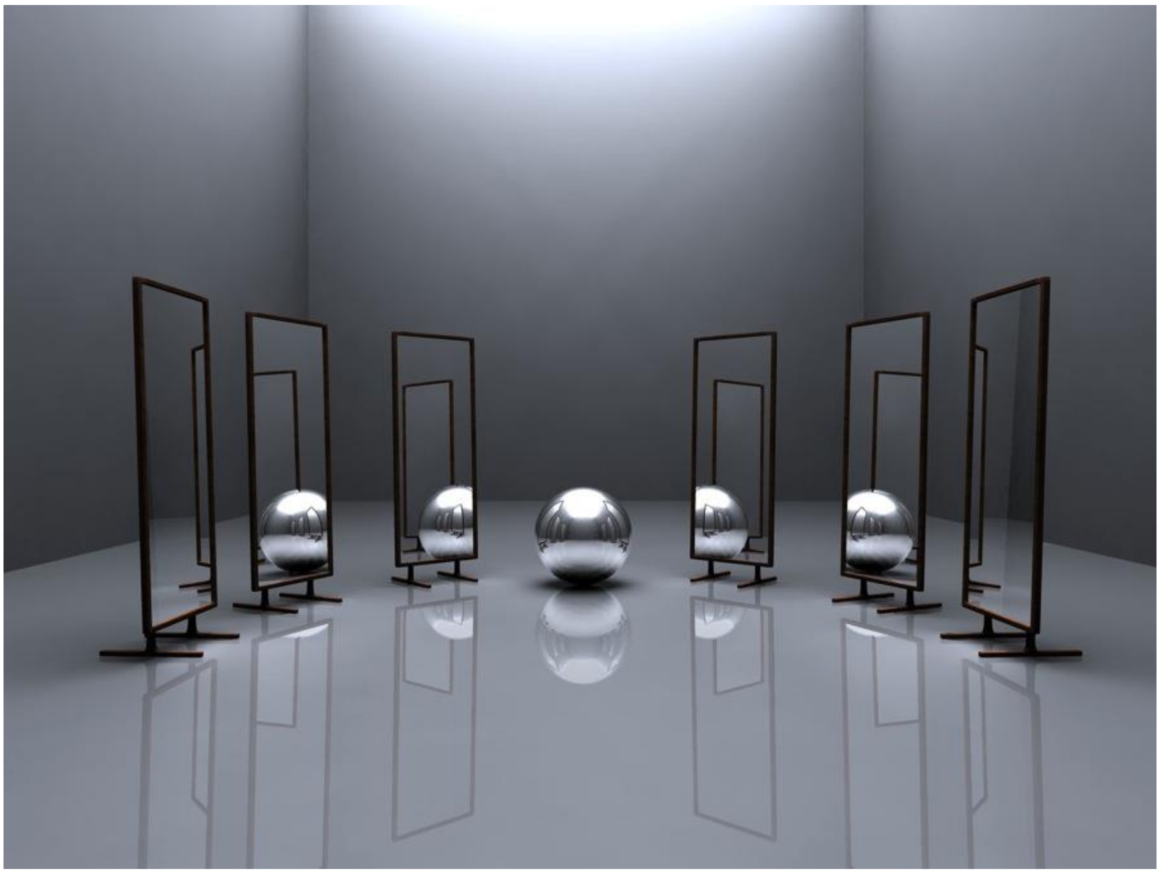
Matric Revision



Getting a different perspective
on Mathematics exams

Prepared and presented
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Functions, relations and inverses

Definition: A relation is when a variable gets related to another variable by use of any formulae or equation, ie x gets related to y. How y reacts to x gives us the different graphs.

A function is a special relation for which each x only has one y. The moment that that is not the case, we have a non-function.

There are 9 different graphs you have to be able to analyse, interpret and sketch:

- Straight line
- Parabola
- Cubic
- Hyperbola
- Exponential
- Logarithmic
- Sine
- Cosine
- Tangent

There are certain questions, which you must be able to answer for all 9 graphs:

1. **x-intercept:**
    - Let $y = 0$.

2. **y-intercept:**
    - Let $x = 0$.
3. **Definition/Domain:**
    - All x-values for which the function is defined.
4. **Rang**
    - All y-values for which the function is defined.
5. **Intersection between graphs:**
    - Set the y-values equal to each other or
    - Use substitution to solve simultaneously.
6. **Inverse relations:**
    - An inverse function is a reflection of a graph over the line $y = x$.
    - **Determining the equation of an inverse:** Swop the x and y's around in your equation, then, preferrably, you make y the subject.
    - **Sketching an inverse**: Take the original graph's coordinates and swop the x and y values around and draw the sketch.
    - **REMEMBER**: Domain of original is Range of inverse and Range of original is Domain of the inverse.

Specific graphs and their unique questions

- **Straight lines** will be handled during analitical geometry

- **Parabola**
  Equation:

$$y = a(x-p)^2 + q \text{ with (p;q) as stationary point;}$$

OR

$$y = ax^2 + bx + c$$

  - The stationary point can be calculated either by **differentiating and setting the derivative equal to 0** or by calcuting the **axis of symmetry** $\boldsymbol{x = -\frac{b}{2a}}$ and substituting the x-value into the original equation.

  - **Finding equation:** To find the equation depends on what has been given to you. It will always be given either:
    - **Turning point(p;q):** $y = a(x-p)^2 + q$
    - **2 x-intercepts(x_1 and x_2):** $y = a(x-x_1)(x-x_2)$

- **Hyperbola**
  - **Equation:**

$$y = \frac{a}{x-p} + q$$

  - **Equations of assymptotes:**
    - Horizontal $y = q$; and
    - Vertical $x = p$

  - A very important thing to remember is that hyperbolas are symmetrical.

  - Hyperbolas have 2 axes of symmetry:
    - One with positive gradient: $y = x - p + q$
    - One with negative gradient: $y = -x + p + q$

- **Exponential**
  - **Equation:**

$$y = b.a^{x-p} + q$$

  - **Equation of asymptote:**
    - Horizontal $y = q$

  - The p-value is the number of units the graph has been moved left or right
  - The q-value is the number of units the graph has been moved up or down

- **Logarithmic**
  - **Equation:**

$$y = \log_a(x - p) + q$$

  - **Equation of asymptote:**
    - Vertical $x = p$
  - The p-value is the number of units the graph has been moved left or right
  - The q-value is the number of units the graph has been moved up or down

- **Trigonometrical graphs**

  - The standard **sine and cosine** graphs are very similar:
    - Period= 360˚
    - Amplitude=1

  - The standard **tangent** is a strange graph
    - Period= 180˚
    - Amplitude=∞
    - Asymptotes of **standard tangent graph** at $x = 90^\circ + k.180°; k \in \mathbb{Z}$

  - There are several alterations you are going to be asked

1. **Period:** $y = \sin ax; y = \cos ax$ or $y = \tan ax$ then the new period is:

$$\textbf{Original period} \div \boldsymbol{a}$$

2. **Amplitude:** $y = b \sin x$ and $y = b \cos x$ then the new amplitude is b, if b is positive. If b is negative then we make b positive and that will be the new amplitude.

3. **Move of graph:**
   a. Left $y = \cos(x + 30^\circ)$ moved graph left by 30°
   b. Right $y = \sin(x - 40^\circ)$ moved graph right by 40°
   c. Up $y = \tan x + 2$ moved graph up by 2 units
   d. Down $y = \sin x - 1$ moved graph down by 1 unit

4. Very important in sketching the graph is finding the Critical points by dividing the period by 4. This gives you the interval between "special" happenings on the graph.

Sequences and Series

Three types of sequences

- Quadratic
- Arithmetic
- Geometric

Quadratic patterns:

Definition: Second differences are equal where the first term form an arithmetic sequence.

General Term: $T_n = an^2 + bn + c$

To calculate the values of a,b and c:

$$\mathbf{2a = \text{Second difference}}$$
$$\mathbf{3a + b = \text{First first difference, i.e. Term1} - \text{Term2}}$$
$$\mathbf{a + b + c = \text{Term1}}$$

Arithmetic patterns:

Definition: All first differences are equal, i.e. you always add or subtract a constant difference

NB: $T_2 - T_1 = T_3 - T_2$

General Term: $T_n = a + (n-1)d$

$d = T_2 - T_1$

Sum of n Terms: $S_n = \frac{n}{2}(2a + (n-1)d)$

OR $S_n = \frac{n}{2}(a + l)$

l =Last term or n-th Term

Geometric patterns:

Definition: There exists a constant ratio, i.e. you multiply by the same ratio.

NB: $\boldsymbol{{}^{T_3}/_{T_2} = {}^{T_2}/_{T_1}}$

General Term: $T_n = a.r^{n-1}$

$r = {}^{T_2}/_{T_1}$

Sum of n Terms:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

OR

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Sum to infinity: $S_\infty = \frac{a}{(1-r)}$

NB: **Terms for convergence:** $-1 < r < 1$

Differentiation

Definition: Instantaneous rate of change

Formula: First Principles/ Ground Principles/ Definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Short formula:

1. $f(x) = ax^n$ then $f'(x) = anx^{n-1}$
2. $y = kx$ then $\frac{dy}{dx} = kx$
3. $D_x[k] = 0$

Your way of writing is very important

There are some restrictions on when you are allowed to differentiate; at university level you are going to learn strategies to handle them straight-forward, but for now you must manipulate.

NOTATION WHEN DIFFERENTIATING

1. $f(x) \to f'(x)$
2. $y \to \frac{dy}{dx}$
3. $D_x[\ldots] = \cdots \{Derivative\ of\ inside\ brackets\}$
4. $\frac{d}{dx}(\ldots) = \cdots \{Derivative\ of\ inside\ brackets\}$

Restrictions:	Solution:
1. Brackets	Multiply out
2. Roots	Exponential laws e.g. $\sqrt[3]{x^2} = x^{\frac{2}{3}}$
3. x in denominator	If x is alone, split up
	If x is not alone, factorize top and cancel

Applications of differentiation

Graphs

The differentiate of f shows the exact form of f

If $f'(x) < 0$ then f is decreasing

If $f'(x) > 0$ then f is increasing

If $f'(x) = 0$ then f is non-moving/stationary

The last point is very important, since it gives us the formula to find any stationary or turning point, namely you differentiate and put equal to 0.

$f'(x) = 0$ **or** $\frac{dy}{dx} = 0$

The second derivative gives you the concavity of the graph.

If $f''(x) < 0$ then f is concave down, for a maximum
If $f''(x) > 0$ then f is concave up, for a minimum
If $f''(x) = 0$ then f is at a changing point

Real life applications of differentiation

Most probably the most important tool for which we use differentiation is to find the minimum and/or maximum for a problem by

$$f'(x) = 0 \text{ or } \frac{dy}{dx} = 0.$$

This applies for all problems that we can model with an equation.

Differentiation is also used to measure rate of change, i.e. the rate of change in distance gives speed and the rate of change in speed gives acceleration.

Determining the equation of a cubic graph:

- Given 3 x-intercepts:

$$y = a(x - x_1)(x - x_2)(x - x_3)$$

- Given 2 x-intercepts where 1 of the x-intercepts is also a turning point:

$$y = a(x - x_1)(x - x_2)^2$$

  where x_2 is the x-value that is both x-intercept **AND** turning point.

- Given turning points:
  1. Get the derivative of the given function;
  2. Put the derivative equal to 0;
  3. Substitute the different x-values of the turning points in to determine 2 different equations;
  4. Solve these equations simultaneously;

If you have any other unknowns in the functions**, substitute** coordinate(s) into the equation and **solve the equation(s)**.

Exponents

The reason for exponents is to shorten the way of writing any number.

Exponents laws

1. $3^2 \times 3^4 = 3^{4+2} = 3^6$
2. $x^6 \div x^3 = x^{6-3} = x^3$
3. $(xy^2)^2 = x^{1\times 2} y^{2\times 2} = x^2 y^4$
4. $\sqrt[3]{x^2 y} = x^{2/3} y^{1/3}$

Basic definitions

1. $x^0 = 1$ except for $x = 0$
2. $a^{-2} = \frac{1}{a^2}$ or $\frac{1}{a^{-1}} = a^1 = a$ a shortcut for this is, if you have $a^{-2}bc^4 = \frac{bc^4}{a^2}$
3. $\left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)$

How to handle exponential sums:

Simplification:

- Only multiply and divide:
    - i. Change the bases into prime numbers
    - ii. Apply law 3 or 4
    - iii. Apply laws 1 and 2
    - iv. Find an answer
- Add or subtract of terms with exponents
    - i. Split the terms with exponents up
    - ii. Factorise
    - iii. Cancel out
    - iv. Find an answer

Equations:

- You handle each of the different types of equations the same as in simplification, but your fourth steps change, instead of finding an answer, you :
    - Get the bases the same;
    - Leave away the bases;
    - Solve the sum.

- When you have an x as base, you simply:
    - Get x and its exponent alone on 1 side;
    - Put both sides to the power of the reciprocal, inverse, of the exponent.

Logarithms

The madness behind logarithms is to solve exponential equations where it is nearly impossible to get the bases the same. For this we must realise that logarithms are the inverse of exponents, ie. if you have an equation $y = 2^x$ and you take the inverse, which means that x and y change positions, we have $x = 2^y$. To get y alone we must use logs:

$$x = 2^y \text{ becomes } y = \log_2 x$$

Logarithmic laws

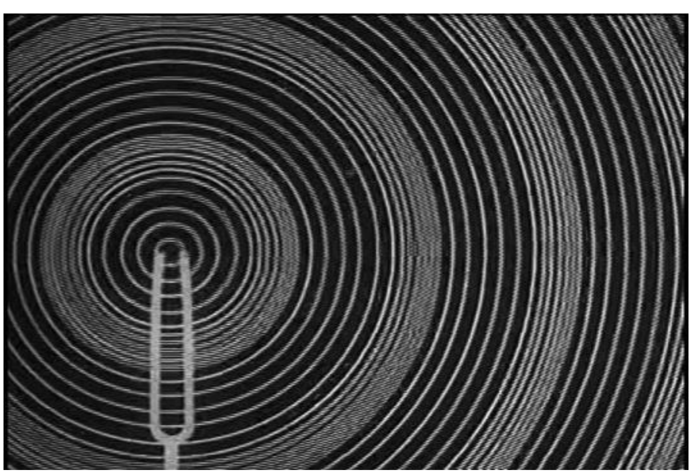
1. $\log_a 2 + \log_a 3 = \log_a (2 \times 3) = \log_a 6$
2. $\log_b 8 - \log_b 2 = \log_b (8 \div 2) = \log_b 4$
3. $\log_b x^3 = 3\log_b x$
4. $\log_b a = \dfrac{\log_x a}{\log_x b}$

Definitions

1. $\log_a a = 1$
2. $\log_b 1 = 0$
3. $\log_b a = \dfrac{1}{\log_a b}$
4. $\log_b \left(\dfrac{1}{a}\right) = -\log_b a$

   $\log_{\frac{1}{b}} a = -\log_b a$

Log laws are simply there to get rid of the logs, so that we can work with all the previous years' methods.



Financial Mathematics

Financial Mathematics is simply about the loan and investment of money in banks or financial institutions. There are 2 types of investments which appears in your syllabus:

- Once-off and
- Periodic transactions

Once-off financial transactions include all transactions, which occur once and then interest is added on the Principal investment or loan amount. The following will be seen as once-off transactions:

- Normal investments
- Loans at a bank(Not home loans)
- Inflation
- Depreciation

Normal investments

Simple Interest $A = P(1 + i.n)$

Compound interest $A = P(1 + i)^n$

Depreciation

Cost price/ Straight line method $A = P(1 - i.n)$

Reducing balance method $A = P(1 - i)^n$

Inflation $A = P(1 + i)^n$

Converting between nominal interest rate and effective interest:

A beloved question to be asked is to switch between nominal and effective interest. The formula is as follows:

$$1 + i_e = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

In the formula: i_e = Effective interest rate

m= How many times a year the interest is compounded

$i^{(m)}$= Nominal interest rate

Periodic financial transactions are also known as annuities. Here there are 2 choices, either we take the value at the end of the period or at the beginning of the period.

- **Future value** $F = \frac{x((1+i)^n - 1)}{i}$
- **Present value** $P = \frac{x(1-(1+i)^{-n})}{i}$

In the exam there will are usually one of two questions asked:

- **Sinking funds or**
- **Home loans**

Sinking funds

A sinking fund is a fund set up to replace an asset (Vehicle or equipment) after a period of time. Sinking funds questions consists of four parts:

1.	Replacing value	How much the will cost to replace after the period (Inflation)
2.	Bookvalue	The value of the car(Depreciation)
3.	Sinking fund value	Replacing value – Bookvalue
4.	Monthly installment	Annuity(Sinking fund = F)

Home loans

Home loans are relatively self explanatory. You want to buy a house and are willing to pay off the debt over a period of time, usually 20 to 30 years. The calculations are relatively simple since we use the Present value annuity to calculate the monthly repayments.

NB!! They are going to ask of you to calculate the outstanding balance after a number of years or payments

To calculate the balance we have a look at how many payments must be made after the time at which the balance is asked, an example could be that if you repay a house over 20 years and we want the balance outstanding after 11 years, then there are 9 years of monthly repayments left and hence the n-value will be $9 \times 12 = 108$. We use this n-value and the present value formula to get the balance:

$$P = \frac{x(1-(1+i)^{-n}}{i}$$

Trigonometry

CAST diagram

Sin $180^\circ - \theta$ $90^\circ + \theta$	**All** $90^\circ - \theta$
Tan $180^\circ + \theta$	**Cos** $360^\circ - \theta$ $\theta - 90^\circ$

Negative angles

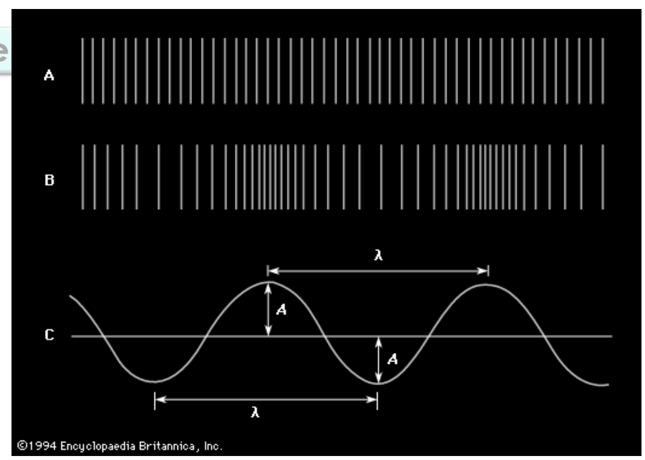
The best strategy you can follow is just adding 360˚ to the angle, since adding a revolution doesn't change the angle.

Angles greater than 360°

You simply subtract 360° until your angle is between 0° and 360°.

Co-functions

$$\cos(90^\circ - x) = \sin x$$
$$\sin(90^\circ - x) = \cos x$$



Identities

Quotient identity

$$\tan x = \frac{\sin x}{\cos x}$$

Square identities

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

Compound angles

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$
$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

Double angles

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 2\cos^2 x - 1 \\ 1 - 2\sin^2 x \end{cases}$$

The big problem is manipulation, the secret is changing your sum so that you can get to a stage where you can solve your question.A very important part of your matric syllabus is being able to factorise trigonometrical expressions. There are 2 methods you can use:

1. Substitute the $\cos x$ and/or $\sin x$ with another variable, like a or b; or
2. Straight factorising

Trigonomentic equations

When it comes to solving trigonometrical equations, the first step would be to get an identity alone or factorise to solve the equation. To be able to factorise you will be expected to manipulate your sum to get it in factorisable form.

After you have the identity alone, you must find the reference angle. You use shift/2nd function to find the reference angle. After finding the reference angle you check in which quadrant you want to work.

Now you must see what the exact question is, if asked for:

General solutions

- sine and cosine, you add $k.360^\circ$
- tangent is special, since you only have to work in 1 quad and then add $k.180^\circ$
- Remember: $k \in \mathbb{Z}$

Specific solutions

- Find the general solutions
- Choose values for k such that your answer falls in the desired interval

Non-right angled triangles

Sine Rule

Formula	**When is the formula used**
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Working with 2 Angles

Cosine Rule

Formula	**When is the formula used**
$a^2 = b^2 + c^2 - 2bc.\cos A$	Working with 3 Sides

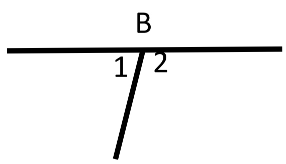
Area Rule

Formula	**When is the formula used**
Area of ΔABC= $\frac{1}{2}ab.\sin C$	When asked

Euclidean Geometry

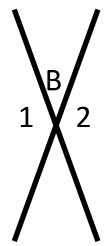
Euclidean Geometry is that special section in Mathematics where we study the different shapes and their properties, specifically the properties relating to the sizes of the angles and the lengths of the sides. Definitions, theorems, axioms and proofs form the basics of Euclidean geometry. In this section we will look at the basic theorems that you need to know to answer the questions asked in the exams.

Straight lines



$\hat{B}_1 + \hat{B}_2 = 180^\circ$

Reason: Angles on a straight line

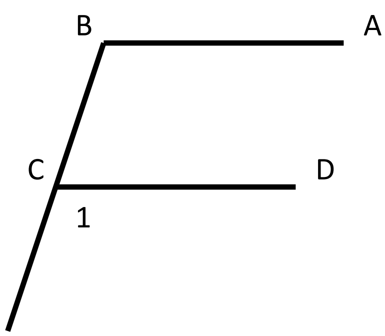


$\hat{B}_1 = \hat{B}_2$

Reason: Vertically opposite angles

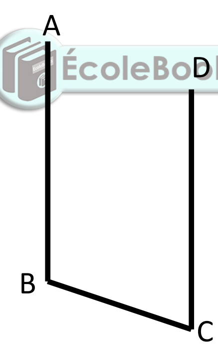
Parallel lines

Assume in the theorems that AB ∥ CD:



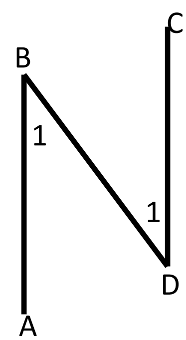
$\hat{C}_1 = \hat{B}$

Reason: Corresponding angles



$\hat{C} + \hat{B} = 180^\circ$

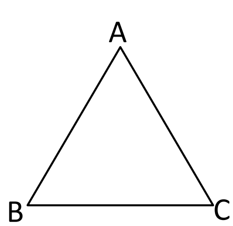
Reason: Co-interior angles



$\hat{D}_1 = \hat{B}_1$

Reason: Alternate angles

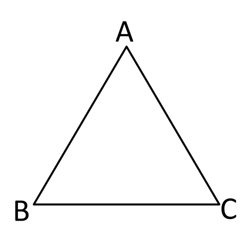
Triangles



$\hat{A} + \hat{B} + \hat{C} = 180^\circ$

Reason: Interior $\angle s$ of Δ

Isosceles Δ

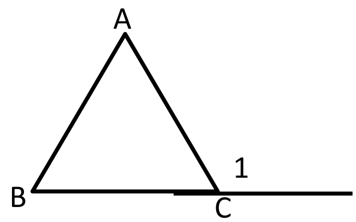


If $\hat{B} = \hat{C}$ then $AB = AC$

Reason: Sides opp = ∠s

If $AB = AC$ then $\hat{B} = \hat{C}$

Reason: $\angle s$ opp =sides



$\hat{B} + \hat{A} = \hat{C}_1$

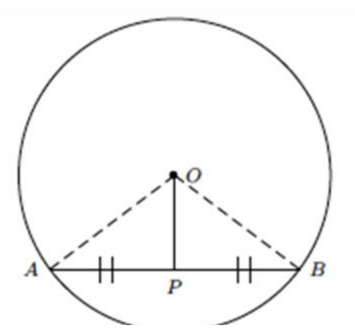
Reason: Exterior ∠ of Δ

Circle theorems

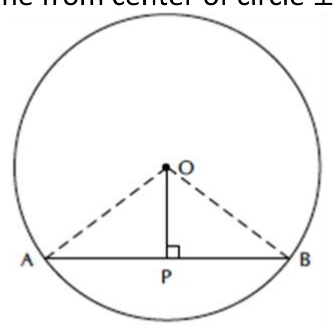
There are 9 theorems in total with quite a few of them also having converses. When answering a question in an exam, it is quite handy to think of the theorems in 3 sections:

Center of circle theorems

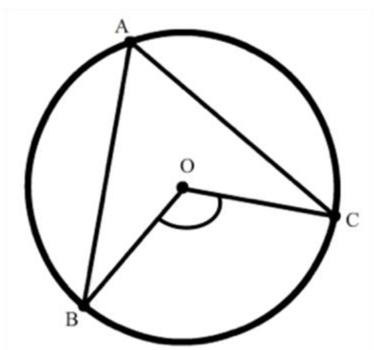
Theorem 1: If AP=PB then $OP \perp AB$.
Reason: Line from center of circle bisects chord



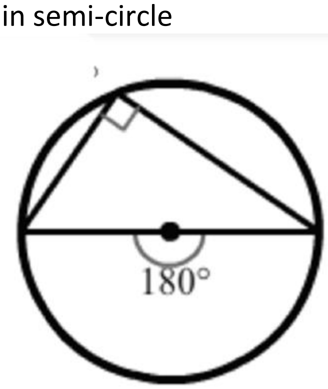
Theorem 1 converse: If $OP \perp AB$ then AP=PB
Reason: Line from center of circle ⊥ to chord



Theorem 2: $B\hat{O}C = 2 \times \hat{A}$.
Reason: ∠ at center= 2 × ∠ at circ

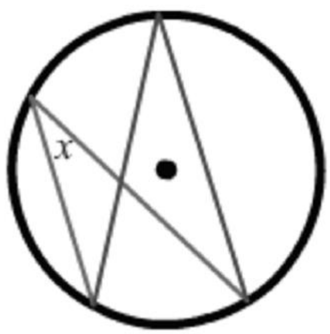


Theorem 3:
Reason: ∠ in semi-circle

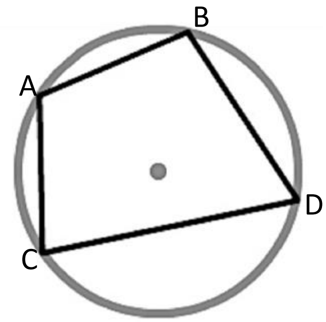


Cyclic Quadrilateral theorems

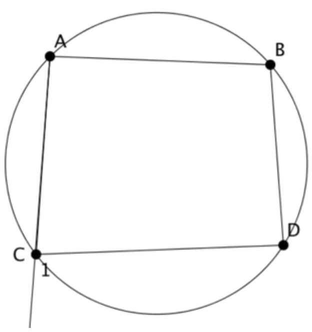
Theorem 4:
Reason: ∠s on same segment



Theorem 5: $\hat{B} + \hat{C} = 180^{\circ}$ and $\hat{A} + \hat{D} = 180^{\circ}$
Reason: Opp $\angle s$ of cyclic quad

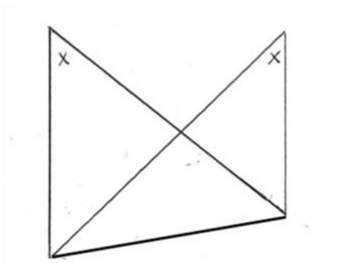


Theorem 6: $\angle C_1 = \angle B$
Reason: Ext ∠ of cyclic quad

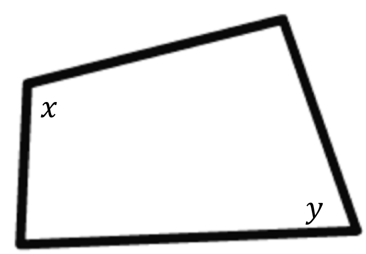


Cyclic Quadrilateral Converse Theorems

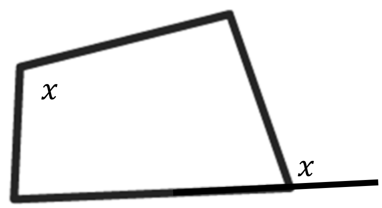
Theorem 4 Converse: If the same chord subtends equal angles in the quadrilateral, then the quadrilateral is cyclic.
Reason: Equal ∠s subtended by chord.



Theorem 5 converse: If $x + y = 180^{\circ}$ then the quadrilateral is cyclic
Reason: Opp $\angle s$ are supplementary

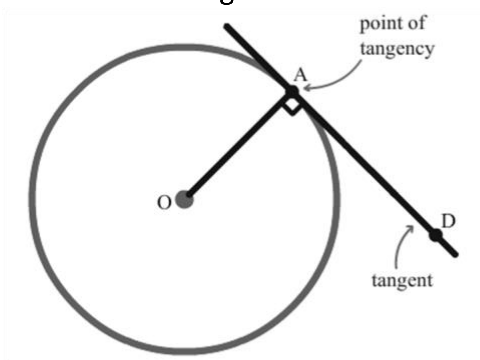


Theorem 6 Converse: If the exterior angle of a quadrilateral equals the interior opposite angle, then the quadrilateral is cyclic.
Reason: Ext ∠=Interior opp ∠

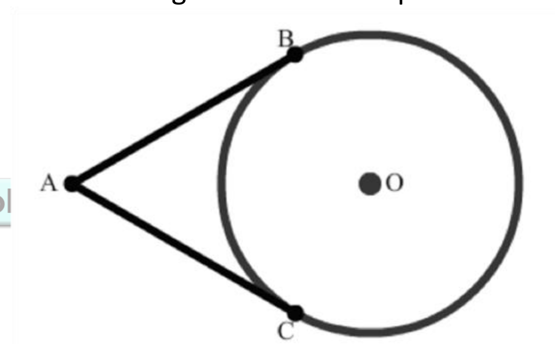


Tangent to circle theorems

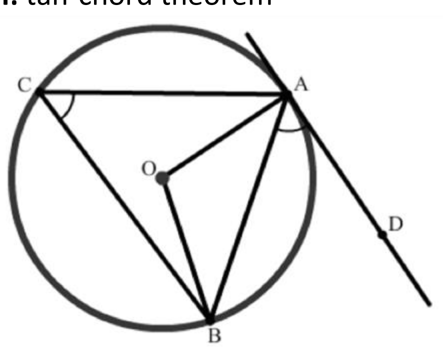
Theorem 7:
Reason: Radius ⊥ tangent



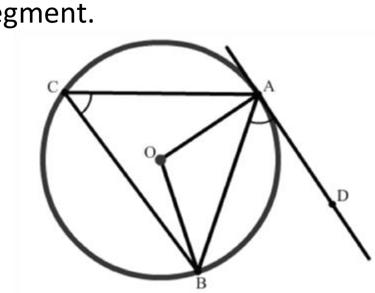
Theorem 8: $AB = AC$
Reason: Tangents from same point



Theorem 9: $D\hat{A}B = \hat{C}$
Reason: tan-chord theorem



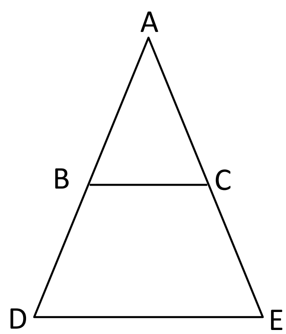
Theorem 9 Converse: If $D\hat{A}B = \hat{C}$, then AD is a tangent
Reason: ∠ between line and chord=∠ in opp circle segment.



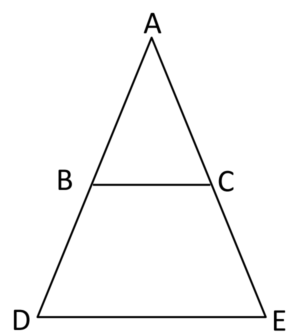
Advanced triangle Geometry

This section of work consists of working with the theorems that deal with triangles, specifically proportions, similarity and the theorem of Pythagoras.

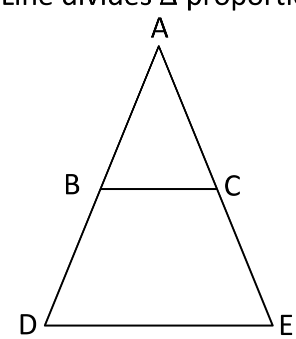
Theorem 1: Proportionality in triangles
If a line is drawn parallel to a side of a triangle, then the other sides are divided proportionally, i.e. if BC∥DE, $\frac{AB}{BD} = \frac{AC}{CE}$.
Reason: Line parallel to one side of Δ



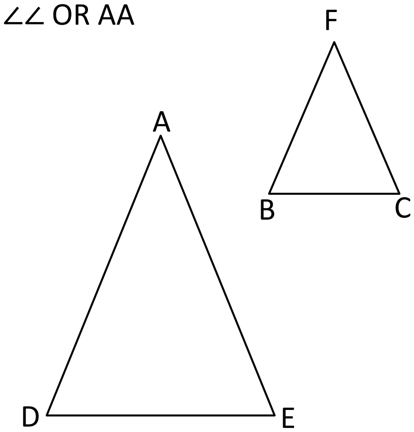
Theorem 1 Converse: Parallel lines in triangles
If a line drawn in a triangle divides the sides proportionally, then that line is parallel to the other side of the triangle, i.e. if $\frac{AB}{BD} = \frac{AC}{CE}$, BC∥DE.
Reason: Line divides Δ proportionally



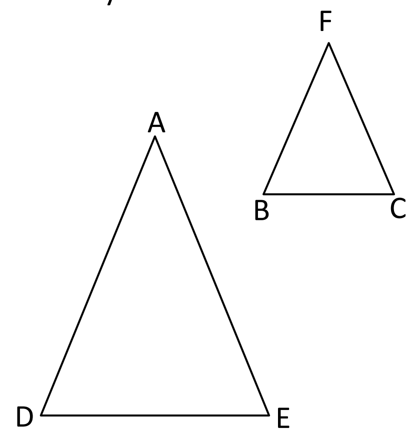
Theorem 1 Special case: Midpoint theorem
If a line drawn in a triangle divides the sides in half, then that line is parallel to the other side of the triangle and half of the third side, i.e. if $AB = BD$ and $AC = CE$, BC∥DE and $BC = \frac{1}{2}DE$.
Reason: Line divides Δ proportionally



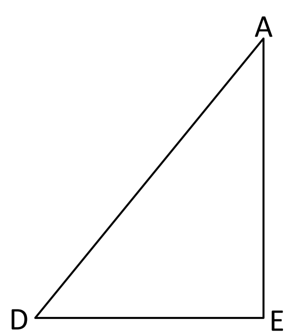
Theorem 2: Equiangular triangles are similar
If two triangles have pairs corresponding angles that are equal, then the triangles are similar, i.e. if $\hat{A} = \hat{F}$ and $\widehat{D} = \hat{B}$, then $\Delta ADE /// \Delta FBC$.
Reason: ∠∠ OR AA



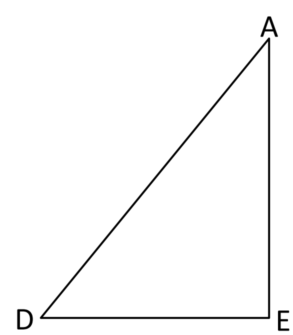
Theorem 3: Proportionality of similar triangles
If two triangles are similar, then their sides are in proportion, i.e. if $\Delta ADE /// \Delta FBC$, $\frac{AD}{FB} = \frac{AE}{FC} = \frac{DE}{BC}$.
Reason: Similarity



Theorem 4: Pythagorean theorem
In a right-angled triangle the sum of the square of the hypotenuse is equal to the sum of the square of the other two sides, i.e. if $\hat{E} = 90^\circ$, then $AD^2 = AE^2 + DE^2$.
Reason: Pythagoras



Theorem 4 Converse: Pythagorean theorem
If the sum of the square of the hypotenuse is equal to the sum of the square of the other two sides, then the triangle is right-angled, i.e. if $AD^2 = AE^2 + DE^2$, then $\hat{E} = 90^\circ$.
Reason: Pythagoras



Analytical Geometry

The basic formulae for this section of work are:

Distance:

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint:

Formula: $M = \left(\frac{x_2+x_1}{2}; \frac{y_2+y_1}{2}\right)$

OR

$x_M = \frac{x_2+x_1}{2}$ and $y_M = \frac{y_2+y_1}{2}$

Gradient:

To find gradient there are 4 different methods:

1. Two coordinates: $m = \frac{y_2-y_1}{x_2-x_1}$
2. Parallel lines: $m_1 = m_2$
3. Perpendicular line: $m_1 \times m_2 = -1$
4. Inclination angle: $m = \tan\theta$

Equation of straight line

Formula: $y - y_1 = m(x - x_1)$where $(x_1; y_1)$ is a coordinate on the straight line

Inclination angle

Formula: $m = \tan\theta$ if m is negative then $\theta = 180^\circ - \text{reference angle}$

Very important, is to find ways to calculate θ by use of Grade 8-10 geometry theorems

Special straight lines

There are special lines which you must be able to find the equation for:

1. **Altitudes** – a line from a point of a triangle, perpendicular to the opposite side
   a. Here we use the formula for perpendicular lines to calculate the second gradient
   b. We then use the formula and the coordinate on the line to find the equation of the straight line

2. **Median** – a line from a point of triangle, which bisects the opposite side
   a. Calculate the midpoint of the bisecting side
   b. Find the gradient between the midpoint and the opposite coordinate
   c. Use the formula and the coordinate on the line to find the equation of the straight line

3. **Perpendicular bisector** – ANY line which bisects a side and is perpendicular to that side
   a. Calculate the midpoint of the bisecting side
   b. Calculate the gradient of the side which is bisected
   c. Now use the formula for perpendicular lines to calculate the second gradient

Use the formula and the coordinate on the line to find the equation of the straight line

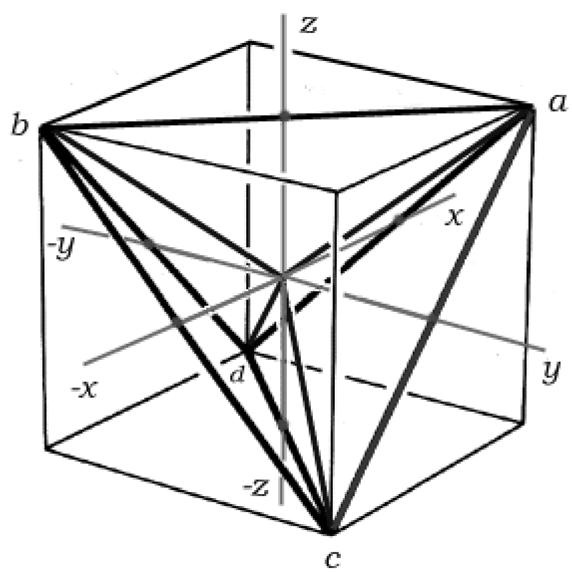
Circles:

Formula: $(x-a)^2+(y-b)^2=r^2$ with the centre at $(a;b)$

Tangents to a circle

A theorem to remember is that a tangent is perpendicular to the radius where is touches the circle

a. Calculate the gradient of the radius
b. Use the formula for perpendicular lines to calculate the tangent gradient
c. Find the equation of the tangent



Statistics and linear regression

Definition of Statistics: The art of making sense of numbers.

Basic measurements used:

Individual stats

Individual statistics is where each number is given.

Average/Mean

The average number

Formula: $\bar{x} = \frac{\sum x}{n}$ or with frequencies(f) $\bar{x} = \frac{\sum fx}{n}$

Median

The number in the middle

Formula: $\frac{n+1}{2}$-th number

Mode:

The number that occurs most

Range:

The width of the population

Formula: Maximum – minimum

Lower Quartile(Q1)

The number on a quarter of the population

Formula: $\frac{n+1}{4}$-th number

Upper Quartile(Q3)

The number on 3 quarters of the population

Formula: $\frac{3(n+1)}{4}$-th number

Inter-Quartile Range(IQR)

The width between the upper and lower quartile

Formula: $Q_3 - Q_1$

Variance:

The square of the average number by which the numbers differ from average

Formula: $\sigma^2 = \frac{\sum(x-\bar{x})^2}{n}$ or if you work with frequencies: $\sigma^2 = \frac{\sum f(x-\bar{x})^2}{n}$

Standard deviation:

The actual number by which a number varies from the average

Formula: $\sigma = \sqrt{variance} = \sqrt{\sigma^2}$

The best way to find variance and/or standard deviation is by filling in the following table:

Number(x)	**Frequency**(f)	$x - \bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
(1)	(2)	$(1) - \bar{x} = (3)$	$(3)^2 = (4)$	$(4) \times (2)$

Interval Stats:

When all the numbers are given in intervals instead of individually

All the above stays the same except the class midpoint is now used as x, instead of the actual number.

Class Midpoint

Formula: $\text{midpoint} = \frac{\text{Start value}+\text{End value}}{2}$

Linear Regression

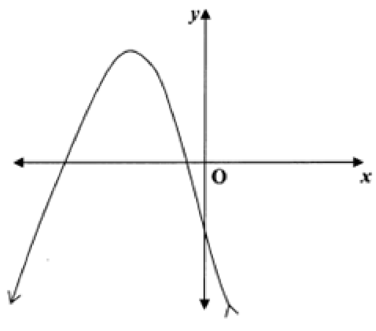
The main object of regression is to see the relationship that exists between two variables. We start off by plotting the graph on a scatter plot diagram. If the relationship is linear then we need to plot a straight line through the coordinates, i.e. draw a line of best fit.

Line of best fit

Equation: $\hat{y} = a + bx$ with $b = \frac{\sum_{i=1}^{n}\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$ and $a = \frac{1}{n}\sum_{i=1}^{n} y_i - b \times \frac{1}{n}\sum_{i=1}^{n} x_i$

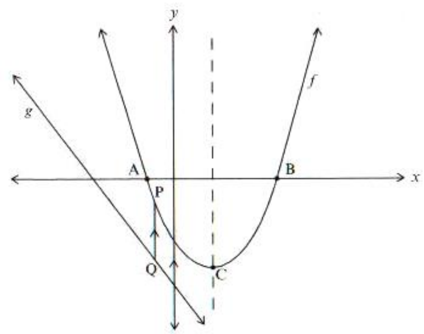
Functions, relations and inverses - Questions

1. Sketch the following graphs:
   a. $y = 3x^2 - 2x - 5$
   b. $y = 2x^2 - 2$
   c. $y = -3x^2 - 2x$
   d. $y = \dfrac{2}{x-1}$
   e. $y = 3.2^{x-1} + 1$
2. In the following sketch you've been given the graph of $f(x) = -x^2 - 6x - 4$



   a. Change the graph into the form $y = a(x-p)^2 + q$
   b. Prove that $f(x) \leq 5$ for all values of x

3. The sketch gives the graphs of f and g where $f(x) = x^2 - 2x - 3$ and $g(x) = -4x - 6$ and $PQ \| y - axis$.

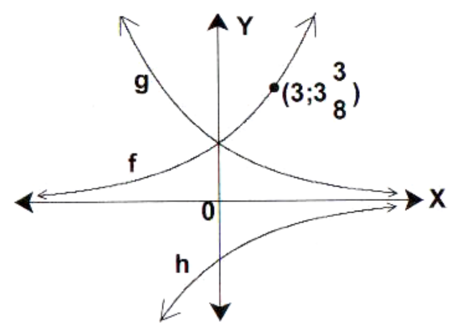


Determine:
   a. The length of AB
   b. The coordinate of C
   c. For which value(s) of x is f(x)>0?
   d. The equation of g^{-1}
   e. Sketch f^{-1}
   f. The equation of f^{-1}
   g. The minimum length of PQ

4. Find the inverses of the following graphs:
   a. $f(x) = 3x - 1$
   b. $f(x) = \dfrac{x}{2x-1}$

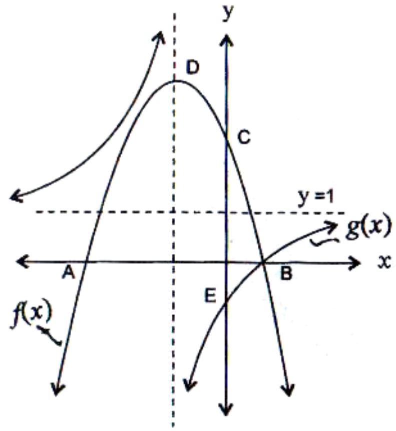
5. Given $f(x) = 4.2^{x+1} - 2$. Determine the following:
   a. X-intercept
   b. Y-intercept
   c. Equation of the asymptote
   d. f(-1)
   e. Sketch the graph of f
   f. Give the equation of g(x) if g is f moved 2 units right.

6. The following sketch represents the graph of f with $f(x) = a^x$; g is the reflection of f in the y-axis and h, the reflection of g in the x-axis.

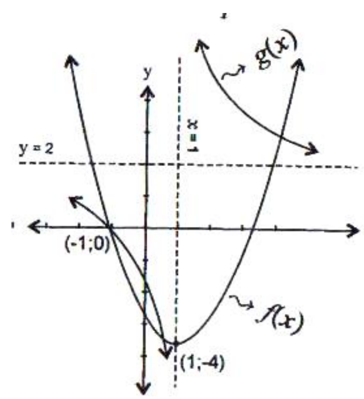


   a. Calculate the value of a
   b. Write down the equation of g and h
   c. Find the equation of $f^{-1}(x)$ in the form y=...
   d. Sketch the graph of $f^{-1}(x)$
   e. What is the domain of $f^{-1}(x)$

7. Given the graphs of f(x) and g(x) with $f(x) = -x^2 - 2x + 3$ and $g(x) = \frac{-2}{x-p} + q$
   a. Find the value of p and q
   b. Hence, find the coordinate of E



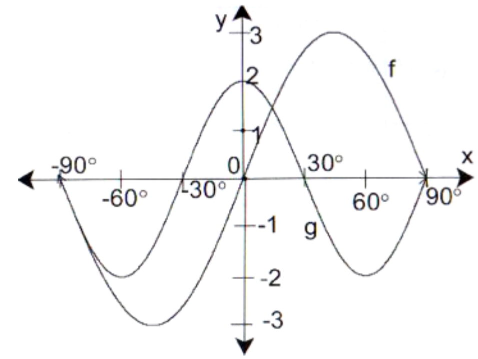
8. Determine the equation of $f(x)$ and $g(x)$



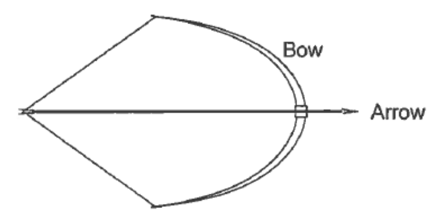
9. Given that $h(x) = \cos(x + 30^\circ)$ and $g(x) = -2\sin x$
   a. Determine without the use of a calculator the general solution of $h(x) = g(x)$
   b. Sketch the graphs of h and g for all $x \in [-120^\circ; 180^\circ]$

10. Given that $f(x) = \tan x$ and $g(x) = \sin 2x$
   a. Sketch f and g for $x \in [-180^\circ; 90^\circ]$
   b. For which value(s) of x is both f and g increasing for $x \in [-90^\circ; 90^\circ]$

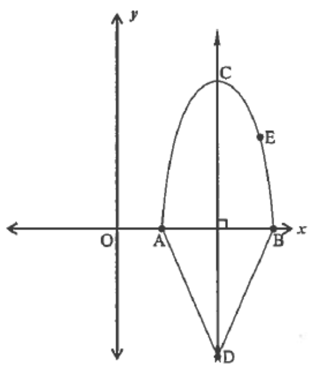
11. The graphs represented are:
   $f(x) = a\sin bx$
   and
   $g(x) = d\cos cx$
   a. Write down the values of a, b, c and d
   b. Write down 2 values of x where $\sin 2x - \frac{2}{3}\cos 3x = 0$
   c. What is the period of g?
   d. For which negative values of x will g(x) decrease in value if x increases?



12.
(b) The diagram below shows a picture of a bow and arrow.



This picture is represented on the set of axes below.
Points A(3 ; 0), B(7 ; 0) and E(6 ; 6) are given. CD is perpendicular to AB.



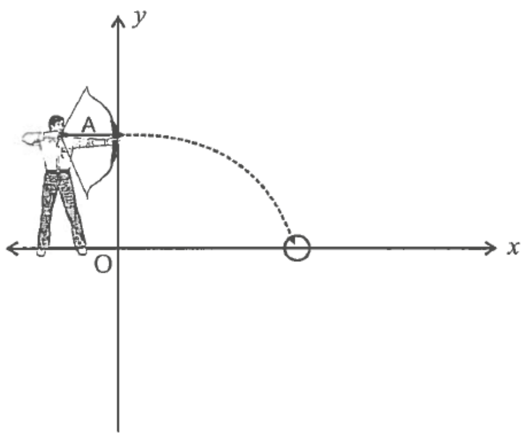
(1) Determine the equation of the parabola in the form $y = ax^2 + bx + c$. (4)

(2) Determine the equation of the line AD if the gradient of AD $= -2$. (3)

(3) Hence determine the length of CD. (4)

(c) The arrow will follow a parabolic path, with maximum at the point of release.

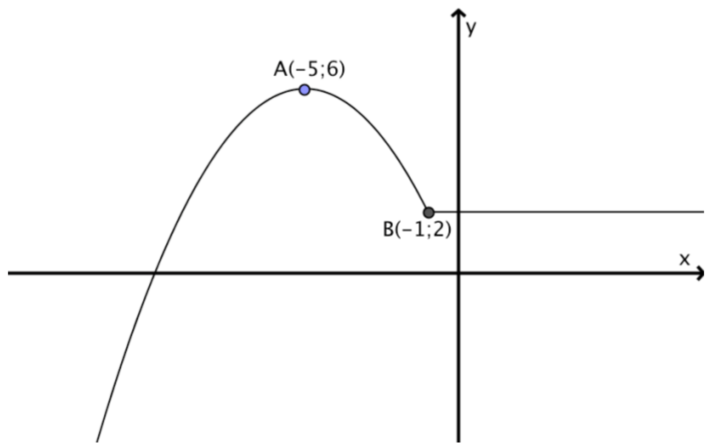
It is given that the equation of its path is $f(x) = -\frac{1}{50}(x^2 - 100)$; $0 \le x \le 10$.



Write down the equation of the inverse function $f^{-1}(x)$, and state its domain. (3)

[17]

13. The sketch below shows k(x), which is formed using part of a parabola and a horizontal line.



(a) Is $k^{-1}(x)$ a function? Explain.
(b) Give the domain of k^{-1}.

Sequences and series - Questions

1. 5; x; y is an arithmetic sequence and x; y; 81 is a geometric sequence. All the terms in the sequence are integers.
   Find the values of x and y

2. The sum of a geometric series is 100 times the value of its first term, while the last term is 9 times the first term.
   Calculate the number of terms in the series, if the first term is non-zero

3. Calculate the value of n if:

$$\sum_{k=1}^{n} 3^k = 1092$$

4. The first term of a geometric sequence is 3 and the sum of the first 4 terms is 5 times the sum of the first 2 terms. The common difference is greater than 1

   Calculate:
   a. The first 3 terms of the sequence
   b. The value of n for which the sum of n terms are 765

5. Sum of the first 50 terms of an arithmetic series is 1275. Calculate the sum of the 25th and the 26th term of this series.

6. The sum of the first n-terms of an arithmetic series is:

$$S_n = \frac{3n^2 - n}{2}$$

   a. Calculate S_{10}
   b. Calculate the value of $\sum_{r=5}^{10} T_r$, where T_r is the r-th term of the series

7. The sum of the first and second term of an unending geometric series is 11. The sum to infinity is 36 and the common ratio is r. Determine the possible values of r.

8. The first two terms of a geometric series is: $x + 3$ and $x^2 - 9$
   a. Determine the values of x for which the series will converge
   b. Determine the value of x if the sum to infinity is 13

9. A ball falls from a height of 10 meters; it bounces 6 meters and then continues to fall $3/5$ of its previous height.

   Determine after how many bounces the ball's height will be less than 1 cm.

10. A fitness test requires that athletes repeatedly run a distance of 20m. They finish the distance 5 times in the first min, 6 times in the second min and 7 times in the third min. They carry on in this manner. Determine after how minutes have the athletes ran 2200m.

11. Write down the next two terms and the general term (Tn) in the following sequences:
    a. 3; 12; 35; 52; …
    b. 100; 80; 58; 34; …

12. Given: 2;3;2;5;2;7;…. Answer the following questions:
    a. Write down the next three terms
    b. Determine the 43rd term
    c. Calculate the sum of the first 40 terms

Differentiation - Questions

1. Determine $f'(x)$ from first principles if
   a. $f(x) = x^2 - 6x$
   b. $f(x) = \frac{1}{2x}$

2. Determine $\frac{dy}{dx}$:
   a. $y = (2x)^2 - \frac{1}{3x}$
   b. $y = \frac{2\sqrt{x} - 5}{\sqrt{x}}$
   c. $xy - 5 = \sqrt{x^3}$
   d. $y = x^{\pi} - 4\sqrt{2x} + t$

3. Given: $g(x) = \frac{-2x}{\sqrt{x}} - x^{10}$ and

   $h(x) = (x^5 + 5x^{-1})(x^5 - 5x^{-1})$

   Determine:
   a. $g'(x)$
   b. $h'(x)$
   c. $\frac{d}{dx}[2g(x) + h(x)]$

4. Given: $f(x) = -x^3 + 3x^2 - 4$
   a. Calculate the x- and y-intercepts of f
   b. Determine the turning points of f
   c. Draw the sketch
   d. For which values of x will f be increasing?
   e. What is the maximum value of $-x^3 + 3x^2 - 4$ if $0 \le x \ge 3$
   f. How many solutions do $f(x) = -5$ have?

5. A clothing producer estimates that the cost (in rand) to produce x shirts are given by the function

   $C(x) = 10 + 5x + 0.001x^3$

   Determine the rate at which the cost when the 100th shirt is produced is changing.

6. There are 40 fruit trees in an orchard. The average earnings per tree in a season are 580 fruit. The farmer calculates that for every extra tree you plant in the orchard, the earnings per tree will decrease by 10 fruit. If the number of extra trees planted in the orchard is x the total earnings in a season is N, then:

   $N = (40 + x)(580 - 10x)$

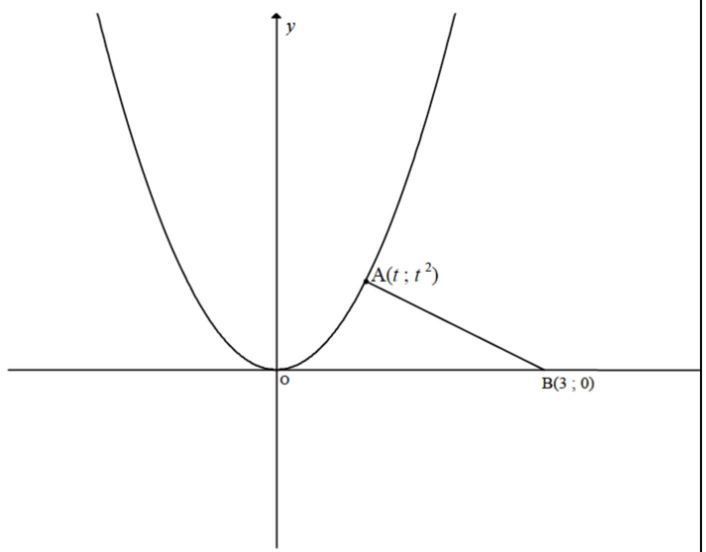
   Calculate how many extra trees must be planted such that the earnings will be at a maximum

7. If given: $f(x) = x^3 - 12x + 11$. Calculate the following:
   a. Intercepts with the axes
   b. Stationary points
   c. Point of inflection
   d. Sketch the graph

8. Repeat question 7 with the following graphs:
   a. $y = x^3 - 3x + 2$
   b. $f(x) = -x^2 + x^3 - x$
   c. $g(x) = -2x^3 + 6x - 4$
   d. $h(x) = x^3 + x^2 - 5x + 3$

9.

Sketched is the graph of $y = x^2$. A$(t\,;\,t^2)$ and B(3 ; 0) are shown.



1 A$(t\,;\,t^2)$ is a point on the curve $y = x^2$ and the point B(3 ; 0) lies on the x-
Show that $\text{AB}^2 = t^4 + t^2 - 6t + 9$.

2 Hence, determine the value of t which minimises the distance AB.

10.

(a) The strength of the reaction of a person's body to x units of a drug is given by:

$$R(x) = x^2\left(\frac{C}{2} - \frac{x}{3}\right)$$

where C is the maximum number of units that can be given.

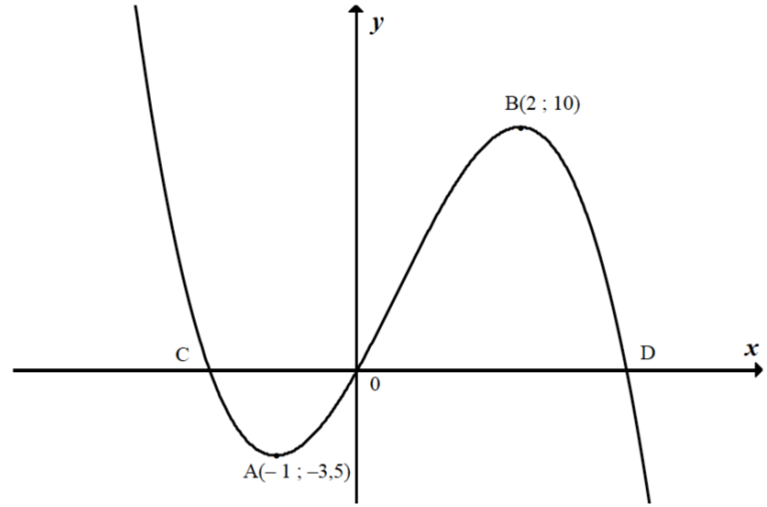
(1) Show that $R(C) = 2R\left(\frac{C}{2}\right)$. (4)

(2) $R'(x)$ is used as a measure of the sensitivity of the body to the drug. Determine x such that the sensitivity is maximised. (5)

(b) Show that the curve $y = 4x^2 + \frac{1}{x}$ has only one turning point and determine whether it is a maximum or minimum. (6)

11.

The graph of $h(x) = -x^3 + ax^2 + bx$ is shown below. A(– 1 ; 3,5) and B(2 ; 10) are the turning points of h. The graph passes through the origin and further cuts the x-axis at C and D.



1 Show that $a = \frac{3}{2}$ and $b = 6$.

2 Calculate the average gradient between A and B.

3 Determine the equation of the tangent to h at $x = -2$.

4 Determine the x-value of the point of inflection of h.

5 Use the graph to determine the values of p for which the equation $-x^3 + \frac{3}{2}x^2 + 6x + p = 0$ will have ONE real root.

Exponents and logarithms - Questions

1. Simply the following:
    a. $\left(\frac{5}{4^{-1}-9^{-1}}\right)^{\frac{1}{2}} + \log_3 9^{2,12}$
    b. $\frac{\sqrt{27}+\sqrt{12}}{\sqrt{75}}$
    c. $\left[\frac{81x^3}{16x^{-1}}\right]^{\frac{3}{4}}$
    d. $3\log\sqrt[3]{40} - 2\log\frac{1}{5}$
    e. $\log_m \sqrt{3}.\log_9 m$
    f. $\frac{36^{x-1}.49^{x}.8^{x}}{81^{\frac{1}{2}x}.16^{x-1}.98^{x-1}}$
    g. $\frac{10.2^{p-1}-24.2^{p-3}}{2^{p+1}.3-2^{p}}$

2. If $\log_2 5 = a$, express $\log_8 \sqrt{10}$ in terms of a

3. Given: $5^n = x$ and $n = \log_2 y$
    a. Write y in terms of n
    b. Express $\log_8 4y$ in terms of n
    c. Determine 50^{n+1} in terms of x and y

4. Solve for x in each of the following equations:
    a. $\log_{\frac{1}{2}} x + \log_{\frac{1}{2}}(x+1) \geq -1$
    b. $3^{x-1} + 3^{x+1} = \sqrt{300}$
    c. $2^{2x+1} - 3.2^{x} + 1 = 0$
    d. $2^{x}.3^{x+1} = 10$
    e. $\sqrt{3x^{\frac{3}{4}}} - \sqrt{24} = 0$
    f. $5^{x+1} + 4 = 5^{-x}$

Financial Mathematics - Questions

1. You invest R 5000 and it doubles in a certain amount of years @ 12% p.a. compounded monthly. Answer the following questions:
    a. How much is the effective interest rate?
    b. For how many years was the investment?

2. At which monthly compounded interest rate should I invest to double my principal amount in 4 years time?

3. You invest R 60 000 for 5 years @ an effective interest rate of 10,8% pa. Calculate the following:
    a. The monthly compounded interest rate
    b. Amount necessary to accumulate to the same value in 5 years, but at an interest rate of 9% pa compounded quarterly?

4. Which is a better investment option, 9% pa compounded monthly or 9,3% pa compounded quarterly?

5. To buy a camera in 5 years time, you will need R 20 000. What will your monthly installments be if your interest rate that FNB can offer you is 7,2% pa compounded monthly?

6. A basic car is valued at R 80 000 at present. In 3 years time the same car will cost you R95 000. Market norm for depreciation is 15% pa on reducing balance method. Answer the following questions:

   a. What will the book value be after 3 years?
   b. Calculate the inflation rate
   c. You want to replace the car in 3 years time, so you set up a sinking fund. The bank offers you 6,6% pa compounded monthly. How much will your monthly installments be?

7. A Chev Aveo is available for you. In 4 years time the car will be valued at R 125 000. Market norm for depreciation is 10% pa on reducing balance method and inflation is at 5% pa. Answer the following questions:

   a. Calculate the cost price?
   b. Calculate the replacing value
   c. You want to replace the car in 4 years time, so you set up a sinking fund. The bank offers you 8,4% pa compounded monthly. How much will your monthly installments be?

8. How many years will you have to save if you invest R 550 pm in an annuity which yields 10% pa compounded monthly if you want to receive R 250 000?

9. You want to buy a house of R 890 000 and the current rate of interest is 11% pa compounded monthly and you want to repay the loan in 20 years. Answer the following questions:

   a. Calculate the monthly installments
   b. Balance after 11 years
   c. How long will it take you to repay the loan if you pay an additional R700 pm?

10. In this stage of your life you can only pay an installment of R 7 000 pm on a new house. The bank offers you 13% pa compounded monthly. Will you be able to buy a house of R 620 000 if you plan on repaying the loan over a 20 year period?

11. A Flat is on the market for R 4500 pm and the current rate of interest is 10,4% pa compounded monthly and the repayment period is over 20 years. Answer the following questions:

   a. What is the cost of the flat?
   b. Balance after 9 years

Trigonometry - Questions

1. Simplify each of the following expressions:

   a. $\dfrac{\tan(-420^\circ).\cos 156^\circ.\cos 294^\circ}{\sin 492^\circ}$

   b. $\dfrac{\cos(\theta - 720^\circ)}{\sin^2(180^\circ + \alpha).\cos(\theta - 90^\circ)}$ if $\alpha + \theta = 90^\circ$

   c. $\cos(\theta - 90^\circ).\sin(\theta - 180^\circ) + \dfrac{\cos(720^\circ + \theta)}{\sin(90^\circ - \theta)}$

   d. $\sqrt{\dfrac{\tan(-207^\circ)}{\tan 333^\circ} - \dfrac{\sin^2(x - 360^\circ)}{\cos x.\sin(x - 90^\circ)}}$

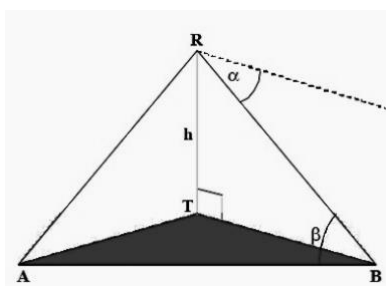
e. $\dfrac{\sin(\theta+45^\circ)}{\cos(585^\circ).\sin(\theta-90^\circ)}$

2. Given that $\cos 61^\circ = p$, express the following in terms of p:
   a. $\sin 209^\circ$
   b. $\dfrac{1}{\sin(-421^\circ)}$
   c. $\cos 1^\circ$

3. Prove the following identities and state where the identity is undefined:
   a. $\dfrac{\cos 2x+1}{\sin 2x.\tan x} = \dfrac{1}{\tan^2 x}$
   b. $\dfrac{1+\cos 2A}{\cos 2A} = \dfrac{\tan 2A}{\tan A}$
   c. $\dfrac{\sin x+\sin 2x}{1+\cos x+\cos 2x} = \tan x$

4. Solve the following equations, find the general solutions:
   a. $2\sin x + \dfrac{1}{\sin x} - 3 = 0$
   b. $2\sin 2x - 2\sin x + 3\cos x = 6\cos^2 x$
   c. $\cos 54^\circ.\cos x + \sin 54^\circ \sin x = \sin 2x$

5. In the diagram RT is the height of a vertical tower, with T the foot of the tower. A and B are 2 points, with equidistance from T and they lie in the same horizontal space as T. The height of the triangle is h.



The angle of depression to B from R is α . $R\hat{B}A = \beta$
   a. Give the magnitude of $A\hat{R}B$ in terms of β
   b. Show that $AB = \dfrac{2h\cos\beta}{\sin\alpha}$, and calculate h if AB=5,4 units, $\alpha = 51^\circ$ and $\beta = 65^\circ$

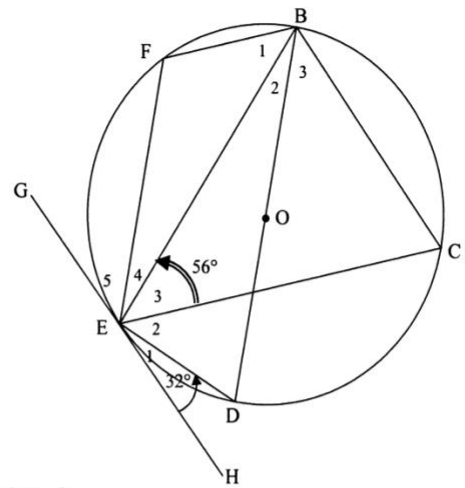
Euclidean Geometry – Questions

All the Euclidean Geometry questions come from previous year's Grade 12 Examination papers set up by the Department of Education.

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$.



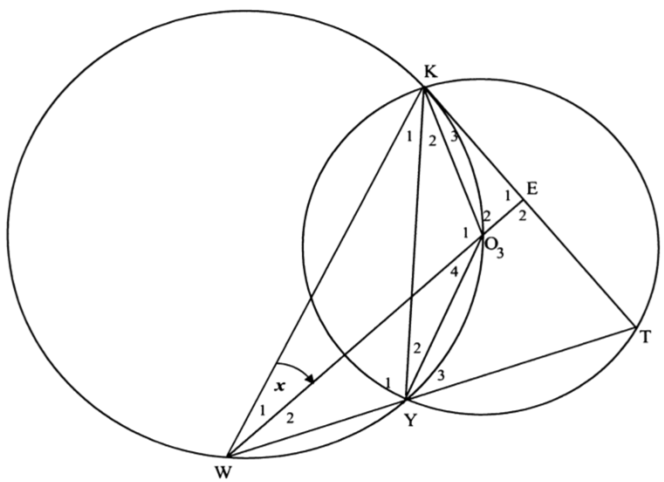
Calculate, with reasons, the values of:

10.1	$\hat{E}_2$	(2)
10.2	$E\hat{B}C$	(3)
10.3	$\hat{F}$	(4)
		[9]

QUESTION 12

In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W.
WO produced meets KT at E.

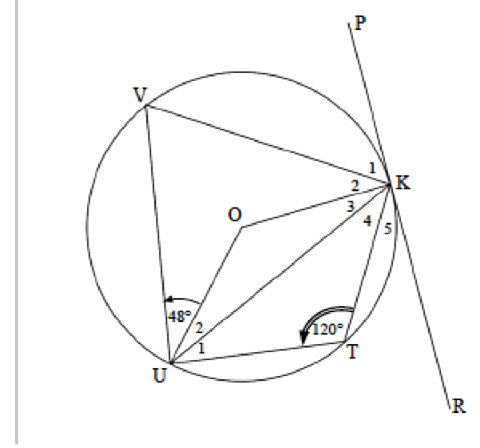
Let $\hat{W}_1 = x$



12.1	Determine FOUR other angles, each equal to x.	(8)
12.2	Prove that $\hat{T} = 90° - x$.	(3)
12.3	Prove that KE = ET.	(3)

QUESTION 8

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K.
$O\hat{U}V = 48°$ and $K\hat{T}U = 120°$.

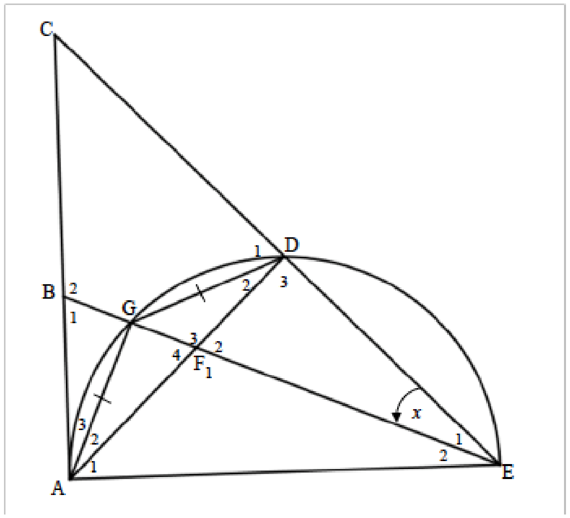


Calculate, with reasons, the sizes of the following angles:

8.1	$\hat{V}$	(2)
8.2	$K\hat{O}U$	(2)
8.3	$\hat{U}_2$	(2)
8.4	$\hat{K}_1$	(2)
8.5	$\hat{K}_2$	(2)
		[10]

QUESTION 10

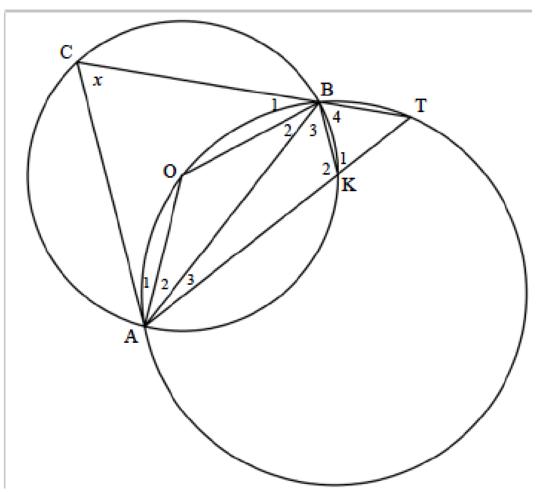
In the figure AGDE is a semicircle. AC is the tangent to the semicircle at A and EG produced intersects AC at B. AD intersects BE in F.
AG = GD. $\hat{E}_1 = x$.



10.1 Write down, with reasons, FOUR other angles each equal to x. (8)

QUESTION 9

O is the centre of the circle CAKB.
AK produced intersects circle AOBT at T.
$A\hat{C}B = x$



9.1 Prove that $\hat{T} = 180° - 2x$. (3)

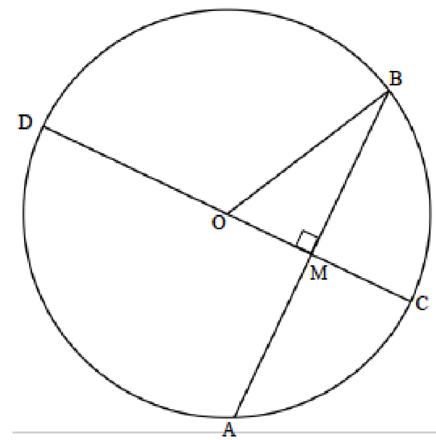
9.2 Prove AC || KB. (5)

9.3 Prove ΔBKT ||| ΔCAT (3)

9.4 If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$ (3)
[14]

QUESTION 10

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC.
CM : MD = 4 : 9 and AB = 24 units.



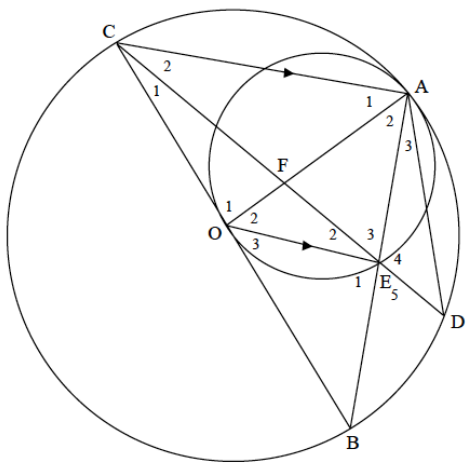
10.1 Determine an expression for DC in terms of x if CM = $4x$ units. (1)

10.2 Determine an expression for OM in terms of x. (2)

10.3 Hence, or otherwise, calculate the length of the radius. (4)
[7]

QUESTION 10

Two circles touch each other at point A. The smaller circle passes through O, the centre of the larger circle. Point E is on the circumference of the smaller circle. A, D, B and C are points on the circumference of the larger circle.
OE || CA.



10.1 Prove, with reasons, that AE = BE. (2)

10.2 Prove that ΔAED ||| ΔCEB. (3)

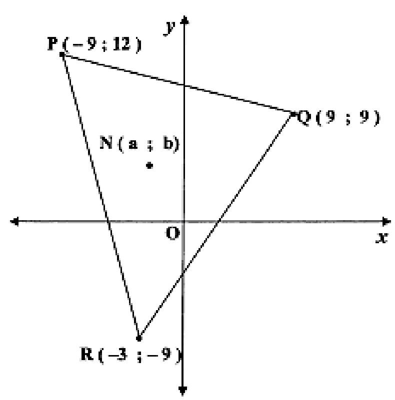
10.3 Hence, or otherwise, show that $AE^2 = DE.CE$. (2)

10.4 If AE.EB = EF.EC, show that E is the midpoint of DF. (3)
[10]

Analytical Geometry - Questions

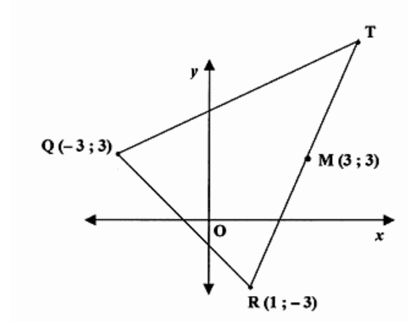
1. In the diagram we have P (-9 ; 12), Q(9 ; 9) and R(-3; -9) the corner of ΔPQR. N(a ; b) is a coordinate in the second quadrant.

   Determine the following:
   a. Gradient of PQ



   b. Magnitude of $\hat{Q}$
   c. The coordinate of M, the midpoint of QR
   d. The equation of the median PM
   e. The coordinate of N if P, N and M are co-linear and QN=$5\sqrt{5}$ units

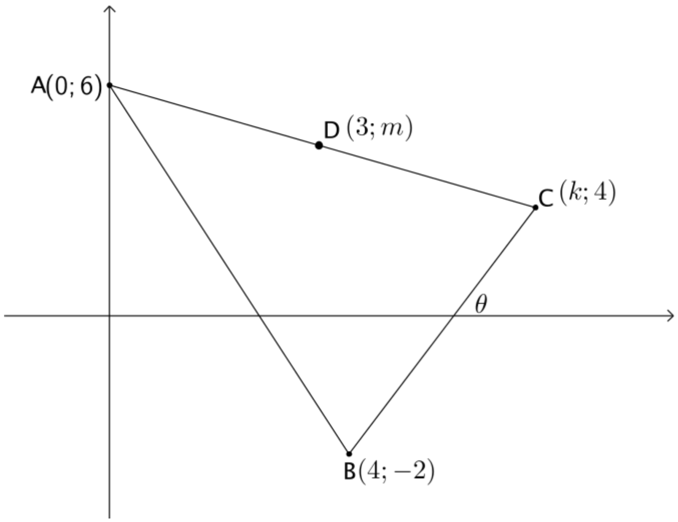
2. The diagram shows ΔTQR, where Q(-3 ; 3) and R(1 ; -3). M(3 ; 3) is the midpoint of RT.



   a. Calculate:
      i. Length of TR
      ii. Size of $\hat{R}$
      iii. Size of
   b. Find the following:
      i. Equation of the median from T to RQ
      ii. Hence, or otherwise, determine the intersection point of the medians of the ΔTQR
   c. Find the equation of the perpendicular bisector of RQ

Question 3

In the sketch below you are given the coordinates of A $(0;6)$, B$(4;-2)$ and C $(3k;k)$. Furthermore it is given that AB⊥BC.



a) Determine the gradient of AB

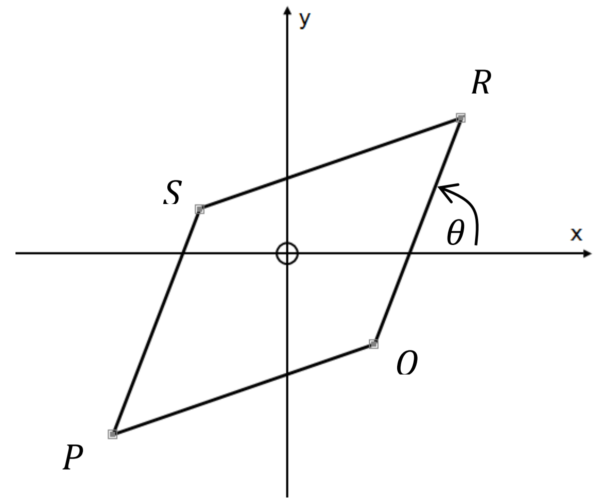
b) Determine the equation of the line parallel to AB passing through (0;-4).

c) If D$(3;m)$ is the midpoint of AC, determine thevalue of k and m.

d) If M is the midpoint of AB, prove by using analytical methods that $BC = 2MD$.

Question 4

In the diagram $PQRS$ is a parallelogram. With vertices $P(-2;-4)$, $Q(1;-2)$, $R(2;3)$ and $S(x;y)$.



Determine:

(a) The distance PQ, in simplest surd form.

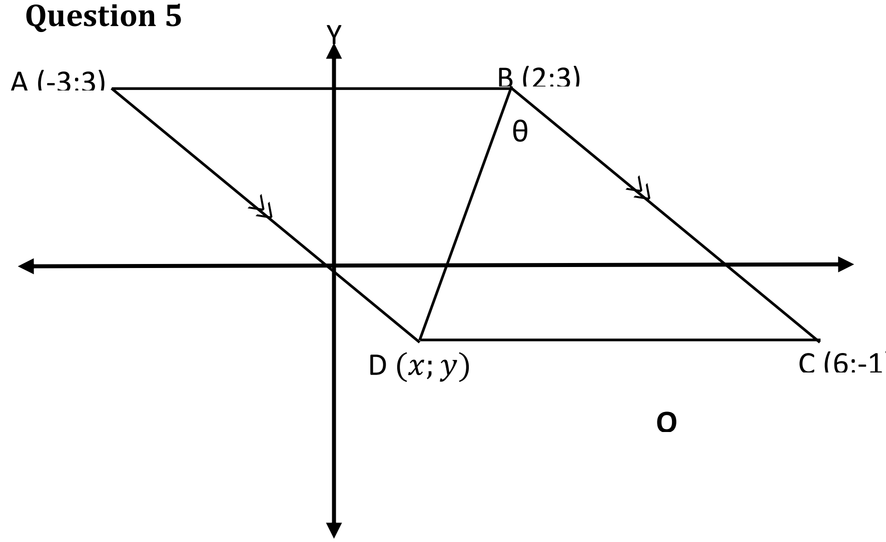
(b) The midpoint of PR.

(c) The equation of the line QR.

(d) The angle of inclination (θ) of line QR.

(e) The point S.

Question 5



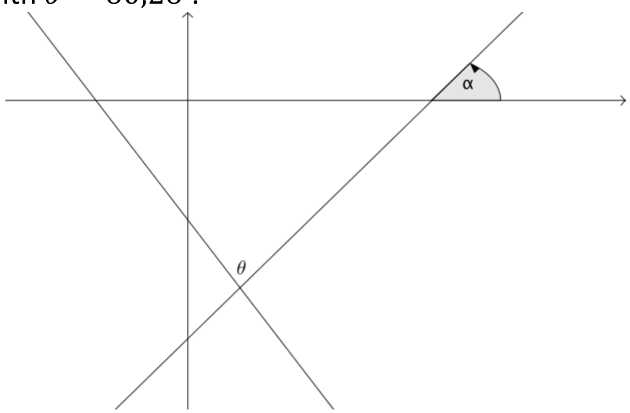
A (-3;3); B(2;3); C (6;-1) and D$(x;y)$ are vertices of quadrilateral ABCD in a Cartesian plane.

(a) Determine the equation of AD

(b) Determine the coordinates of D if D is equidistant from B and C.

(c) If it is given that the coordinates of D are $\left(\frac{3}{2};\ -\frac{3}{2}\right)$, determine the size of θ, the angle between BD and BC, rounded off to one decimal place.

6. The sketch shows the lines $2y - 3x = -20$ and $y = kx + p$ with $\theta = 60{,}26^\circ$.



Determine the value of:

3.1) α, to 2 decimal places.

3.2) k, to nearest whole number.

3.3) For the graph to be on scale, what are the possible values of p?

7. The equation of the circle is give by:

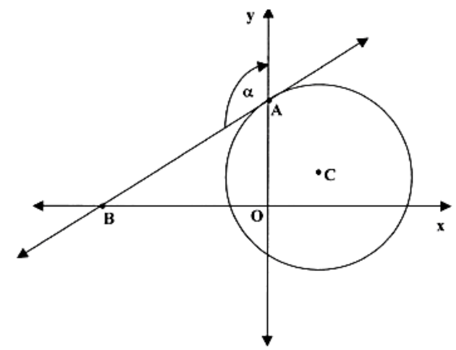
$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Determine the following:

a. The coordinate of the midpoint and radius of the circle

b. The equation of the tangent to the circle at point T(5 ; -1).

8. In the circle C(1;2) and A(0;4):



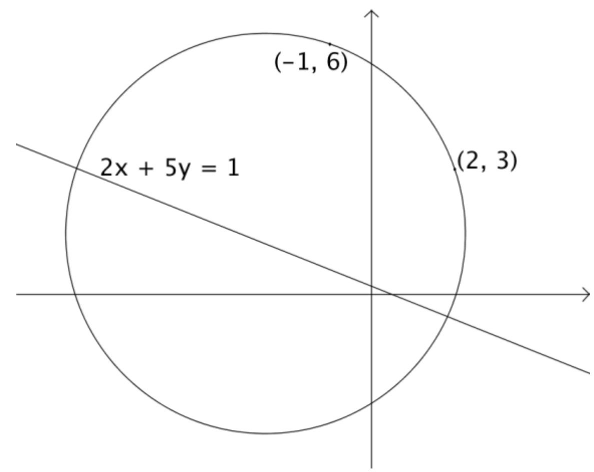
Determine the following:

a. Equation of circle

b. Equation of BA

c. Size of α

9. In the sketch A(2 ; 3) and B(-1 ; 6) lie on the circle with midpoint M(c ; d) M is also a point on the line given by $2x + 5y + 1 = 0$.



Determine the coordinate of M

Statistics - Questions

1. Age of 40 people:

20	17	53	65	16	18	33	69	50	45
66	25	43	48	45	53	26	38	19	41
52	60	40	38	48	53	48	27	35	38
50	69	27	29	35	41	36	39	42	53

a. Complete the table:

Interval	Tally	Frequency	Cumulative Frequency
10-19			
20-29			
30-39			
40-49			
50-59			
60-69			

b. Draw an ogive for the data

c. Use the letters A and B and indicate the first quartile and median is read off

d. Determine the mean and the deviation of the grouped data

2. The following is the number of marbles by certain kids on the playground:

4 86 27 21 29 37 29 44 31 42
35 38 41 29 40

Determine the following:

a. Mean

b. Mode

c. Median

d. Q1 and Q3

e. Range

f. IQR

g. Standard deviation

h. Outliers

i. Draw a box and whisker diagram for the data

j. Comment on the shape of the set of data

k. Which of the central values is most appropriate to use? Motivate your answer

3. Complete the table and calculate the variation and standard deviation of the following numbers:

12 32 3 18 14

Number(x)	$x - \bar{x}$	$(x - \bar{x})^2$
12		
32		
3		
18		
14		
		$\sum(x - \bar{x})^2 =$

4. Here is 2 learners' marks:

a. 43 61 31 79

b. 32 22 34 28

i. Which one has the higher average?

ii. Which learner is more consistent?

iii. If you're looking for a stable performer, which child would you choose?