KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

HARRY GWALA DISTRICT

MATHEMATICS

Practice Paper for June Provincial Test 2021

NATIONAL SENIOR CERTIFICATE

GRADE 12

Marks: 75

Time: 1hr 30mins

This question paper consists of 6 pages 1 diagram sheet and an information sheet.

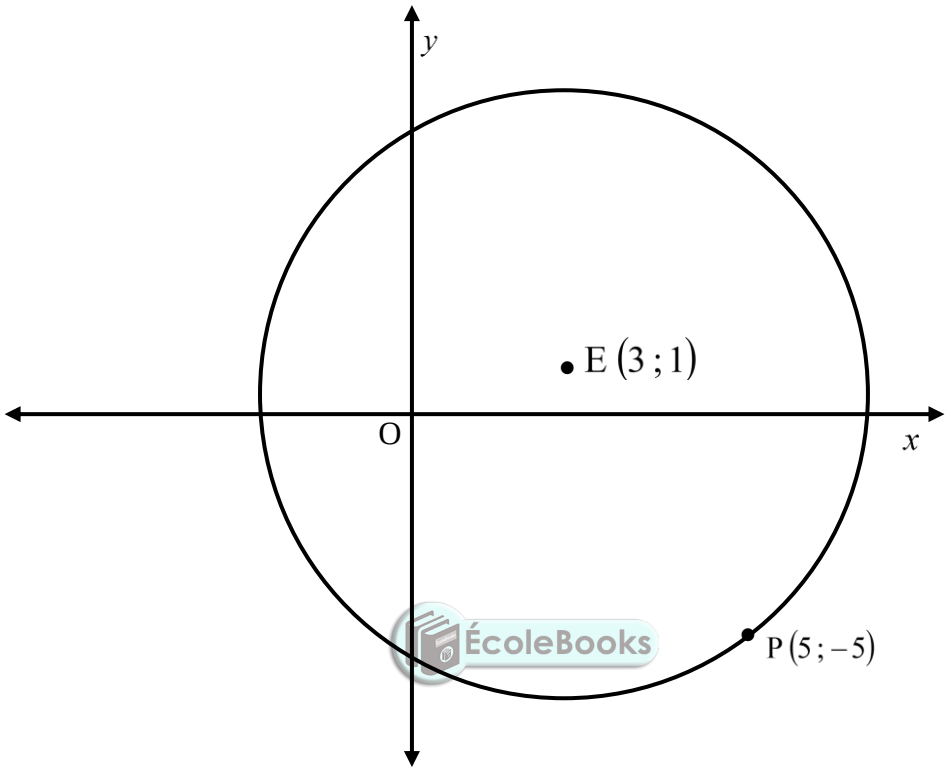
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

In the diagram below, the circle centred at $E(3;1)$ passes through point $P(5;-5)$.



1.1 Determine the equation of:

1.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)

1.1.2 The tangent to the circle at $P(5;-5)$ in the form $y = mx + c$. (5)

1.2 A smaller circle is drawn inside the circle. Line EP is a diameter of the small circle. Determine the:

1.2.1 Coordinates of the centre of the smaller circle. (3)

1.2.2 Length of the radius. (3)

1.3 Hence, or otherwise, determine whether point $C(9;3)$ lies inside or outside the circle centre at E. (3)

[18]

QUESTION 2

Given: $f(x) = \left(\frac{1}{5}\right)^x$

2.1 Determine the equation of f^{-1} in the form $y =$ ..... (1)

2.2 Sketch the graphs of f and f^{-1} on the same system of axes on the diagram sheet. Clearly show all intercepts with the axes. (4)

2.3 Write down the domain of f^{-1}. (2)

2.4 For which values of x will $f(x).f^{-1}(x) \geq 0$? (2)

2.5 Write down the range of $g(x)$ if $g(x) = -f(x) - 3$. (2)

[12]

QUESTION 3

3.1 Given: $f(x) = 2.2^x - 1$

3.1.1 Write down the range of f. (2)

3.1.2 $g(x) = f(x-1) + 1$. Write down the equation of g^{-1}, the inverse of g in the form $y =$... (2)

3.2 Given: $h(x) = -\sqrt{\frac{x}{3}}$; $x \geq 0$

3.2.1 If $k(x)$ is the inverse of h, give the equation of $k(x)$ (2)

3.2.2 Give the coordinates of the point of intersection of $h(x)$ and $k(x)$ (2)

[8]

QUESTION 4

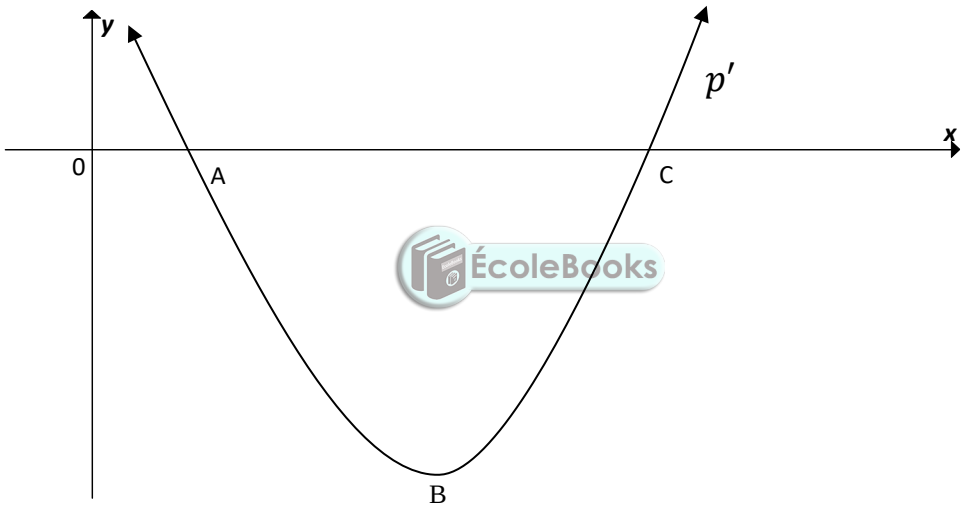
4.1 From first principles, determine $f'(x)$ if $f(x) = 4x^2 - x$. (5)

4.2 Determine: $D_x\left[x^2 - \frac{1}{2x^3} + \sqrt{x}\right]$ (3)

[8]

QUESTION 5

The sketch below shows the graph of $p'(x)$ where $p(x) = x^3 + bx^2 + 24x + c$. A(2;0) is an x-intercept of both $p(x)$ and $p'(x)$. C is the other x-intercept of $p'(x)$.



5.1 Show that the numerical value of b is equal to -9.
Clearly show all your calculations. (3)

5.2 Calculate the coordinates of C. (3)

5.3 For which value(s) of x will $p(x)$ be increasing? (3)

5.4 Calculate the value(s) of x for which the graph of p is concave up. (2)

5.5 Sketch a possible graph of $p(x)$. Clearly indicate the x-coordinates of the turning points and the point of inflection. (4)

[15]

QUESTION 6

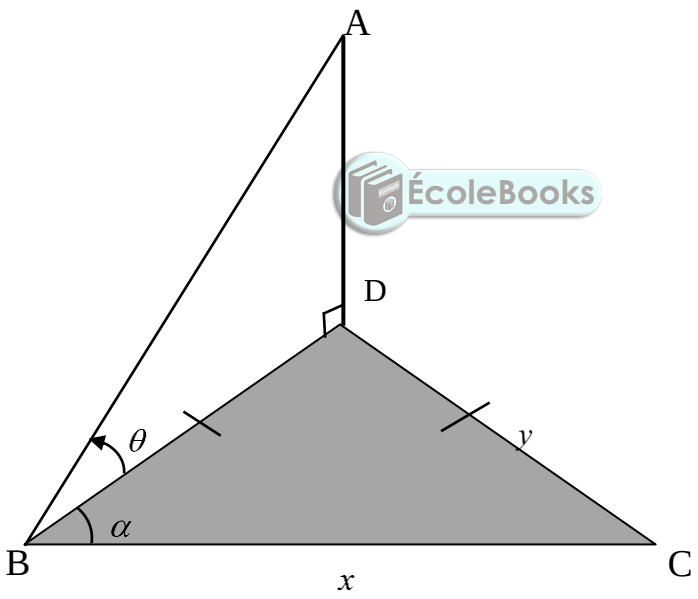
6.1 If a car valued at R255 000 depreciates on a reducing balance method at an interest rate of 12,5 % p.a., calculate the book value of the car after 7 years. (3)

6.2 How long will it take for a motor car to double in value if the annual inflation rate is 8,5 % ? (4)

[7]

QUESTION 7

In the diagram below, B, C and D are three points on the same horizontal plane such that BD = DC = y . C$\hat{B}$D = α and A$\hat{B}$D = θ. Line BC = x .



Prove that AB = $\dfrac{x}{2\cos\alpha\cos\theta}$ **[7]**

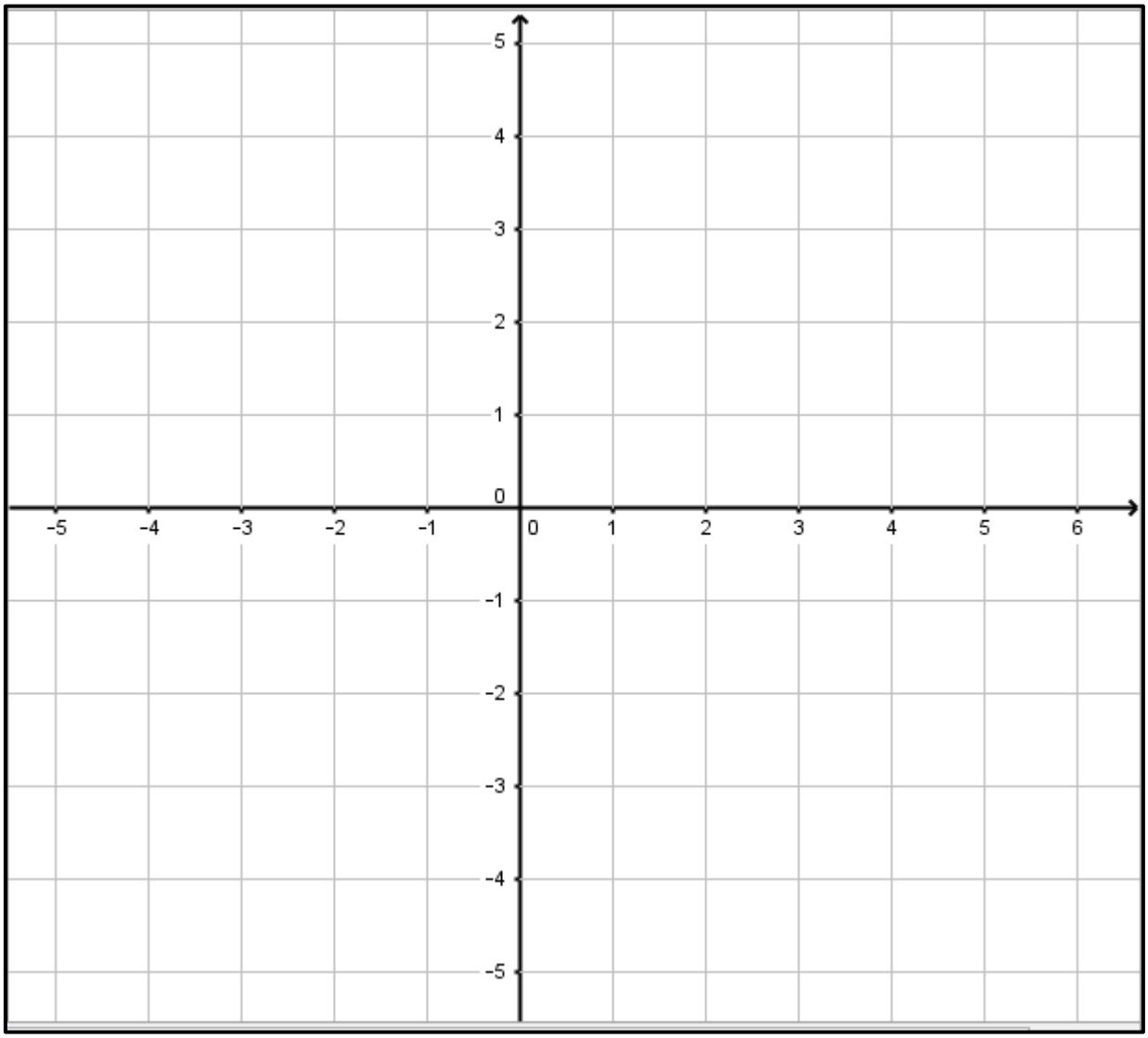
TOTAL = 75

DIAGRAM SHEET

Name and Surname: …….

Class: …….

QUESTION 2.2



Please hand in this page with your Answer Script

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad area\ \Delta ABC = \dfrac{1}{2}ab.\sin C$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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MARKING GUIDELINE

For June Practice for Control Test

NATIONAL SENIOR CERTIFICATE

GRADE 12

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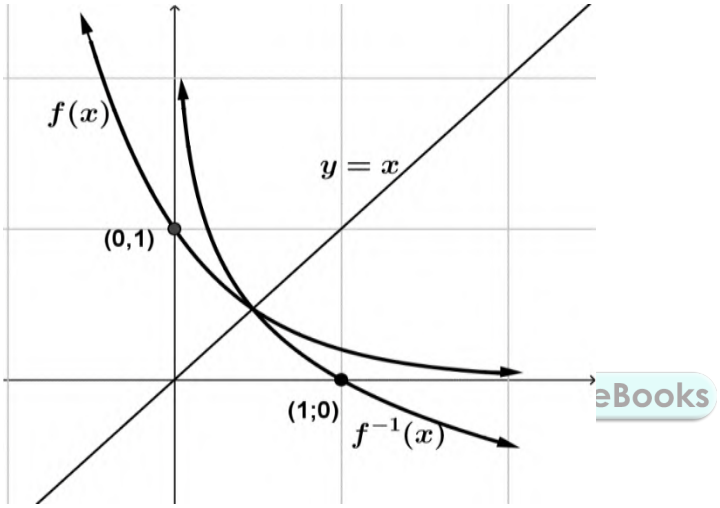
QUESTION 1

1.1.1	centre E(3;1) $(x-3)^2+(y-1)^2=r^2$ $(5-3)^2+(-5-1)^2=r^2$ $40=r^2$ $(x-3)^2+(y-1)^2=40$ $x^2-6x+9+y^2-2y+1=40$ $x^2-6x+y^2-2y=30$	✓ substitution E(3; 1) ✓ substitution P(5; –5) ✓$40=r^2$ ✓standard form (4)
1.1.2	$m_{rad}=\frac{1-(-5)}{3-5}$ $=\frac{6}{-2}\quad=-3$ $m\frac{1}{3}[\text{radius}\perp\tan]_{tan}$ $y=\frac{1}{3}x+c$ $-5=\frac{1}{3}(5)+c$ $c=\frac{-20}{3}$ $y=\frac{1}{3}x-\frac{20}{3}$	✓ correct substitution ✓ $m_{rad}=-3$ ✓ $m_{rad}=\frac{1}{3}$ ✓ substitution (5; –5) ✓ equation (5)
1.2.1	centre $=\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}$ $=\frac{3+5}{2};\frac{1-5}{2}$ centre(4; -2)	✓ method ✓x value ✓y value (3)
1.2.2	$(x-4)^2+(y+2)^2=r^2$ $(3-4)^2+(1+2)^2=r^2$ $10=r^2$ $r=\sqrt{10}$	✓substitution of centre ✓ substitution $(3;1)$ ✓ $=\sqrt{10}$ (3)

1.3	$r = \sqrt{40}$ $EC = \sqrt{(9-3)^2 + (3-1)^2}$ $= 2\sqrt{10}$ C is on the circumference	✓ $r = \sqrt{40}$ ✓distance EC ✓motivation (3)
		[18]

QUESTION 2

2.1	$y = \log_{\frac{1}{5}} x$	✓ ✓answer	(2)
2.2		f: ✓ (0;1) ✓ shape f^{-1}: ✓ (1;0) ✓ shape	(4)
2.3	$x > 0; x \in R$	✓✓ answer **DO NOT PENALIZE IF** $x \in R$ **IS OMITTED**	(2)
2.4	$0 < x \leq 1; x \in R$	✓✓ answer **DO NOT PENALIZE IF** $x \in R$ **IS OMITTED**	(2)
2.5	$y < -3; y \in R$	✓✓ answer **DO NOT PENALIZE IF** $x \in R$ **IS OMITTED**	(2)
			[12]

QUESTION 3

3.1.1	$y > -1; y \in \mathbb{R}$	$y > 0; y \in \mathbb{R}$	2

3.1.2	$g(x) = 2^x$ $\therefore g^{-1}: y = \log_2 x$	$g(x) = 2^x$ $y = \log_2 x$	2
3.2.1	$k(x) = 3x^2 \ ; x \leq 0$	$k(x) = 3x^2$ $x \leq 0$	2
3.2.2	$(0; 0)$ OR	Answer	2
			[8]

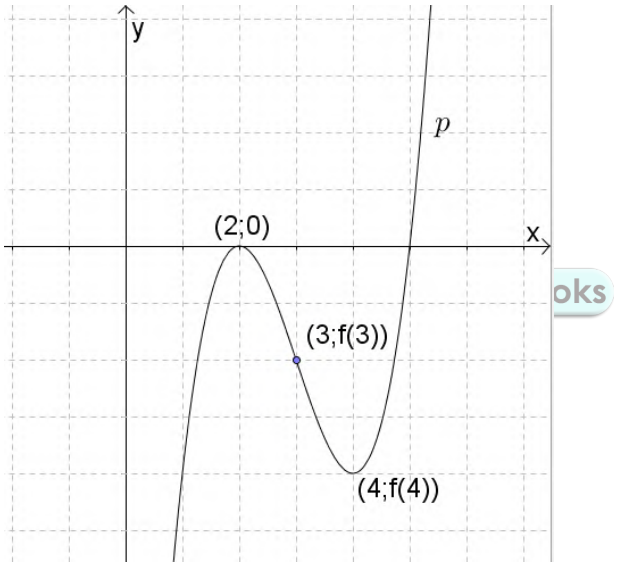
QUESTION 4

Penalise for notation: Only once in this question.

4.1	$f(x) = 4x^2 - x$ $f(x+h) = 4(x+h)^2 - (x+h)$ $= 4(x^2 + 2xh + h^2) - x - h$ $= 4x^2 + 8xh + 4h^2 - x - h$ $f(x+h) - f(x) = 8xh + 4h^2 - h$ $f'(x) = \lim_{h \to 0} \frac{h(8x + 4h - 1)}{h}$ $= \lim_{h \to 0} (8x + 4h - 1)$ $= 8x - 1$	✓ $4x^2 + 8xh + 4h^2 - x - h$ ✓ $8xh + 4h^2 - h$ ✓ factorising ✓simplify ✓ answer CA	(5)
4.2	$D_x\left[x^2 - \frac{1}{2x^3} + \sqrt{x}\right]$ $= D_x[x^2 - \frac{1}{2}x^{-3} + x^{\frac{1}{2}}]$ $= 2x + \frac{3}{2}x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$	✓ $2x$ ✓$\frac{3}{2}x^{-4}$ only CA if index is negative integer. ✓$\frac{1}{2}x^{-\frac{1}{2}}$ only CA if index is rational	(3)

QUESTION 5

5.1	$p'(x) = 3x^2 + 2bx + 24$ $subst\ A(2; 0)$ $0 = 3(2)^2 + 2b(2) + 24$ $-36 = 4b$ $-9 = b$	✓ $p'(x)$ ✓ $subst$ ✓ $answer$	(3)C
5.2	$p'(x) = 0$ $3x^2 - 18x + 24 = 0$	✓ $p'(x) = 0$	(3)R

	$x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ $x = 2\,; x = 4$ $C(4;0)$	✓ $factors$ ✓ $C(4;0)$	
5.3	p increasing : $p'(x) > 0$. $x < 2$ *or* $x > 4$	✓ $p'(x) > 0$ ✓ $x < 2$ *or* $x > 4$ **CA from 8.4.2**	(3)C
5.4	p concave up : $p''(x) > 0$ $x > 3$	✓ $p''(x) > 0$ ✓ $x > 3$ **CA from 8.4.2 (middle of 2 x − *intercepts*)**	(2)C
5.5		✓(2; 0) ✓$infl\ x = 3$ ✓ TP $x = 4$ ✓ $shape$	(4)C

QUESTION 6

6.1	$A = P(1-i)^n$ $A = 255000(1-0{,}125)^7$ $= \text{R}\quad 100\,137{,}45$	A✓ formula A✓correct substitution CA✓answer	(3)
6.2	$A = P(1+i)^n$ $2x = x(1+0{,}085)^n$ $n = \dfrac{\log 2}{\log(1+0{,}085)}$ $= 9\quad \text{years}$	✓✓correct substitution into correct formula ✓making n the subject ✓answer	(4)

QUESTION 7

7.1	$\hat{BDC} = 180^\circ - 2\alpha$ $\frac{y}{\sin\alpha} = \frac{x}{\sin 180^\circ - 2\alpha}$ $y = \frac{x\sin\alpha}{\sin(180^\circ - 2\alpha)}$ $y = \frac{x\sin\alpha}{2\sin\alpha\cos\alpha}$ $y = \frac{x}{2\cos\alpha}$ $\cos\theta = \frac{BD}{AB}$ $AB\cos\theta = BD$ $AB = \frac{BD}{\cos\theta}$ $AB = \frac{x}{2\cos\alpha} \div \cos\theta$ $AB = \frac{x}{2\cos\alpha} \times \frac{1}{\cos\theta}$ $AB = \frac{x}{2\cos\alpha\cos\theta}$	✓ $\hat{BDC} = 180^\circ - 2\alpha$ ✓ method ✓ substitution ✓ $2\sin\alpha\cos\alpha$ ✓ $AB = \frac{BD}{\cos\theta}$ ✓ substitution BD ✓ simplification (7)
		[7]