



MATHEMATICS

REVISION BOOKLET 1 OF 2020 (PAPER 1 QUESTIONS AND MEMORANDA)

A COLLECTION OF 2017 – 2019 NSC EXAM QUESTIONS GROUPED ACCORDING TO TOPICS



- # MASTER THE BASICS FIRST!
- HARDWORK NEVER KILLS!
- PRACTICE MAKES PERFECT!

YES
YOU
CAN!



PLEASE READ:

<p>Dear Grade 12 Mathematics learner</p> <p>Your final school exam result in Mathematics is extremely important. Good result in Mathematics will surely open doors for you that will influence the quality of your future life. So, Practice Mathematics regularly, not only before tests and exams but daily!</p> <p>Always tell yourself that ‘I Can Do Mathematics!’.</p> <p>1. REQUIRED RESOURCES</p> <ul style="list-style-type: none"> ● A Mathematics textbook ● workbooks/ Revision Material ● Past Examination Question Papers ● A scientific calculator, etc. <p>2. CONTENT CHECKLIST</p> <p>Below is a checklist you should use to ensure that you have covered the content for Grade 12 Mathematics in full:</p> <p><u>Paper 1</u></p> <p>Equations and inequalities</p> <ul style="list-style-type: none"> ● Quadratic equations and inequalities ● Simultaneous equations ● Exponents and Surds ● Nature of Roots <p>Number patterns and sequences</p> <ul style="list-style-type: none"> ● General patterns (linear and quadratic) ● Sigma notation ● Arithmetic and geometric sequences and series (formulae for n^{th} term & Sum) ● Sum to infinity; Convergence 	<p>Functions and graphs</p> <ul style="list-style-type: none"> ● Linear, parabola; hyperbola; ● exponential and logarithmic; ● and their transformations ● Inverse functions <p>Financial mathematics</p> <ul style="list-style-type: none"> ● Simple and compound interest ● Logarithms in the context ● Present value and future value ● annuities (investments, sinking funds, loans and bond repayments) ● Nominal and effective interest rates ● Depreciation (reducing balance and straight line) <p>Differential Calculus</p> <ul style="list-style-type: none"> ● Limits and average gradient ● First principles and differentiation rules ● Gradient at a point and tangents to curves ● Polynomials (Remainder and Factor theorems) ● Cubic functions ● Applications (maxima and minima; rate of change) <p>Probability</p> <ul style="list-style-type: none"> ● Probability rules (identity, mutually exclusive events, independent events and complementary events). ● Venn-diagram, Tree diagram, Contingency table ● Counting principles
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ENJOY MATHEMATICS... BECAUSE YOU CAN!

ALGEBRA, EQUATIONS AND INEQUALITIES

NOV 2017

QUESTION 1

1.1 Solve for x :

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:

$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)
[24]

FEB 2018



QUESTION 1

1.1 Solve for x :

1.1.1 $x^2 - 6x - 16 = 0$ (3)

1.1.2 $2x^2 + 7x - 1 = 0$ (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to $x^2 - 25 < 0$. (4)1.3 Solve for x and y :

$-2y + x = -1$ and $x^2 - 7 - y^2 = -y$ (6)

1.4 Evaluate: $\frac{3^{2018} + 3^{2016}}{3^{2017}}$ (2)1.5 Given: $t(x) = \frac{\sqrt{3x-5}}{x-3}$ 1.5.1 For which values of x will $\frac{\sqrt{3x-5}}{x-3}$ be real? (3)1.5.2 Solve for x if $t(x) = 1$. (4)
[26]

NOV 2018

QUESTION 1

- 1.1 Solve for x :
- 1.1.1 $x^2 - 4x + 3 = 0$ (3)
- 1.1.2 $5x^2 - 5x + 1 = 0$ (correct to TWO decimal places) (3)
- 1.1.3 $x^2 - 3x - 10 > 0$ (3)
- 1.1.4 $3\sqrt{x} = x - 4$ (4)
- 1.2 Solve simultaneously for x and y :
- $3x - y = 2$ and $2y + 9x^2 = -1$ (6)
- 1.3 If $3^{2x} = 64$ and $5^{\sqrt{x}} = 64$, calculate, WITHOUT the use of a calculator, the value of: $\frac{3^{x-1}}{\sqrt{5^{\sqrt{x}}}}$ (4)
- [23]

NOV 2019

QUESTION 1

- 1.1 Solve for x :
- 1.1.1 $x^2 + 5x - 6 = 0$ (3)
- 1.1.2 $4x^2 + 3x - 5 = 0$ (correct to TWO decimal places) (3)
- 1.1.3 $4x^2 - 1 < 0$ (3)
- 1.1.4 $(\sqrt{\sqrt{32+x}})(\sqrt{\sqrt{32-x}}) = x$ (4)
- 1.2 Solve simultaneously for x and y :
- $y + x = 12$ and $xy = 14 - 3x$ (5)
- 1.3 Consider the product $1 \times 2 \times 3 \times 4 \times \dots \times 30$.
Determine the largest value of k such that 3^k is a factor of this product. (4)
- [22]

PATTERNS, SEQUENCES AND SERIES**NOV 2017****QUESTION 2**

- 2.1 Given the following quadratic number pattern: $5 ; -4 ; -19 ; -40 ; \dots$
- 2.1.1 Determine the constant second difference of the sequence. (2)
- 2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)
- 2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)
- 2.2 The first three terms of an arithmetic sequence are $2k - 7 ; k + 8$ and $2k - 1$.
- 2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)
- 2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)
- [18]**

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a , constant ratio

r and n^{th} term, T_n , such that $\sum_{n=1}^{\infty} T_n = \frac{1}{4}$.

- 3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r . (2)
- 3.2 Calculate the values of a and r . (6)
- [8]**

FEB 2018**QUESTION 2**

- 2.1 Given the following geometric sequence: $30 ; 10 ; \frac{10}{3} ; \dots$
- 2.1.1 Determine n if the n^{th} term of the sequence is equal to $\frac{10}{729}$. (4)
- 2.1.2 Calculate: $30 + 10 + \frac{10}{3} + \dots$ (2)
- 2.2 Derive a formula for the sum of the first n terms of an arithmetic sequence if the first term of the sequence is a and the common difference is d . (4)
- [10]**

QUESTION 3

The first three terms of an arithmetic sequence are -1 ; 2 and 5 .

3.1 Determine the n^{th} term, T_n , of the sequence. (2)

3.2 Calculate T_{43} . (2)

3.3 Evaluate $\sum_{k=1}^n T_k$ in terms of n . (3)

3.4 A quadratic sequence, with general term T_n , has the following properties:

- $T_{11} = 125$
- $T_n - T_{n-1} = 3n - 4$

Determine the first term of the sequence. (6)
[13]

NOV 2018

QUESTION 2

2.1 Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; ...

2.1.1 Write down the next term of the sequence. (1)

2.1.2 Determine the n^{th} term of the sequence. (4)

2.1.3 Calculate the 20^{th} term of the sequence. (2)

2.2 Given the arithmetic sequence: 35 ; 28 ; 21 ; ...

Calculate which term of the sequence will have a value of -140 . (3)

2.3 For which value of n will the sum of the first n terms of the arithmetic sequence in QUESTION 2.2 be equal to the n^{th} term of the quadratic sequence in QUESTION 2.1? (6)
[16]

QUESTION 3

A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6.

3.1 Calculate the first term of the series. (2)

3.2 Calculate the 8th term of the series. (2)

3.3 Given: $\sum_{k=1}^n 3(2)^{1-k} = 5,8125$ Calculate the value of n . (4)

3.4 If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{1-k}$ in terms of p . (3)
[11]

NOV 2019**QUESTION 2**

2.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ;

2.1.1 Write down the values of the next TWO terms of the sequence. (2)

2.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

2.1.3 Which term(s) of the sequence will have a value of 74? (4)

2.1.4 Which term in the sequence has the least value? (2)

2.2 Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \dots = K$

2.2.1 Determine the value of K if the series has 21 terms. (3)

2.2.2 Determine the largest value of n for which $T_n > \frac{5}{8192}$ (4)

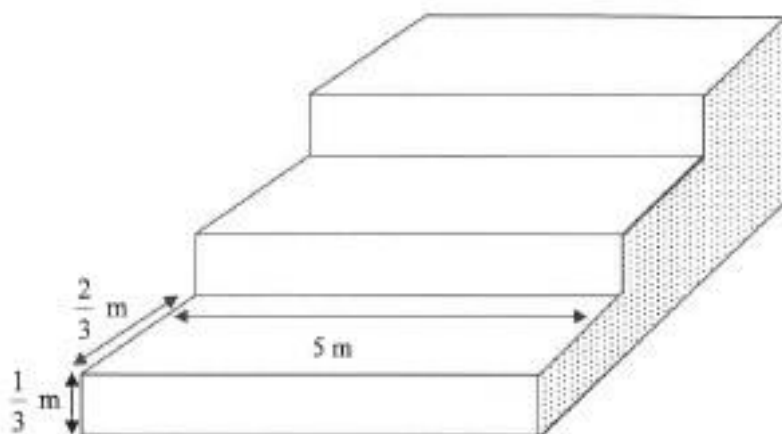
[19]

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ (3)

3.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.



The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in m^2) of metal sheeting needed to cover both open sides.


(6)
[9]

FUNCTIONS AND INVERSES

NOV 2017

QUESTION 4

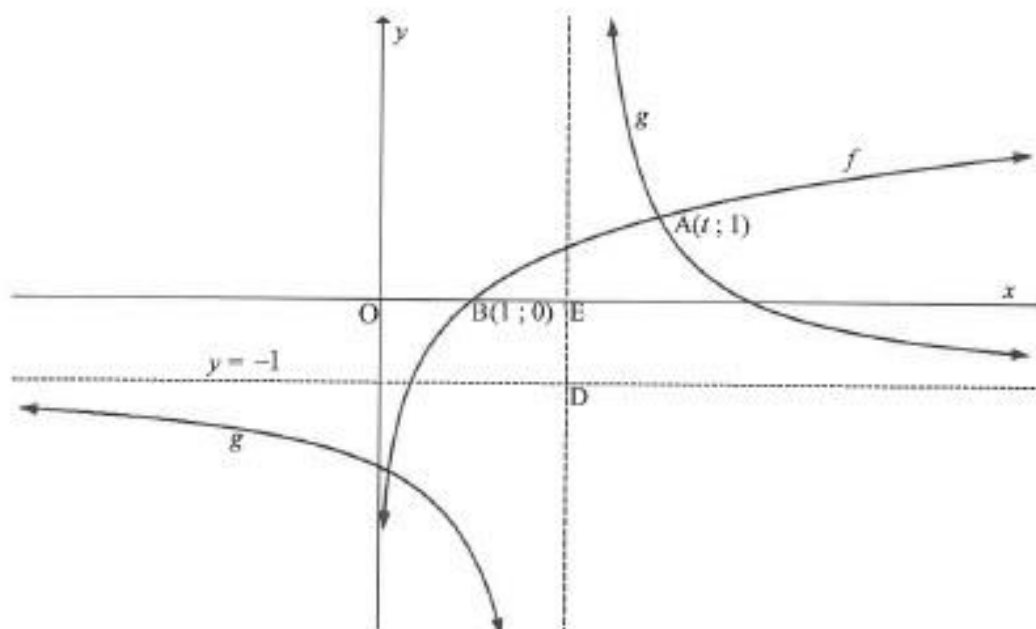
Given: $f(x) = -ax^2 + bx + 6$

- 4.1 The gradient of the tangent to the graph of f at the point $\left(-1; \frac{7}{2}\right)$ is 3.
Show that $a = \frac{1}{2}$ and $b = 2$. (5)
- 4.2 Calculate the x -intercepts of f . (3)
- 4.3 Calculate the coordinates of the turning point of f . (3)
- 4.4 Sketch the graph of f . Clearly indicate ALL intercepts with the axes and the turning point. (4)
- 4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)
- 4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f . Clearly indicate ALL intercepts with the axes. (2)
- 4.7 Write down the values of x for which $f(x), g(x) \leq 0$. (3)
- [23]
- 

QUESTION 5

The diagram below shows the graphs of $g(x) = \frac{2}{x+p} + q$ and $f(x) = \log_3 x$.

- $y = -1$ is the horizontal asymptote of g .
- $B(1; 0)$ is the x -intercept of f .
- $A(t; 1)$ is a point of intersection between f and g .
- The vertical asymptote of g intersects the x -axis at E and the horizontal asymptote at D .
- $OB = BE$.



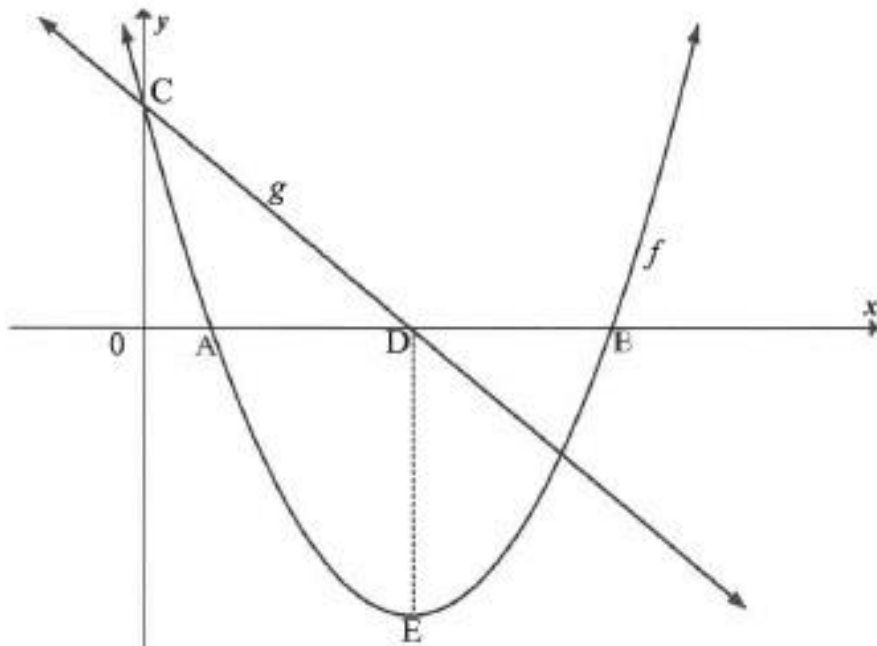
- | | | |
|-----|--|-------------|
| 5.1 | Write down the range of g . | (2) |
| 5.2 | Determine the equation of g . | (2) |
| 5.3 | Calculate the value of t . | (3) |
| 5.4 | Write down the equation of f^{-1} , the inverse of f , in the form $y = \dots$ | (2) |
| 5.5 | For which values of x will $f^{-1}(x) < 3$? | (2) |
| 5.6 | Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. | (3) |
| | | [14] |

FEB 2018

QUESTION 4

Below are the graphs of $f(x) = (x-4)^2 - 9$ and a straight line g .

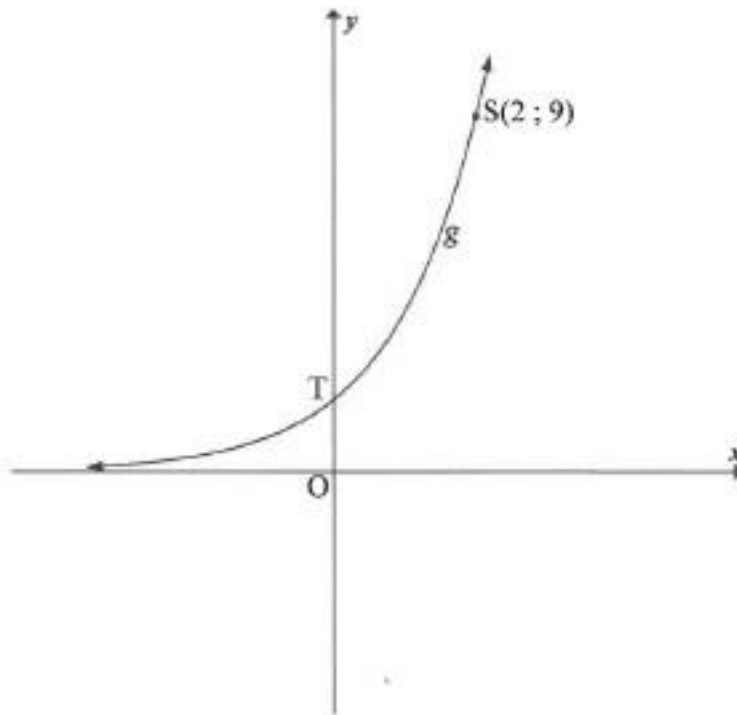
- A and B are the x -intercepts of f and E is the turning point of f .
- C is the y -intercept of both f and g .
- The x -intercept of g is D. DE is parallel to the y -axis.



- 4.1 Write down the coordinates of E. (2)
- 4.2 Calculate the coordinates of A. (3)
- 4.3 M is the reflection of C in the axis of symmetry of f . Write down the coordinates of M. (3)
- 4.4 Determine the equation of g in the form $y = mx + c$. (3)
- 4.5 Write down the equation of g^{-1} in the form $y = \dots$ (3)
- 4.6 For which values of x will $x(f(x)) \leq 0$? (4)
- [18]**

QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point $S(2 ; 9)$ lies on g . T is the y -intercept of g .



- | | | |
|-----|--|------------|
| 5.1 | Write down the coordinates of T . | (2) |
| 5.2 | Calculate the value of a . | (2) |
| 5.3 | The graph h is obtained by reflecting g in the y -axis. Write down the equation of h . | (2) |
| 5.4 | Write down the values of x for which $0 < \log_3 x < 1$. | (2) |
| | | [8] |

QUESTION 6

The function f , defined by $f(x) = \frac{a}{x+p} + q$, has the following properties:

- The range of f is $y \in R, y \neq 1$.
- The graph f passes through the origin.
- $P(\sqrt{2} + 2; \sqrt{2} + 1)$ lies on the graph f .

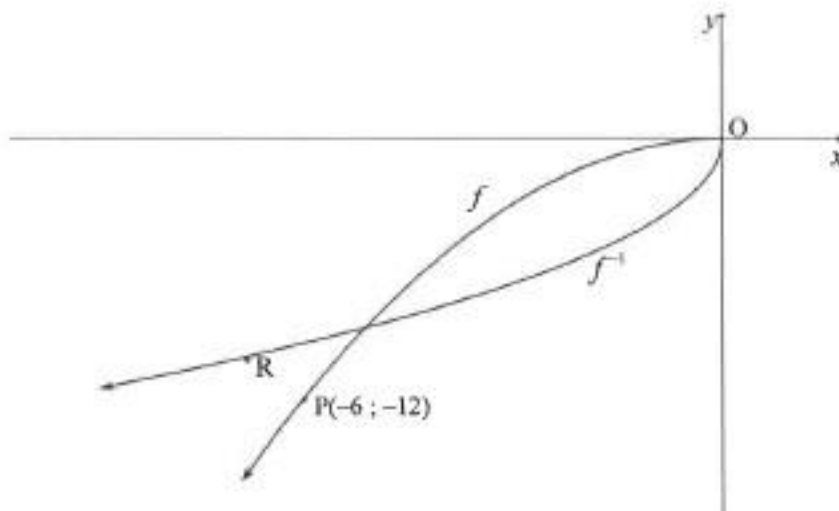
- 6.1 Write down the value of q . (1)
- 6.2 Calculate the values of a and p . (5)
- 6.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4)
- [10]**

NOV 2018

QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.

The graph of f^{-1} is also drawn. $P(-6; -12)$ is a point on f and R is a point on f^{-1} .



- 4.1 Is f^{-1} a function? Motivate your answer. (2)
- 4.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R . (1)
- 4.3 Calculate the value of a . (2)
- 4.4 Write down the equation of f^{-1} in the form $y = \dots$ (3)
- [8]**

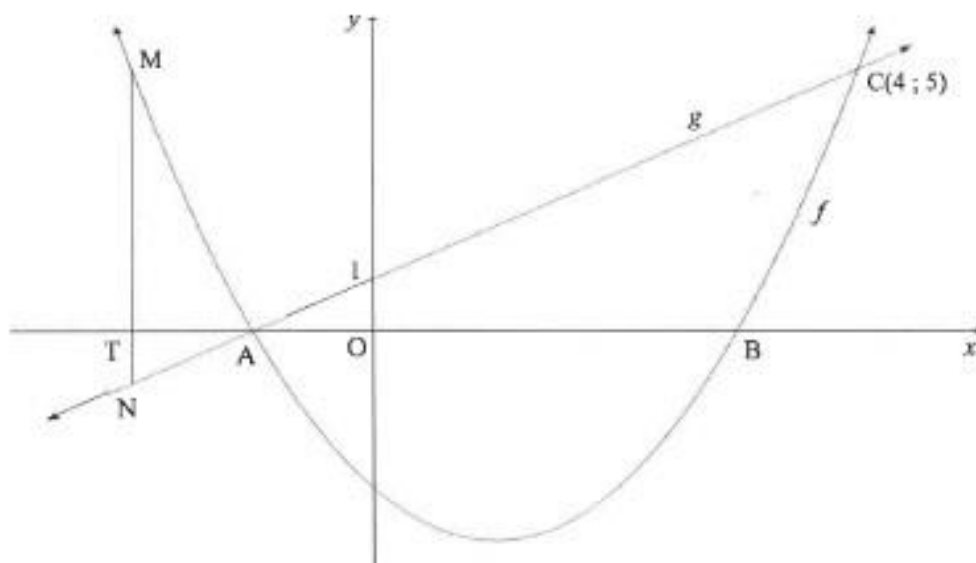
QUESTION 5

Given: $f(x) = \frac{-1}{x-1}$

- 5.1 Write down the domain of f . (1)
- 5.2 Write down the asymptotes of f . (2)
- 5.3 Sketch the graph of f , clearly showing all intercepts with the axes and any asymptotes. (3)
- 5.4 For which values of x will $x.f'(x) \geq 0$? (2)
- [8]**

QUESTION 6

In the diagram below, A and B are the x -intercepts of the graph of $f(x) = x^2 - 2x - 3$. A straight line, g , through A cuts f at $C(4; 5)$ and the y -axis at $(0; 1)$. M is a point on f and N is a point on g such that MN is parallel to the y -axis. MN cuts the x -axis at T.



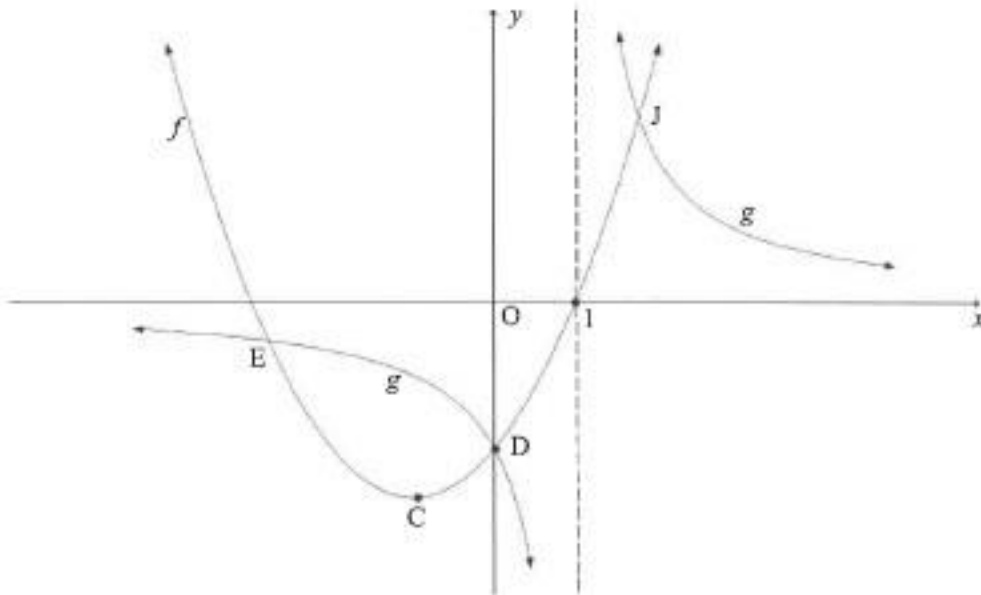
- 6.1 Show that $g(x) = x + 1$. (2)
- 6.2 Calculate the coordinates of A and B. (3)
- 6.3 Determine the range of f . (3)
- 6.4 If $MN = 6$:
- 6.4.1 Determine the length of OT if T lies on the negative x -axis. Show ALL your working. (4)
- 6.4.2 Hence, write down the coordinates of N. (2)
- 6.5 Determine the equation of the tangent to f drawn parallel to g . (5)
- 6.6 For which value(s) of k will $f(x) = x^2 - 2x - 3$ and $h(x) = x + k$ NOT intersect? (1)
- [20]**

NOV 2019

QUESTION 4

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \frac{a}{x+p}$.

- f has a turning point at C and passes through the x -axis at $(1; 0)$.
- D is the y -intercept of both f and g . The graphs f and g also intersect each other at E and J .
- The vertical asymptote of g passes through the x -intercept of f .

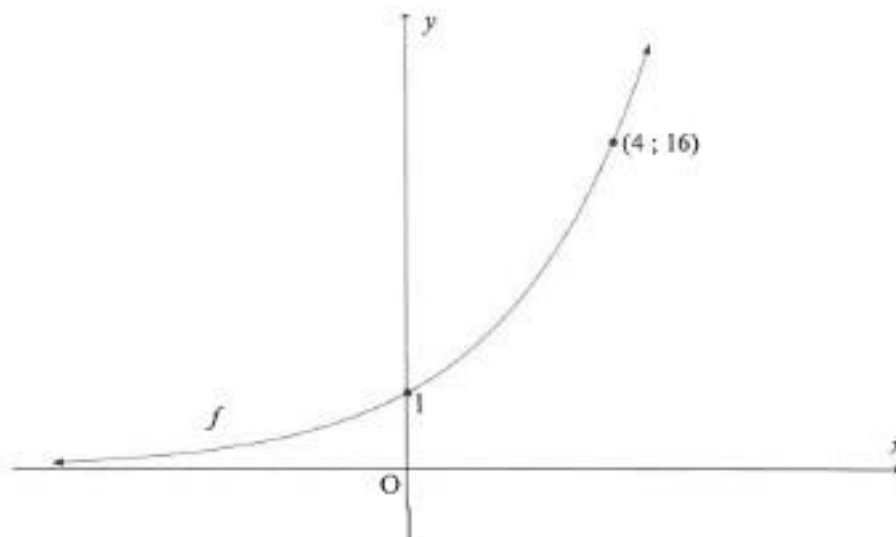


- 4.1 Write down the value of p . (1)
- 4.2 Show that $a = 3$ and $b = 2$. (3)
- 4.3 Calculate the coordinates of C . (4)
- 4.4 Write down the range of f . (2)
- 4.5 Determine the equation of the line through C that makes an angle of 45° with the positive x -axis. Write your answer in the form $y = \dots$ (3)
- 4.6 Is the straight line, determined in QUESTION 4.5, a tangent to f ? Explain your answer. (2)
- 4.7 The function $h(x) = f(m-x) + q$ has only one x -intercept at $x = 0$. Determine the values of m and q . (4)

[19]

QUESTION 5

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $(4; 16)$ lies on f .



- 5.1 Determine the value of k . (2)
- 5.2 Graph g is obtained by reflecting graph f about the line $y = x$. Determine the equation of g in the form $y = \dots$. (2)
- 5.3 Sketch the graph g . Indicate on your graph the coordinates of two points on g . (4)
- 5.4 Use your graph to determine the value(s) of x for which:
- 5.4.1 $f(x) \times g(x) > 0$ (2)
- 5.4.2 $g(x) \leq -1$ (2)
- 5.5 If $h(x) = f(-x)$, calculate the value of x for which $f(x) - h(x) = \frac{15}{4}$. (4)
- [16]**

FINANCE, GROWTH AND DECAY**NOV 2017****QUESTION 6**

- 6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r , correct to ONE decimal place. (5)
- 6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.
- 6.2.1 Calculate Piet's monthly instalment. (4)
- 6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)
- [15]**

FEB 2018**QUESTION 7**

- 7.1 On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018.
- Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (3)
- 7.2 On 1 February 2018, Genevieve took a loan of R82 000 from the bank to pay for her studies. She will make her first repayment of R3 200 on 1 February 2019 and continue to make payments of R3 200 on the first of each month thereafter until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.
- 7.2.1 Calculate how much Genevieve will owe the bank on 1 January 2019. (3)
- 7.2.2 How many instalments of R3 200 must she pay? (5)
- 7.2.3 Calculate the final payment, to the nearest rand, Genevieve has to pay to settle the loan. (5)
- [16]**

NOV 2018

QUESTION 7

- 7.1 Selby decided today that he will save R15 000 per quarter over the next four years. He will make the first deposit into a savings account in three months' time and he will make his last deposit at the end of four years from now.
- 7.1.1 How much will Selby have at the end of four years if interest is earned at 8,8% per annum, compounded quarterly? (3)
- 7.1.2 If Selby decides to withdraw R100 000 from the account at the end of three years from now, how much will he have in the account at the end of four years from now? (3)
- 7.2 Tshepo takes out a home loan over 20 years to buy a house that costs R1 500 000.
- 7.2.1 Calculate the monthly instalment if interest is charged at 10,5% p.a., compounded monthly. (4)
- 7.2.2 Calculate the outstanding balance immediately after the 144th payment was made. (5)
- [15]**

NOV 2019

QUESTION 6

- 6.1 Two friends, Kuda and Thabo, each want to invest R5 000 for four years. Kuda invests his money in an account that pays simple interest at 8,3% per annum. At the end of four years, he will receive a bonus of exactly 4% of the accumulated amount. Thabo invests his money in an account that pays interest at 8,1% p.a., compounded monthly.
- Whose investment will yield a better return at the end of four years? Justify your answer with appropriate calculations. (5)
- 6.2 Nine years ago, a bank granted Mandy a home loan of R525 000. This loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Mandy's monthly repayments commenced exactly one month after the loan was granted.
- 6.2.1 Mandy decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan? (5)
- 6.2.2 After making monthly repayments of R6 000 for nine years, Mandy required money to fund her daughter's university fees. She approached the bank for another loan. Instead, the bank advised Mandy that the extra amount repaid every month could be regarded as an investment and that she could withdraw this full amount to fund her daughter's studies. Calculate the maximum amount that Mandy may withdraw from the loan account. (4)
- [14]**

DIFFERENTIAL CALCULUS**NOV 2017****QUESTION 7**

- 7.1 Given: $f(x) = 2x^2 - x$
Determine $f'(x)$ from first principles. (6)
- 7.2 Determine:
- 7.2.1 $D_x[(x+1)(3x-7)]$ (2)
- 7.2.2 $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$ (4)
[12]

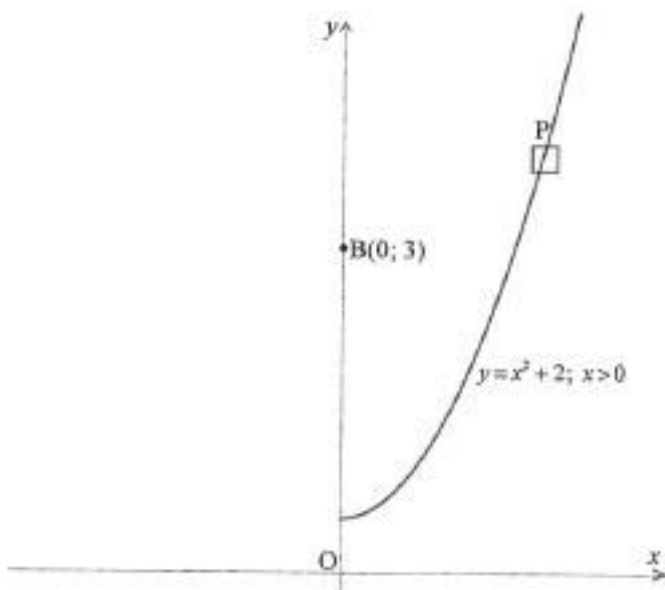
QUESTION 8Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

- 8.1 Show that f has a point of inflection at $x = 2$. (5)
- 8.2 Sketch the graph of f , clearly indicating the intercepts with the axes and the turning points. (4)
- 8.3 For which values of x will $y = -f(x)$ be concave down? (2)
- 8.4 Use your graph to answer the following questions:
- 8.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2) + 3$. (2)
- 8.4.2 Claire claims that $f'(2) = 1$.
Do you agree with Claire? Justify your answer. (2)
[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point $B(0; 3)$ and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny.

[7]

FEB 2018

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 4x^2$. (5)

8.2 Determine:

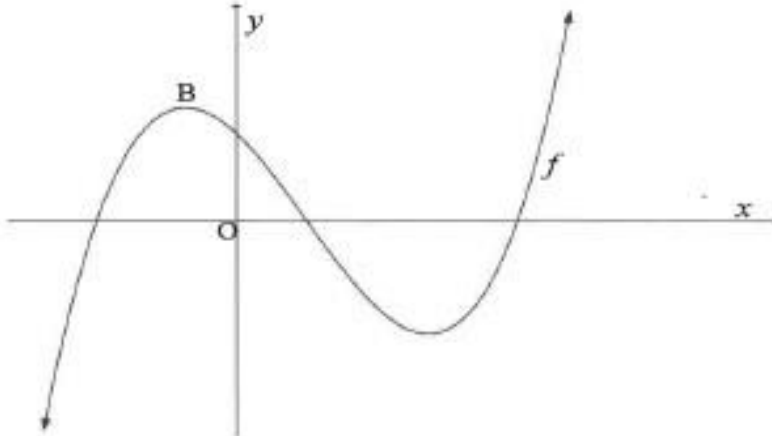
8.2.1 $D_x \left[\frac{x^2 - 2x - 3}{x + 1} \right]$ (3)

8.2.2 $f''(x)$ if $f(x) = \sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x) = x^3 + bx^2 + cx + d$. The solutions of the equation $f(x) = 0$ are -2 , 1 and 4 .



- 9.1 Calculate the values of b , c and d . (4)
 - 9.2 Calculate the x -coordinate of B , the maximum turning point of f . (4)
 - 9.3 Determine an equation for the tangent to the graph of f at $x = -1$. (4)
 - 9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x - and y -intercepts on your sketch. (3)
 - 9.5 For which value(s) of x is $f(x)$ concave upwards? (2)
- [17]**

QUESTION 10

Given: $f(x) = -3x^3 + x$.

Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$. (6)

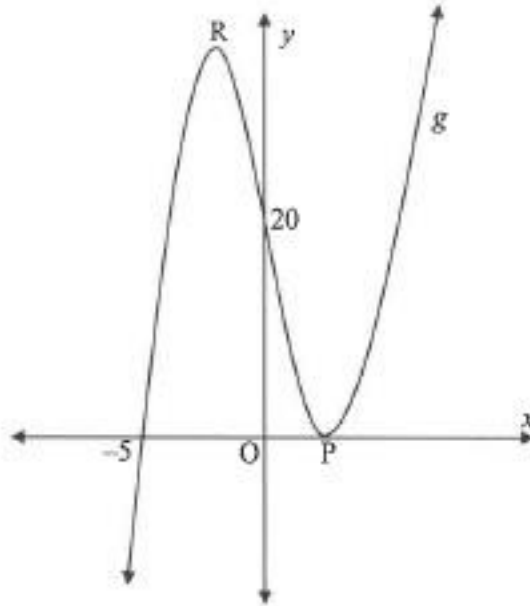
NOV 2018

QUESTION 8

- 8.1 Determine $f'(x)$ from first principles if it is given $f(x) = x^2 - 5$. (5)
 - 8.2 Determine $\frac{dy}{dx}$ if:
 - 8.2.1 $y = 3x^3 + 6x^2 + x - 4$ (3)
 - 8.2.2 $yx - y = 2x^2 - 2x$; $x \neq 1$ (4)
- [12]**

QUESTION 9

- 9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
 The graph of g intersects the x -axis at $(-5; 0)$ and at P , and the y -axis at $(0; 20)$.
 P and R are turning points of g .

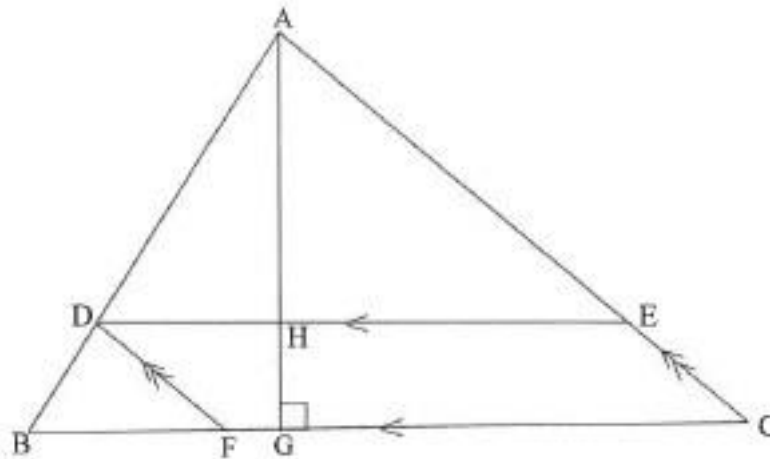


- 9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)
- 9.1.2 Calculate the coordinates of P and R . (5)
- 9.1.3 Is the graph concave up or concave down at $(0; 20)$? Show ALL your calculations. (3)
- 9.2 If g is a cubic function with:
- $g(3) = g'(3) = 0$
 - $g(0) = 27$
 - $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,
- draw a sketch graph of g indicating ALL relevant points. (3)
- [15]**

QUESTION 10

In $\triangle ABC$:

- D is a point on AB, E is a point on AC and F is a point on BC such that DECF is a parallelogram.
- $BF : FC = 2 : 3$.
- The perpendicular height AG is drawn intersecting DE at H.
- $AG = t$ units.
- $BC = (5 - t)$ units.



- 10.1 Write down $AH : HG$. (1)
- 10.2 Calculate t if the area of the parallelogram is a maximum.
(NOTE: Area of a parallelogram = base \times \perp height) (5)
- [6]**

NOV 2019

QUESTION 7

- 7.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 4 - 7x$. (4)
- 7.2 Determine $\frac{dy}{dx}$ if $y = 4x^8 + \sqrt{x^3}$. (3)
- 7.3 Given: $y = ax^2 + a$
- Determine:
- 7.3.1 $\frac{dy}{dx}$ (1)
- 7.3.2 $\frac{dy}{da}$ (2)
- 7.4 The curve with equation $y = x + \frac{12}{x}$ passes through the point $A(2 ; b)$. Determine the equation of the line perpendicular to the tangent to the curve at A. (4)
- [14]**

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t - 6)(-2t^2 + 3t - 6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)

[8]

QUESTION 9

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f , f' and f'' all pass through the point $(0; 0)$.

9.2.1 For which of the graphs will $(0; 0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)

[13]

PROBABILITY**NOV 2017****QUESTION 10**

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

- 10.1 Draw a Venn diagram to illustrate the information above. (4)
- 10.2 Calculate the value of x . (2)
- 10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)
- [8]**

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

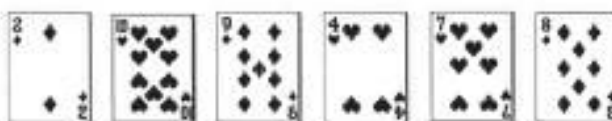
- 11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)
- 11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)
- [6]**

FEB 2018

QUESTION 11

- 11.1 Veli and Bongki are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongki will arrive late on any day is 0,7.
- 11.1.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1)
- 11.1.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongki will arrive late for school on that day. (3)
- 11.1.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongki arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3)

11.2 The cards below are placed from left to right in a row.



- 11.2.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)
- 11.2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)
- 11.2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)
- [15]**

NOV 2018

QUESTION 11

Given the digits: 3 ; 4 ; 5 ; 6 ; 7 ; 8 and 9

- 11.1 Calculate how many unique 5-digit codes can be formed using the digits above, if:
- 11.1.1 The digits may be repeated (2)
- 11.1.2 The digits may not be repeated (2)
- 11.2 How many unique 3-digit codes can be formed using the above digits, if:
- Digits may be repeated
 - The code is greater than 400 but less than 600
 - The code is divisible by 5
- (3)
- [7]**

QUESTION 12

12.1 Given: $P(A) = 0,45$; $P(B) = y$ and $P(A \text{ or } B) = 0,74$

Determine the value(s) of y if A and B are mutually exclusive. (3)

12.2 An organisation decided to distribute gift bags of sweets to a Grade R class at a certain school. There is a mystery gift in exactly $\frac{1}{4}$ of the total number of bags.

Each learner in the class may randomly select two gift bags of sweets, one after the other. The probability that a learner selects two bags of sweets with a mystery gift is $\frac{7}{118}$. Calculate the number of gift bags of sweets with a mystery gift inside.

(6)
[9]

NOV 2019**QUESTION 10**

The school library is open from Monday to Thursday. Anna and Ben both studied in the school library one day this week. If the chance of studying any day in the week is equally likely, calculate the probability that Anna and Ben studied on:

10.1 The same day (2)

10.2 Consecutive days (3)
[5]

QUESTION 11

11.1 Events **A** and **B** are independent. $P(A) = 0,4$ and $P(B) = 0,25$.

11.1.1 Represent the given information on a Venn diagram. Indicate on the Venn diagram the probabilities associated with each region. (3)

11.1.2 Determine $P(A \text{ or NOT } B)$. (2)

11.2 Motors Incorporated manufacture cars with 5 different body styles, 4 different interior colours and 6 different exterior colours, as indicated in the table below.

BODY STYLES	INTERIOR COLOURS	EXTERIOR COLOURS
Five body styles	Blue	Silver
	Grey	Blue
	Black	White
	Red	Green
		Red
	Gold	

The interior colour of the car must NOT be the same as the exterior colour.

Motors Incorporated wants to display one of each possible variation of its car in their showroom. The showroom has a floor space of 500 m^2 and each car requires a floor space of 5 m^2 .

Is this display possible? Justify your answer with the necessary calculations. (6)
[11]

ANNEXURE A: MATHEMATICS INFORMATION SHEET

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + m)^n$$

$$A = P(1 - m)^n$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

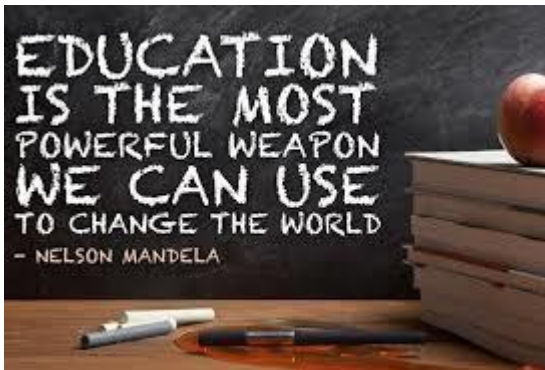
$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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MEMORANDA GROUPED ACCORDING TO TOPICS

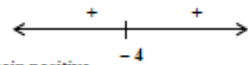
EQUATIONS AND INEQUALITIES

NOV 2017

QUESTION/PRAAG 1

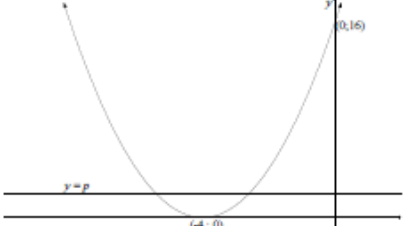
1.1.1	$x^2 + 9x + 14 = 0$ $(x+7)(x+2) = 0$ $x = -7$ or $x = -2$	✓ factors ✓ $x = -7$ ✓ $x = -2$	(3)
1.1.2	$4x^2 + 9x - 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-9 \pm \sqrt{9^2 - 4(4)(-3)}}{2(4)}$ $= \frac{-9 \pm \sqrt{129}}{8}$ $x = 0,29$ or $x = -2,54$ OR/OF $x^2 + \frac{9}{4}x + \frac{81}{64} = \frac{3}{4} + \frac{81}{64}$ $\left(x + \frac{9}{8}\right)^2 = \frac{129}{64}$ $x + \frac{9}{8} = \pm \frac{\sqrt{129}}{8}$ $x = \frac{-9 \pm \sqrt{129}}{8}$ $x = 0,29$ or $x = -2,54$	✓ substitution ✓ simplification ✓ $x = 0,29$ ✓ $x = -2,54$ OR/OF ✓ for adding $\frac{81}{64}$ on both sides ✓ simplification ✓ $x = 0,29$ ✓ $x = -2,54$	(4)
1.1.3	$\sqrt{x^2 - 5} = 2\sqrt{x}$ $x^2 - 5 = 4x$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = 5$ or $x = -1$ $x = 5$	✓ $x^2 - 5 = 4x$ ✓ standard form ✓ both answers ✓ select $x = 5$	(4)



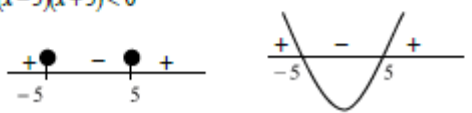
1.2	$3x - y = 4$ $y = 3x - 4$ $x^2 + 2xy - y^2 = -2$ $x^2 + 2x(3x - 4) - (3x - 4)^2 = -2$ $x^2 + 6x^2 - 8x - (9x^2 - 24x + 16) = -2$ $7x^2 - 8x - 9x^2 + 24x - 16 = -2$ $-2x^2 + 16x - 14 = 0$ $x^2 - 8x + 7 = 0$ $(x-7)(x-1) = 0$ $x = 1$ or $x = 7$ $y = 3(1) - 4$ or $y = 3(7) - 4$ $y = -1$ or $y = 17$ OR/OF $3x - y = 4$ $x = \frac{y+4}{3}$ $x^2 + 2xy - y^2 = -2$ $x^2 + 2xy - y^2 = -2$ $\left(\frac{y+4}{3}\right)^2 + 2\left(\frac{y+4}{3}\right)y - y^2 = -2$ $y^2 + 8y + 16 + 6y^2 + 24y - 9y^2 = -18$ $-2y^2 + 32y + 34 = 0$ $y^2 - 16y - 17 = 0$ $(y-17)(y+1) = 0$ $y = -1$ or $y = 17$ $x = \frac{-1+4}{3}$ or $x = \frac{17+4}{3}$ $x = 1$ or $x = 7$	✓ y subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓ x-values ✓ y-values OR/OF ✓ x subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓ y-values ✓ x-values	(6)
1.3.1	$x^2 + 8x + 16 > 0$ $(x+4)(x+4) > 0$ $x \in \mathbb{R}, x \neq -4$ or $x \in (-\infty; -4)$ or $x \in (-4; \infty)$ or $x < -4$ or $x > -4$ OR/OF $x^2 + 8x + 16 > 0$ $(x+4)(x+4) > 0$  <p>The function values remain positive $x \in \mathbb{R}, x \neq -4$</p>	✓ $(x+4)(x+4)$ ✓ any one of the solutions OR/OF ✓ $(x+4)(x+4)$ ✓ any one of the solutions	(3)

FEB 2018

QUESTION/VR44G 1

1.3.2	 <p>For two negative unequal roots: $0 < p < 16$</p> <p>OR/OF $x^2 + 8x + 16 = p$ $x^2 + 8x + 16 - p = 0$ $0 < 16 - p < 16$ $-16 < -p < 0$ $0 < p < 16$</p> <p>OR/OF $x^2 + 8x + 16 - p = 0$ $x = \frac{-8 \pm \sqrt{64 - 4(16 - p)}}{2}$ $0 < 64 - 4(16 - p) < 64$ $0 < 4p < 64$ $0 < p < 16$</p> <p>OR/OF $x^2 + 8x + 16 = p$ $x^2 + 8x + 16 - p = 0$ Roots are real and unequal: $8^2 - 4(16 - p) > 0$ $4p > 0$ $p > 0$ Roots are: $\frac{-8 \pm \sqrt{4p}}{2}$</p> <p>For both roots to be negative: $\sqrt{4p} < 8$ $4p < 64$ $p < 16$ $0 < p < 16$</p>	<p>✓ 0 ✓ 16</p> <p>✓ ✓ $0 < p < 16$ (4)</p> <p>OR/OF</p> <p>✓ 0 ✓ 16</p> <p>✓ ✓ $0 < p < 16$ (4)</p> <p>✓ 0 ✓ 16</p> <p>✓ ✓ $0 < p < 16$ (4)</p> <p>✓ 0 ✓ 16</p> <p>✓ ✓ $0 < p < 16$ (4)</p> <p>✓ 0 ✓ 16</p> <p>✓ ✓ $0 < p < 16$ (4)</p>	<p>(4)</p> <p>(4)</p> <p>(4)</p> <p>(4)</p> <p>(4)</p> <p>(4)</p> <p>(4)</p> <p>(4)</p>
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1.1.1	$x^2 - 6x - 16 = 0$ $(x - 8)(x + 2) = 0$ $x = -2$ or $x = 8$	<p>✓ factors ✓ $x = -2$ ✓ $x = 8$</p>	(3)	
1.1.2	$2x^2 + 7x - 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-7 \pm \sqrt{7^2 - 4(2)(-1)}}{2(2)}$ $= \frac{-7 \pm \sqrt{57}}{4}$ $x = 0,14$ or $x = -3,64$	<p>OR/OF</p> <p>NOTE: Penalise 1 mark if the rounding to TWO decimal places is incorrect.</p>	<p>✓ subs into correct formula</p> <p>✓ $\frac{-7 \pm \sqrt{57}}{4}$ ✓ $x = 0,14$ ✓ $x = -3,64$</p> <p>OR/OF</p> <p>✓ for adding $\frac{49}{16}$ on both sides</p> <p>✓ $\frac{-7 \pm \sqrt{57}}{4}$ ✓ $x = 0,14$ ✓ $x = -3,64$</p>	(4)
1.2	$x^2 - 25 < 0$ $(x - 5)(x + 5) < 0$  $-5 < x < 5$ $x = \{-4; -3; -2; -1; 0; 1; 2; 3; 4\}$	<p>✓ factors</p> <p>✓ ✓ inequality</p> <p>✓ answer</p>	(4)	

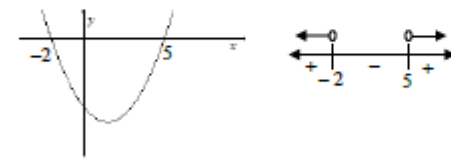
<p>1.3</p>	$x = 2y - 1$ $(2y - 1)^2 - 7 - y^2 = -y$ $4y^2 - 4y + 1 - 7 - y^2 = -y$ $3y^2 - 3y - 6 = 0$ $y^2 - y - 2 = 0$ $(y - 2)(y + 1) = 0$ $y = 2 \text{ or } y = -1$ $x = 2(2) - 1 \text{ or } x = 2(-1) - 1$ $x = 3 \text{ or } x = -3$ <p>OR/OF</p> $y = \frac{x+1}{2}$ $x^2 - 7 - y^2 = -y$ $x^2 - 7 - \left(\frac{x+1}{2}\right)^2 = -\left(\frac{x+1}{2}\right)$ $x^2 - 7 - \left(\frac{x^2 + 2x + 1}{4}\right) = \frac{-x-1}{2}$ $4x^2 - 28 - x^2 - 2x - 1 = -2x - 2$ $3x^2 - 27 = 0$ $x^2 - 9 = 0$ $(x - 3)(x + 3) = 0$ $x = -3 \text{ or } x = 3$ $y = \frac{-3+1}{2} \text{ or } y = \frac{3+1}{2}$ $y = -1 \text{ or } y = 2$	<ul style="list-style-type: none"> ✓ $x = 2y - 1$ ✓ substitution ✓ correct standard form ✓ factors ✓ y - values ✓ x - values OR/OF ✓ $y = \frac{x+1}{2}$ ✓ substitution ✓ correct standard form ✓ factors ✓ x - values ✓ y - values <p style="text-align: right;">(6)</p>
<p>1.4</p>	$\frac{3^{2018} + 3^{2016}}{3^{2017}}$ $= \frac{3^{2017}(3^1 + 3^{-1})}{3^{2017}}$ $= 3 + \frac{1}{3}$ $= 3\frac{1}{3} \text{ or } \frac{10}{3}$ <p>OR/OF</p>	<ul style="list-style-type: none"> ✓ common factor 3^{2017} ✓ answer OR/OF

	$\frac{3^{2018} + 3^{2016}}{3^{2017}}$ $= \frac{3^{2016}(3^2 + 1)}{3^{2017}}$ $= \frac{10}{3}$ <p>OR/OF</p> $\frac{3^{2018} + 3^{2016}}{3^{2017}}$ $= \frac{3^{2018}}{3^{2017}} + \frac{3^{2016}}{3^{2017}}$ $= 3 + \frac{1}{3}$ $= 3\frac{1}{3} \text{ or } \frac{10}{3}$	<ul style="list-style-type: none"> ✓ common factor 3^{2016} ✓ answer OR/OF ✓ dividing by 3^{2017} ✓ answer <p style="text-align: right;">(2)</p>
<p>1.5.1</p>	$3x - 5 \geq 0 \text{ and } x \neq 3$ $x \geq \frac{5}{3} \text{ and } x \neq 3$	<ul style="list-style-type: none"> ✓ $3x - 5 \geq 0$ ✓ $x \geq \frac{5}{3}$ ✓ $x \neq 3$ <p style="text-align: right;">(3)</p>
<p>1.5.2</p>	$\frac{\sqrt{3x-5}}{x-3} = 1$ $\sqrt{3x-5} = x-3$ $3x-5 = (x-3)^2$ $3x-5 = x^2 - 6x + 9$ $x^2 - 9x + 14 = 0$ $(x-7)(x-2) = 0$ $x \neq 2 \text{ or } x = 7$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>NOTE: If $x = 2$ is not rejected, then maximum 3 / 4 marks</p> </div>	<ul style="list-style-type: none"> ✓ $\sqrt{3x-5} = x-3$ ✓ $3x-5 = (x-3)^2$ ✓ factors ✓ $x = 7$ <p style="text-align: right;">(4) [26]</p>



NOV 2018

QUESTION/VRAAG 1

1.1.1	$x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 3$ or $x = 1$	✓ factors/correct sub in formula ✓ $x = 3$ ✓ $x = 1$	(3)
1.1.2	$5x^2 - 5x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{5 \pm \sqrt{25 - 4(5)(1)}}{2(5)}$ $= \frac{5 \pm \sqrt{5}}{10}$ $x = 0,72$ or $x = 0,28$	✓ substitution into the correct formula ✓ $x = 0,72$ ✓ $x = 0,28$	(3)
1.1.3	$x^2 - 3x - 10 > 0$ $(x-5)(x+2) > 0$ OR/OF 	✓ factors/ critical values ✓✓ $x < -2$ or $x > 5$	(3)
1.1.4	$3\sqrt{x} = x - 4$ $9x = x^2 - 8x + 16$ $x^2 - 17x + 16 = 0$ $(x-16)(x-1) = 0$ $x = 16$ or $x = 1$ NA	✓ squaring both sides ✓ $x^2 - 17x + 16 = 0$ ✓ factors ✓ answer with selection	(4)



	OR/OF $\frac{1}{3x^2} - x - 4$ $x - 3x^2 - 4 = 0$ $\left(\frac{1}{x^2} - 4\right)\left(\frac{1}{x^2} + 1\right) = 0$ $\frac{1}{x^2} = 4$ or $\frac{1}{x^2} = -1$ $x = 16$ NA	✓ standard form ✓ recognize $x = \left(\frac{1}{x^2}\right)^2$ ✓ factors ✓ answer with selection (4)	
1.2	$2y + 9x^2 = -1$(1) $3x - y = 2$ (2) $y = 3x - 2$(3) $2(3x - 2) + 9x^2 = -1$ $6x - 4 + 9x^2 = -1$ $9x^2 + 6x - 3 = 0$ $3x^2 + 2x - 1 = 0$ $(3x-1)(x+1) = 0$ $x = \frac{1}{3}$ or $x = -1$ $y = -1$ or $y = -5$ OR/OF $2y + 9x^2 = -1$(1) $3x - y = 2$ (2) $x = \frac{y+2}{3}$ $2y + 9\left(\frac{y+2}{3}\right)^2 = -1$ $2y + 9\left(\frac{y^2 + 4y + 4}{9}\right) = -1$ $2y + y^2 + 4y + 4 = -1$ $y^2 + 6y + 5 = 0$ $(y+5)(y+1) = 0$ $y = -1$ or $y = -5$ $x = \frac{1}{3}$ or $x = -1$	✓ $y = 3x - 2$ ✓ substitution ✓ standard form ✓ factors ✓ both x values ✓ both y values OR/OF ✓ $x = \frac{y+2}{3}$ ✓ substitution ✓ standard form ✓ factors ✓ both y values ✓ both x values	(6)

NOV 2019

QUESTION/VR44G 1

1.3	$3^{9x} = 64$ $(3^{3x})^3 = (4)^3$ $3^{3x} = 4$ $5^{\sqrt{p}} = 64$ $\sqrt{5^{\sqrt{p}}} = \sqrt{64}$ $\sqrt{5^{\sqrt{p}}} = 8$ $\frac{[3^{x-1}]^3}{\sqrt{5^{\sqrt{p}}}} = \frac{3^{3x-3}}{\sqrt{5^{\sqrt{p}}}}$ $= \frac{3^{3x}}{27 \times \sqrt{5^{\sqrt{p}}}}$ $= \frac{4}{27 \times 8}$ $= \frac{1}{54}$ OR/OF $\frac{(3^{x-1})^3}{\sqrt{5^{\sqrt{p}}}} = \frac{3^{3x-3}}{(5^{0.5})^{\sqrt{p}}}$ $= \frac{3^{3x-3}}{(5^{\sqrt{p}})^{0.5}}$ $= \frac{4 \cdot 3^{-3}}{\sqrt{64}}$ $= \frac{4}{8} \cdot \frac{1}{27} = \frac{1}{54}$	$\checkmark 3^{3x} = 4$ $\checkmark \sqrt{5^{\sqrt{p}}} = 8$ $\checkmark 3^{3x-3}$ or $3^{3x} \cdot 3^{-3}$ $\checkmark \frac{3^{3x}}{27 \times \sqrt{5^{\sqrt{p}}}}$ $\checkmark \frac{1}{54}$ OR/OF $\checkmark 3^{3x-3}$ or $3^{3x} \cdot 3^{-3}$ $\checkmark 3^{3x} = 4$ $\checkmark \sqrt{5^{\sqrt{p}}} = 8$ $\checkmark \text{answer}$	(4)
			[23]



1.1.1	$x^2 + 5x - 6 = 0$ $(x+6)(x-1) = 0$ $x = -6$ or $x = 1$	$\checkmark \text{factors}$ $\checkmark x = -6$ $\checkmark x = 1$ (3)
1.1.2	$4x^2 + 3x - 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-5)}}{2(4)}$ $x = \frac{-3 \pm \sqrt{89}}{8}$ $x = -1,55$ or $x = 0,8$	$\checkmark \text{substitution into the correct formula}$ $\checkmark x = -1,55$ $\checkmark x = 0,8$ (3)
1.1.3	$4x^2 - 1 < 0$ $(2x+1)(2x-1) < 0$ $-\frac{1}{2} < x < \frac{1}{2}$	$\checkmark \text{factors}$ $\checkmark \text{method}$ $\checkmark \text{answer}$ (3)
1.1.4	$(\sqrt{\sqrt{32} + x})(\sqrt{\sqrt{32} - x}) = x$ $\sqrt{32 - x^2} = x$ $32 - x^2 = x^2$ $-2x^2 = -32$ $x^2 = 16$ $x = \pm 4$ $\therefore x = 4$	$\checkmark \sqrt{32 - x^2}$ $\checkmark \text{squaring both sides}$ $\checkmark x^2 = 16$ $\checkmark x = 4$ (selection) (4)

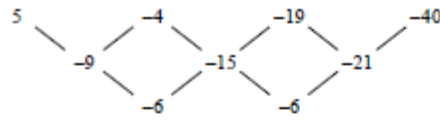
<p>1.2</p>	<p>$y + x = 12$ $y = -x + 12 \dots\dots(1)$ $xy = 14 - 3x \dots\dots(2)$ Sub (1) into (2) $x(-x + 12) = 14 - 3x$ $-x^2 + 12x - 14 + 3x = 0$ $-x^2 + 15x - 14 = 0$ $x^2 - 15x + 14 = 0$ $(x - 14)(x - 1) = 0$ $x = 14$ or $x = 1$ $y = -2$ or $y = 11$ OR/OF $y + x = 12$ $x = -y + 12 \dots\dots(1)$ $xy = 14 - 3x \dots\dots(2)$ Sub (1) into (2) $y(-y + 12) = 14 - 3(-y + 12)$ $12y - y^2 - 14 + 36 - 3y = 0$ $-y^2 + 9y + 22 = 0$ $y^2 - 9y - 22 = 0$ $(y + 2)(y - 11) = 0$ $y = -2$ or $y = 11$ $x = 14$ or $x = 1$</p>	<p>✓y subject of the formula ✓ substitution ✓simplification ✓both values of x ✓both values of y (5) OR/OF ✓x subject of the formula ✓substitution ✓simplification ✓both values of y ✓both values of x (5)</p>
<p>1.3</p>	<p>3 6 9 12 15 18 21 24 27 30 3 3 3² 3 3 3² 3 3 3³ 3 $\therefore k = 14$</p>	<p>✓identifying multiples of 3 ✓ten multiples of 3 ✓ powers of 3 ✓ answer (4)</p>
<p>[22]</p>		



NUMBER PATTERNS, SEQUENCES AND SERIES

NOV 2017

QUESTION/VRAAG 2

2.1.1	 <p>first differences: -9; -15; -21 second difference = -6</p>	<p>✓ first differences ✓ -6</p> <p>(2)</p>
2.1.2	$T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2} = -3$ $3a + b = -9$ $3(-3) + b = -9$ $b = 0$ $a + b + c = 5$ $-3 + 0 + c = 5$ $c = 8$ $T_n = -3n^2 + 8$ <p>OR/OF</p> $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 5 + (n-1)(-9) + \frac{(n-1)(n-2)(-6)}{2}$ $= 5 - 9n + 9 - 3n^2 + 9n - 6$ $T_n = -3n^2 + 8$	<p>✓ $a = -3$</p> <p>✓ $b = 0$</p> <p>✓ $c = 8$</p> <p>✓ $T_n = -3n^2 + 8$</p> <p>OR/OF</p> <p>✓ $a = -3$</p> <p>✓ $b = 0$</p> <p>✓ $c = 8$</p> <p>✓ $T_n = -3n^2 + 8$</p> <p>(4)</p>
2.1.3	$-3n^2 + 8 = -25\ 939$ $-3n^2 = -25947$ $n^2 = 8649$ $n = -93 \text{ or } n = 93$ <p>The 93rd term has a value of -25 939</p>	<p>✓ $T_n = -25\ 939$</p> <p>✓ $n^2 = 8649$</p> <p>✓ answer</p> <p>(3)</p>



2.2.1	$2k - 7; k + 8 \text{ and } 2k - 1$ $k + 8 - (2k - 7) = 2k - 1 - (k + 8)$ $-k + 15 = k - 9$ $2k = 24$ $k = 12$ $2k - 7; k + 8 \text{ and } 2k - 1$ $17; 20; 23, \dots$ $d = 3$ $T_{15} = 17 + 14(3)$ $= 59$	<p>✓ $k + 8 - (2k - 7) = 2k - 1 - (k + 8)$</p> <p>✓ $k = 12$</p> <p>✓ 17 ✓ $d = 3$</p> <p>✓ $T_{15} = 59$</p> <p>(5)</p>
2.2.2	<p>Sequence is 17; 20; 23; 26; 29; 32,</p> <p>Every alternate term of the sequence will be even / Elke tweede term van die ry sal ewe wees</p> $20 + 26 + 32 + \dots$ $S_{30} = \frac{30}{2} [2(20) + (29)(6)]$ $= 15[40 + 174]$ $= 3210$ <p>OR/OF</p> $T_{30} = 20 + 29(6)$ $= 94$ $S_{30} = \frac{30}{2} (20 + 194)$ $= 3210$	<p>✓ $20 + 26 + 32 + \dots$</p> <p>✓ $a = 20$ $d = 6$ ✓ subst into correct formula</p> <p>✓ answer</p> <p>(4)</p> <p>✓ $a = 20$ $d = 6$</p> <p>✓ $T_{30} = 94$</p> <p>✓ $S_{30} = \frac{30}{2} (20 + 194)$</p> <p>✓ answer</p> <p>(4)</p> <p>[18]</p>

QUESTION/VRAAG 3

<p>3.1</p> $a + ar = 2$ $a(1+r) = 2$ $a = \frac{2}{1+r}$ <p>OR/OF</p> $\frac{a}{1-r} - 2 = \frac{1}{4}$ $4a - 8(1-r) = 1 - r$ $4a - 8 + 8r = 1 - r$ $4a - 9 = 9r$ $a = \frac{9-9r}{4}$ <p>OR/OF</p> $S_n = \frac{a(r^n - 1)}{r - 1}$ $2 = \frac{a(r^2 - 1)}{r - 1}$ $2 = \frac{a(r-1)(r+1)}{r-1}$ $2 = a(r+1)$ $a = \frac{2}{r+1}$ <p>OR/OF</p> $\frac{ar^2}{1-r} = \frac{1}{4}$ $a = \frac{1-r}{4r^2}$	<p>$\checkmark a + ar = 2$</p> <p>$\checkmark a = \frac{2}{1+r}$ (2)</p> <p>$\checkmark \frac{a}{1-r} - 2 = \frac{1}{4}$</p> <p>$\checkmark a = \frac{9-9r}{4}$ (2)</p> <p>OR/OF</p> <p>$\checkmark 2 = \frac{a(r^2-1)}{r-1}$</p> <p>$\checkmark a = \frac{2}{r+1}$ (2)</p> <p>OR/OF</p> <p>$\checkmark \frac{ar^2}{1-r} = \frac{1}{4}$</p> <p>$\checkmark a = \frac{1-r}{4r^2}$ (2)</p>
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<p>3.2</p> $S_n = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$ $S_n = 2 + \frac{1}{4}$ $\frac{a}{1-r} = 2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$ $\frac{2}{1-r^2} = \frac{9}{4}$ $8 = 9 - 9r^2$ $9r^2 = 1$ $r = \frac{1}{3}$ $a = \frac{3}{2}$ <p>OR/OF</p> $S_n = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$ $S_n = 2 + \frac{1}{4}$ $\frac{a}{1-r} = 2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $4a = 9 - 9r$ $r = \frac{9-4a}{9}$ $a + a\left(\frac{9-4a}{9}\right) = 2$ $9a + 9a - 4a^2 = 18$ $2a^2 - 9a + 9 = 0$ $(a-3)(2a-3) = 0$ $a = \frac{3}{2} \text{ or } a = 3$ $r = \frac{1}{3} \text{ or } r = -\frac{1}{3}$ <p>N/A</p>	<p>$\checkmark S_n = 2 + \frac{1}{4}$</p> <p>$\checkmark \frac{a}{1-r} = \frac{9}{4}$</p> <p>$\checkmark$ substitution of a into the correct formula</p> <p>$\checkmark 9r^2 = 1$</p> <p>$\checkmark r = \frac{1}{3}$</p> <p>$\checkmark a = \frac{3}{2}$ (6)</p> <p>OR/OF</p> <p>$\checkmark S_n = 2 + \frac{1}{4}$</p> <p>$\checkmark \frac{a}{1-r} = \frac{9}{4}$</p> <p>$\checkmark r = \frac{9-4a}{9}$</p> <p>$\checkmark$ substitution of a into the correct formula</p> <p>$\checkmark a = \frac{3}{2}$</p>
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FEB 2018

QUESTION/PRAAG 2

<p>OR/OF</p> $r = \frac{2-a}{a}$ $\frac{ar^2 - 1}{1-r} = \frac{1}{4}$ $4ar^2 - 1 - r$ $4a\left(\frac{2-a}{a}\right)^2 - 1 - \frac{2-a}{a}$ $16 - 16a + 4a^2 - 2a + 2$ $2a^2 - 9a + 9 = 0$ $(2a-3)(a-3) = 0$ $a = \frac{3}{2} \quad a = 3$ $r = \frac{1}{3} \quad r = -\frac{1}{3}$ <p>OR/OF</p> $S_n = T_1 + T_2 + \sum_{i=1}^n T_i$ $S_n = -2 + \frac{1}{4}$ $\frac{a}{1-r} = -2 + \frac{1}{4}$ $\frac{a}{1-r} = \frac{9}{4}$ $\left(\frac{1-r}{4r^2}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$ $\frac{1}{4r^2} = \frac{9}{4}$ $4 = 36r^2$ $9r^2 = 1$ $r = \frac{1}{3}$ $a = \frac{3}{2}$	<p>✓ $r = \frac{1}{3}$ (6)</p> <p>OR/OF</p> $\checkmark r = \frac{2-a}{a}$ $\checkmark \frac{ar^2 - 1}{1-r} = \frac{1}{4}$ <p>✓ substitution of a</p> <p>✓ $(2a-3)(a-3) = 0$</p> <p>✓ $a = \frac{3}{2}$</p> <p>✓ $r = \frac{1}{3}$</p> <p>OR/OF (6)</p> <p>✓ $S_n = -2 + \frac{1}{4}$</p> <p>✓ $\frac{a}{1-r} = \frac{9}{4}$</p> <p>✓ substitution of a</p> <p>✓ $9r^2 = 1$</p> <p>✓ $r = \frac{1}{3}$</p> <p>✓ $a = \frac{3}{2}$ (6)</p> <p>[8]</p>
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<p>2.1.1</p> <p>30; 10; $\frac{10}{3}$.....</p> <p>$a = 30 \quad r = \frac{1}{3}$</p> <p>$T_n = ar^{n-1}$</p> <p>$\frac{10}{729} = 30\left(\frac{1}{3}\right)^{n-1}$</p> <p>$\frac{1}{2187} = 3^{1-n}$</p> <p>$3^{-7} = 3^{1-n}$</p> <p>$-7 = 1 - n$</p> <p>$n = 8$</p> <p>OR/OF</p> <p>$\frac{1}{2187} = \left(\frac{1}{3}\right)^{n-1}$</p> <p>$\left(\frac{1}{3}\right)^7 = \left(\frac{1}{3}\right)^{n-1}$</p> <p>$7 = n - 1$</p> <p>$n = 8$</p>	<p>✓ $r = \frac{1}{3}$</p> <p>✓ substitution into correct formula</p> <p>✓ $3^{-7} = 3^{1-n}$ or $\left(\frac{1}{3}\right)^7 = \left(\frac{1}{3}\right)^{n-1}$ or use of logs</p> <p>✓ $n = 8$ (4)</p>
<p>2.1.2</p> <p>$S_n = \frac{a}{1-r}$</p> <p>$= \frac{30}{1-\frac{1}{3}}$</p> <p>$= 45$</p>	<p>✓ substitution into correct formula</p> <p>✓ answer (2)</p>
<p>2.2</p> <p>$S_n = a + (a+d) + \dots + (a+(n-2)d) + (a+(n-1)d)$ (1)</p> <p>$S_n = (a+(n-1)d) + (a+(n-2)d) + \dots + (a+d) + a$ (2)</p> <p>Adding both equations/Tel die twee vergelykings bymekaar:</p> <p>$2S_n = 2a + (n-1)d + 2a + (n-1)d + 2a + (n-1)d + \dots$</p> <p>$= n[2a + (n-1)d]$</p> <p>$S_n = \frac{n}{2}[2a + (n-1)d]$</p> <p>OR/OF</p> <p>$S_n = a + (a+d) + \dots + (a+(n-2)d) + T_n$ (1)</p> <p>$S_n = T_n + (T_n - d) + (T_n - 2d) + \dots + a$ (2)</p> <p>Adding both equations/Tel die twee vergelykings bymekaar:</p> <p>$2S_n = (a+T_n) + (a+T_n) + (a+T_n) + \dots + (a+T_n)$</p> <p>$S_n = \frac{n}{2}(a+T_n)$</p> <p>but $T_n = a + (n-1)d$</p> <p>$S_n = \frac{n}{2}[2a + (n-1)d]$</p>	<p>✓ expanding S_n</p> <p>✓ reverse writing</p> <p>✓ $2S_n = n[2a + (n-1)d]$</p> <p>✓ $S_n = \frac{n}{2}[2a + (n-1)d]$ (4)</p> <p>✓ expanding S_n</p> <p>✓ reverse writing</p> <p>✓ $2S_n = n(a+T_n)$</p> <p>✓ $S_n = \frac{n}{2}[2a + (n-1)d]$ (4)</p> <p>[10]</p>

QUESTION/VRAAG 3

3.1	$-1; 2; 5$ $T_n = -1 + (n-1)(3)$ $= 3n - 4$	$\checkmark 3n$ $\checkmark -4$	(2)
3.2	$T_{43} = 3(43) - 4$ OR/OF $T_{43} = -1 + (43-1)(3)$ $= 125$ OR/OF $= 125$	\checkmark subs of 43 \checkmark answer	(2)
3.3	$T_n = 3n - 4$ $S_n = \sum_{k=1}^n T_k = -1 + 2 + 5 + \dots + 3n - 4$ $S_n = \frac{n}{2}[-1 + 3n - 4]$ OR $S_n = \frac{n}{2}[-2 + (n-1)3]$ $= \frac{n}{2}[3n - 5]$ $= \frac{3n^2 - 5n}{2}$ OR/OF $T_n = 3n - 4$ $\sum_{k=1}^n T_k = 3(1) - 4 + 3(2) - 4 + 3(3) - 4 + \dots + 3n - 4$ $= 3(1 + 2 + 3 + \dots + n) - 4n$ $= \frac{3n(n+1)}{2} - 4n$ $= \frac{3n^2 - 5n}{2}$	$\checkmark S_n = \sum_{k=1}^n T_k$ \checkmark substitution into correct formula $\checkmark \frac{n}{2}[3n - 5]$ OR $\frac{3n^2 - 5n}{2}$ OR/OF $\checkmark (1) - 4 + 3(2) - 4 + 3(3) - 4 + \dots + 3n - 4$ $\checkmark 3(1 + 2 + 3 + \dots + n) - 4n$ $\checkmark \frac{3n^2 - 5n}{2}$	(3)



3.4	$T_{11} = (T_{11} - T_{10}) + (T_{10} - T_9) + (T_9 - T_8) + \dots + (T_3 - T_2) + (T_2 - T_1) + T_1$ $125 = 29 + 26 + 23 + \dots + 2 + T_1$ $= \frac{10}{2}(29 + 2) + T_1$ $= 155 + T_1$ $T_1 = -30$	$\checkmark \checkmark$ generating sum $\checkmark 29 + 26 + 23 + \dots + 2$ $\checkmark \frac{10}{2}(29 + 2)$ $\checkmark 155$ $\checkmark -30$	(6)
	OR/OF $T_n = an^2 + bn + c$ $\therefore T_{11} = 121a + 11b + c = 125$ $T_n - T_{n-1} = an^2 + bn + c - [a(n-1)^2 + b(n-1) + c]$ $= an^2 + bn + c - an^2 + 2an - a - bn + b - c$ $= 2an + b - a$ $T_n - T_{n-1} = 3n - 4$ $2a = 3$ and $b - a = -4$ $a = \frac{3}{2}$ and $b = -\frac{5}{2}$ $121a + 11b + c = 125$ $121\left(\frac{3}{2}\right) + 11\left(-\frac{5}{2}\right) + c = 125$ $c = -29$ $T_n = \frac{3}{2}n^2 - \frac{5}{2}n - 29$ $T_1 = \frac{3}{2}(1)^2 - \frac{5}{2}(1) - 29$ $= -30$	NOTE: Answer only 1 / 6 If they only use $3n - 4$ breakdown 0 / 6 $\checkmark 121a + 11b + c = 125$ \checkmark calculating $T_n - T_{n-1}$ in terms of a, b and c $\checkmark a = \frac{3}{2}$ $\checkmark b = -\frac{5}{2}$ $\checkmark c = -29$ $\checkmark -30$	(6)
			[13]

NOV 2018

QUESTION/VRAAG 2

2.1.1	42	✓ answer	(1)
2.1.2	$2a = 6$ $a = 3$ $T_n = 3n^2 - 8n + 7$ OR/OF $2a = 6$ $a = 3$ $T_n = 3n^2 + bn + c$ $T_1 : 3 + b + c = 2$ $T_2 : 12 + 2b + c = 3$ $T_2 - T_1 : b = -8$ Subst. in (1): $-8 + c = -1$ $c = 7$ $T_n = 3n^2 - 8n + 7$	$3a + b = 1$ $3(3) + b = 1$ $b = -8$ $a + b + c = 2$ $(3) + (-8) + c = 2$ $c = 7$ $\checkmark T_n = an^2 + bn + c$	(4)
2.1.3	$T_{20} = 3(20)^2 - 8(20) + 7$ $= 1047$	✓ substitution ✓ answer	(2)
2.2	$T_n = -7n + 42$ $-7n + 42 = -140$ $-7n = -182$ $n = 26$	$\checkmark T_n = -7n + 42$ $\checkmark -7n + 42 = -140$ $\checkmark n = 26$	(3)
2.3	$S_n = \frac{n}{2}(a + 1)$ $S_n = \frac{n}{2}(35 - 7n + 42)$ $S_n = \frac{n}{2}(-7n + 77)$ $S_n = -\frac{7}{2}n^2 + \frac{77}{2}n$ $-\frac{7}{2}n^2 + \frac{77}{2}n = 3n^2 - 8n + 7$ $13n^2 - 93n + 14 = 0$ $(n - 7)(13n - 2) = 0$ $n = 7$ or $n = \frac{2}{13}$ NA $\therefore n = 7$	$S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_n = \frac{n}{2}(70 - 7n + 7)$ $\checkmark S_n = \frac{n}{2}(35 - 7n + 42)$ or $S_n = \frac{n}{2}(70 - 7n + 7)$ ✓ simplification of S_n ✓ equating ✓ standard form ✓ factors ✓ answer with selection	(6)
			[16]

QUESTION/VRAAG 3

3.1	$r = \frac{1}{2}$ and $S_n = 6$ $S_n = \frac{a}{1 - r}$ $6 = \frac{a}{1 - \frac{1}{2}}$ $a = 3$	✓ substitution ✓ answer	(2)
3.2	$T_n = ar^{n-1}$ $T_7 = 3\left(\frac{1}{2}\right)^7$ $T_8 = \frac{3}{128}$	$\checkmark \checkmark T_8 = 3\left(\frac{1}{2}\right)^7$	(2)
3.3	$\sum_{k=1}^n 3(2)^{1-k} = 5,8125$ $3 + \frac{3}{2} + \frac{3}{4} + \dots = 5,8125$ $S_n = \frac{a(1 - r^n)}{1 - r} = 5,8125$ $\frac{3\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}} = 5,8125$ $6\left[1 - \left(\frac{1}{2}\right)^n\right] = 5,8125$ $\left(\frac{1}{2}\right)^n - \frac{1}{32} = 0,03125$ $2^{-n} - 2^{-5} = 0,03125$ $n = 5$ or $n \log \frac{1}{2} = \log \frac{1}{32}$ $n = 5$	$\checkmark r = \frac{1}{2}$ ✓ substitution ✓ simplification ✓ answer	(4)



NOV 2019

QUESTION/PRAAG 2

<p>3.4</p> $\sum_{k=1}^{20} 3(2)^{1-k} - p$ $3 + \frac{3}{2} + \frac{3}{4} + \dots + 3 \cdot 2^{-19} = p$ $\sum_{k=1}^{20} 24(2)^{-k}$ $= 12 + 6 + 3 + \dots + 24 \cdot 2^{-20}$ $= 4 \left(3 + \frac{3}{2} + \frac{3}{4} + \dots + 3 \cdot 2^{-19} \right)$ $= 4p$ <p>OR/OF</p> $\sum_{k=1}^{20} 3(2)^{1-k} - p$ $\sum_{k=1}^{20} 6(2)^{-k} - p$ $\therefore \sum_{k=1}^{20} 24(2)^{-k} = 4p$ <p>OR/OF</p> $\sum_{k=1}^{20} 24(2)^{-k} = \sum_{k=1}^{20} 4 \times 3 \times 2(2)^{-k}$ $= 4 \sum_{k=1}^{20} 3 \times 2(2)^{-k}$ $= 4 \sum_{k=1}^{20} 3 \times (2)^{1-k} = 4p$ <p>OR/OF</p> $S_{20} = \frac{3 \left(\left(\frac{1}{2} \right)^{20} - 1 \right)}{\frac{1}{2} - 1} = 6 - p$ $S_{20} = \frac{12 \left(\left(\frac{1}{2} \right)^{20} - 1 \right)}{\frac{1}{2} - 1} = 24$ $24 = 4 \times 6 = 4p$	<p>✓ expansion</p> <p>✓ expansion</p> <p>✓ answer (3)</p> <p>OR/OF</p> <p>✓ $\sum_{k=1}^{20} 6(2)^{-k} - p$</p> <p>✓ $\sum_{k=1}^{20} 4 \times 6(2)^{-k}$</p> <p>✓ 4p (3)</p> <p>OR/OF</p> <p>✓ $\sum_{k=1}^{20} 4 \times 3 \times 2(2)^{-k}$</p> <p>✓ $4 \sum_{k=1}^{20} 3 \times 2(2)^{-k}$</p> <p>✓ 4p (3)</p> <p>OR/OF</p> <p>✓ substitution and answer</p> <p>✓ substitution and answer</p> <p>✓ 4p (3)</p>
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2.1.1	209 ; 186	✓ 209 ✓ 186 (2)
2.1.2	$ \begin{array}{cccc} 321 & ; & 290 & ; & 261 & ; & 234 \\ & \swarrow & & \swarrow & & \swarrow & \\ & -31 & & -29 & & -27 & \\ & & \swarrow & & \swarrow & & \\ & & 2 & & 2 & & \end{array} $ <p>1st diff</p> <p>2nd diff</p> $ \begin{array}{ccc} 2a = 2 & 3a + b = -31 & a + b + c = 321 \\ a = 1 & 3(1) + b = -31 & 1 + (-34) + c = 321 \\ & b = -34 & c = 354 \end{array} $ $T_n = n^2 - 34n + 354$	<p>✓ 2nd diff = 2</p> <p>✓ a = 1</p> <p>✓ b = -34</p> <p>✓ c = 354</p> <p>(4)</p>
2.1.3	$n^2 - 34n + 354 = 74$ $n^2 - 34n + 280 = 0$ $(n - 14)(n - 20) = 0$ $n = 14 \text{ or } n = 20$	<p>✓ equating T_n to 74</p> <p>✓ standard form</p> <p>✓ 14 ✓ 20 (4)</p>
2.1.4	$f'(n) = 0$ $2n - 34 = 0$ $2n = 34$ $n = 17$ <p>Term 17 will have the smallest value</p> <p>OR/OF</p> $n = \frac{-b}{2a}$ $n = \frac{34}{2}$ $n = 17$ <p>Term 17 will have the smallest value</p> <p>OR/OF</p> $n = \frac{14 + 20}{2} = 17$ <p>Term 17 will have the smallest value</p>	<p>✓ 2n - 34 = 0</p> <p>✓ answer (2)</p> <p>OR/OF</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>OR/OF</p> <p>✓ substitution</p> <p>✓ answer (2)</p>

2.2.1	$a = \frac{5}{8} ; r = \frac{1}{2} ; n = 21$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_{21} = \frac{\frac{5}{8} \left(1 - \left(\frac{1}{2} \right)^{21} \right)}{1 - \frac{1}{2}}$ $= 1,2499\dots$ $= 1,25$	<p>✓ r</p> <p>✓ substitution into the correct formula</p> <p>✓ answer (3)</p>
2.2.2	$T_n > \frac{5}{8192}$ $ar^{n-1} > \frac{5}{8192}$ $\frac{5}{8} \left(\frac{1}{2} \right)^{n-1} > \frac{5}{8192}$ $\left(\frac{1}{2} \right)^{n-1} > \frac{1}{1024}$ $\left(\frac{1}{2} \right)^{n-1} > \left(\frac{1}{2} \right)^{10} \quad \text{or} \quad 2^{-n+1} > 2^{-10}$ $\therefore n-1 < 10 \quad -n+1 > -10$ $n < 11 \quad n < 11$ $\therefore n = 10 \quad \therefore n = 10$ <p>OR/OF</p> $8; 16; 32; \dots; 8192$ $8 \cdot 2^{n-1} < 8192$ $2^{n-1} < 1024$ $2^{n-1} < 2^{10}$ $n-1 < 10$ $n < 11$ $\therefore n = 10$	<p>✓ substitution into the correct formula</p> <p>✓ method /same base or log</p> <p>✓ calculating n</p> <p>✓ answer (4)</p> <p>OR/OF</p> <p>✓ substitution into the correct formula</p> <p>✓ method</p> <p>✓ calculating n</p> <p>✓ answer (4)</p>
[19]		



QUESTION/VRAAG 3		
3.1	$\sum_{y=1}^{10} \frac{1}{y-2} - \sum_{y=1}^{10} \frac{1}{y-1}$ $= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} \right) - \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} + \frac{1}{9} \right)$ $= 1 - \frac{1}{9}$ $= \frac{8}{9}$	<p>✓ $\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} \right)$</p> <p>✓ $\left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{8} + \frac{1}{9} \right)$</p> <p>✓ answer (3)</p>
3.2	$\left(\frac{1}{3} \times \frac{2}{3} \right) + \left(\frac{2}{3} \times \frac{2}{3} \right) + \left(1 \times \frac{2}{3} \right) + \dots + \left(4 \times \frac{2}{3} \right)$ $= \frac{2}{9} + \frac{4}{9} + \frac{2}{3} + \dots + \frac{8}{3}$ $a = \frac{2}{9} \quad \text{and} \quad d = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$ $S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{OR} \quad S_n = \frac{n}{2} (a+l)$ $S_{12} = \frac{12}{2} \left[2 \left(\frac{2}{9} \right) + (12-1) \left(\frac{2}{9} \right) \right] \quad S_{12} = \frac{12}{2} \left(\frac{2}{9} + \frac{8}{3} \right)$ $= \frac{52}{3} \text{ m}^2 \quad = \frac{52}{3} \text{ m}^2$ $\therefore \text{for both sides} = 2 \times \frac{52}{3} = \frac{104}{3} = 34,67 \text{ m}^2$ <p>OR/OF</p> $\frac{2}{9} \times (1+2+3+4+5+6+7+8+9+10+11+12) \times 2$ $= 34,67 \text{ m}^2$ <p>OR/OF</p> $T_1 = \frac{2}{9} \times 12 = \frac{8}{3} \quad l = \frac{2}{9} \times 12 = \frac{8}{3}$ $2S_{12} = 2 \left(\frac{12}{2} \right) \left(\frac{8}{3} + \frac{8}{3} \right)$ $= 34,67 \text{ m}^2$	<p>✓ ✓ a</p> <p>✓ d</p> <p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>✓ answer for both sides (6)</p> <p>OR/OF</p> <p>✓ ✓ a</p> <p>✓ ✓ (1 + ... + 12)</p> <p>✓ × 2</p> <p>✓ answer (6)</p> <p>OR/OF</p> <p>✓ ✓ a</p> <p>✓ $T_1 = \frac{8}{3}$ ✓ $l = \frac{8}{3}$</p> <p>✓ substitution into correct formula</p> <p>✓ answer (6)</p>
[9]		

FUNCTIONS AND INVERSES

NOV 2017

QUESTION/PRAAG 4

4.1	$f(x) = -ax^2 + bx + 6$ $f'(x) = -2ax + b$ $-2ax + b = 3$ <p>at $x = -1$</p> $2a + b = 3 \quad [1]$ $f(-1) = \frac{7}{2}$ $-a - b + 6 = \frac{7}{2}$ $-2a - 2b + 12 = 7$ $2a + 2b = 5 \quad [2]$ <p>[2] - [1]</p> $b = 2$ $2a + 2 = 3$ $a = \frac{1}{2}$ <p>OR/OF</p> $f'(x) = -2ax + b$ $3 = 2a + b$ $b = 3 - 2a$ $\frac{7}{2} = -a(-1)^2 + (3 - 2a)(-1) + 6$ $a + 3 = \frac{7}{2}$ $a = \frac{1}{2}$ $b = 2$	$\checkmark -2ax + b$ $\checkmark \checkmark 2a + b = 3$ $\checkmark -a - b + 6 = \frac{7}{2}$ <p>✓ solve simultaneously (5)</p> $\checkmark -2ax + b$ $\checkmark \checkmark 2a + b = 3$ $\checkmark \frac{7}{2} = -a(-1)^2 + (3 - 2a)(-1) + 6$ <p>✓ solve simultaneously (5)</p>
4.2	$f(x) = -\frac{1}{2}x^2 + 2x + 6$ <p>x - intercepts:</p> $-\frac{1}{2}x^2 + 2x + 6 = 0$ $-x^2 + 4x + 12 = 0$ $x^2 - 4x - 12 = 0$ $(x - 6)(x + 2) = 0$ <p>(-2; 0) (6; 0)</p>	$\checkmark -\frac{1}{2}x^2 + 2x + 6 = 0$ $\checkmark (-2; 0)$ $\checkmark (6; 0)$ <p>(3)</p>



4.3	$f(x) = -\frac{1}{2}x^2 + 2x + 6$ $f'(x) = 0 \quad \text{or} \quad x = -\frac{b}{2a} \quad \text{or} \quad x = \frac{-2+6}{2}$ $-x + 2 = 0 \quad x = -\frac{2}{2\left(-\frac{1}{2}\right)} \quad x = 2$ $x = 2 \quad x = 2$ $y = -\frac{1}{2}(2)^2 + 2(2) + 6$ $= -2 + 4 + 6$ $= 8$ <p>TP (2; 8)</p> <p>OR/OF</p> $y = -\frac{1}{2}(x^2 - 4x - 12)$ $= -\frac{1}{2}[(x - 2)^2 - 4 - 12]$ $= -\frac{1}{2}(x - 2)^2 + 8$ <p>TP (2; 8)</p>	$\checkmark -x + 2 = -\frac{2}{2\left(-\frac{1}{2}\right)}$ $\frac{-2+6}{2}$ $\checkmark x = 2$ $\checkmark y = 8$ <p>OR/OF</p> $\checkmark -\frac{1}{2}(x - 2)^2 + 8$ $\checkmark x = 2$ $\checkmark y = 8$ <p>(3)</p>
4.4 4.6		<p>4.4: f: ✓ shape ✓ x- intercepts ✓ y- intercept ✓ (2; 8)</p> <p>(4)</p> <p>4.6: g: ✓ x- intercept ✓ y- intercept</p> <p>(2)</p>
4.5	$0 < x < 4 \quad \text{or} \quad (0; 4)$	$\checkmark 4$ $\checkmark 0 < x < 4 \quad (3)$
4.7	$x \leq -2 \quad \text{or} \quad -1 \leq x \leq 6$ <p>OR/OF</p> $(-\infty; -2] \quad \text{or} \quad [-1; 6]$	$\checkmark x \leq -2$ $\checkmark -1 \leq x \leq 6$ <p>(3) [23]</p>

FEB 2018

QUESTION/VRAAG 5

5.1	$y \in R; y \neq -1$ OR/OF $y < -1$ or $y > -1$ OR/OF $y \in (-\infty; -1)$ or $y \in (-1; \infty)$ OR/OF $R - \{-1\}$	✓✓ answer (2)
5.2	$D(2; -1)$ $g(x) = \frac{2}{x-2} - 1$	✓ $D(2; -1)$ ✓ $\frac{2}{x-2} - 1$
5.3	$f(x) = \log_3 x$ $\log_3 t = 1$ OR/OF $g(x) = \frac{2}{x-2} - 1$ $t = 3$	✓ correct substitution of A ✓✓ $t = 3$
5.4	$x = \log_3 y$ $y = 3^x$	✓ interchange x and y ✓ $y = 3^x$
5.5	$3^x < 3^1$ $x < 1$ OR/OF $3^x < 3^1$ $x \in (-\infty; 1)$	✓ $3^x < 3^1$ ✓ $x < 1$ ✓ $3^x < 3^1$ ✓ $x \in (-\infty; 1)$
5.6	Equation of the axis of symmetry: $y = -x + 1$ x-intercept of the axis of symmetry is at $x = 1$ f has an x-intercept at $B(1; 0)$ which is the same as the x-intercept of the axis of symmetry Point of intersection: $B(1; 0)$ OR/OF Since $BE = ED = 1$ and D lies on the axis of symmetry and the gradient of the axis of symmetry is -1 , B will also lie on the axis of symmetry. But B also lies on f. Therefore $B(1; 0)$ is the point of intersection between f and the axis of symmetry with a negative gradient. <i>Omdat $BE = ED = 1$ en D op die simmetrie-as lê en die simmetrie-as se gradiënt -1 is, sal B ook op die simmetrie-as lê. Maar B lê ook op f. Dus is $B(1; 0)$ die snypunt van f en die simmetrie-as met negatiewe gradiënt.</i>	✓✓ equation of axis of symmetry ✓ B or $(1; 0)$ OR/OF ✓✓ $BE = ED = 1$ ✓ B or $(1; 0)$

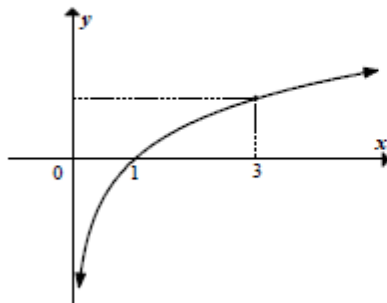
QUESTION/VRAAG 4

4.1	$E(4; -9)$	✓ $x = 4$ ✓ $y = -9$
4.2	$f(x) = (x-4)^2 - 9$ $(x-4)^2 - 9 = 0$ $(x-4)^2 = 9$ $x-4 = \pm 3$ $x = 7$ or $x = 1$ $A(1; 0)$ OR/OF $f(x) = (x-4)^2 - 9$ $0 = x^2 - 8x + 16 - 9$ $0 = x^2 - 8x + 7$ $(x-7)(x-1) = 0$ $x = 7$ or $x = 1$ $A(1; 0)$	✓ $y = 0$ ✓ $x - 4 = \pm 3$ ✓ $A(1; 0)$ OR/OF ✓ $y = 0$ ✓ $(x-7)(x-1)$ ✓ $A(1; 0)$
4.3	$C(0; 7)$ $M(8; 7)$	NOTE: Answer only 3 / 3
4.4	$C(0; 7)$ $D(4; 0)$ $m = \frac{7-0}{0-4}$ or $m = \frac{0-7}{4-0}$ or $0 = 4m + 7$ $m = -\frac{7}{4}$ $m = -\frac{7}{4}$ $m = -\frac{7}{4}$ $y - 0 = -\frac{7}{4}(x - 4)$ $y = -\frac{7}{4}x + 7$	✓ $C(0; 7)$ ✓ $x = 8$ ✓ $y = 7$
4.5	$g: y = -\frac{7}{4}x + 7$ $g^{-1}: x = -\frac{7}{4}y + 7$ $4x = -7y + 28$ $7y = -4x + 28$ $y = -\frac{4}{7}x + 4$	✓ $D(4; 0)$ ✓ $m = -\frac{7}{4}$ ✓ $y = -\frac{7}{4}x + 7$
4.5	OR/OF	✓ interchange x and y ✓ simplification ✓ $y = -\frac{4}{7}x + 4$ OR/OF

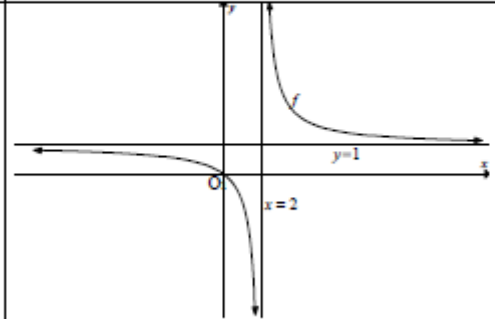


	g^{-1} is the straight line through (0 ; 4) and (7 ; 0) $y = mx + 4$ $0 = 7m + 4$ $y = -\frac{4}{7}x + 4$	✓ straight line through (0 ; 4) and (7 ; 0) ✓ substitution ✓ $y = -\frac{4}{7}x + 4$	(3)
4.6	$x \cdot f(x) \leq 0$ $\therefore x \leq 0$ or $1 \leq x \leq 7$	✓✓ $x \leq 0$ ✓✓ $1 \leq x \leq 7$	(4) [18]

QUESTION/VRAAG 5

5.1	$a^0 = 1$ $T(0 ; 1)$	✓ $x = 0$ ✓ $y = 1$	(2)
5.2	$g(x) = a^x$ $9 = a^2$ $a = 3$ $a > 0$	✓ substitution ✓ $a = 3$	(2)
5.3	$y = \left(\frac{1}{3}\right)^x$ or $y = 3^{-x}$	✓✓ $y = \left(\frac{1}{3}\right)^x$	(2)
5.4	$3^0 < 3^{\log_3 x} < 3^1$ $1 < x < 3$ OR  $1 < x < 3$	✓ $1 < x$ ✓ $x < 3$	(2) [8]

QUESTION/VRAAG 6

6.1	$q = 1$	✓ $q = 1$	(1)
6.2	Subs (0;0) $0 = \frac{a}{0+p} + 1$ $\frac{a}{p} = -1$ $a = -p$ Subs P: $\sqrt{2} + 1 = \frac{a}{\sqrt{2} + 2 + p} + 1$ $\sqrt{2} = \frac{a}{\sqrt{2} + 2 + p}$ $2 + 2\sqrt{2} + \sqrt{2}p = a$ $2 + 2\sqrt{2} - a - p\sqrt{2} = a + a\sqrt{2}$ $2(1 + \sqrt{2}) = a(1 + \sqrt{2})$ $a = 2$; $p = -2$	✓ $0 = \frac{a}{0+p} + 1$ ✓ $a = -p$ ✓ substitution NOTE: Answer only 2 / 5	(5)
6.3		✓ $y = 1$ ✓ $x = 2$ ✓ shape ✓ (0 ; 0)	(4) [10]

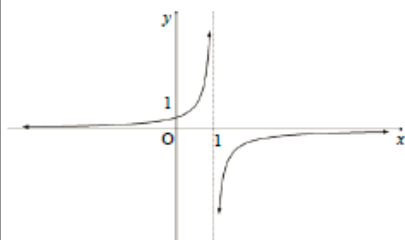


NOV 2018

QUESTION/VRAAG 4

4.1	Yes For every x -value there is only one corresponding y value OR/OF One to one mapping (vertical line test)	✓ answer ✓ reason	(2)
4.2	$R(-12; -6)$	✓ answer	(1)
4.3	$f(x) = ax^2$ substitute $(-6; -12)$ $-12 = a(-6)^2$ $a = \frac{-1}{3}$	✓ substitution ✓ answer	(2)
4.4	$f: y = -\left(\frac{1}{3}\right)x^2$ $f^{-1}: x = -\left(\frac{1}{3}\right)y^2$ $y^2 = -3x$ $y = \pm\sqrt{-3x}$ Only $y = -\sqrt{-3x}$ and $x \leq 0$	✓ swapping x and y ✓ $y^2 = -3x$ ✓ $y = -\sqrt{-3x}$	(3)
			[8]

QUESTION/VRAAG 5

5.1	Domain: $x \in R; x \neq 1$ OR/OF $x \in (-\infty; 1) \cup (1; \infty)$	✓ answer	(1)
5.2	$x = 1$ $y = 0$	✓ $x = 1$ ✓ $y = 0$	(2)
5.3		✓ y intercept ✓ vertical asymptote ✓ shape	(3)
5.4	$x \geq 0; x \neq 1$ OR/OF $0 \leq x < 1$ or $x > 1$ OR/OF $x \in [0; 1) \cup (1; \infty)$	✓ $x \geq 0$ ✓ $x \neq 1$ OR/OF ✓ $0 \leq x < 1$ ✓ $x > 1$	(2)
			[8]

QUESTION/VRAAG 6

6.1	$y = mx + c$ $m = \frac{5-1}{4-0}$ $m = 1$ $c = 1$ $g(x) = x + 1$ OR/OF $y = mx + c$ $5 = m(4) + 1$ $m = 1$ $g(x) = x + 1$	✓ substitution into gradient formula ✓ y-intercept $(0; 1)$ OR/OF ✓ substitute $(4; 5)$ ✓ $c = 1$	(2)
6.2	$x^2 - 2x - 3 = 0$ $(x+1)(x-3) = 0$ $x = -1$ or $x = 3$ A $(-1; 0)$ B $(3; 0)$	✓ $y = 0$ ✓ factors ✓ x-values	(3)
6.3	$x = \frac{-1+3}{2}$ or $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)}$ or $f'(x) = 2x - 2 = 0$ $x = 1$ $f(x) = x^2 - 2x - 3$ $y = (1)^2 - 2(1) - 3$ or $y = (x^2 - 2x + (-1)^2) - 3 - 1$ $y = -4$ or $y = (x-1)^2 - 4$ $y \geq -4$ or $[-4; \infty)$	✓ x-value ✓ substitution/ completing the square ✓ answer	(3)
6.4.1	MIN: $y = (x^2 - 2x - 3) - (x+1)$ $= x^2 - 3x - 4$ $6 = x^2 - 3x - 4$ $0 = x^2 - 3x - 10$ $0 = (x-5)(x+2)$ $x = 5$ or $x = -2$ OT = 2 or OT = 5 NA	✓ $x^2 - 3x - 4$ ✓ substituting $y = 6$ ✓ values of x ✓ OT = 2	(4)
6.4.2	$y = x + 1$ substitute $x = -2$ $= (-2) + 1$ $= -1$ N $(-2; -1)$	✓ substituting $x = -2$ ✓ answer	(2)



NOV 2019

QUESTION/PRAAG 4

6.5	$f'(x) = 2x - 2$ $2x - 2 = 1$ $x = \frac{3}{2}$ $f\left(\frac{3}{2}\right) = \frac{-15}{4}$ $y + \frac{15}{4} = 1\left(x - \frac{3}{2}\right)$ or $-\frac{15}{4} = \frac{1}{2} + c$ $y = x - \frac{21}{4}$ OR/OF $x^2 - 2x - 3 = x + p$ $x^2 - 2x - 3 - x - p = 0$ This equation will have equal roots, therefore: $b^2 - 4ac = 0$ $(-3)^2 - 4(1)(-3 - p) = 0$ $9 + 12 + 4p = 0$ $p = \frac{-21}{4}$ $y = x - \frac{21}{4}$	$\checkmark f'(x) = 2x - 2$ $\checkmark 2x - 2 = 1$ $\checkmark x = \frac{3}{2}$ $\checkmark f\left(\frac{3}{2}\right) = \frac{-15}{4}$ \checkmark answer (5) OR/OF \checkmark equating \checkmark equal roots \checkmark substitution \checkmark simplification \checkmark answer (5)
6.6	$k < \frac{-21}{4}$	\checkmark answer (1)
[20]		



4.1	$p = -1$	$\checkmark p = -1$ (1)
4.2	$y = \frac{a}{x-1}$ $-3 = \frac{a}{0-1}$ $a = 3$ $y = x^2 + bx - 3$ $0 = (1)^2 + (1)b - 3$ $b = 2$	\checkmark coordinates D(0 ; -3) \checkmark substitute (0 ; -3) \checkmark substitute (1 ; 0) (3)
4.3	$y = x^2 + 2x - 3$ axis of sym: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)}$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4) OR/OF $\frac{dy}{dx} = 0$ $2x + 2 = 0$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4)	\checkmark substitution $\checkmark x = -1$ \checkmark substitution $\checkmark y = -4$ (4) OR/OF \checkmark derivative $\checkmark x = -1$ \checkmark substitution $\checkmark y = -4$ (4)
4.4	$y \in [-4; \infty)$ or $y \geq -4$	$\checkmark -4$ \checkmark answer (2)
4.5	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	\checkmark gradient \checkmark subs m and $(-1; -4)$ \checkmark equation (3)
4.6	No, the line passes through C and D OR/OF No, a tangent through turning point C will have a gradient of 0	\checkmark No \checkmark reason (2) OR/OF \checkmark No \checkmark reason (2)

4.7	$f(m-x) = f[-(x-m)]$ f is reflected in the y -axis and translated 1 unit to the left and 4 units upwards. Therefore: $m = -1$ $q = 4$	✓✓ value of m ✓✓ value of q (4)
	OR/OF Substitute $x = 0$ and $q = 4$ for one x -intercept $h(x) = (m-x)^2 + 2(m-x) - 3 + q$ $h(0) = (m-0)^2 + 2(m-0) - 3 + 4$ $0 = m^2 + 2m + 1$ $0 = (m+1)^2$ $m = -1$ $q = 4$	OR/OF ✓✓ value of m ✓✓ value of q (4)
[19]		



QUESTION/PRAAG 5

5.1	$f(x) = k^x$ $16 = k^4$ $k = 2$	✓ substitution (4; 16) ✓ answer (2)
5.2	$f: y = 2^x$ $f^{-1}: x = 2^y$ $y = \log_2 x$	✓ $x = 2^y$ ✓ $y = \log_2 x$ (2)
5.3		✓ asymptote ✓ shape ✓ for any two valid points eg. (16; 4) or (2; 1) or (4; 2) or (1; 0) (4)
5.4.1	$x \in (1; \infty)$ or $x > 1$	✓ 1 ✓ answer (2)
5.4.2	$0 < x \leq \frac{1}{2}$ or $x \in \left(0; \frac{1}{2}\right]$	✓ $\frac{1}{2}$ ✓ answer (2)

<p>5.5</p>	$2^x - 2^{-x} = \frac{15}{4}$ $2^x - \frac{1}{2^x} = \frac{15}{4}$ $2^{2x} - 1 = \frac{15}{4} \times 2^x$ $4 \cdot 2^{2x} - 4 = 15 \times 2^x$ $4 \cdot 2^{2x} - 15 \cdot 2^x - 4 = 0$ $(4 \cdot 2^x + 1)(2^x - 4) = 0$ $4 \cdot 2^x + 1 = 0 \text{ or } 2^x - 4 = 0$ $2^x = \frac{-1}{4} \text{ or } 2^x = 2^2$ <p>N/A $x = 2$</p> <p>OR/OF</p> $2^x - 2^{-x} = \frac{15}{4}$ $2^x - \frac{1}{2^x} = \frac{15}{4}$ <p>Let $k = 2^x$</p> $k^2 - 1 = \frac{15}{4} \times k$ $4 \cdot k^2 - 4 = 15 \times k$ $4 \cdot k^2 - 15 \cdot k - 4 = 0$ $(4 \cdot k + 1)(k - 4) = 0$ $k = \frac{-1}{4} \text{ or } k = 4$ $2^x = \frac{-1}{4} \text{ or } 2^x = 2^2$ <p>N/A $x = 2$</p>	$\checkmark 2^x - 2^{-x} = \frac{15}{4}$ $\checkmark \text{standard form}$ $\checkmark \text{factors}$ $\checkmark \text{answer}$ <p>(4)</p> <p>OR/OF</p> \checkmark $2^x - 2^{-x} = \frac{15}{4}$ $\checkmark \text{standard form}$ $\checkmark \text{factors}$ $\checkmark \text{answer}$ <p>(4)</p>
		[16]



FINANCE, GROWTH AND DECAY

NOV 2017

QUESTION/PRAAG 6

6.1	$A = P(1+i)^n$ $12\ 146,72 = 10\ 000 \left(1 + \frac{r}{12}\right)^{36}$ $\left(1 + \frac{r}{12}\right)^{36} = 1,214672$ $1 + \frac{r}{12} = \sqrt[36]{1,214672}$ $= 1,005416$ $\frac{r}{12} = 0,005416$ $r = 0,06500$ $r = 6,5\%$	✓ $\frac{r}{12}$ ✓ $n = 36$ ✓ correct substitution into formula ✓ $1 + \frac{r}{12} = \sqrt[36]{1,214672}$ ✓ 6,5%
6.2.1	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $235\ 000 = \frac{x \left[1 - \left(1 + \frac{0,11}{12}\right)^{-54}\right]}{\frac{0,11}{12}}$ $x = \frac{235\ 000 \times \frac{0,11}{12}}{\left[1 - \left(1 + \frac{0,11}{12}\right)^{-54}\right]}$ $= R\ 5\ 536,95$ <p>His monthly instalment is R 5 536,95</p>	✓ $i = \frac{0,11}{12}$ ✓ $n = 54$ ✓ correct substitution in P ✓ answer
6.2.2	<p>Amount paid for the year : $(5\ 536,95 \times 12) = R\ 66\ 443,40$</p> $\text{Balance} = 235\ 000 \left(1 + \frac{0,11}{12}\right)^{12} - \frac{5\ 536,95 \left[\left(1 + \frac{0,11}{12}\right)^{12} - 1\right]}{\frac{0,11}{12}}$ $= 192\ 296,17$ <p>Interest = $(5\ 536,95 \times 12) - (235\ 000 - 192\ 296,17)$</p> $= 66\ 443,40 - 42\ 703,83$ $= 23\ 739,57$ <p>OR/OF</p>	✓ R.66 443,40 ✓ $235\ 000 \left(1 + \frac{0,11}{12}\right)^{12}$ ✓ $5\ 536,95 \left[\left(1 + \frac{0,11}{12}\right)^{12} - 1\right]$ ✓ $\frac{0,11}{12}$ ✓ R192 296,17 ✓ R42 703,83 ✓ R23 739,57 OR/OF



	<p>Total amount paid in first year = $R\ 5\ 536,95 \times 12$</p> $= R\ 66\ 443,40$ <p>Balance on loan after 1 year = P of remaining installments</p> $P = \frac{x[1 - (1+i)^{-n}]}{i}$ $= \frac{5\ 536,95 \left[1 - \left(1 + \frac{0,11}{12}\right)^{-42}\right]}{\frac{0,11}{12}}$ $= R\ 192\ 296,20$ <p>Amount paid off in the first year:</p> $R\ 235\ 000 - R\ 192\ 296,20 = R\ 42\ 703,80$ <p>Amount of interest = $R\ 66\ 443,40 - R\ 42\ 703,80$</p> $= R\ 23\ 739,60$ <p>OR/OF</p> $P = \frac{5536,95 \left[1 - \left(1 + \frac{0,11}{12}\right)^{-42}\right]}{\frac{0,11}{12}}$ $= R\ 62\ 648,18$ $235\ 000 - 62\ 648,18 = R\ 172\ 351,82$ <p>After 12 months, money owed on house is</p> $172\ 351,82 \left(1 + \frac{0,11}{12}\right)^{12}$ $= 192\ 296,17$ <p>Amount paid after 12 months is</p> $5\ 536,95 \times 12 = R\ 66\ 443,40$ <p>Amount of interest paid:</p> $R\ 66\ 443,40 - (235\ 000 - 192\ 296,17)$ $= R\ 23\ 739,57$	✓ R.66 443,40 ✓ $n = -42$ ✓ substitution into correct formula ✓ R.192 296,20 ✓ R.42 703,80 ✓ R.23 739,60 OR/OF ✓ R.62 648,18 ✓ R.172 351,82 ✓ R.192 296,17 ✓ R.66 443,40 ✓ 235 000 – 192 296,17 ✓ R.23 739,57 (6) [15]
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FEB 2018

QUESTION/VRAAG 7

7.1	$F = \frac{x[(1+i)^n - 1]}{i}$ $= \frac{2500 \left[\left(1 + \frac{0,06}{12}\right)^{60} - 1 \right]}{\frac{0,06}{12}}$ $= R174\,425,08$	<ul style="list-style-type: none"> ✓ $n = 60$ and $i = \frac{0,06}{12} / 0,005$ ✓ correct substitution into correct formula ✓ answer <p style="text-align: right;">(3)</p>
7.2.1	<p>After eleven months, Genevieve will owe/ Na elf maande skuld Genevieve</p> $A = 82\,000 \left(1 + \frac{0,15}{12}\right)^{11}$ $= R\,94\,006,79$	<ul style="list-style-type: none"> ✓ $n = 11$ ✓ correct substitution into correct formula ✓ answer <p style="text-align: right;">(3)</p>
7.2.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $94\,006,79 = \frac{3\,200 \left[1 - \left(1 + \frac{0,15}{12}\right)^{-n} \right]}{\frac{0,15}{12}}$ $\frac{94\,006,79 \times 0,15}{3\,200 \times 12} = 1 - \left(1 + \frac{0,15}{12}\right)^{-n}$ $\left(1 + \frac{0,15}{12}\right)^{-n} = 1 - 0,3672147\dots$ $-n \log \left(1 + \frac{0,15}{12}\right) = \log 0,6327852\dots$ $-n = -36,8382\dots$ $n = 36,84$ <p>Genevieve will have to pay 36 installments of R3 200</p>	<ul style="list-style-type: none"> ✓ 94006,79 ✓ substitute into correct formula ✓ correct use of logs (logs to be defined) ✓ $n = 36,84$ ✓ 36 installments <p style="text-align: right;">(5)</p>

7.2.3	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $= \frac{3200 \left[1 - \left(1 + \frac{0,15}{12}\right)^{-0,83826912} \right]}{\frac{0,15}{12}}$ $P = 2652$ <p>Outstanding balance after 36 installments is R2 652 Final payment will be:</p> $A = 2\,652,00 \left(1 + \frac{0,15}{12}\right)^1$ $= R\,2\,685,00$ <p>OR/OF</p> $\text{Balance} : 94\,006,79 \left(1 + \frac{0,15}{12}\right)^{36} - \frac{3\,200 \left[\left(1 + \frac{0,15}{12}\right)^{36} - 1 \right]}{\frac{0,15}{12}}$ $= R\,2\,651,72$ <p>Final payment will be:</p> $A = 2\,651,72 \left(1 + \frac{0,15}{12}\right)^1$ $= R\,2\,685,00$	<ul style="list-style-type: none"> ✓ $n = -0,83826912$ ✓ substitute into correct formula ✓ answer ✓ $2\,652,00 \left(1 + \frac{0,15}{12}\right)^1$ ✓ answer OR/OF ✓ $94\,006,79 \left(1 + \frac{0,15}{12}\right)^{36}$ ✓ $\frac{3\,200 \left[\left(1 + \frac{0,15}{12}\right)^{36} - 1 \right]}{\frac{0,15}{12}}$ ✓ 2 651,72 ✓ $2\,651,72 \left(1 + \frac{0,15}{12}\right)^1$ ✓ answer <p style="text-align: right;">(5)</p>
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NOV 2018

QUESTION/PRAAG 7

7.1.1	$F = \frac{x[(1+i)^n - 1]}{i}$ $15\,000 \left[\left(1 + \frac{0,088}{4}\right)^{16} - 1 \right]$ $F = \frac{0,088}{4}$ $F = R283\,972,28$	<ul style="list-style-type: none"> ✓ $\frac{0,088}{4}$ and $n = 16$ ✓ substitution into correct formula ✓ answer <p style="text-align: right;">(3)</p>
7.1.2	$A = R283\,972,28 - 100\,000 \left(1 + \frac{0,088}{4}\right)^4$ $= R\,174\,877,60$ <p>OR/OF Amount at end of 3 years:</p> $F = \frac{15\,000 \left[\left(1 + \frac{0,088}{4}\right)^{12} - 1 \right]}{0,088} - 100\,000$ $= R103\,459,12$ <p>Amount at end of 4 years:</p> $P(1+i)^n + \frac{x[(1+i)^n - 1]}{i}$ $-103\,459,12 \left(1 + \frac{0,088}{4}\right)^4 + \frac{15\,000 \left[\left(1 + \frac{0,088}{4}\right)^4 - 1 \right]}{0,088}$ $= R\,174\,877,60$	<ul style="list-style-type: none"> ✓ future value – amount including interest ✓ $100\,000 \left(1 + \frac{0,088}{4}\right)^4$ ✓ answer <p>OR/OF</p> <ul style="list-style-type: none"> ✓ R15 000 including interest – R100 000 ✓ $\left(1 + \frac{0,088}{4}\right)^4$ on P and x in F, ✓ method <p style="text-align: right;">(3)</p>
7.2.1	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $1500\,000 = \frac{x \left[1 - \left(1 + \frac{0,105}{12}\right)^{-12 \cdot 20} \right]}{0,105/12}$ $x = R14\,975,70$	<ul style="list-style-type: none"> ✓ $i = \frac{0,105}{12}$ ✓ $n = 240$ ✓ substitution into correct formula ✓ answer <p style="text-align: right;">(4)</p>

7.2.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $14\,975,70 \left[1 - \left(1 + \frac{0,105}{12}\right)^{-12 \cdot 9} \right]$ $P = \frac{0,105}{12}$ $P = R969\,927,74$ <p>OR/OF</p> <p>Balance outstanding = A – F</p> $-1\,500\,000 \left(1 + \frac{0,105}{12}\right)^{144} - \frac{14\,975,70 \left[\left(1 + \frac{0,105}{12}\right)^{144} - 1 \right]}{0,105/12}$ $= R5\,259\,229,61 - R4\,289\,302,47$ $= R969\,927,14$	<ul style="list-style-type: none"> ✓ R14 975,70 in P_v-formula ✓ $n = 96$ ✓ substitution into correct formula ✓ answer <p style="text-align: right;">(5)</p> <p>OR/OF</p> <ul style="list-style-type: none"> ✓ $n = 144$ in A-formula ✓ $n = 144$ in F_v-formula ✓ R14 975,70 ✓ A – F ✓ answer <p style="text-align: right;">(5)</p> <p style="text-align: right;">[15]</p>
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NOV 2019

QUESTION/VRAG 6

6.1	<p>Kuda: $A = P(1+in)$ $= 5\,000(1+0,083 \times 4)$ $= R6\,660,00$</p> <p>Final Answer: $R6\,660,00 + R266,40$ $= R6\,926,40$</p> <p>OR/OF</p> <p>Kuda: $A = P(1+in) \times 1,04$ $= 5\,000(1+0,083 \times 4) \times 1,04$ $= R6\,926,40$</p> <p>Thabo: $A = P(1+i)^n$ $= 5\,000 \left(1 + \frac{0,081}{12}\right)^{12n}$ $= R6\,905,71$</p> <p>Kuda will have a better investment</p>	<p>✓ substitution into the correct formula</p> <p>✓ final answer</p> <p>OR/OF</p> <p>✓ substitution into the correct formula</p> <p>✓ final answer</p> <p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>✓ conclusion (5)</p>
6.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $525\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,1}{12}\right)^{-n}\right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1 - \left(1 + \frac{0,1}{12}\right)^{-n}$ $-n \log \left(1 + \frac{0,1}{12}\right) = \log \frac{13}{48}$ $-n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12}\right)}$ $n = 157,40$ $n = 158 \text{ payments}$ <p>OR/OF</p>	<p>✓ $\frac{0,1}{12}$</p> <p>✓ substitution into the correct formula</p> <p>✓ simplification</p> <p>✓ use of logs</p> <p>✓ answer (5)</p> <p>OR/OF</p>



	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $525\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,1}{12}\right)^{-12n}\right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1 - \left(1 + \frac{0,1}{12}\right)^{-12n}$ $-12n \log \left(1 + \frac{0,1}{12}\right) = \log \frac{13}{48}$ $-12n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12}\right)}$ $n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12}\right)} \times \frac{1}{12}$ $n = 13,11686841$ <p>Number of payments = $13,11686841 \times 12 = 157,40$ $n = 158 \text{ payments}$</p>	<p>✓ $\frac{0,1}{12}$</p> <p>✓ substitution into the correct formula</p> <p>✓ simplification</p> <p>✓ use of logs</p> <p>✓ answer (5)</p>
6.2.2	<p>Difference: $R6\,000 - R5\,066,36 = R933,64$</p> $F = \frac{x[(1+i)^n - 1]}{i}$ $933,64 = \frac{108 \left[\left(1 + \frac{0,1}{12}\right)^{108} - 1 \right]}{\frac{0,1}{12}}$ $= R162\,503,51$ <p>OR/OF</p>	<p>✓ R.933,64</p> <p>✓ $n = 108$</p> <p>✓ substitution into the correct formula</p> <p>✓ answer (4)</p> <p>OR/OF</p>

$F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{6000 \left[\left(1 + \frac{0,1}{12}\right)^{108} - 1 \right]}{\frac{0,1}{12}}$ <p>= R1 044 322,28</p> $F = \frac{5\,066,36 \left[\left(1 + \frac{0,1}{12}\right)^{108} - 1 \right]}{\frac{0,1}{12}}$ <p>F = R.881 818,77.....</p> <p>Amount available for withdrawal = R1 044 322,28 – R 881 818,77 = R162 503,51</p> <p>OR/OF</p> <p>Outstanding balance with monthly repayment of R5 066,35</p> $= 525\,000 \left(1 + \frac{0,1}{12}\right)^{108} - \frac{5\,066,36 \left[\left(1 + \frac{0,1}{12}\right)^{108} - 1 \right]}{\frac{0,1}{12}}$ <p>= R404 666,23</p> <p>Outstanding balance with monthly repayment of R6 000</p> $= 525\,000 \left(1 + \frac{0,1}{12}\right)^{108} - \frac{6\,000 \left[\left(1 + \frac{0,1}{12}\right)^{108} - 1 \right]}{\frac{0,1}{12}}$ <p>= R242 162,72</p> <p>Amount available for withdrawal R404 666,23 – R242 162,72 = R162 512,18</p>	<p>✓ n = 108 ✓ substitution into correct formula</p> <p>✓ substitution into correct formula</p> <p>✓ final answer (4)</p> <p>OR/OF</p> <p>✓ n = 108 ✓ substitution into the correct formula</p> <p>✓ substitution into the correct formula</p> <p>✓ final answer (4)</p>
	<p>[14]</p>



QUESTION/FRAG 9

$y = x^2 + 2$ $P(x; x^2 + 2)$ $B(0; 3)$ $PB^2 = (x-0)^2 + (x^2 + 2 - 3)^2$ $= x^2 + x^4 - 2x^2 + 1$ $= x^4 - x^2 + 1$ <p>PB will be a minimum if PB^2 is a minimum</p> $\frac{d(PB^2)}{dx} = 4x^3 - 2x$ $4x^3 - 2x = 0$ $x(2x^2 - 1) = 0$ $x = 0 \text{ or } x^2 = \frac{1}{2}$ $x = \frac{1}{\sqrt{2}}$ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= \frac{1}{4} - \frac{1}{2} + 1$ $= \frac{3}{4}$ $PB = \frac{\sqrt{3}}{2} = 0,87$ <p>OR/OF</p>	$\checkmark (x-0)^2 + (x^2 + 2 - 3)^2$ $\checkmark x^4 - x^2 + 1$ $\checkmark 4x^3 - 2x$ $\checkmark \frac{d(PB^2)}{dx} = 0$ $\checkmark x = \frac{1}{\sqrt{2}}$ $\checkmark PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $\checkmark \text{answer}$ <p>OR/OF</p>
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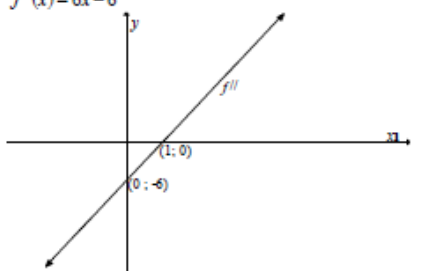
<p>Gradient of tangent to curve = $2x$</p> <p>Gradient of line joining B and the curve = $\frac{x^2 + 2 - 3}{x - 0}$</p> $= \frac{x^2 - 1}{x}$ <p>Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve.</p> $\frac{x^2 - 1}{x} = -\frac{1}{2x}$ $2x(x^2 - 1) = -x$ $2x^3 - 2x = 0$ $x(2x^2 - 1) = 0$ $x = 0 \text{ or } x^2 = \frac{1}{2}$ $x = \frac{1}{\sqrt{2}}$ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= \frac{1}{4} - \frac{1}{2} + 1$ $= \frac{3}{4}$ $PB = \frac{\sqrt{3}}{2} = 0,87$ <p>OR/OF</p> <p>$P(k; k^2 + 2)$ and $B(0; 3)$</p> <p>BP \perp tangent passing through $y = x^2 + 2$ at P.</p> $m_{\text{tangent at P}} = 2k$ $m_{BP} = -\frac{1}{2k}$ <p>Equation of BP: $y = \left(-\frac{1}{2k}\right)x + 3$</p> $y_p = \left(-\frac{1}{2k}\right)(k) + 3 = 2,5$ $\Rightarrow k^2 + 2 = 2,5 \text{ and so } k = \sqrt{0,5} \text{ and } P(\sqrt{0,5}; 2,5)$ $BP = \sqrt{(\sqrt{0,5} - 0)^2 + (2,5 - 3)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = 0,87$	$\checkmark = 2x$ $\checkmark = \frac{x^2 - 1}{x}$ $\checkmark \frac{x^2 - 1}{x} = -\frac{1}{2x}$ $\checkmark 2x^3 - 2x = 0$ $\checkmark x = \frac{1}{\sqrt{2}}$ $\checkmark PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $\checkmark \text{answer}$ <p>OR/OF</p> $\checkmark P(k; k^2 + 2)$ $\checkmark m_{\text{tangent at P}} = 2k$ $\checkmark m_{BP} = -\frac{1}{2k}$ $\checkmark y = \left(-\frac{1}{2k}\right)x + 3$ $\checkmark \text{value of } y \text{ at P}$ $\checkmark \text{value of } k$ $\checkmark \text{answer}$
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FEB 2018

QUESTION/VRAAG 8

8.1	$f(x+h) = 4x^2$ $f(x+h) - f(x) = 4(x+h)^2 - 4x^2$ $= 4(x^2 + 2xh + h^2) - 4x^2$ $= 4x^2 + 8xh + 4h^2 - 4x^2$ $= 8xh + 4h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \left[\frac{8xh + 4h^2}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{h(8x + 4h)}{h} \right]$ $= 8x$ <p>OR/OF</p> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \left[\frac{4(x+h)^2 - 4x^2}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{4x^2 + 8xh + 4h^2 - 4x^2}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{8xh + 4h^2}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{h(8x + 4h)}{h} \right]$ $= 8x$	$\checkmark 4(x+h)^2$ $\checkmark 8xh + 4h^2$ $\checkmark \frac{f(x+h) - f(x)}{h}$ $\checkmark \frac{h(8x + 4h)}{h}$ $\checkmark 8x$ <p>OR/OF</p> $\checkmark \frac{f(x+h) - f(x)}{h}$ $\checkmark 4(x+h)^2$ $\checkmark 8xh + 4h^2$ $\checkmark \frac{h(8x + 4h)}{h}$ $\checkmark 8x$
8.2.1	$D_1 \left[\frac{x^2 - 2x - 3}{x-1} \right]$ $= D_1 \left[\frac{(x-3)(x+1)}{x+1} \right]$ $= D_1(x-3)$ $= 1$	$\checkmark \frac{(x-3)(x+1)}{x+1}$ $\checkmark (x-3)$ $\checkmark 1$
8.2.2	$f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$	$\checkmark x^{\frac{1}{2}}$ $\checkmark \frac{1}{2}x^{-\frac{1}{2}}$ $\checkmark -\frac{1}{4}x^{-\frac{3}{2}}$

QUESTION/VRAAG 9

9.1	$f(x) = (x+2)(x-1)(x-4)$ $= (x^2 + x - 2)(x-4)$ $= x^3 + x^2 - 2x - 4x^2 - 4x + 8$ $= x^3 - 3x^2 - 6x + 8$ $b = -3; c = -6; d = 8$	$\checkmark \checkmark f(x) = (x+2)(x-1)(x-4)$ $\checkmark \text{ expansion}$ $\checkmark x^3 - 3x^2 - 6x + 8$
9.2	$f(x) = x^3 - 3x^2 - 6x + 8$ $f'(x) = 0$ $3x^2 - 6x - 6 = 0$ $x^2 - 2x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$ $= \frac{2 \pm \sqrt{12}}{2}$ $x = -0,73$	$\checkmark f'(x) = 0$ $\checkmark 3x^2 - 6x - 6$ <p>substitution into correct formula</p> $\checkmark x = -0,73$
9.3	$f(x) = x^3 - 3x^2 - 6x + 8$ $f(-1) = (-1)^3 - 3(-1)^2 - 6(-1) + 8$ $= 10$ <p>or $f(-1) = (1)(-2)(-5) = 10$</p> $f'(-1) = 3(-1)^2 - 6(-1) - 6$ $= 3$ $y - 10 = 3(x + 1)$ $y = 3x + 13$	$\checkmark f(-1) = 10$ $\checkmark f'(-1) = 3$ <p>substitution</p> $\checkmark y = 3x + 13$
9.4	$f''(x) = 6x - 6$ 	$\checkmark f''(x) = 6x - 6$ <p>x- intercept</p> <p>y- intercept</p>



NOV 2018

QUESTION/VRAAG 8

9.5	f concave upwards $f''(x) > 0$ $6x - 6 > 0$ $x > 1$	NOTE: Answer only 2 / 2	$\checkmark f''(x) > 0$ $\checkmark x > 1$	(2) [17]
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QUESTION/VRAAG 10

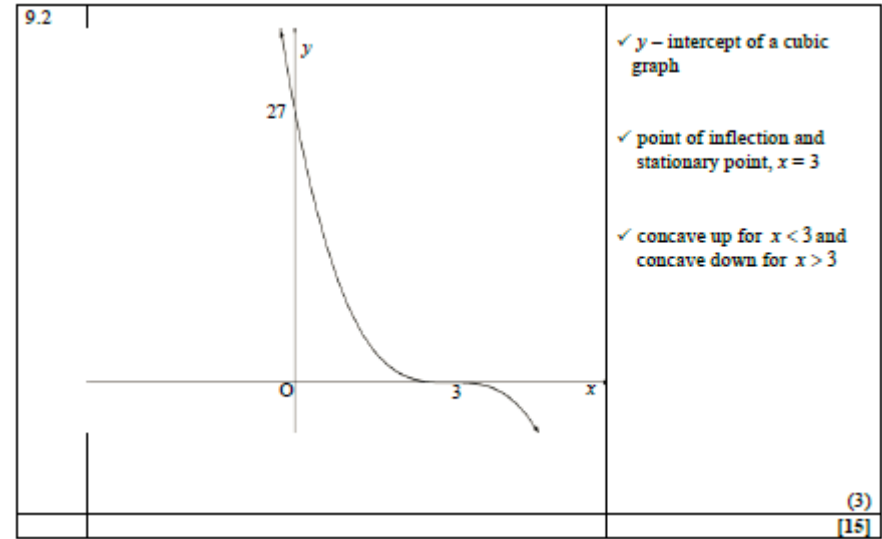
$f(x) = -3x^3 + x$ $-9x^2 + 1 = 0$ $x = \frac{1}{3}$ or $x = -\frac{1}{3}$ Maximum of f will be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$ $= -\frac{2}{9}$ Maximum of $f(x) + q$ will also be at $x = \frac{1}{3}$ $f\left(\frac{1}{3}\right) + q = -\frac{8}{9}$ $\frac{2}{9} + q = -\frac{8}{9}$ $q = -\frac{6}{9}$ $= -\frac{2}{3}$ For $f(x) + q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.	$\checkmark -9x^2 + 1 = 0$ $\checkmark x = \frac{1}{3}$ or $x = -\frac{1}{3}$ \checkmark Maximum at $x = \frac{1}{3}$ $\checkmark f\left(\frac{1}{3}\right) = -\frac{2}{9}$ $\checkmark \frac{2}{9} + q = -\frac{8}{9}$ $\checkmark q = -\frac{2}{3}$	[6]
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8.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$ $= \lim_{h \rightarrow 0} (2x+h)$ $= 2x$	$\checkmark x^2 + 2xh + h^2 - 5$ \checkmark simplification \checkmark factorisation $\checkmark \lim_{h \rightarrow 0} (2x+h)$ $\checkmark 2x$	(5)
	OR/OF $f(x+h) = (x+h)^2 - 5$ $= x^2 + 2xh + h^2 - 5$ $f(x+h) - f(x) = x^2 + 2xh + h^2 - 5 - (x^2 - 5)$ $= 2xh + h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$ $= \lim_{h \rightarrow 0} (2x+h)$ $= 2x$	$\checkmark x^2 + 2xh + h^2 - 5$ \checkmark simplification \checkmark factorisation $\checkmark \lim_{h \rightarrow 0} (2x+h)$ $\checkmark 2x$	(5)
8.2.1	$y = 3x^3 + 6x^2 + x - 4$ $\frac{dy}{dx} = 9x^2 + 12x + 1$	$\checkmark 9x^2$ $\checkmark 12x$ $\checkmark 1$	(3)
8.2.2	$y(x-1) = 2x(x-1)$ $y = \frac{2x(x-1)}{x-1}$ if $x \neq 1$ $y = 2x$ $\frac{dy}{dx} = 2$	$\checkmark y(x-1)$ $\checkmark 2x(x-1)$ $\checkmark y = 2x$ \checkmark answer	(4)
[12]			

QUESTION/VRAAG 9

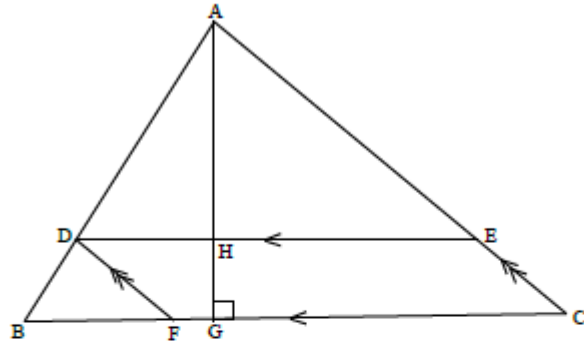
9.1.1	$g(x) = (x+5)(x-x_1)^2$ $20 = 5(x_1)^2$ $x_1^2 = 4$ $x_1 = 2$ $g(x) = (x+5)(x-2)^2$ $g(x) = (x+5)(x^2 - 4x + 4)$ $g(x) = x^3 + x^2 - 16x + 20$	✓ $(x+5)$ ✓ repeated root ✓ $x_1 = 2$ ✓ $g(x) = (x+5)(x^2 - 4x + 4)$ (4)
9.1.2	$g(x) = x^3 + x^2 - 16x + 20$ $g'(x) = 3x^2 + 2x - 16$ $3x^2 + 2x - 16 = 0$ $(3x+8)(x-2) = 0$ $x = -\frac{8}{3}$ or $x = 2$ $R\left(-\frac{8}{3}; \frac{1372}{27}\right)$ or $R(-2,67; 50,81)$ $P(2; 0)$	✓ derivative ✓ equating to zero ✓ factors ✓ co-ordinates of R ✓ co-ordinates of P (5)
9.1.3	$g''(x) = 6x + 2$ $g''(0) = 2$ \therefore concave up OR/OF $g'(x) = 6x + 2$ $6x + 2 = 0$ $x = -\frac{1}{3}$ is the point of inflection \therefore concave up	✓ $g''(x) = 6x + 2$ ✓ $g''(0) = 2$ ✓ conclusion (3) OR/OF ✓ $g''(x) = 6x + 2$ ✓ $x = -\frac{1}{3}$ ✓ conclusion (3)



NOV 2019

QUESTION/VRAAG 7

QUESTION/VRAAG 10



10.1	$\frac{AH}{HG} = \frac{3}{2}$	✓ answer (1)
10.2	<p>Area of a parallelogram = base \times \perp height</p> <p>Area = $\frac{3}{5}(5-t) \cdot \frac{2}{5}t$</p> <p>Area = $\frac{6}{25}(5-t)t$</p> <p>$A(t) = -\frac{6}{25}t^2 + \frac{6}{5}t$</p> <p>$A'(t) = -\frac{12}{25}t + \frac{6}{5}$</p> <p>$-\frac{12}{25}t + \frac{6}{5} = 0$</p> <p>$12t - 30 = 0$</p> <p>$t = \frac{30}{12}$ or $\frac{5}{2}$</p>	<p>✓ $\frac{2}{5}t$</p> <p>✓ $\frac{3}{5}(5-t)$</p> <p>✓ $A(t) = -\frac{6}{25}t^2 + \frac{6}{5}t$</p> <p>✓ $-\frac{12}{25}t + \frac{6}{5}$</p> <p>✓ answer (5)</p>
		(6)




7.1	$f(x) = 4 - 7x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4 - 7(x+h) - (4 - 7x)}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-7)}{h}$ $= -7$	<p>✓ $4 - 7(x+h)$</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ answer (4)</p>
7.2	$y = 4x^8 + \sqrt{x^3}$ $= 4x^8 + x^{\frac{3}{2}}$ $\frac{dy}{dx} = 32x^7 + \frac{3}{2}x^{\frac{1}{2}}$	<p>✓ $x^{\frac{3}{2}}$</p> <p>✓ $32x^7$</p> <p>✓ $\frac{3}{2}x^{\frac{1}{2}}$ (3)</p>
7.3.1	$y = ax^2 + a$ $\frac{dy}{dx} = 2ax + 0$ $\frac{dy}{dx} = 2ax$	<p>✓ $2ax$ (1)</p>
7.3.2	$y = ax^2 + a$ $\frac{dy}{da} = x^2 + 1$	<p>✓ ✓ answer (2)</p>

7.4	<p>Substitute (2 ; b) in $y = x + \frac{12}{x}$</p> $b = 2 + \frac{12}{2}$ $b = 8$ <p>$m_{\text{tangent}} = \frac{dy}{dx}$</p> $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ $m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$ <p>$m_{\text{perp}} = \frac{1}{2}$</p> <p>Equation of perpendicular line: $y - y_1 = m(x - x_1)$ OR $y = mx + c$</p> $y - 8 = \frac{1}{2}(x - 2) \quad 8 = \frac{1}{2}(2) + c$ $y = \frac{1}{2}x + 7 \quad c = 7$ $y = \frac{1}{2}x + 7$	<p>✓ value of b</p> <p>✓ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$</p> <p>✓ gradient of perpendicular line</p> <p>✓ equation (4)</p>
		[14]

QUESTION/VRAAG 8

8.1	36cm	✓ answer (1)
8.2	<p>$\therefore t = 6$ ($-2t^2 + 3t - 6$) have no real roots</p> <p>Insect reaches the floor only once.</p>	✓✓✓ only once (3)
8.3	<p>$h(t) = -2t^2 + 15t - 24 + 36$</p> <p>$h'(t) = -6t + 30t - 24$</p> <p>$-6t + 30t - 24 = 0$</p> <p>$t^2 - 5t + 4 = 0$</p> <p>$(t - 4)(t - 1) = 0$</p> <p>$t = 4$ or $t = 1$</p> <p>Only $t = 4$ because maximum value required</p> <p>$h = -2(4)^2 + 15(4) - 24(4) + 36 = 52 \text{ cm}$</p>	<p>✓ expansion</p> <p>✓ $-6t + 30t - 24 = 0$</p> <p>✓ both values</p> <p>✓ answer (4)</p>
		[8]

QUESTION/VRAAG 9

9.1	<p>$f'(x) = 9x^2$</p> <p>$3x^3 = 9x^2$</p> <p>$3x^3 - 9x^2 = 0$</p> <p>$3x^2(x - 3) = 0$</p> <p>$x = 0$ or $x = 3$</p>	<p>✓ $f'(x) = 9x^2$</p> <p>✓ $x = 0$</p> <p>✓ $x = 3$ (3)</p>
9.2.1	For f and f'	✓ answer (1)
9.2.2	<p>The point (0 ; 0) is :</p> <p>A point of inflection of f</p> <p>A turning point of f'</p>	<p>✓ f: inflection point</p> <p>✓ f': turning point (2)</p>
9.3	<p>$f''(x) = 18x$</p> <p>Distance = $f''(1) - f'(1)$</p> <p>$= 18(1) - 9(1)^2$</p> <p>$= 9$</p>	<p>✓ $f''(x) = 18x$</p> <p>✓ substitution</p> <p>✓ answer (3)</p>
9.4	<p>$3x^3 - 9x^2 < 0$</p> <p>$3x^2(x - 3) < 0$</p> <p>but $3x^2 > 0$</p>  <p>$\therefore x - 3 < 0$</p> <p>$\therefore x < 3, x \neq 0$</p>	<p>✓ $3x^3 - 9x^2 < 0$</p> <p>✓ factors</p> <p>✓ $x < 3$</p> <p>✓ $x \neq 0$ (4)</p>
		[13]



PROBABILITY
NOV 2017

QUESTION/VRAAG 10

10.1		<p>8 values need to be placed in correct position:</p> <p>2 or 3 correct: 1 mark 4 or 5 correct: 2 marks 6 or 7 correct: 3 marks 8 correct: 4 marks</p>
10.2	$(49 - x) + x + 8 + 4 + 5 + 2 + (60 - x) + 14 = 100$ $-x + 142 = 100$ $x = 42$	<p>✓ setting up equation</p> <p>✓ answer</p>
10.3	<p>P (use only one application) = $\frac{7+2+18}{100}$</p> $= \frac{27}{100} \text{ or } 27\%$	<p>✓ $\frac{7+2+18}{100}$</p> <p>✓ answer</p>

QUESTION/VRAAG 11

11.1	$5 \times 5 \times 10 \times 9 = 2250$	<p>✓ 5×5 ✓ 10×9 ✓ 2250</p>																								
11.2	<table border="1"> <thead> <tr> <th>No of digits used</th> <th>Letters</th> <th>Digits</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>5×5</td> <td>10</td> <td>250</td> </tr> <tr> <td>2</td> <td>5×5</td> <td>10×9</td> <td>2 250</td> </tr> <tr> <td>3</td> <td>5×5</td> <td>$10 \times 9 \times 8$</td> <td>18 000</td> </tr> <tr> <td>4</td> <td>5×5</td> <td>$10 \times 9 \times 8 \times 7$</td> <td>126 000</td> </tr> <tr> <td>5</td> <td>5×5</td> <td>$10 \times 9 \times 8 \times 7 \times 6$</td> <td>756 000</td> </tr> </tbody> </table> <p>Codes of two letters and five digits will ensure unique numbers for 700 000 clients.</p>	No of digits used	Letters	Digits	Total	1	5×5	10	250	2	5×5	10×9	2 250	3	5×5	$10 \times 9 \times 8$	18 000	4	5×5	$10 \times 9 \times 8 \times 7$	126 000	5	5×5	$10 \times 9 \times 8 \times 7 \times 6$	756 000	<p>✓ $5 \times 5 \times 10 \times 9 \times 8 \times 7 \times 6$ ✓ five digits</p>
No of digits used	Letters	Digits	Total																							
1	5×5	10	250																							
2	5×5	10×9	2 250																							
3	5×5	$10 \times 9 \times 8$	18 000																							
4	5×5	$10 \times 9 \times 8 \times 7$	126 000																							
5	5×5	$10 \times 9 \times 8 \times 7 \times 6$	756 000																							

FEB 2018

QUESTION/VRAAG 11

11.1.1	<p>Let the event Veli arrive late for school be V. Let the event Bongri arrive late for school be B. / Laat V die gebeurtenis wees dat Veli Laat B die gebeurtenis wees dat Bongri laatkom $P(V \text{ or } B) = 1 - 0,7 = 0,3$</p>	<p>✓ answer</p>
11.1.2	$P(V \text{ or } B) = P(V) + P(B) - P(V \text{ and } B)$ $0,3 = 0,25 + P(B) - 0,15$ $P(B) = 0,2$	<p>✓ $P(V \text{ or } B) = P(V) + P(B) - P(V \text{ and } B)$ ✓ substitution ✓ 0,2</p>
11.1.3	$P(V) \times P(B) = 0,25 \times 0,2 = 0,05$ $P(V) \times P(B) \neq P(V \text{ and } B)$ V and B are NOT independent/ V en B is NIE onafhanklik nie.	<p>✓ $P(V) \times P(B) = 0,05$</p> <p>✓ $P(V) \times P(B) \neq P(V \text{ and } B)$ ✓ NOT independent</p>
11.2.1	$6! = 720$	<p>✓ $6!$ or 720</p>
11.2.2	<p>Number of arrangements</p> $3! \times 3! \times 2 = 72$	<p>✓ $3! \times 3!$ ✓ $\times 2$ ✓ answer</p>
11.2.3	<p>P(hearts next to each other) = $\frac{3! \times 4!}{6!} = \frac{144}{720} = \frac{1}{5}$ or 0,2 or 20%</p> <p>OR/OF</p> <p>P(hearts next to each other) = $\frac{4 \times 3! \times 3!}{6!} = \frac{144}{720} = \frac{1}{5}$ or 0,2 or 20%</p>	<p>✓ ✓ $3! \times 4!$</p> <p>✓ $\frac{1}{5}$ or 0,2 or 20%</p> <p>OR/OF</p> <p>✓ ✓</p> <p>✓ $\frac{1}{5}$ or 0,2 or 20%</p>

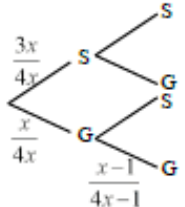


NOV 2018

QUESTION/VRAAG 11

11.1.1	$7^5 = 16\ 807$	✓✓ answer (2)
11.1.2	$7 \times 6 \times 5 \times 4 \times 3$ $= \frac{7!}{2!} = 2520$	✓ $7 \times 6 \times 5 \times 4 \times 3$ or $\frac{7!}{2!}$ ✓ answer (2)
11.2	$2 \times 7 \times 1 = 14$	✓✓✓ $2 \times 7 \times 1$ (3)
		[7]

QUESTION/VRAAG 12

12.1	$P(A \text{ or } B) = P(A) + P(B)$ $0,74 = 0,45 + y$ $y = 0,29$	✓ $P(A \text{ or } B) = P(A) + P(B)$ ✓ substitution ✓ answer (3)
12.2	 <p>Let the number of mystery gift bags = x The total number of bags = $4x$</p> $\left(\frac{x}{4x}\right) \times \left(\frac{x-1}{4x-1}\right) = \frac{7}{118}$ $\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{7}{118}$ $\frac{x-1}{4x-1} = \frac{28}{118}$ $118x - 118 = 112x - 28$ $x = 15$	<p>✓ $4x$</p> <p>✓ $\left(\frac{x}{4x}\right)$ or $\left(\frac{1}{4}\right)$</p> <p>✓ $\left(\frac{x-1}{4x-1}\right)$</p> <p>✓ $\frac{1}{4} \times \frac{x-1}{4x-1}$</p> <p>✓ equating to $\frac{7}{118}$</p> <p>✓ answer (6)</p>

<p>OR/OF</p> <p>$P(\text{gift and gift}) = P(\text{gift at first draw}) \times P(\text{gift at second draw})$</p> $\frac{7}{118} = \frac{1}{4} \times P(\text{gift at second draw})$ $P(\text{gift at second draw}) = \frac{7}{118} \div \frac{1}{4}$ $= \frac{14}{59}$ <p>Therefore: $P(\text{gift at first draw}) = \frac{15}{60}$</p> <p>And: 15 bags had mystery gifts inside</p>	<p>OR/OF</p> <p>✓ $\frac{1}{4}$</p> <p>✓ $\frac{1}{4} \times P(\text{gift at 2}^{\text{nd}} \text{ draw})$</p> <p>✓ $\frac{7}{118} = \frac{1}{4} \times P(\text{gift at 2}^{\text{nd}} \text{ draw})$</p> <p>✓ $\frac{14}{59}$</p> <p>✓ $\frac{15}{60}$</p> <p>✓ answer (6)</p>
[9]	

ecoleBooks

NOV 2019

QUESTION/VR44G 10

10.1	$P(\text{same day}) = \frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%	✓4 numerator ✓16 denominator (2)
10.2	$P(2 \text{ consecutive days}) = \frac{3 \times 2}{16} = \frac{3}{8}$	✓3 ✓×2 ✓ answer (3)
		[5]

QUESTION/VR44G 11

11.1.1	$P(A) \times P(B)$ independent events $= 0,40 \times 0,25 = 0,1$	✓0,1 ✓0,15 and 0,3 ✓0,45 (3)
11.1.2	$P(A \text{ or not } B) = P(A) + P(\text{not } B) - P(A \text{ and not } B)$ $= 0,4 + 0,75 - 0,3$ $= 0,85$ OR/OF $P(A \text{ or not } B) = 1 - P(\text{only } B)$ $= 1 - 0,15$ $= 0,85$ OR/OF From Venn diagram: $0,3 + 0,1 + 0,45 = 0,85$	✓ substitution ✓ answer (2) OR/OF ✓ 1 - 0,15 ✓ answer (2) OR/OF ✓ substitution ✓ answer (2)
11.2	$(5 \times 1 \times 5) + (5 \times 1 \times 6) + (5 \times 1 \times 6) + (5 \times 1 \times 5) = 110$ $110 \times 5 = 550 > 500$ Not possible, because not enough space OR/OF $(5 \times 2 \times 5) + (5 \times 2 \times 6) = 110$ $110 \times 5 = 550 > 500$ Not possible because not enough space OR/OF	✓ $5 \times 1 \times 5$ ✓ $5 \times 1 \times 6$ ✓ $5 \times 1 \times 6$ ✓ $5 \times 1 \times 5$ ✓ 110 ✓ conclusion (6) OR/OF ✓ $5 \times 2 \times 5$ ✓ $5 \times 2 \times 6$ ✓ 110 ✓ conclusion (6) OR/OF

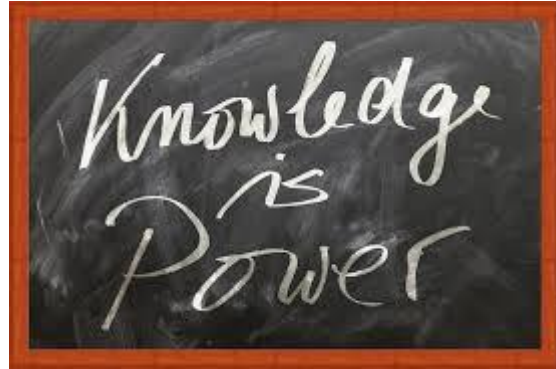


$5 \times 4 \times 6 = 120$ $5 \times 2 = 10$ $\therefore 120 - 10 = 110$ $110 \times 5 = 550 > 500$ Not possible because not enough space	$\checkmark \checkmark 5 \times 4 \times 6 = 120$ $\checkmark 5 \times 2 = 10$ $\checkmark 120 - 10 = 110$ $\checkmark 110$ \checkmark conclusion (6)
	[11]





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