GAUTENG DEPARTMENT OF EDUCATION

JOHANNESBURG NORTH DISTRICT

2021
GRADE 12
CONTROL TEST

MATHEMATICS
TERM1

MARKS : 100
TIME : 2 hours

This question paper consist of 17 pages

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which was used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. **ANSWER Question 7 on Annexure 7.1 - 7.2.2**
9. **ANSWER Question 8 on Annexure 8.1 - 8.1.3**
10. **ANSWER Question 9 on Annexure 9.1 - 9.2**
11. **Tear off page 12 till page 17 . AND SUBMIT theses pages with your answer scripts .**
12. An **information sheet** is on page 11 of the question paper.
13. Number the questions correctly according to the numbering used in the question paper.
14. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-5)(x+1)=0$ (2)

1.1.2 $2x^2-11x+7=0$ (correct to two decimal places) (3)

1.1.3 $x-5x^{\frac{1}{2}}=-6$ (4)

1.2 Calculate a and b if $\sqrt{\dfrac{5^{2014}-5^{2012}}{6}}=a(5^b)$ and a is not a multiple of 5. (4)

1.3 Solve for x and y:

$1=3y-x$ and $y^2+2xy=3x^2-7$ (7)

[20]

QUESTION 2

Given the arithmetic series: 3 + 10 + 17 + …. + 150.

2.1 Write down the fourth term in the series. (1)

2.2 Determine the general term of the series. (2)

2.3 Express the series in sigma notation. (1)

[4]

QUESTION 3

3.1 Consider the progression : $3;\ \frac{1}{2};\ 3;\ \frac{4}{10};\ 3;\ \frac{16}{50};$……….

3.1.1 Write down the next TWO terms of the progression. (1)

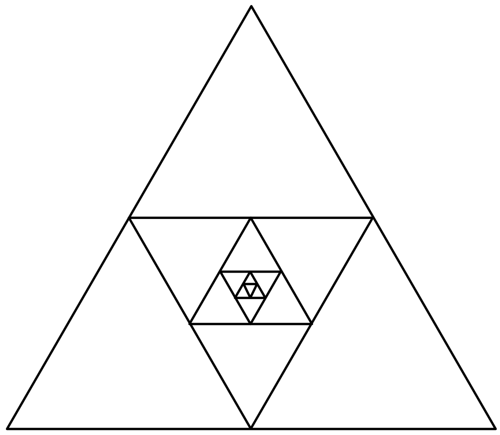
3.1.2 Calculate the sum of the first thirty-five terms of the progression. (5)

3.2 Calculate : $\sum_{n=3}^{\infty} 5(3)^{1-n}$ (4)

[10]

QUESTION 4

In the diagram below, the 1^{st} (outer) triangle is an equilateral triangle with sides of $8cm$. A 2^{nd} triangle is drawn within this triangle by joining the midpoints of the sides of the 1^{st} triangle. This process is continued without end.



4.1 What his the perimeter of the 4^{th} triangle ? (2)

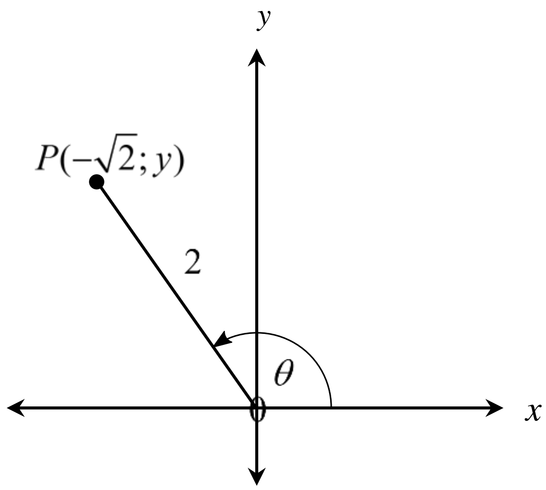
4.2 Whats is the perimeter of the n^{th} triangle? (3)

[5]

QUESTION 5

5.1 In the sketch below, P is a point on the Cartesian plane, with $P\hat{O}X = \theta$.

Use the sketch to determine the following:



5.1.1 The value of y. (2)

5.1.2 The value of $\dfrac{2sin\theta cos\theta}{cos^2\theta - 1}$ (5)

5.2 Simplify the following, WITHOUT USING A CALCULATOR:

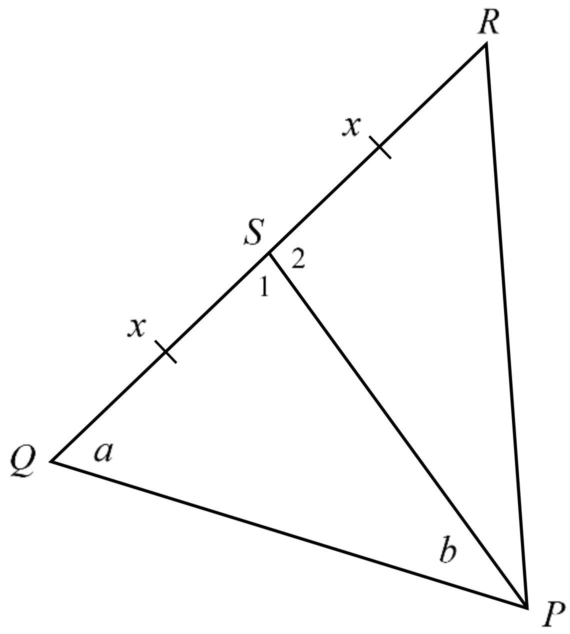
$$\frac{cos(180^\circ + \theta).tan(720^\circ - \theta).sin^2(90^\circ - \theta)}{sin(180^\circ - \theta)} + sin^2\theta$$ (7)

5.3 If $6sin^2\theta - 4cos^2\theta = -5sin\theta.cos\theta$, determine the general solution for θ. (8)

[22]

QUESTION 6

In the sketch below, PS is the median of $\triangle PQR$, and thus $QS = SR = x$. $\hat{Q} = a$ and $Q\hat{P}S = b$.



6.1 Show that $PS = \frac{x\sin a}{\sin b}$ (2)

6.2 Express the size of S_2 , in terms of a and b, without reasons. (1)

6.3 Hence, show that: Area of $\triangle PSR = \frac{x^2 \sin a \times \sin(a+b)}{2\sin b}$ (3)

6.4 Determine the area of $\triangle PSR$, rounded to two decimal places, if $x = 14{,}2cm$, $a = 34°$ and $b = 41°$. (3)

[9]

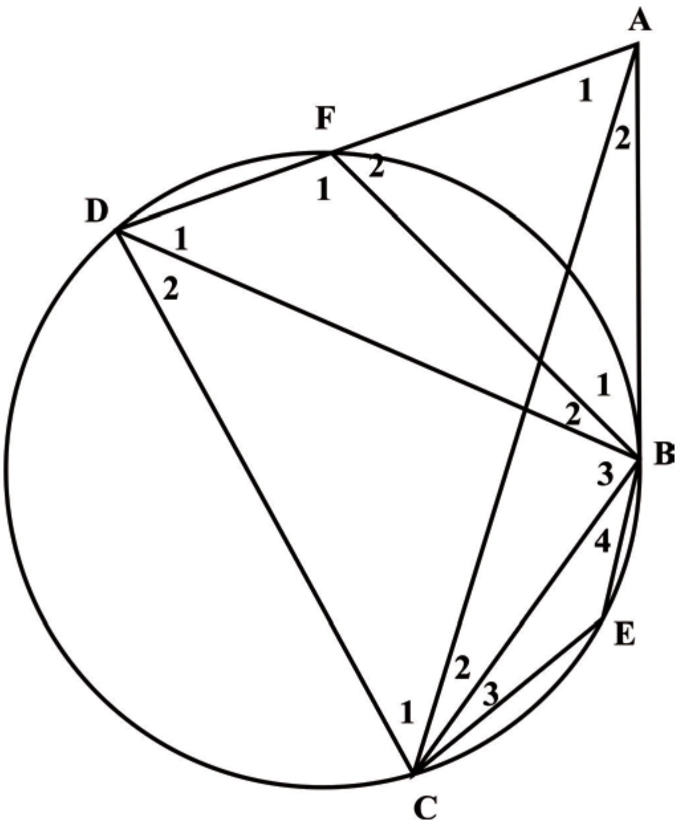
Give reasons for your statements and calculations in QUESTIONS 7, 8 and 9

Use the Annexure's provided to answer QUESTIONS 7, 8 and 9

QUESTION 7

7.1 In the diagram below, AB is a tangent to the circle passing though B, E, C and D

AD cuts the circle at F. AC is drawn.



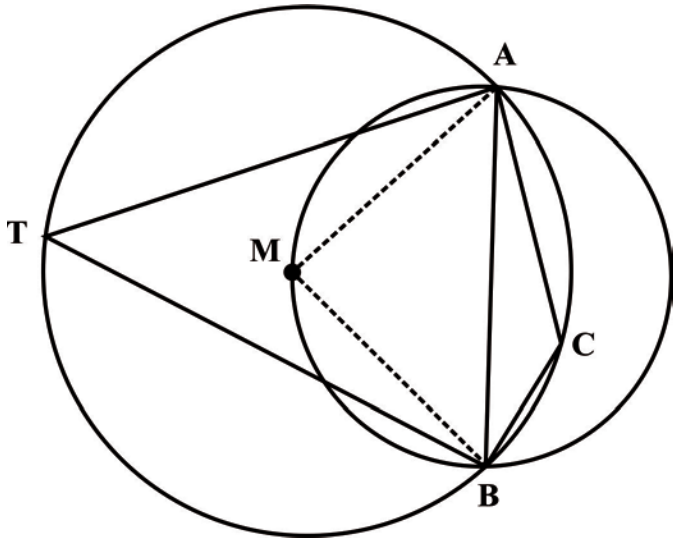
Give reasons for the following statements: (5)

STATEMENT	REASONS
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	
$\hat{D}_2 + \hat{E} = 180°$	
$\hat{B}_1 = \hat{D}_1$	
$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180°$	
$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	

7.2 In the diagram below, circle centre M intersects a second smaller circle at A and B.

A, C, B and T are points on circle M.

AB is the diameter of the smaller circle.



7.2.1 Determine the size of $\hat{C}$. (6)

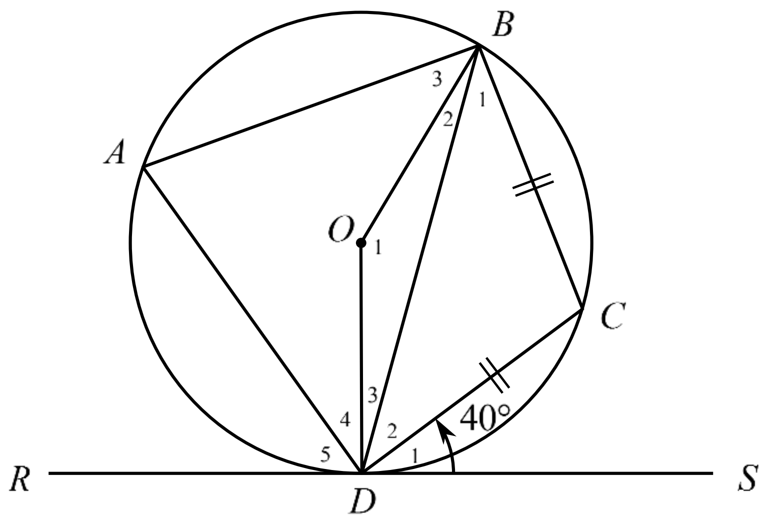
7.2.2 Explain why AMBC is not a cyclic quadrilateral. (1)

[12]

QUESTION 8

In the figure below, RDS is a tangent to circle O at D. BC = DC, and $C\hat{D}S = 40°$.

Thus, calculate the size of the following angles, with reasons.



8.1 $B\hat{D}C$ (2)

8.2 $\hat{C}$ (2)

8.3 $\hat{A}$ (2)

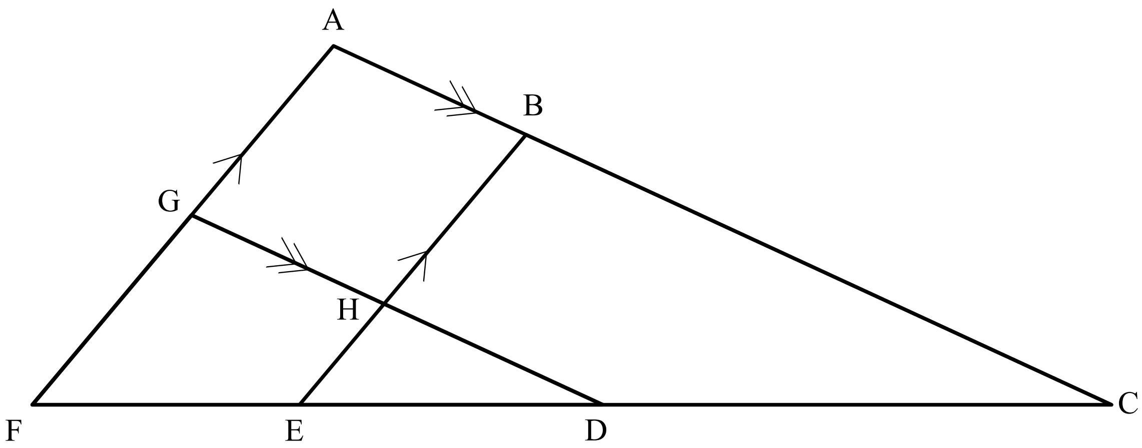
8.4 $\hat{O}_1$ (1)

[7]

QUESTION 9

The diagram below is the top view design of a new railway system. There are eight stations being built and these are labelled with letters from A- H.You have been asked to do some calculations fro the railway company. As the engineer you know that:

- $AF \parallel BE$ and $AC \parallel GD$.
- $\frac{AB}{BC} = \frac{4}{7}$ and $\frac{AG}{AF} = \frac{9}{17}$.



9.1 Calculate

9.1.1 $\frac{FE}{FC}$. (3)

9.1.2 $\frac{CD}{DF}$. (2)

9.2 If the straight line distance of the track from F to C is 374 kilometres and its takes 50 hours to build one kilometre of the track, determine the number of hours it will take to build the section from E to D. (6)

[11]

TOTAL 100 MARKS

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x; y) \rightarrow (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

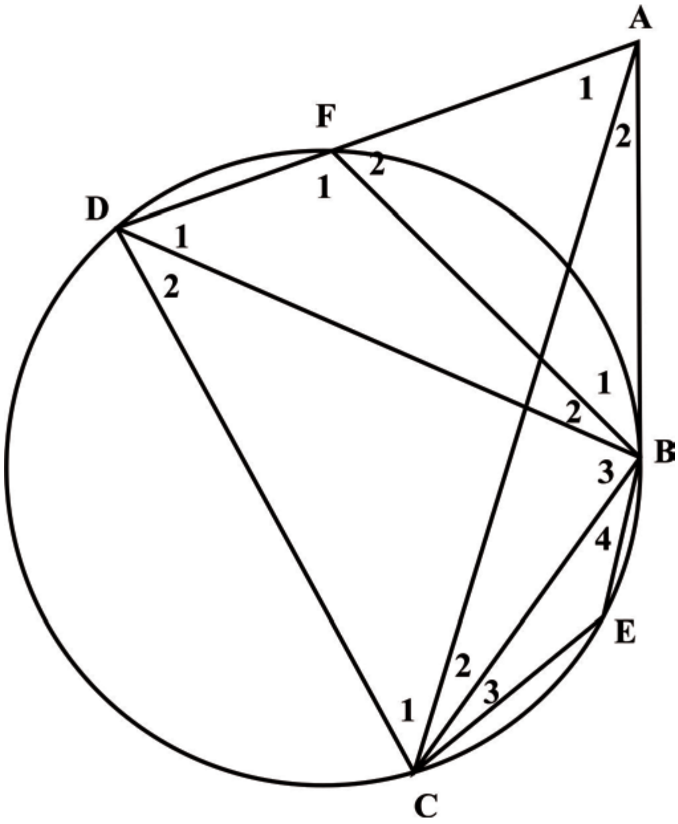
$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Name:________________________________ Class:________________

ANNEXURE 7.1 - 7. 2 .2

QUESTION 7

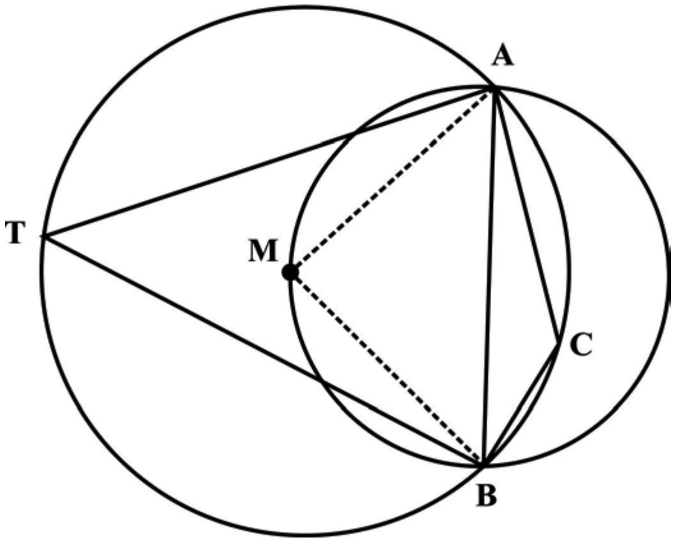
7.1



Give reasons for the following statements: (5)

STATEMENT	REASONS
$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	
$\hat{D}_2 + \hat{E} = 180°$	
$\hat{B}_1 = \hat{D}_1$	
$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180°$	
$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	

7.2



7.2.1 Determine the size of $\hat{C}$. (6)

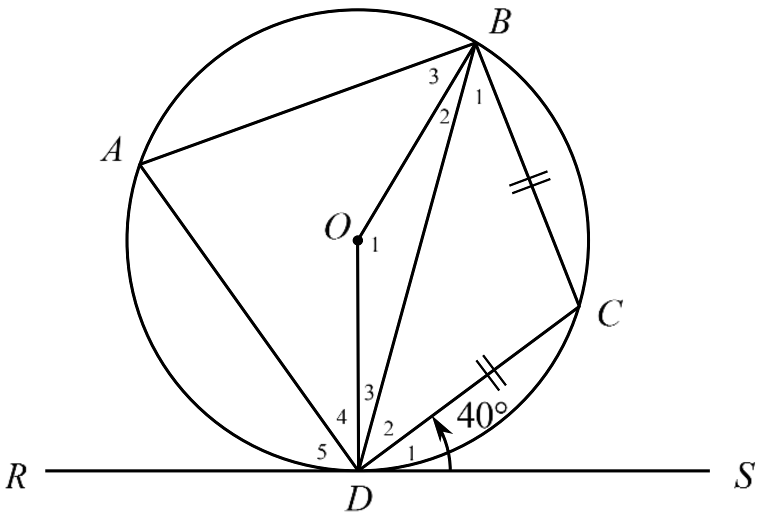
7.2.2 Explain why AMBC is not a cyclic quadrilateral. (1)

[12]

Name:_________________________________ Class:_________________

ANNEXURE 8.1 - 8.1.3

QUESTION 8



8.1 $B\hat{D}C$ (2)

____

____

____

____

8.2 $\hat{C}$ (2)

____

____

____

____

8.3 $\hat{A}$ (2)

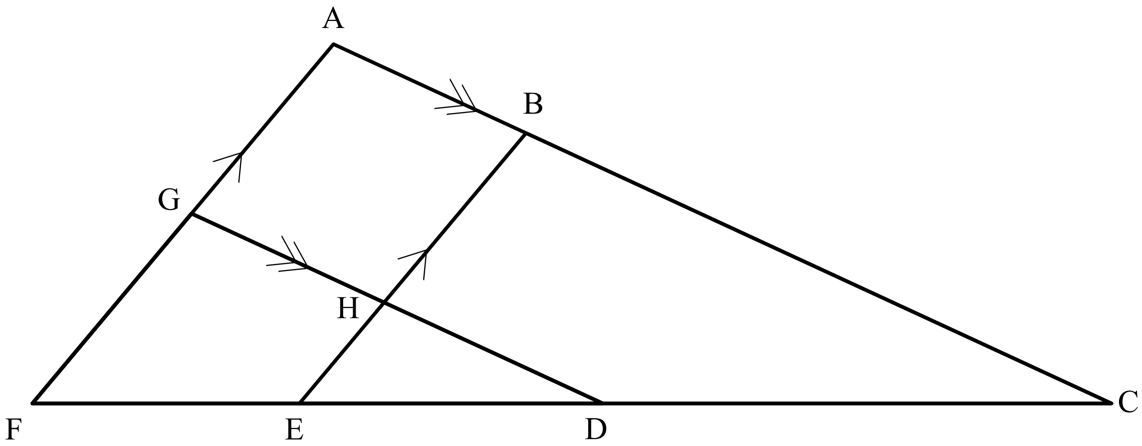
8.4 $\hat{O}_1$ (1)

[7]

Name:________________________________ Class:________________

ANNEXURE 9.1 - 9.2

QUESTION 9



9.1 Calculate

9.1.1 $\frac{FE}{FC}$. (3)

9.1.2 $\frac{CD}{DF}$. (2)

9.2 If the straight line distance of the track from F to C is 374 kilometres and its takes 50 hours to build one kilometre of the track, determine the number of hours it will take to build the section from E to D. (6)

[11]

GAUTENG DEPARTMENT OF EDUCATION

JOHANNESBURG NORTH DISTRICT

2021
GRADE 12
CONTROL TEST

MATHEMATICS
TERM 1

MARKING GUIDELINES

MARKS : 100
TIME : 2 hours

QUESTION 1		
1.1.1	$(x-5)(x+1)=0$ $x=5$ or $x=-1$	✓ $x=5$ ✓ $x=-1$ (2)
1.1.2	$2x^2-11x+7=0$ $x=\dfrac{-(-11)\pm\sqrt{(-11)^2-4(2)(7)}}{2(2)}$ $x=4{,}77$ or $x=0{,}73$	✓ Sub ✓ $x=4{,}77$ ✓ $x=0{,}73$ (3)
1.1.3	$x-5x^{\frac{1}{2}}=-6$ $x-5x^{\frac{1}{2}}+6=0$ Let $k^2=x$ and $k=x^{\frac{1}{2}}$ $\therefore k^2-5k+6=0$ $(k-3)(k-2)=0$ $k=3$ or $k=2$ But: $x^{\frac{1}{2}}=3$ or. $x^{\frac{1}{2}}=2$ $(x^{\frac{1}{2}})^2=(3)^2$ or. $(x^{\frac{1}{2}})^2=(2)^2$ $x=9$ or $x=4$	✓ Standard form ✓ factors ✓ squaring ✓ x values (4)
1.2	$\sqrt{\dfrac{5^{2014}-5^{2012}}{6}}$ $=\sqrt{\dfrac{5^{2012}.5^2-5^{2012}}{6}}$ $=\sqrt{\dfrac{5^{2012}(25-1)}{6}}$ $=\sqrt{4.5^{2012}}$ $=2(5^{1006})$ $\therefore a=2.$ and. $b=1006$	✓ $\sqrt{\dfrac{5^{2012}.5^2-5^{2012}}{6}}$ ✓ $\sqrt{4.5^{2012}}$ ✓ $a=2.$ ✓ . $b=1006$ (4)

1.3	$1 = 3y - x$ ………………(1) $y^2 + 2xy = 3x^2 - 7$ ……..(2) $x = 3y - 1$ ……………….(3) Sub (3) into (2) $y^2 + 2y(3y - 1) = 3(3y - 1)^2 - 7$ $y^2 + 6y^2 - 2y = 27y^2 - 18y + 3 - 7$ $0 = 20y^2 - 16y - 4$ $0 = 5y^2 - 4y - 1$ $0 = (y - 1)(5y + 1)$ $y = 1$ or $y = \frac{-1}{5}$ Sub y-values into (3) $x = 3(1) - 1$ or $x = 3(\frac{-1}{5}) - 1$ $x = 2$ or $x = \frac{-8}{5}$	✓ Equation 3 ✓ Sub ✓ Simplification ✓ Standard form ✓ Factors ✓ y-values ✓ x-values (7)
20 MARKS		

QUESTION 2		
2.1	$T_n = 24$	✓ Ans (1)
2.2	$T_n = 3 + (n - 1)(7)$ $T_n = 7n - 4$	✓ Sub ✓ Ans (2)
2.3	$\sum_{k=1}^{22} (7k - 4)$	✓ Ans (1)
4 MAKRS		

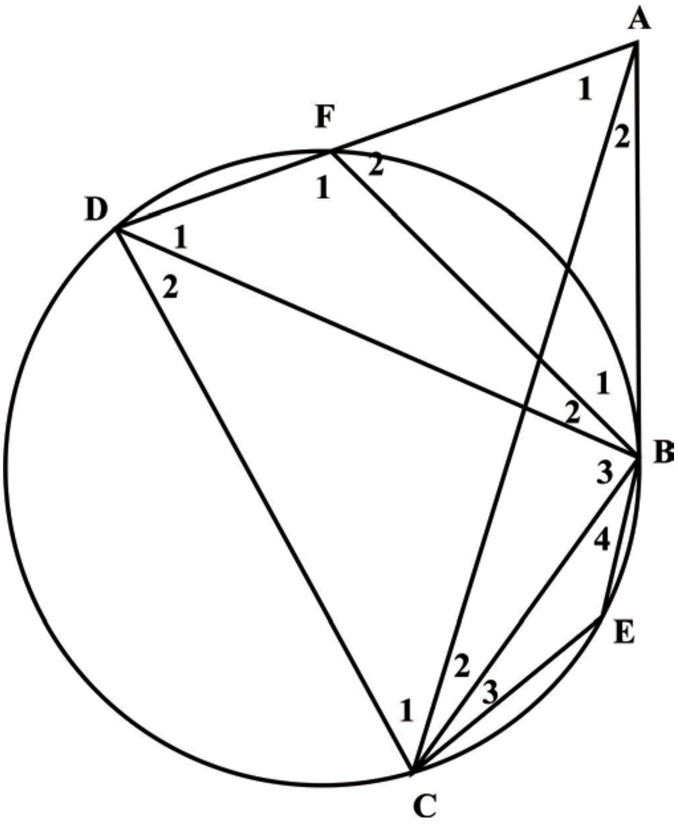
QUESTION 3		
3.1.1	$3;\ \frac{64}{250}$	✓ Ans (1)
3.1.2	$3 \times 18 = 54$ $S_{17} = \frac{\frac{1}{2}[(\frac{4}{5})^{17} - 1]}{(\frac{4}{5}) - 1}$ $= 2{,}44$ $\therefore S_{35} = 56{,}44$	✓ Odd terms ✓ Sub ✓ 2,44 ✓ ✓ $S_{35} = 56{,}44$ (5)
3.2	$T_1 = \frac{5}{9}$ $T_2 = \frac{5}{27}$ $\therefore r = \frac{1}{3}$ $S_\infty = \frac{\frac{5}{9}}{1 - \frac{1}{3}}$ $= \frac{5}{6}$ or 0,83	✓ a ✓ r ✓ Sub ✓ Ans (4)
10 MARKS		

QUESTION 4		
4.1	3cm	✓ ✓ Ans (2)
4.2	24; 12; 6; 3 $r = \frac{1}{2}$ $T_n = 24(\frac{1}{2})^{n-1}$	✓ Sequence ✓ ratio ✓ Ans (3)
5 MAKRS		

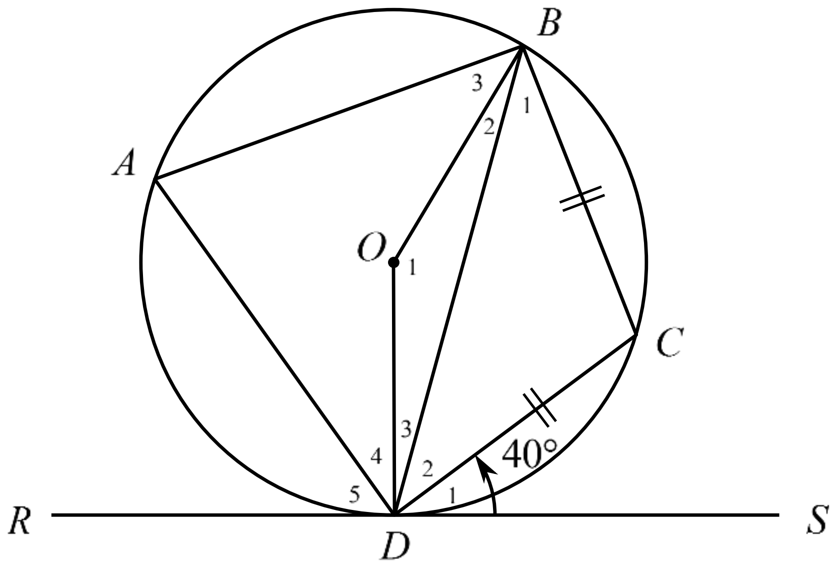
QUESTION 5		
5.1.1	$y^2 = (2)^2 - (-\sqrt{2})^2$ $y = \sqrt{2}$	✓ method ✓ Answer (2)
5.1.2	$\dfrac{2\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)^2 - 1} = \dfrac{2\left(-\frac{1}{2}\right)}{\frac{1}{2} - 1} = 2$	✓✓✓substitution ✓sign ✓answer (5)
5.2	$\dfrac{\cos(180^\circ+\theta)\cdot\tan(720^\circ-\theta)\cdot\sin^2(90^\circ-\theta)}{\sin(180^\circ-\theta)} + \sin^2\theta$ $= \dfrac{-\cos\theta \times -\tan\theta \times \cos^2\theta}{\sin\theta} + \sin^2\theta$ $= \dfrac{\cos\theta \times \frac{\sin\theta}{\cos\theta} \times \cos^2\theta}{\sin\theta} + \sin^2\theta$ $= \cos^2\theta + \sin^2\theta = 1$	✓ $-\cos\theta$ ✓ $-\tan\theta$ ✓ $\cos^2\theta$ ✓ $\sin\theta$ ✓Identity: $\tan\theta$ ✓ $\cos^2\theta + \sin^2\theta$ ✓$=1$ (7)
5.3	$6\sin^2\theta - 5\sin\theta\cos\theta - 4\cos^2\theta = 0$ $(2\sin\theta - \cos\theta)(3\sin\theta + 4\cos\theta) = 0$ $\therefore 2\sin\theta = \cos\theta$ $\therefore \tan\theta = \frac{1}{2}$ $\text{Ref}\angle = 26,57^\circ$ $\therefore \theta = 26,57^\circ + n180^\circ$ OR: $\theta = 180^\circ + 26,57^\circ + n180^\circ$ $\therefore \theta = 206,57^\circ + n180^\circ$ AND: $3\sin\theta = -4\cos\theta$ $\therefore \tan\theta = -\frac{4}{3}$ $\text{Ref } \angle = 53,13^\circ$ $\therefore \theta = 180^\circ - 53,13^\circ + n180^\circ$ $\theta = 126,87^\circ + n180$ OR: $\theta = 360^\circ - 53,13^\circ + n180^\circ$ $\therefore \theta = 306,87^\circ + n180^\circ$	✓$=0$ ✓factors ✓ $\tan\theta$ ✓solution ✓solution ✓ $\tan\theta$ ✓solution ✓solution (8)
22 MARKS		

QUESTION 6		
6.1	$\frac{PS}{\sin a}=\frac{x}{\sin b}$ $\therefore PS=\frac{x\sin a}{\sin b}$	✓✓method sine rule (2)
6.2	$S_2=a+b$	✓accuracy (1)
6.3	$Area=\frac{1}{2}PS\times SR\sin S_2$ $=\frac{1}{2}\left(\frac{x\sin a}{\sin b}\right)(x)(\sin(a+b))$ $=\frac{x^2\sin a\cdot\sin(a+b)}{2\sin b}$	✓method: Area rule ✓✓sub in (3)
6.4	$Area=\frac{(14,2)^2\times\sin(34^\circ)\times\sin(34^\circ+41^\circ)}{2\sin(41^\circ)}$ $=83,01cm^2$	✓✓sub into formula ✓answer (3)
9 MARKS		

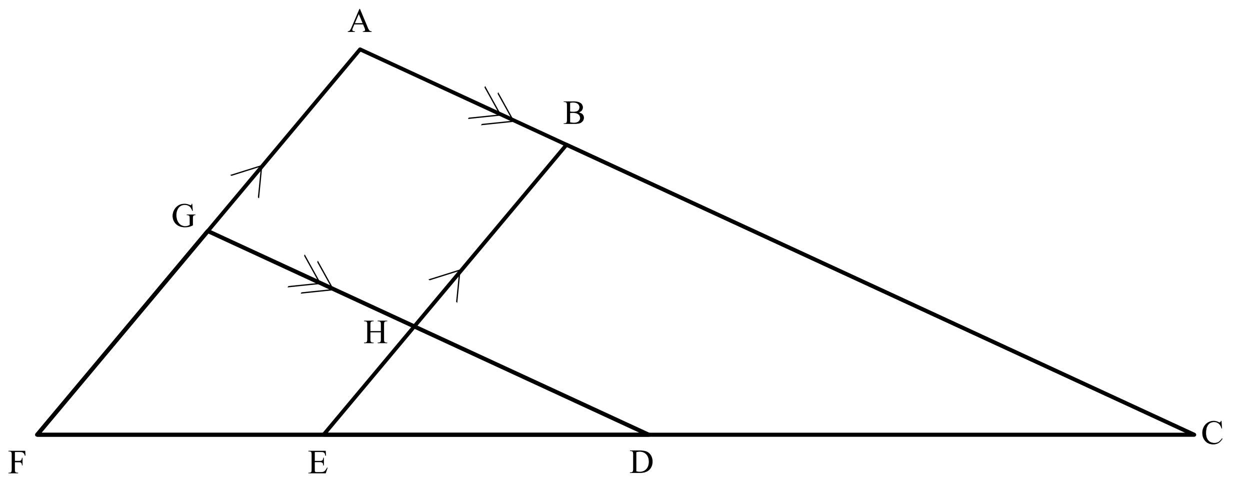
QUESTION 7



	STATEMENT	REASONS
7.1	$\hat{C}_1 + \hat{C}_2 = \hat{F}_2$	✓ [ext $\angle'$ of cyclic quad]
	$\hat{D}_2 + \hat{E} = 180^0$	✓ [opp $\angle'$ of cyclic quad]
	$\hat{B}_1 = \hat{D}_1$	✓ [tan chord]
	$\hat{B}_2 + \hat{B}_3 + \hat{D}_1 + \hat{D}_2 = 180^0$	✓ [opp $\angle'$ of cyclic quad]
	$\hat{B}_2 + \hat{B}_1 = \hat{C}_1 + \hat{C}_2$	✓ [tan chord] (5)
7.2.1	$A\hat{M}B = 90°$ ($\angle's$ in semi circle) $\hat{T} = 45°$ ($\angle'$ at center $= 2 \times \angle$ at circum) $\hat{C} = 135°$ (opp $\angle's$ of cyclic quad $= 180$)	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
7.2.2	$\hat{M} + \hat{C} \neq 180°$; opp $\angle$'s do not add up to $180°$	✓ S & R (1)
	12 MARKS	

QUESTION 8		
		
8.1	$B_1 = 40^\circ$ (tan-chord theorem) $\therefore BDC = 40^\circ$ ($\angle$'s opp = sides)	✓ S & R ✓ S & R (2)
8.2	$C = 100^\circ$ ($\angle$'s in $\triangle$)	✓ S ✓ R (2)
8.3	$\therefore A = 80^\circ$ (opp $\angle$'s in a cyclic quad)	✓ S ✓ R (2)
8.4	$O_1 = 160^\circ$ ($\angle$ at centre = 2$\angle$ at circum)	✓ S & R (1)
7 MARKS		

QUESTION 9

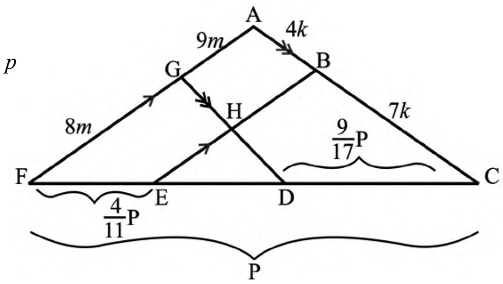


9.1.1	Let AB $= 4k$ and BC $= 7k$ $\therefore \frac{FE}{FC} = \frac{AB}{AC} = \frac{4}{11}$; (Proportionality theorem OR using theorem on diagram)	✓ S ✓ S ✓ R (3)
9.1.2	Let AG $= 9m$ and AF $= 17m$ $\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}$	✓ Reason ✓ $\frac{CD}{DF} = \frac{AG}{GF} = \frac{9}{8}$ (2)

9.2

If $FC = p$ then $ED = p - \frac{9}{17}p - \frac{4}{11}p$

$ED = \frac{20}{187}p$



The length of ED in kilometres is $\frac{20}{187} \times 374$ km $= 40$ kilometres.

It will take 2 000 hours to build the track from E to D.

OR

Alternate:

Let $FE = 4p$ and $EC = 7p$

$FD = 8m$ and $DC = 9m$

$\therefore 11p = 374 \quad \therefore p = 34$

$17m = 374 \quad \therefore m = 22$

$\therefore DC = 374 - 4p - 9m$

$= 40$ km

$\therefore$ 2 000 hours

OR

Alternate:

$FE = \frac{4}{11}(374) = 136$

$CD = \frac{9}{17}(374) = 198$

$\therefore ED = 374 - 136 - 198$

$= 40$ km

$\therefore$ 4 hours $\rightarrow 40 \times 50$

$= 2\ 000$ hours

Marks:

✓ ✓ If $FC = p$ then $ED = p - \frac{9}{17}p - \frac{4}{11}p$

✓ ✓ $ED = \frac{20}{187}p$

✓ The length of ED in kilometres is $\frac{20}{187} \times 374$ km $= 40$ kilometres.

✓ It will take 2 000 hours to build the track from E to D.

or

✓ Let $FE = 4p$ and $EC = 7p$

✓ $FD = 8m$ and $DC = 9m$

✓ $\therefore 11p = 374 \quad \therefore p = 34$

✓ $17m = 374 \quad \therefore m = 22$

✓ $\therefore DC = 374 - 4p - 9m$
$= 40$ km

✓ $\therefore$ 2 000 hours

or

✓ $FE = \frac{4}{11}(374) = 136$

✓ $CD = \frac{9}{17}(374) = 198$

✓ ✓ $\therefore ED = 374 - 136 - 198$
$= 40$ km

✓ ✓ $\therefore$ 4 hours $\rightarrow 40 \times 50$
$= 2\ 000$ hours

(6)

11 MARKS

TOTAL MARKS 100