



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

NOVEMBER 2021

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.



INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

downloaded from stannmorephysics.com



QUESTION 1

A bakery kept a record of the number of loaves of bread a tuck-shop ordered daily over the last 18 days. The information is shown in the table below.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Calculate the:

1.1.1 Mean number of loaves of bread ordered daily (2)

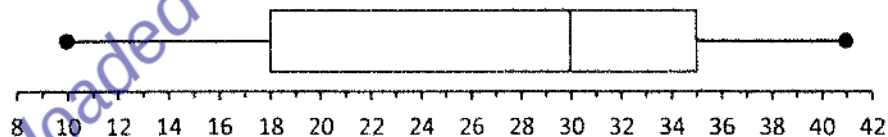
1.1.2 Standard deviation of the data (1)

1.1.3 Number of days on which the number of loaves of bread ordered was more than one standard deviation above the mean (2)

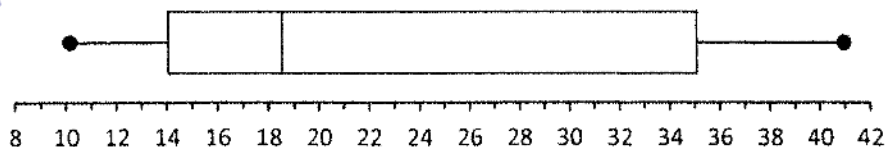
1.2 The tuck-shop owner was not able to sell all the loaves of bread delivered daily. He calculated the mean number of loaves sold over the 18 days to be 20. Calculate the number of loaves of bread which were NOT sold over the 18 days. (2)

1.3 One of the two box and whisker diagrams drawn below represents the data given in the table above.

Graph A:



Graph B:



1.3.1 Which ONE of the two box and whisker diagrams, drawn above, correctly represents the data? Write down a reason for your answer. (2)

1.3.2 Describe the skewness of the data. (1)

[10]

**QUESTION 2**

A farm stall sells milk in 5-litre containers to the local community. The price varies according to the availability of milk at the farm stall. The price of milk, in rands per 5-litre container, and the number of 5-litre containers of milk sold, are recorded in the table below.

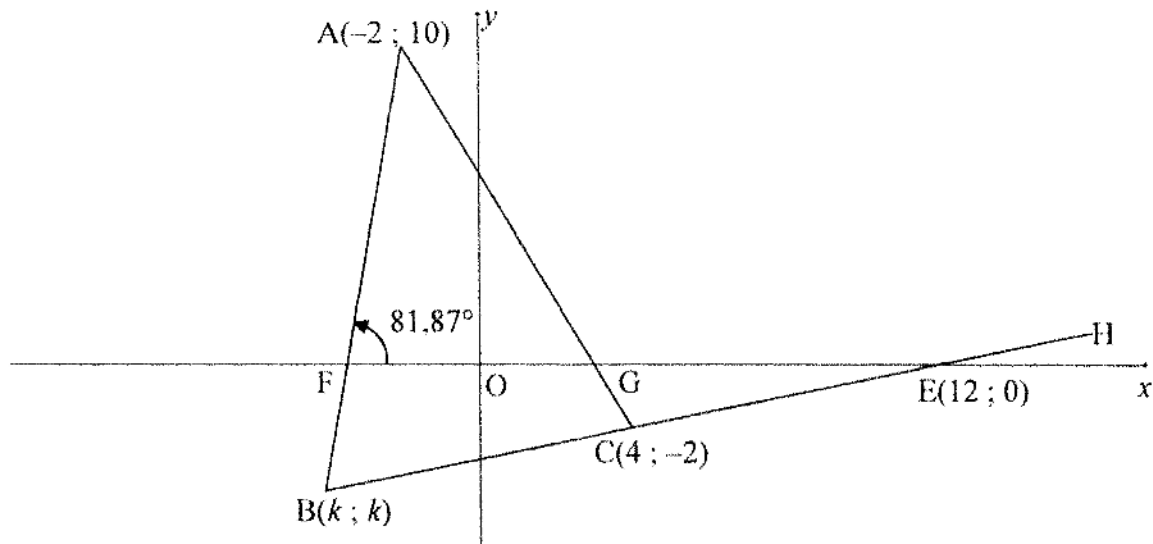
Price of milk in rands per 5-litre container (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)	48	30	26	44	23	32	39	29	42	33

- 2.1 On the grid provided in the ANSWER BOOK, draw the scatter plot to represent the data. (3)
- 2.2 Determine the equation of the least squares regression line for the data. (3)
- 2.3 If the farmer sells a 5-litre container of milk for R38, predict the number of 5-litre containers of milk he will sell. (2)
- 2.4 Refer to the correlation between the price of 5-litre containers of milk and the number of 5-litre containers of milk sold, and comment on the accuracy of your answer to QUESTION 2.3. (2)
- [10]



QUESTION 3

In the diagram, $A(-2; 10)$, $B(k; k)$ and $C(4; -2)$ are the vertices of $\triangle ABC$. Line BC is produced to H and cuts the x -axis at $E(12; 0)$. AB and AC intersect the x -axis at F and G respectively. The angle of inclination of line AB is $81,87^\circ$.



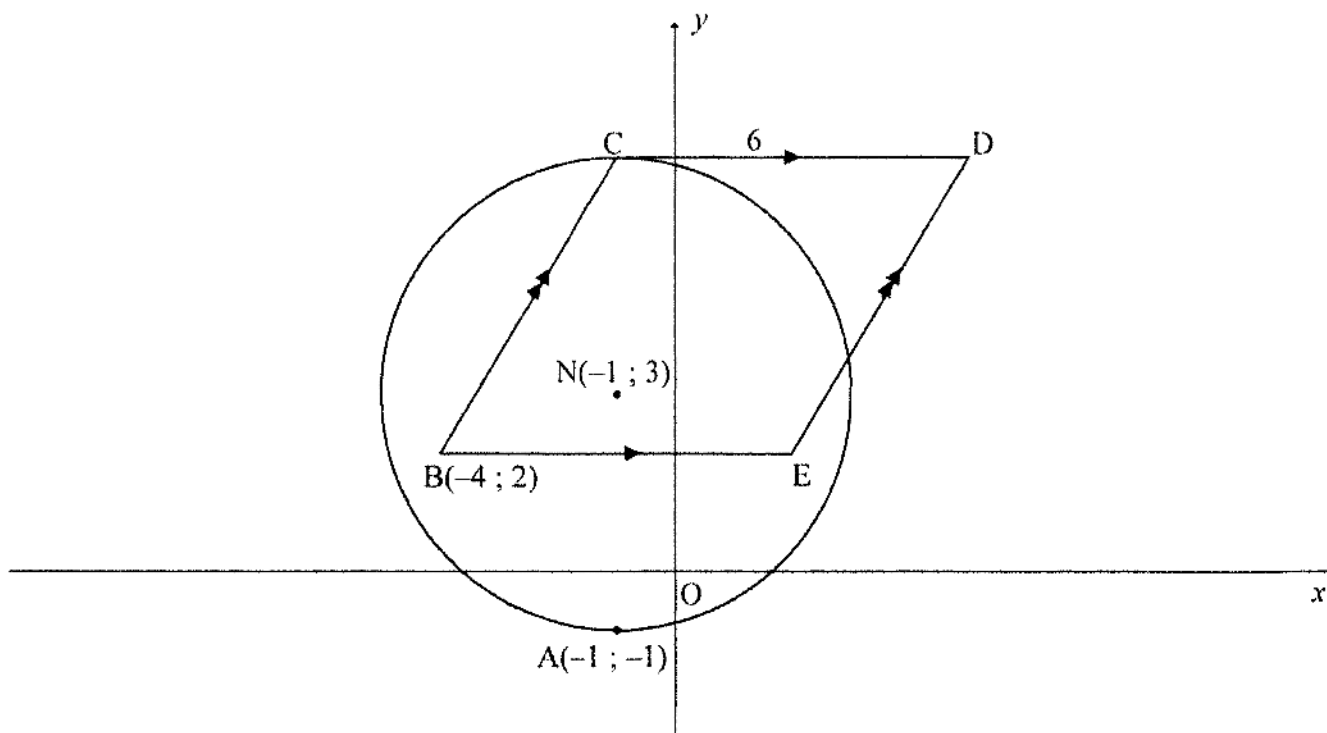
- 3.1 Calculate the gradient of:
- 3.1.1 BE (2)
- 3.1.2 AB (2)
- 3.2 Determine the equation of BE in the form $y = mx + c$ (2)
- 3.3 Calculate the:
- 3.3.1 Coordinates of B , where $k < 0$ (2)
- 3.3.2 Size of \hat{A} (4)
- 3.3.3 Coordinates of the point of intersection of the diagonals of parallelogram $ACES$, where S is a point in the first quadrant (2)
- 3.4 Another point $T(p; p)$, where $p > 0$, is plotted such that $ET = BE = 4\sqrt{17}$ units.
- 3.4.1 Calculate the coordinates of T . (5)
- 3.4.2 Determine the equation of the:
- (a) Circle with centre at E and passing through B and T in the form $(x - a)^2 + (y - b)^2 = r^2$ (2)
- (b) Tangent to the circle at point $B(k; k)$ (3)

[24]



QUESTION 4

In the diagram, the circle centred at $N(-1; 3)$ passes through $A(-1; -1)$ and C . $B(-4; 2)$, C , D and E are joined to form a parallelogram such that BE is parallel to the x -axis. CD is a tangent to the circle at C and $CD = 6$ units.



- 4.1 Write down the length of the radius of the circle. (1)
- 4.2 Calculate the:
- 4.2.1 Coordinates of C (2)
- 4.2.2 Coordinates of D (2)
- 4.2.3 Area of $\triangle BCD$ (3)
- 4.3 The circle, centred at N , is reflected about the line $y = x$. M is the centre of the new circle which is formed. The two circles intersect at A and F .
- Calculate the:
- 4.3.1 Length of NM (3)
- 4.3.2 Midpoint of AF (4)
- [15]



QUESTION 5

- 5.1 Without using a calculator, simplify the following expression to ONE trigonometric ratio:

$$\frac{\sin 140^\circ \cdot \sin(360^\circ - x)}{\cos 50^\circ \cdot \tan(-x)} \quad (6)$$

- 5.2 Prove the identity: $\frac{-2 \sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)} = 2 \cos x - 1$ (4)

- 5.3 Given: $\sin 36^\circ = \sqrt{1 - p^2}$

Without using a calculator, determine EACH of the following in terms of p :

5.3.1 $\tan 36^\circ$ (3)

5.3.2 $\cos 108^\circ$ (4)
[17]

QUESTION 6

- 6.1 Given: $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

6.1.1 Use the given identity to derive a formula for $\cos(\alpha + \beta)$ (3)

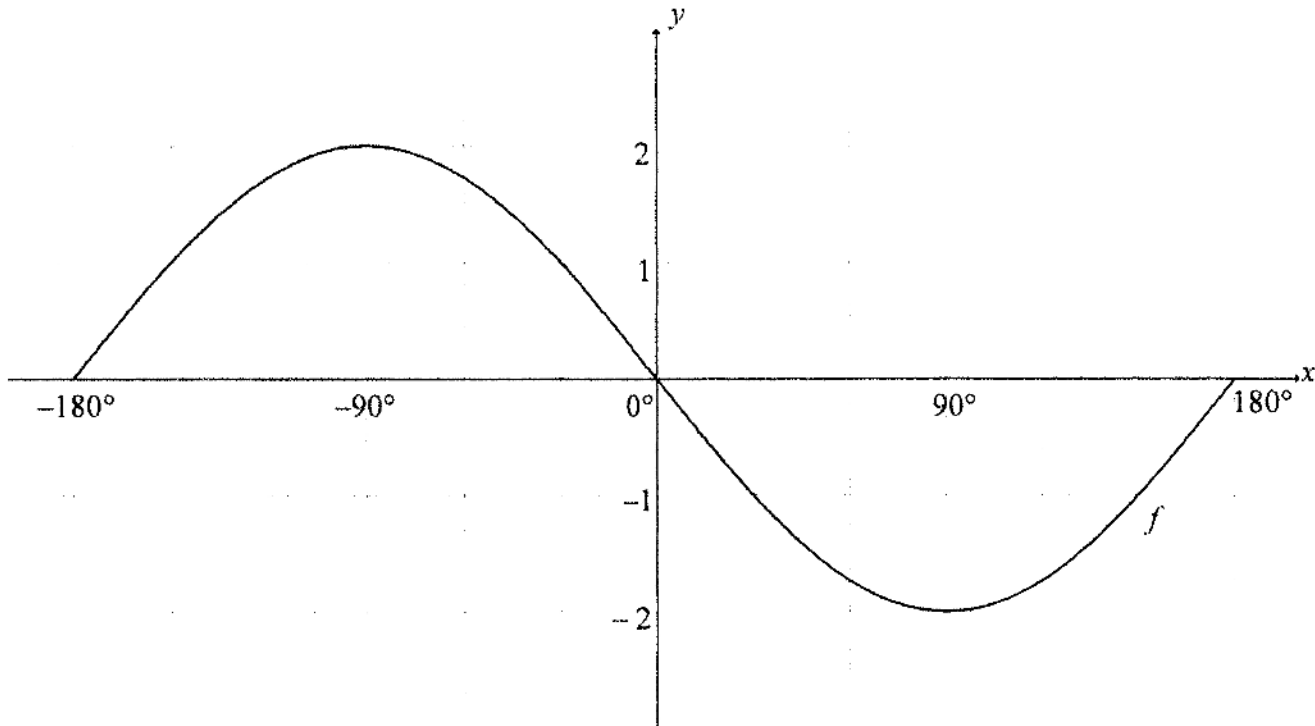
6.1.2 Simplify completely: $2 \cos 6x \cos 4x - \cos 10x + 2 \sin^2 x$ (5)

- 6.2 Determine the general solution of $\tan x = 2 \sin 2x$ where $\cos x < 0$. (7)
[15]



QUESTION 7

In the diagram below, the graph of $f(x) = -2 \sin x$ is drawn for the interval $x \in [-180^\circ; 180^\circ]$.



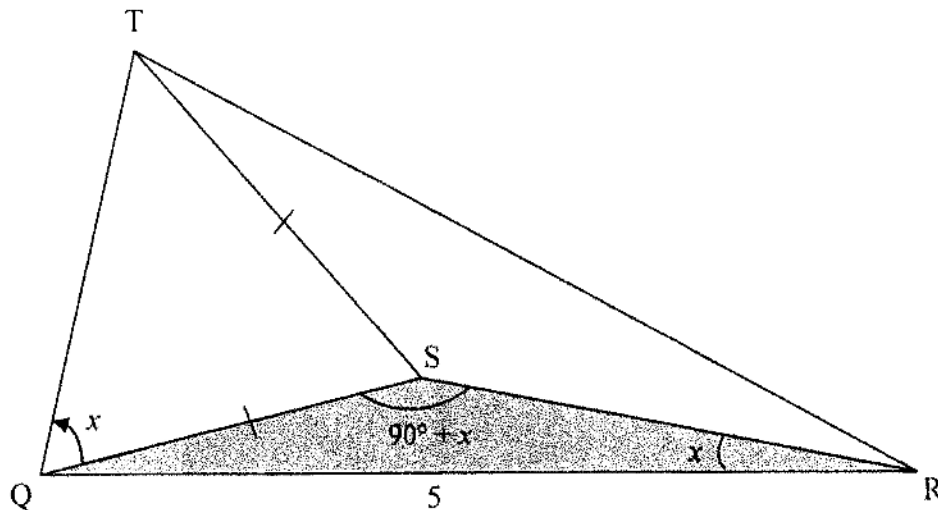
- 7.1 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(x - 60^\circ)$ for $x \in [-180^\circ; 180^\circ]$. Clearly show ALL intercepts with the axes and turning points of the graph. (3)
- 7.2 Write down the period of $f(3x)$. (2)
- 7.3 Use the graphs to determine the value of x in the interval $x \in [-180^\circ; 180^\circ]$ for which $f(x) - g(x) = 1$. (1)
- 7.4 Write down the range of k , if $k(x) = \frac{1}{2}g(x) + 1$. (2)
- [8]**



QUESTION 8

In the diagram below, T is a hook on the ceiling of an art gallery. Points Q, S and R are on the same horizontal plane from where three people are observing the hook T. The angle of elevation from Q to T is x .

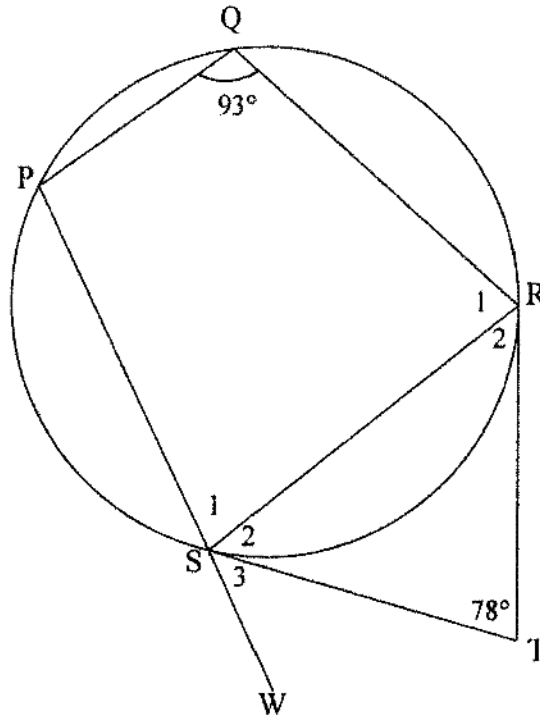
$\hat{QSR} = 90^\circ + x$, $\hat{QRS} = x$, $QR = 5$ units and $TS = SQ$.



- 8.1 Prove that $QS = 5 \tan x$ (3)
- 8.2 Prove that the length of $QT = 10 \sin x$ (5)
- 8.3 Calculate the area of $\triangle TQR$ if $\hat{TQR} = 70^\circ$ and $x = 25^\circ$. (2)
- [10]**

QUESTION 9

In the diagram, PQRS is a cyclic quadrilateral. PS is produced to W. TR and TS are tangents to the circle at R and S respectively. $\hat{T} = 78^\circ$ and $\hat{Q} = 93^\circ$.



9.1 Give a reason why $ST = TR$. (1)

9.2 Calculate, giving reasons, the size of:

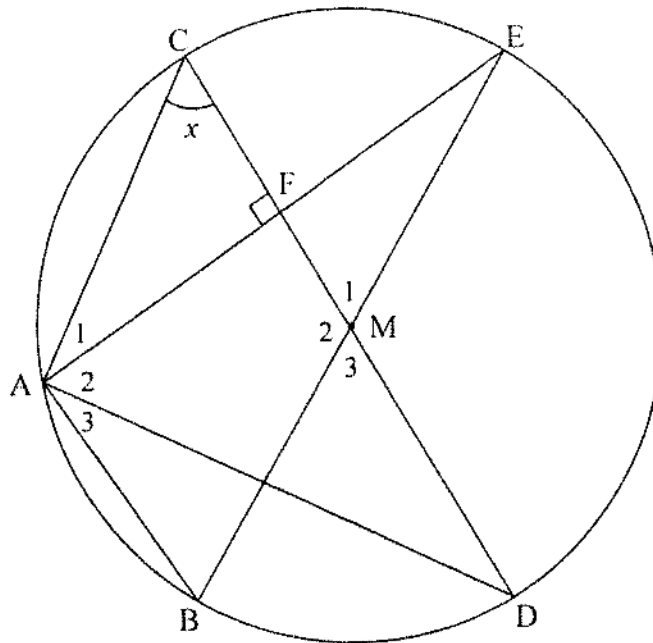
9.2.1 \hat{S}_2 (2)

9.2.2 \hat{S}_3 (2)
[5]



QUESTION 10

In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. $AE \perp CD$. Let $\hat{C} = x$.

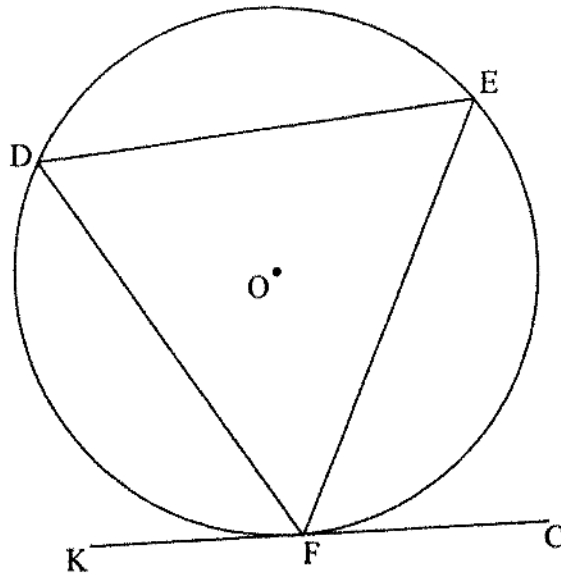


- 10.1 Give a reason why $AF = FE$. (1)
 - 10.2 Determine, giving reasons, the size of \hat{M}_1 in terms of x . (3)
 - 10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)
 - 10.4 Given that $CF = 6$ units and $AB = 24$ units, calculate, giving reasons, the length of AE. (5)
- [13]



QUESTION 11

- 11.1 In the diagram, chords DE, EF and DF are drawn in the circle with centre O. KFC is a tangent to the circle at F.

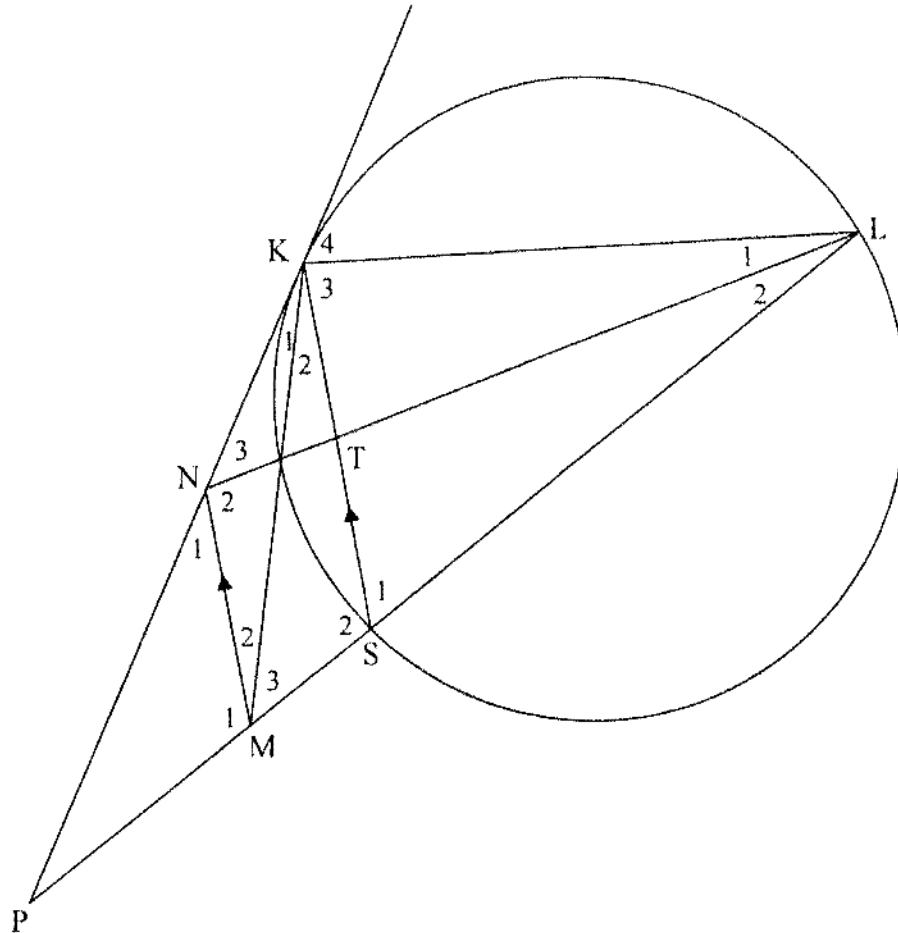


Prove the theorem which states that $\hat{DFK} = \hat{E}$.

(5)



11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that $MN \parallel SK$. Chord KS and LN intersect at T.



11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_1 = \hat{NML}$ (4)

(b) KLMN is a cyclic quadrilateral (1)

11.2.2 Prove, giving reasons, that $\triangle LKN \sim \triangle KSM$. (5)

11.2.3 If $LK = 12$ units and $3KN = 4SM$, determine the length of KS. (4)

11.2.4 If it is further given that $NL = 16$ units, $LS = 13$ units and $KN = 8$ units, determine, with reasons, the length of LT. (4)

[23]

TOTAL: 150



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$




PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	1. <i>Skryf jou eksamennommer en sentrumnummer duidelik in die ruimtes soos verskaf en plak jou stafieskodeplakker in die ruimte soos verskaf.</i>
2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.	2. <i>Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.</i>
3. Answer ALL questions in the spaces provided.	3. <i>Beantwoord ALLE vrae in die ruimtes wat voorsien is.</i>
4. NO pages may be torn from this answer book.	4. <i>GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.</i>
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. <i>Lees die instruksies wat op jou eksamenrooster gedruk is sorgvuldig deur, asook enige ander instruksies wat op elke eksamenvraestel gegee word.</i>
6. Candidates may NOT retain an answer book or remove it from the examination room.	6. <i>GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.</i>
7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.	7. <i>Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.</i>
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	8. <i>Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.</i>
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	9. <i>In geval jy bykomende ruimte benodig vir jou antwoorde:</i> 9.1 <i>Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word.</i> 9.2 <i>As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.</i> 9.3 <i>Trek 'n lyn na elke antwoord.</i>
10. Draw a neat line through any work/rough work that must NOT be marked.	10. <i>Trek 'n netjiese lyn deur enige werk/rofwerk wat NIE nagesien moet word NIE.</i>



QUESTION/VRAAG 1

1.1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

	Solution/Oplissing	Marks Punte
1.1.1		(2)
1.1.2		(1)
1.1.3		(2)
1.2		(2)
1.3.1		(2)
1.3.2		(1)
		[10]



QUESTION/VRAAG 2

Price of milk in rands per 5-litre container (x)/ Prys van melk, in rand, per 5 liter-houer (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)/ Aantal 5 liter-houers melk verkoop (y)	48	30	26	44	23	32	39	29	42	33

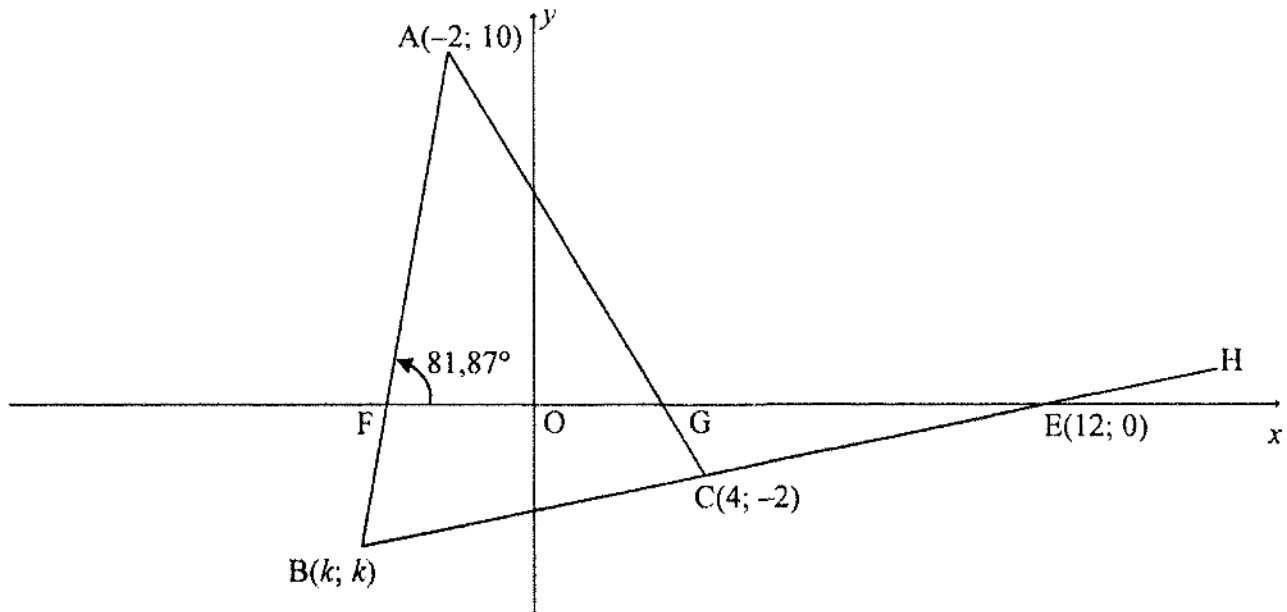
	Solution/Oplissing	Marks Punte
2.1	<p style="text-align: center;">SCATTER PLOT/SPREIDIAGRAM</p> <p style="text-align: center;">Price of milk in rands per 5-litre container/ Prys van melk in rand per 5 liter-houer</p>	(3)



	Solution/Oplissing	Marks Punte
2.2		
		(3)
2.3		
		(2)
2.4		
		(2)
		[10]



QUESTION/VRAAG 3



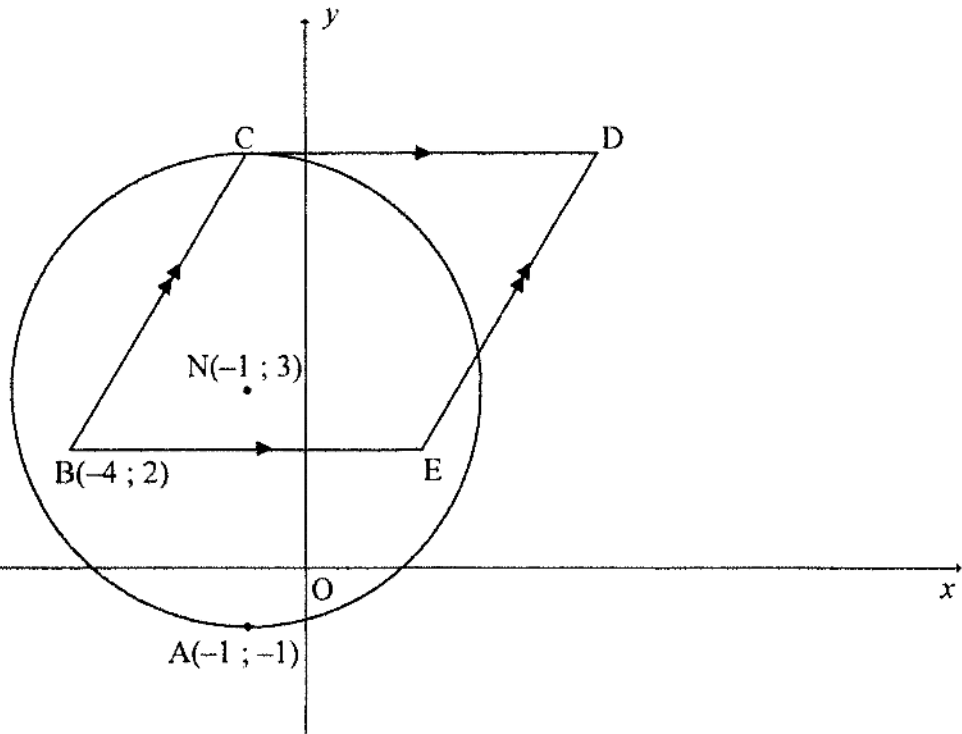
	Solution/Oplissing	Marks Punte
3.1.1		(2)
3.1.2		(2)
3.2		(2)
3.3.1		(2)



	Solution/Oplissing	Marks Punte
3.3.2		(4)
3.3.3		(2)
3.4.1		(5)
3.4.2(a)		(2)
3.4.2(b)		(3)
		[24]



QUESTION/VRAAG 4



	Solution/Oplissing	Marks Punte
4.1		(1)
4.2.1		(2)
4.2.2		(2)
4.2.3		(3)



	Solution/Oplissing	Marks Punte
4.3.1		
4.3.2		(3)
		(4)
		[15]



QUESTION/VRAAG 5

	Solution/Oplissing	Marks Punte
5.1		(6)
5.2		(4)
5.3.1		(3)



5.3.2		
		(4)
		[17]



QUESTION/VRAAG 6

	Solution/Oplissing	Marks Punte
6.1.1		(3)
6.1.2		(5)

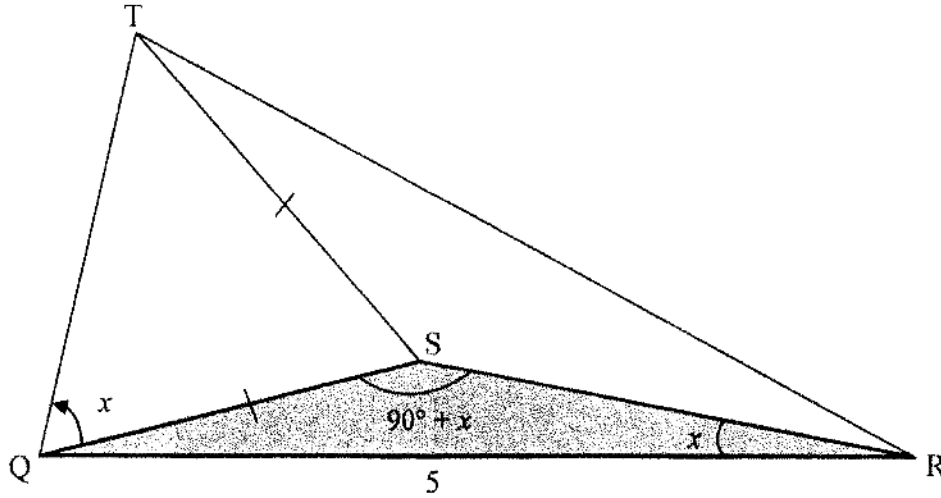


	Solution/ <i>Oplossing</i>	Marks <i>Punte</i>
6.2		
		(7)
		[15]



Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.
 Gee redes vir jou bewerings in VRAAG 8, 9, 10 en 11.

QUESTION/VRAAG 8



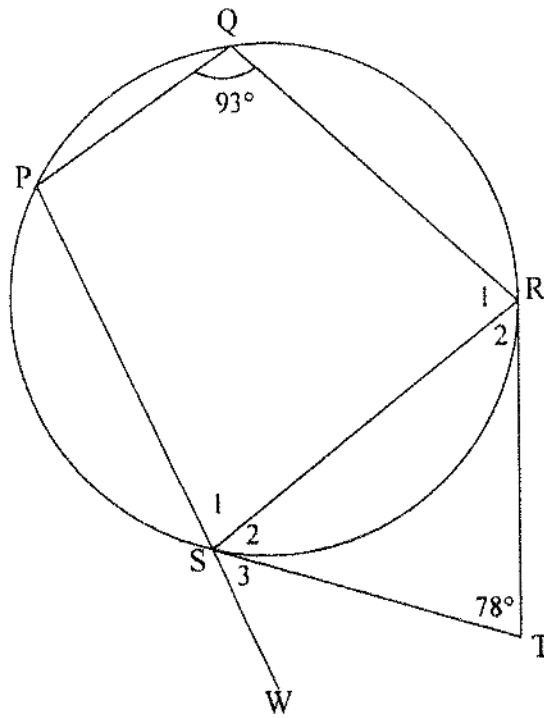
	Solution/Oplissing	Marks Punte
8.1		
		(3)
8.2		
		(5)



8.3		
		(2)
		[10]



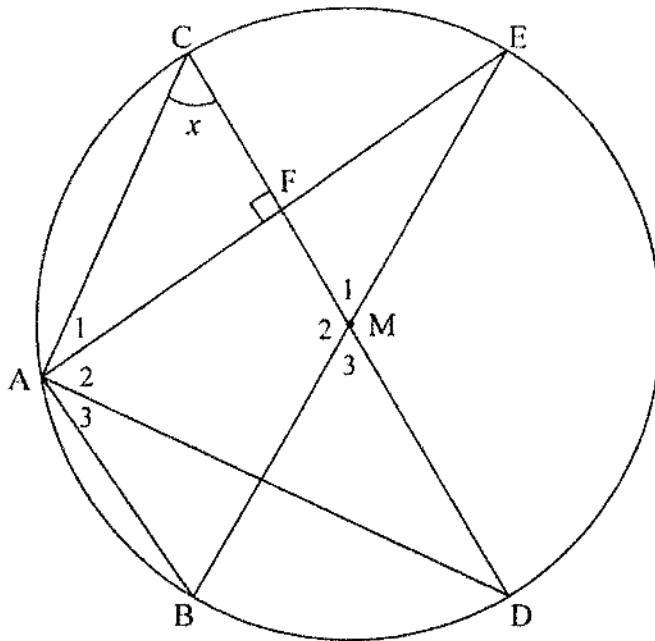
QUESTION/VRAAG 9



	Solution/Oplissing	Marks Punte
9.1		(1)
9.2.1		(2)
9.2.2		(2)
		[5]



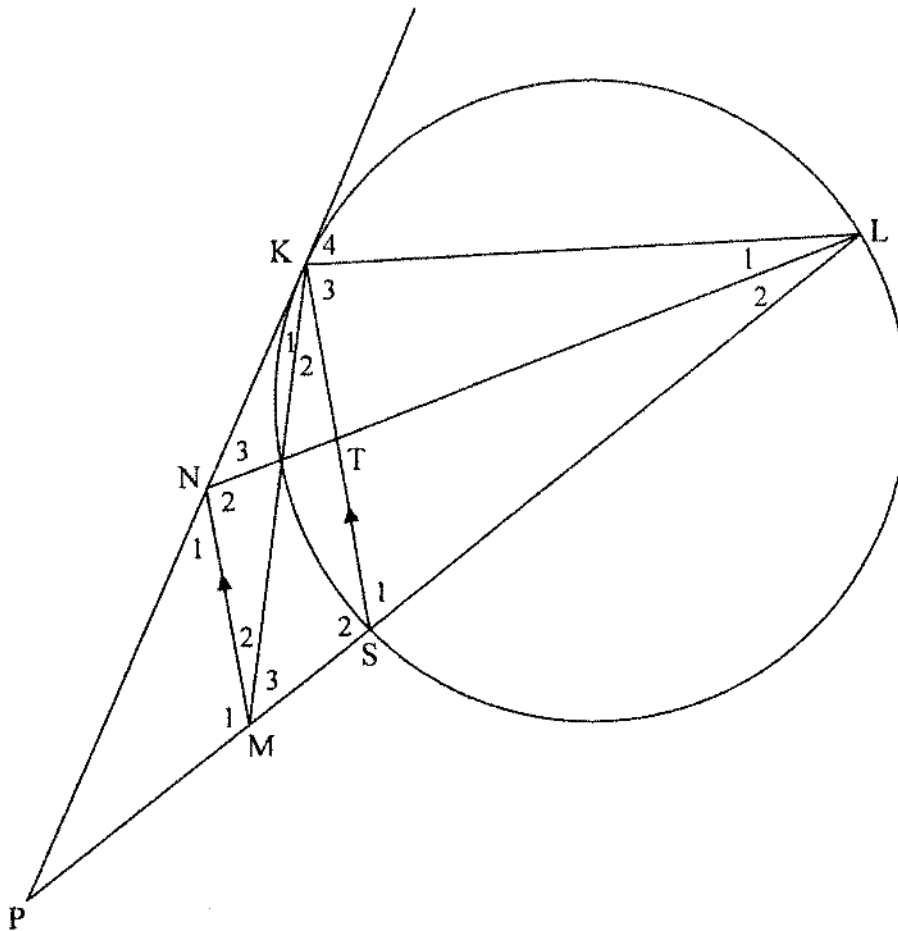
QUESTION/VRAAG 10



	Solution/Oplissing	Marks Punte
10.1		(1)
10.2		(3)
10.3		(4)
10.4		(5)
		[13]



11.2



	Solution/Oplissing	Marks Punte
11.2.1(a)		(4)
11.2.1(b)		(1)



	Solution/Oplissing	Marks Punte
11.2.2		(5)
11.2.3		(4)
11.2.4		(4)
		[23]



Additional space/ <i>Bykomende ruimte</i>	Marks <i>Punte</i>



