GAUTENG DEPARTMENT OF EDUCATION

JOHANNESBURG NORTH DISTRICT

2022
GRADE 12
CONTROL TEST

MATHEMATICS
TERM1

MARKS : 100
TIME : 2 hours

This questions paper consist of 14 pages

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **8 questions.**
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which was used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. **Tear off page 11 till page 14 . AND SUBMIT theses pages with your answer scripts .**
9. An **information sheet** is on page 10 of the question paper.
10. Number the questions correctly according to the numbering used in the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 7x + 10 = 0$ (2)

1.1.2 $3x^2 + 2x + 6 = 10$ (correct to two decimal places) (4)

1.1.3 $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ (4)

1.1.4 $\sqrt{2 - x} = x - 2$ (5)

1.2 Given: $3x^2 + kx - 3x - k = 0$.

For which values of k will the equation have real roots? (4)

1.3 Solve for x and y:

$3y + x = 5$ and $x^2 + y^2 = 100 + 5y$ (6)

[25]

QUESTION 2

2.1 Consider the sequence: 1; 4; 11; 22; 37; . . . .

2.1.1 Calculate the n^{th} term. (4)

2.1.2 Which term in the sequence has a value of 407? (4)

2.2 How many terms are there in the following arithmetic sequence.

40; 46; 52; 58;……202 (3)

[11]

QUESTION 3

3.1 The 4^{th} term of an arithmetic sequence is -3 and the 20^{th} term is -35.

Determine the common difference and the first term. (5)

3.2 Evaluate : $\sum_{k=1}^{20} 3^{k-2}$ (4)

3.3 The following sequence forms a convergent geometric sequence:

$$\frac{3}{(x-1)^2} + \frac{1}{(x-1)} + \frac{1}{3} + \frac{(x-1)}{9} + \ldots$$

3.3.1 Determine the possible values of x. (3)

3.3.2 If $x = 2$, calculate S_∞. (2)

[14]

QUESTION 4

4.1 Given $sin24^\circ = m$ and $cos35^\circ = n$. Determine the following in terms of m or n.

4.1.1 $tan66^\circ$ (3)

4.1.2 $sin70^\circ$ (3)

4.2 Prove that: $sin(45^\circ + x) \,.\, sin(45^\circ - x) = \frac{cos2x}{2}$ (5)

4.3 Given $cos(x + 42^\circ) = sin2x$. Solve for x if $x \in [-180^\circ, 180^\circ]$. (6)

[17]

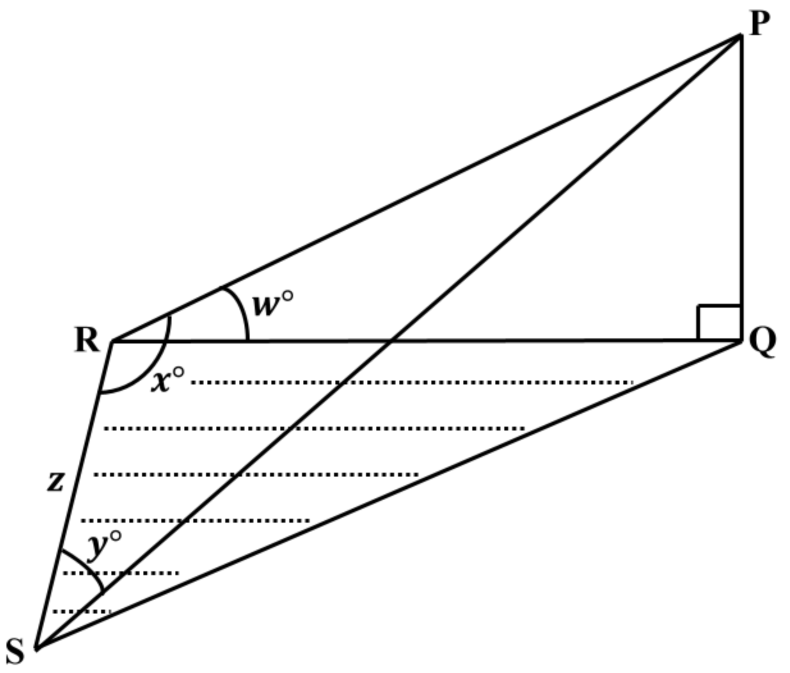
QUESTION 5

A mountain climber wants to determine the height PQ of a mountain. The climber is standing at R on a flat ground. R and S are in the same horizontal plane as the foot of the mountain Q.

From R, he measures the following angles:

- The angle of elevation of the top of the mountain P is $w°$.
- $P\hat{R}S$ is $x°$

He then walks z metres to point S and measures $R\hat{S}P$ which is $y°$



5.1 Show that $PQ = \dfrac{z\sin y . \sin w}{\sin(x + y)}$ (4)

5.2 Determine PQ, if $z = 1000m$, $w = 90° - x$ and $x = y$ (4)

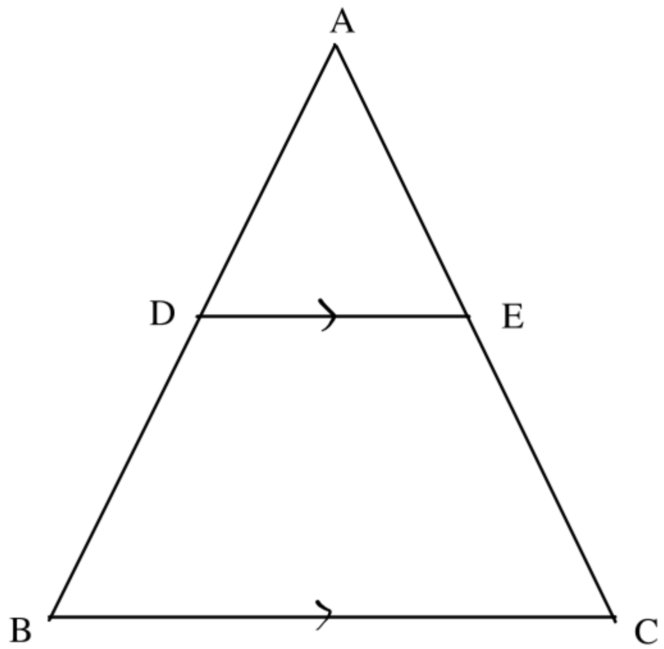
[8]

Give reasons for your statements in Question 6, 7 and 8.

Use the Annexure A provided to answer Question 6, 7 and 8

QUESTION 6

Given $\triangle ABC$ with $DE \parallel BC$ as shown in the figure below:



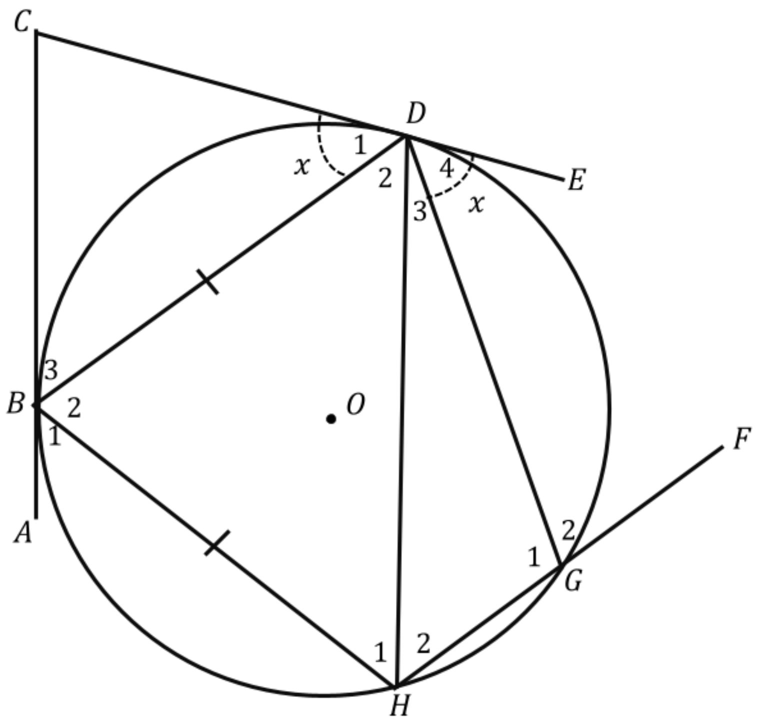
Prove that: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ [5]

QUESTION 7

In the diagram below, AC and CE are tangents to the circle with centre O.

B, D, G and H are points on the circumference of the circle.

HG is produced to F . BD = BH $\hat{D}_1 = \hat{D}_4 = x$.



7.1 Find four other angles equal to x. (4)

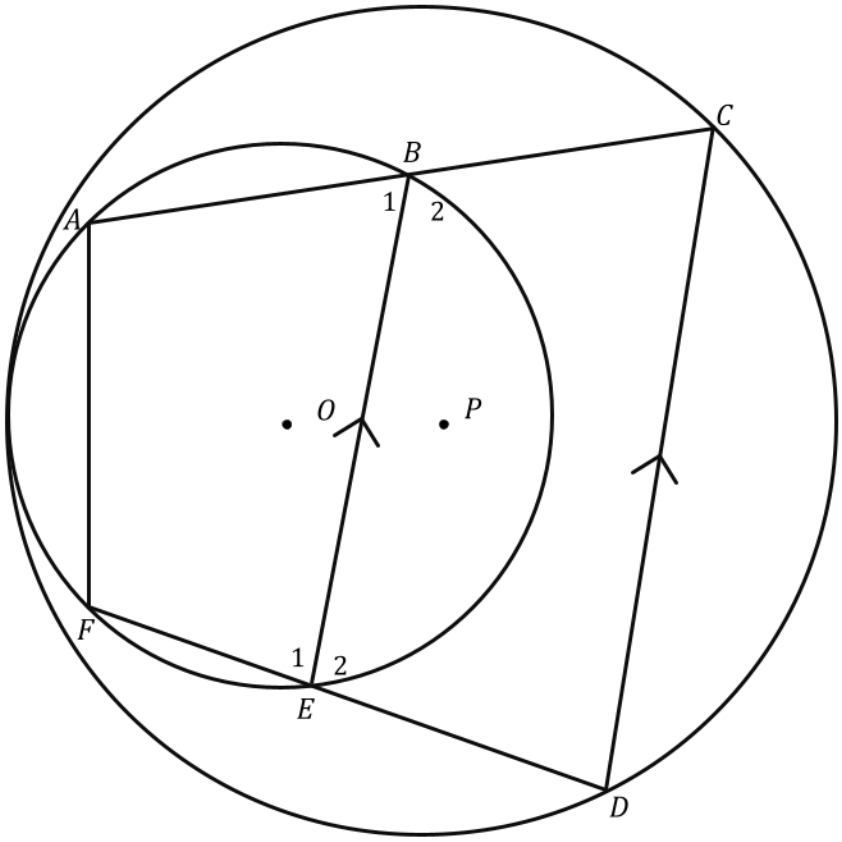
7.2 Hence or otherwise prove that BD || HG. (2)

7.3 Show that $\hat{G}_2 = 180° - 2x$. (2)

[8]

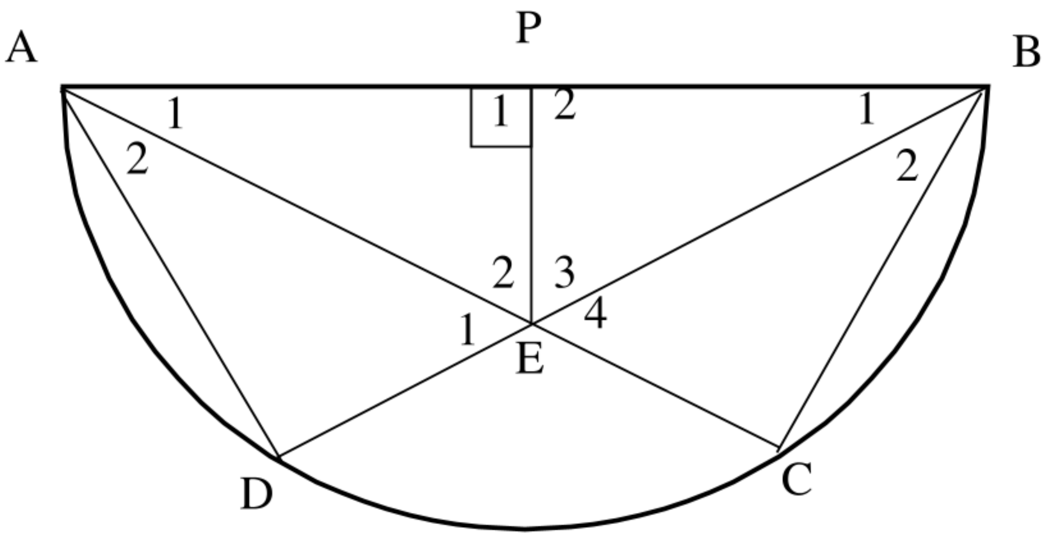
QUESTION 8

8.1 In the diagram below, AB lies on circle with centre O, and is produced to C which lies on circle with centre P. Similarly, FE is produced to D.. $BE \parallel CD$.



Prove that ACDF is a cyclic quadrilateral. (4)

8.2 In the diagram below, AB is the diameter of circle with centre P. $EP \perp AB$.



8.2.1 Prove that $\triangle BPE /// \triangle BDA$ (4)

8.2.2 Hence, prove that $BE = \dfrac{PE^2 . BA . BD}{BP}$ (4)

[12]

TOTAL 100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x; y) \rightarrow (x\cos\theta - y\sin\theta; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

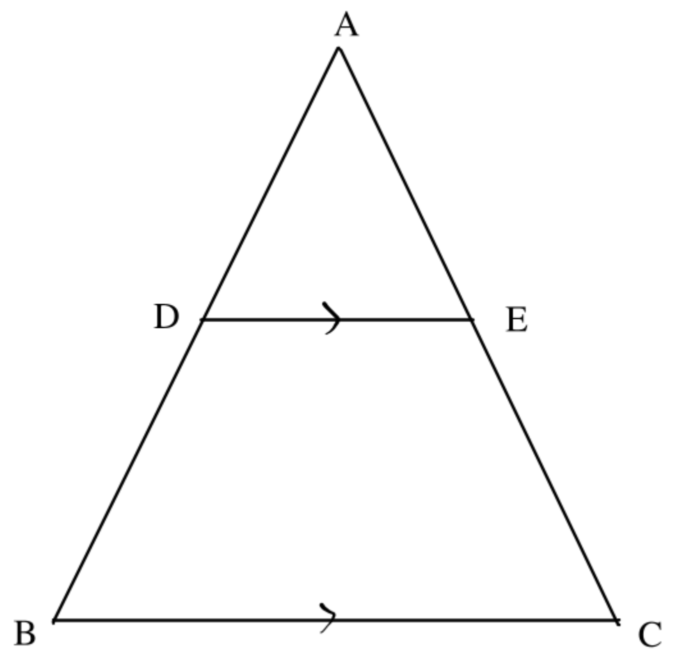
$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

Name: ________________________ **Class:** ________________

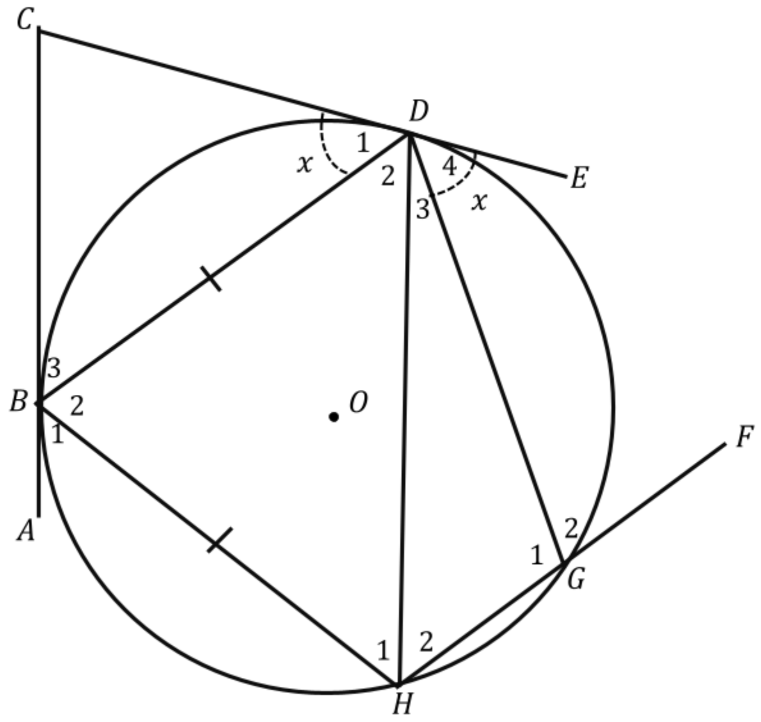
ANNEXTURE A

QUESTION 6



(5)

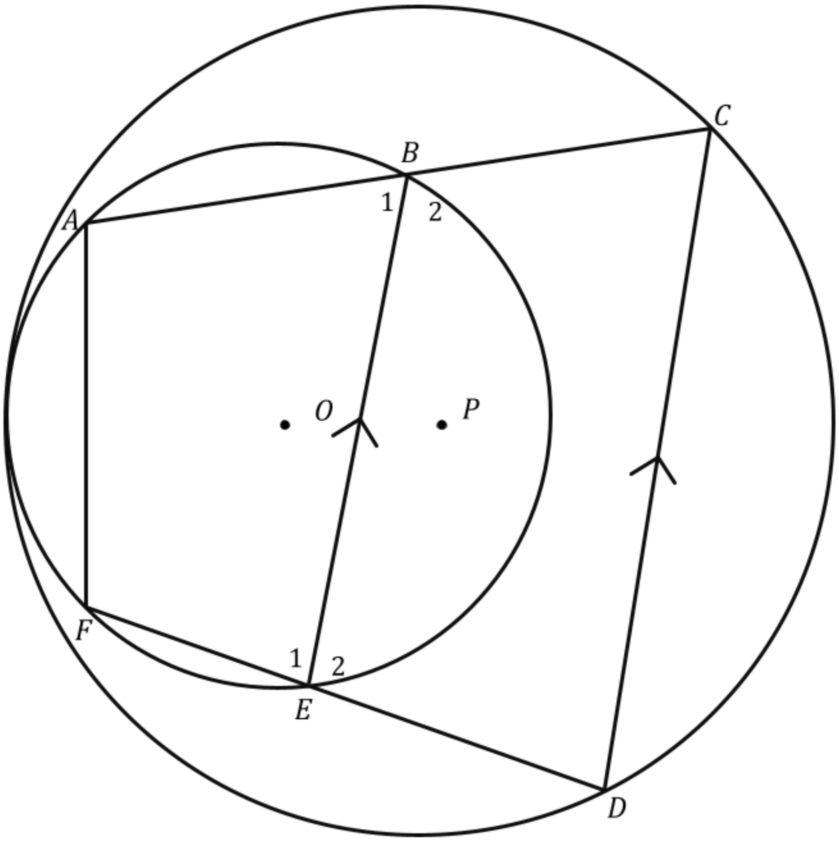
QUESTION 7

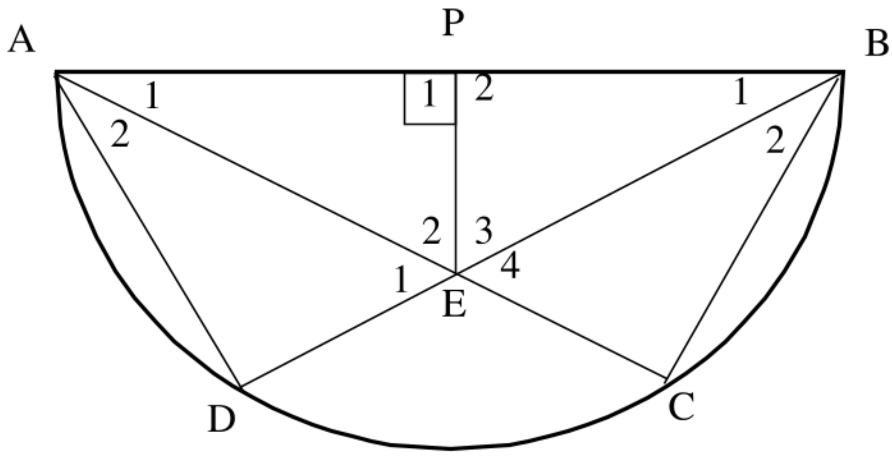


7.1		(4)
7.2		(2)
7.3		(2)

QUESTION 8

8.1 (4)



8.2		
8.2.1		(4)
8.2.2		(4)

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MARKING GUIDELINES

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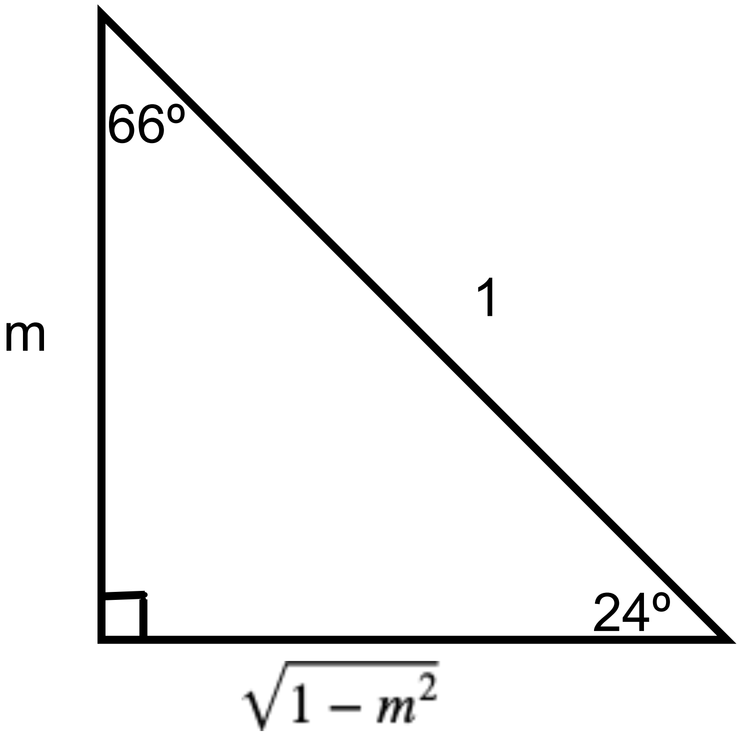
QUESTION 1		
1.1.1	$x^2 - 7x + 10 = 0$ $(x-5)(x-2) = 0$ $x = 5$ or $x = 2$	✓ Factors ✓ Ans (2)
1.1.2	$3x^2 + 2x + 6 = 10$ $3x^2 + 2x - 4 = 0$ $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-4)}}{2(3)}$ **NOTE: penalise 1 mark for incorrect rounding off** $x = 0{,}87$ or $x = -1{,}54$	✓ Standard Form ✓ Correct Sub into formula ✓ $x = 0{,}87$ ✓ $x = -1{,}54$ (4)
1.1.3	$x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ $(x^{\frac{1}{4}} - 4)(x^{\frac{1}{4}} + 7) = 0$ $x^{\frac{1}{4}} = 4$ or. $x^{\frac{1}{4}} = -7$ $x = 256$ or $x = 2401$ OR $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ Let $k^2 = x^{\frac{1}{2}}$ or $k = x^{\frac{1}{4}}$ $\therefore k^2 + 3k - 28 = 0$ $(k-4)(k+7) = 0$ $k = 4$ or. $k = -7$ $x^{\frac{1}{4}} = 4$ or. $x^{\frac{1}{4}} = -7$ $x^{\frac{1}{4} \times 4} = (4)^4$ or. $x^{\frac{1}{4} \times 4} = (-7)^4$ $x = 256$ or $x = 2401$	✓ Factors ✓ $x^{\frac{1}{4}} = \ldots$ ✓ raising both side to reciprocal ✓ x- values (4) OR ✓ Factors ✓ $x^{\frac{1}{4}} = \ldots$ ✓ multiplying by the reciprocal ✓ x- values (4)

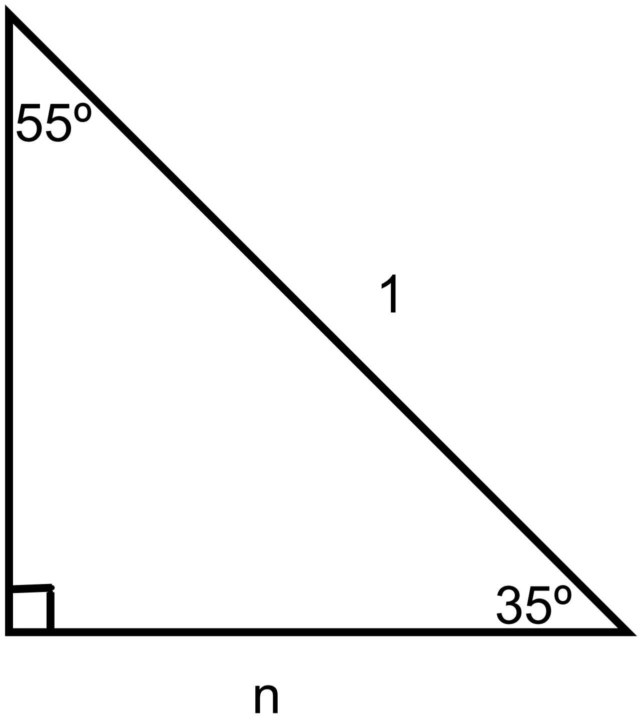
1.1.4	$\sqrt{2-x} = x-2$ $(\sqrt{2-x})^2 = (x-2)^2$ $0 = x^2 - 3x + 2$ $0 = (x-2)(x-1)$ $x = 2$ or $x = 1$ $\therefore x \neq 1$	✓ Squaring ✓ Standard form ✓ Factors ✓ x- values ✓ Selection/ Testing (5)
1.2	$3x^2 + kx - 3x - k = 0.$ $3x^2 + x(k-3) - k = 0$ $\triangle \geq 0$ $b^2 - 4ac \geq 0$ $(k-3)^2 - 4(3)(-k) \geq 0$ $k^2 + 6k + 9 \geq 0$ $(k+3)^2 \geq 0$ $\therefore k \geq -3$	✓ $\triangle \geq 0$ ✓ Subbing correctly ✓ Factors ✓ Ans (4)

1.3	$3y + x = 5$ ……………………(1) $x^2 + y^2 = 100 + 5y$ …………..(2) $x = 5 - 3y$………………(3) Sub (3) into (2) $(5 - 3y)^2 + y^2 = 100 + 5y$ $2y^2 - 7y - 15 = 0$ $(y - 5)(2y + 3) = 0$ $y = 5$ or. \`$y = \frac{-3}{2}$ $x = -10$ or. $x = \frac{19}{2}$	✓ Equation (3) ✓ Subbing correctly ✓ Standard form ✓ Factors ✓ Y-values ✓ X-values (6)
25 MARKS		

QUESTION 2		
2.1.1	$a+b+c=1$……..(1) $3a+b=3$ ………..(2) $2a=4$ $\therefore a=2$ $\therefore b=-3$ $\therefore c=2$ $T_n=2n^2-3n+2$	✓ Value of a ✓ Value of b ✓ Value of c ✓ General Term (4)
2.1.2	$407=2n^2-3n+2$ $0=2n^2-3n-405$ $0=(n-15)(2n+27)$ $\therefore n=15$ or. $n\neq-\frac{27}{2}$	✓ Equating ✓ Standard Form ✓ Both values of n ✓ Rejection (4)
2.2	$d=6$ $202=40+6(n-1)$ $\therefore n=28$	✓ $d=6$ ✓ Sub ✓ Ans (3)
11 MARKS		

QUESTION 3		
3.1	$-3-3d = a$ ....(1) $-35-19d = a$.........(2) (1) = (2) $-3-3d = -35-19d$ $\therefore d = -2$ Sub d-value into (1) $a = -3-(-2)$ $\therefore a = 3$	✓ $a = -3-3d$ ✓ $a = -35-19d$ ✓ Equating ✓ $d = -2$ ✓ $a = 3$ (5)
3.2	$a = \frac{1}{3}$ $r = 3$ $S_{20} = \frac{\frac{1}{3}(3^{20}-1)}{3-1}$ $S_{20} = 581130733{,}3$	✓ $a = \frac{1}{3}$ ✓ $r = 3$ ✓ Sub ✓ Ans (4)
3.3.1	$\frac{3}{(x-1)^2} + \frac{1}{(x-1)} + \frac{1}{3} + \frac{(x-1)}{9} + \dots$ $r = \frac{x-2}{3}$ $-1 < r < 1$ $-1 < \frac{x-2}{3}y < 1$ $-1 < y < 5$	✓ $-1 < r < 1$ ✓ Sub ✓ Ans (3)
3.3.2	$S_\infty = \frac{3}{1-\frac{1}{3}}$ $S_\infty = \frac{9}{2}$	✓ Sub ✓ Ans (2)
14 MARKS		

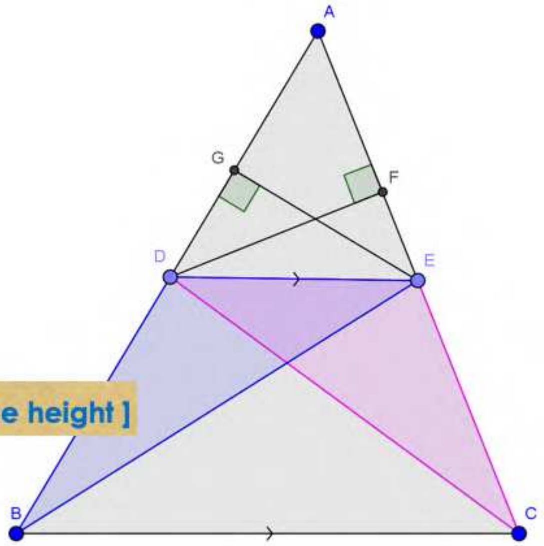
QUESTION 4		
4.1.1	 **Award Full marks for answer only** $\tan 66° = \frac{\sqrt{1-m^2}}{m}$	✓ Sketch ✓ $\sqrt{1-m^2}$ ✓ Ans (3)

4.1.2	 $\sqrt{1-n^2}$ $sin70° = sin(2 \times 35°)$ $= 2sin35°.cos35°$ $= 2(\sqrt{1-n^2})(n)$ or $2n\sqrt{1-n^2}$	✓ Sketch ✓ Double angle identity ✓ Ans (3)
4.2	$sin(45°+x).sin(45°-x) = \frac{cos2x}{2}$ LHS $= sin(45°+x).sin(45°-x)$ $= (sin45°.cosx + cos°.sinx)(sin45°.cosx - cos°.sinx)$ $= sin^2 45°.cos^2 x - cos^2 45° sin^2 x$ $= (\frac{1}{\sqrt{2}})^2 cos^2 x - (\frac{1}{\sqrt{2}})^2 sin^2 x$ $= \frac{1}{2}(cos^2 x - sin^2 x)$ $= \frac{1}{2} cos2x$ $\therefore LHS = RHS$	✓ Compound angle expansion ✓ Simplification ✓ Special angles ✓ Double angle identity ✓ $\frac{1}{2}cos2x$ (5)

4.3	$\cos(x+42°)=\sin 2x$ $\cos(x+42°)=\cos(90°-2x)$ $x+42°=90°-2x+k.360°$ $\therefore 3x=90°-42°+k.360°$ $OR\ x+42°=-(90°-2x)+k.360°$ $\therefore\ x+42°=-90°+2x+k.360°$ $\therefore 3x=48°+k.360°$ OR $-x=-132°+k.360°$ $x=16°+k.120°$ OR $x=132°-k.360°$ $x\in\{-104;16;132;136\}$	✓ cos(90-2x) ✓ ✓ Both gen solution ✓ simplify x= ✓ ✓ Two correct x-values (6)
	17 MARKS	

QUESTION 5		
5.1	In ΔPRS : $R\hat{P}S=180°-(x+y)$ sum ∠'s Δ $\frac{RP}{\sin y}=\frac{z}{\sin[180°-(x+y)]}$ $RP=\frac{z\sin y}{\sin(x+y)}$ In ΔPRQ : $\frac{PQ}{PR}=\sin w$ $PQ=PR\sin w$ $\therefore\ PQ=\frac{z\sin y\,.\,\sin w}{\sin(x+y)}$	✓ $R\hat{P}S$ ✓ Sine rule ✓ Reduction ✓ Ans (4)
5.2	$PQ=\frac{1000\sin x\,.\,\sin(90°-x)}{\sin(x+x)}$ $=\frac{1000\sin x\,.\cos x}{2\sin x\,.\cos x}$ $=500\text{m}$	✓ Substitution ✓ Co-ratio ✓ Double angle ✓ Ans (4)
	8 MARKS	

QUESTION 6



RTP: $\frac{AD}{DB} = \frac{AE}{EC}$

Construct: an altitude DF & GE in ΔADE and join DC & BE ✓

1. $\frac{Area\Delta ADE}{Area\Delta DEB} = \frac{\frac{1}{2} \times AD \times GE}{\frac{1}{2} \times DB \times GE}$ ✓ S/R [Δ's share same height]

$= \frac{AD}{DB}$

2. $\frac{Area\Delta DEA}{Area\Delta DEC} = \frac{\frac{1}{2} \times AE \times DF}{\frac{1}{2} \times EC \times DF}$ ✓ S/R [Δ's share same height]

$= \frac{AE}{EC}$ ✓ S/R

3. BUT $Area\Delta DBE = Area\Delta DEC$ [Δ's lie between parallel lines]

✓ S

$\therefore \frac{Area\Delta ADE}{Area\Delta DBE} = \frac{Area\Delta DEA}{Area\Delta DEC} = \frac{AD}{DB} = \frac{AE}{EC}$

5 MARKS

QUESTION 7		
7.1	$\hat{B}_3 = x$ (BC=CD; tans from the same point) $\hat{H}_1 = \hat{B}_3 = x$ (tan chord th) **or** $\hat{H}_1 = \hat{D}_1 = x$ (tan chord th) $\hat{D}_2 = \hat{H}_1 = x$ (∠'s opp equal sides) $\hat{H}_2 = \hat{D}_4 = x$ (tan chord th)	✓ $\hat{B}_3$ & reason ✓ $\hat{H}_1$ & reason ✓ $\hat{D}_2$ & reason ✓ $\hat{H}_2$ & reason (4)
7.2	$\hat{H}_2 = \hat{D}_2 = x$ (proved above) $\therefore HG//BD$ (alt ∠'s =)	✓ S ✓ R (2)
7.3	$\hat{B}_2 = 180° - 2x$ (sum of ∠s in Δ) $\hat{G}_2 = \hat{B}_2 = 180° - 2x$ (ext ∠ of cyclic quad) **Alternative solution** $\hat{D}_1 + \hat{D}_2 = 2x$ $\hat{G}_1 = 2x$ (tan chord th) $\hat{G}_1 = 180° - 2x$ (∠s on a str line)	✓ $\hat{B}_2$ & reason ✓ $\hat{G}_2$ & reason OR ✓ $\hat{G}_2$ & reason ✓ $\hat{G}_1$ & reason (2)
8 MARKS		

QUESTION 8		
8.1	$\hat{A} = \hat{E}_2$ (ext ∠ of cyclic quad) $\hat{D} = 180° - \hat{E}_2$ (co-int ∠'s BE//CD) $\therefore \hat{D} + \hat{A} = 180°$ $\therefore ACDF$ is a cyclic quad (opp ∠'s quad sup) **Alternative solution** $\hat{D} = \hat{E}_1$ (corres ∠s BE//CD) $\hat{E}_2 = 180° - \hat{E}_1$ (∠s on a str line) $\hat{A} = 180 - \hat{E}_1$ (opp ∠ of cyclic quad $ABEF$) $\therefore \hat{D} + \hat{A} = 180°$ $\therefore ACDF$ is a cyclic quad (opp ∠'s quad sup)	✓ $\hat{A} = \hat{E}_2$ ✓ $\hat{D}$ & reason ✓ $\hat{D}$ +A ✓ conclude OR ✓ $\hat{D} = \hat{E}_1$ ✓ $\hat{A}$ & reason ✓ $\hat{D}$ +A ✓ conclude (4)
8.2.1	$\hat{B} = \hat{B}$ (common ∠) $\hat{D} = 90°$ (∠ in semi circle) $\hat{P}_2 = 90°$ (adj ∠'s on str line) $\therefore \hat{A} = \hat{E}$ (3^{rd}∠) $\triangle BPE /// \triangle BDA$ (∠ ∠ ∠)	✓ S/R ✓ S/R ✓ S/R ✓ S/R (4)

8.2.2	$\frac{BP}{BD}=\frac{PE}{DA}=\frac{BE}{BA}$ (/// $\triangle$' s) $\frac{BP}{BD}=\frac{PE}{DA}$ $\therefore DA=\frac{BP}{BD.PE}$ ....(1) $\frac{PE}{DA}=\frac{BE}{BA}$ $\therefore DA=\frac{PE.BA}{BE}$ ....(2) (2) = (1) $\frac{BP}{BD.PE}=\frac{PE.BA}{BE}$ $\therefore BP.BE=PE^2.BA.BD$ $\therefore BE=\frac{PE^2.BA.BD}{BP}$	✓ S/R ✓ Equation 1 ✓ Equation 2 ✓ $BP.BE=PE^2.BA.BD$ (4)
12 MARKS		