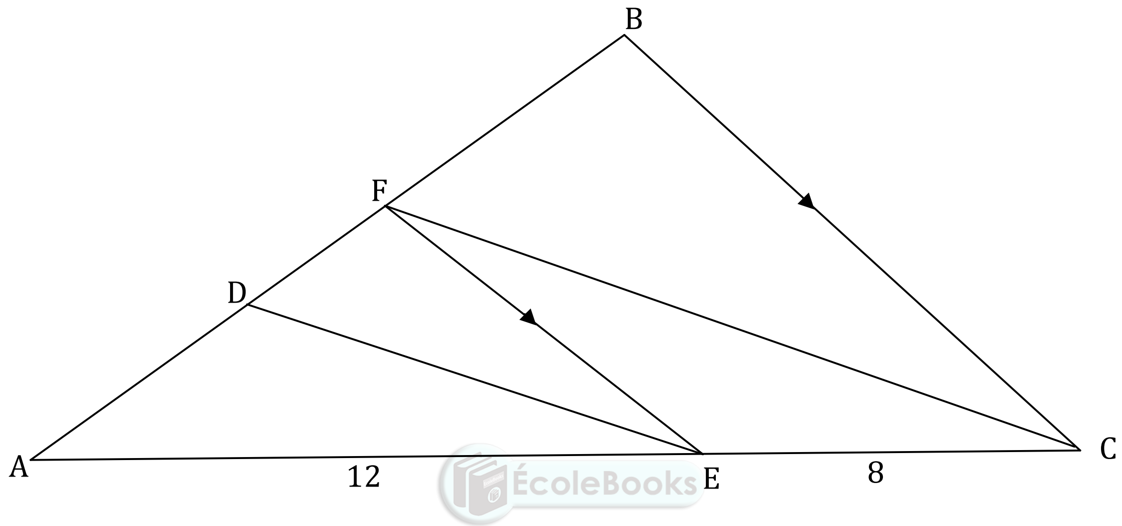
[9]

QUESTION 10

10.1 Complete the following statement of the theorem in the ANSWER BOOK:

If a line divides two sides of a triangle in the same proportion, then … (1)

10.2 In the diagram ABC is a triangle with F on AB and E on AC. BC | | FE. D is on AF with $\frac{AD}{AF} = \frac{3}{5}$. AE = 12 units and EC = 8 units.



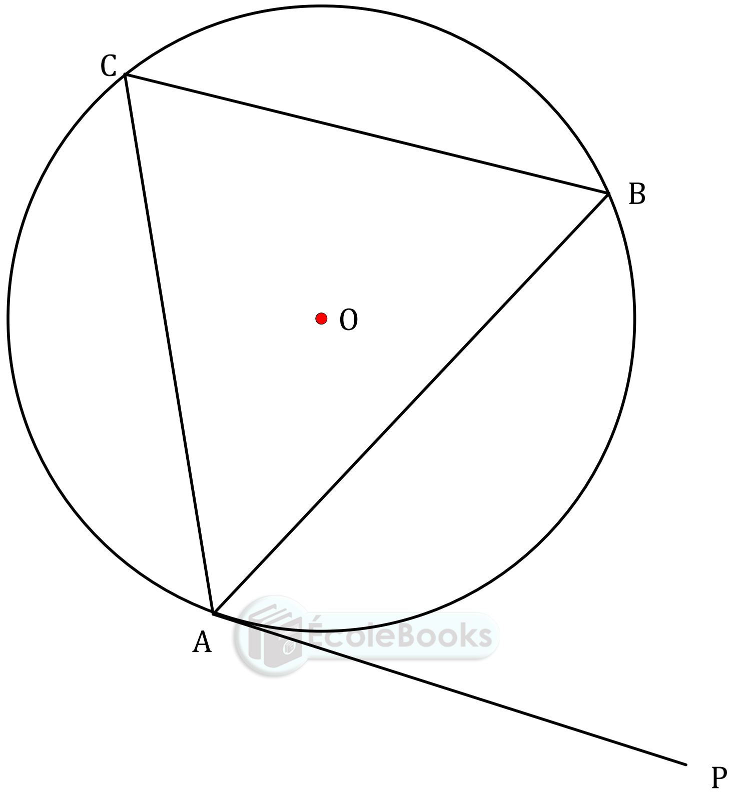
10.2.1 Prove that DE | | FC. (3)

10.2.2 If AB = 14 units, calculate the length of BF. (3)

[7]

QUESTION 11

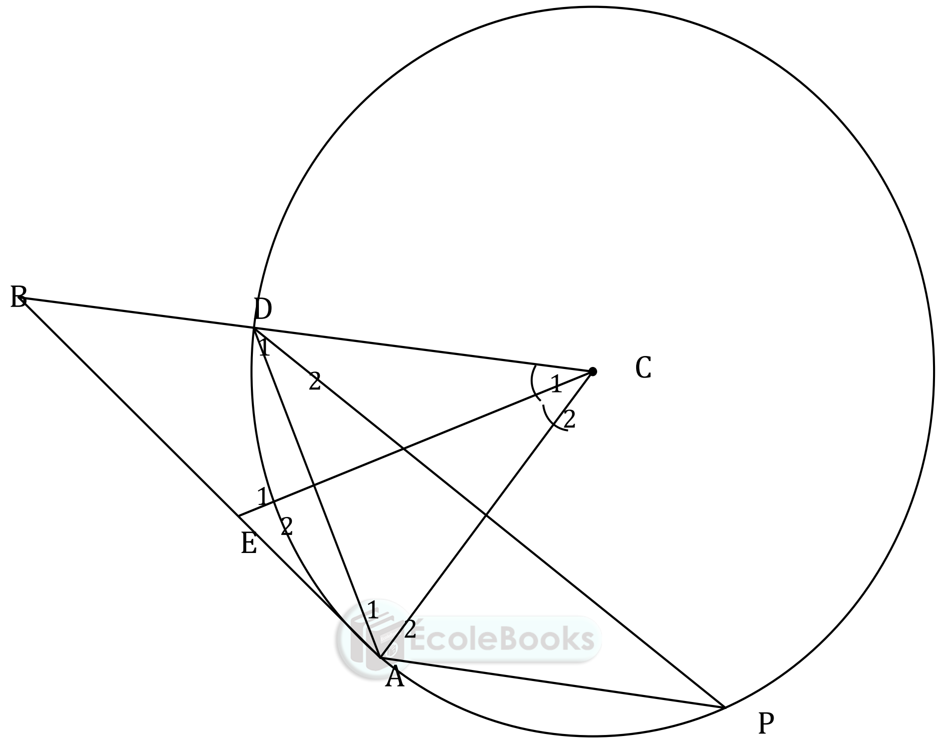
11.1 In the diagram O is the centre of the circle and PA is a tangent to the circle at A. B and C are points on the circumference of the circle.



Use the diagram to prove the theorem that states that $B\hat{A}P = A\hat{C}B$.

(6)

11.2 In the diagram C is the centre of the circle DAP. BA is a tangent to the circle at A. CD is produced to meet the tangent to the circle at B. DP and DA are drawn. E is a point on BA such that EC bisects $D\hat{C}A$. Let $\hat{C}_1 = x$.



11.2.1 Prove that ΔBAD ||| ΔBCE. (7)

11.2.2 If it is also given that AB = 8 units and AC = 6 units, calculate:

(a) The length of BD (5)

(b) The length of BE (3)

(c) The size of x (3)

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