



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

KEREITI 12

LWETSE 2020

DIPALO P2

MATSHWAO: 150

NAKO : Dihora tse 3

Pampiri ena, ena le maqhephe a 15, ho kenyeleditse leqephe le 1 la shiti ya tlhahisoleseding le bukana ya ho arabela dipotso e nang le maqhephe a 25.

DITAELO LE TLHAHISOLESERING

Bala ditaello tse latelang ka hloko pele o araba dipotso.

1. Pampiri ena, ena le dipotso tse leshome (10).
2. Araba dipotso KAOFELA bukaneng ya ho arabela (SPECIAL ANSWER BOOK) eo oe nehilweng.
3. Bontsha mesebetsi (calculations) ya hao hantle, diagrams, graphs, et cetra tseo oka di sebedisang dikarabong tsa hao.
4. Haho bolele hore dikarabo FEELA ditla fumana matshwao kaofela.
5. O ka sebedisa approved scientific calculator (non-programmable and non-graphical), ntle le haeba ho boletswe ka tsela e nngwe.
6. Moo ho hlokahalang teng, atametsa dikarabo ho desimal tse pedi (2), ntle le haeba ho boletswe ka tsela e nngwe.
7. Ditshwantsho (diagrams) ha di a takwa ka sekala (scale).
8. Leqephe la tlhahisolesering (information sheet with formulae) e teng, le ho fumaneha moo pampiri ena e fellang.
9. Ngola ka tsela ya bohleki le ho sebedisa mongolo o bonahalang.



POTSO 1

Tafole e latelang e bontsha papiso sekolong sa Kereiti 12 final marks le School Based Assessment (SBA) marks tsa bana ba sekolo 2019.

LEARNERS	1	2	3	4	5	6	7	8	9	10
SBA MARK	99	93	77	74	63	62	63	63	47	37
FINAL MARK	94	81	73	65	59	58	55	49	43	31

- 1.1 Fumana equation of the least squares regression line ya data. (Atametsa karabo ya hao ho desimal tse 4). (3)
- 1.2 Fumana correlation coefficient pakeng tsa SBA mark le final mark. (1)
- 1.3 Inanathe ka correlation pakeng tsa SBA mark le final mark. (1)
- 1.4 Moithuti wa bo 11 o fumane 51% ya SBA. Ako akanye final mark ya tla e fumana o atametse karabo ho palo e felletseng. (2)
- 1.5 Haeba re nehilwe mean final mark ele 60,8; fumana hore ke baithuti ba bakae ba tlabana within one deviation ya mean. (3)
- [10]**

POTSO 2

Lebelo (speeds), dikilomitara ka hora, tsa bapalami ba dibaesekele (cyclists) o ile wa feta tseleng, jwale Ironman Race e ngotse ntlha tsa lebelo fatshe le ho kgutsufatswa tafoleng e latelang:

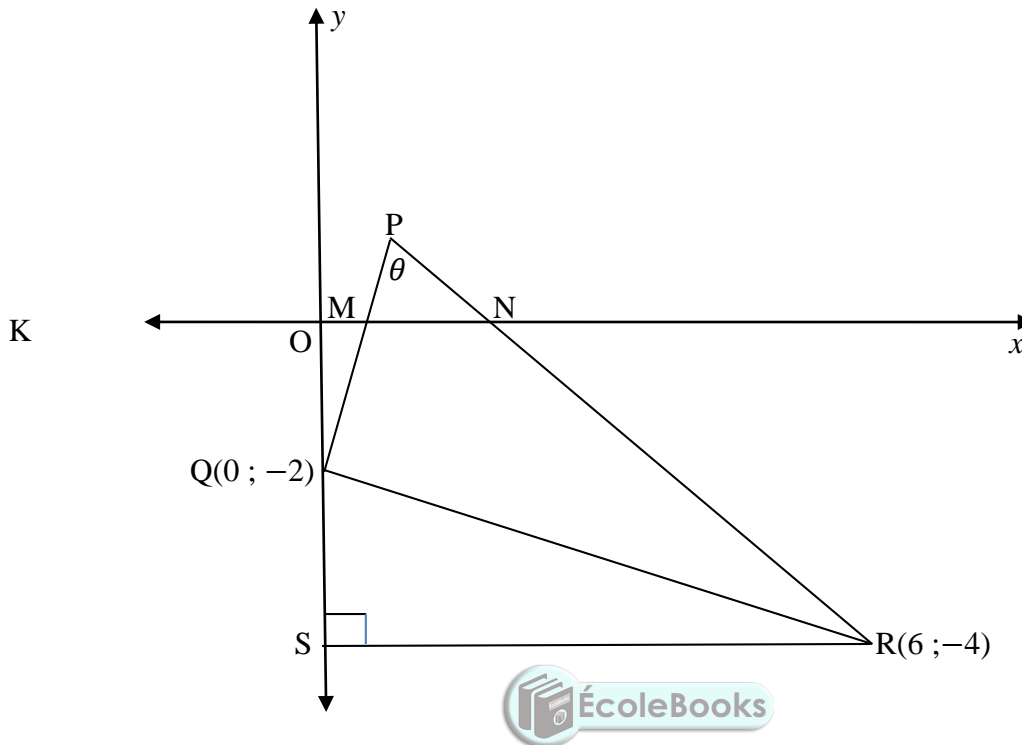


Speed (km/h)	Frequency (f)	Cumulative Frequency
$0 < x \leq 10$	10	10
$10 < x \leq 20$		30
$20 < x \leq 30$	45	
$30 < x \leq 40$	72	
$40 < x \leq 50$		170

- 2.1 Tlatsa tafole eka hodimo, ho ANSWER BOOK e ntshitsweng. (2)
- 2.2 Ela hloko axes tseo ho fanweng ka tsona ho ANSWER BOOK ho taka (draw) cumulative frequency curve ho data eka hodimo. (3)
- 2.3 Bontsha ka ho hlakileng grafong moo the estimates tsa lower quartile (Q_1) le median (M) speeds di tla balwa teng (can be read off). Ngola estimates tsena. (2)
- 2.4 Toroya (draw) box and whisker diagram ya data. Sebedisa molapalo (number line) etla fumaneha ho ANSWER BOOK. (2)
- 2.5 Sebedisa grafo (graph) ho akanya (estimata) lenane la bapalami ba dibaesekele le fetileng point ka lebelo le fetang 35 km/h. (1)
- [10]**

POTSO 3

Ho setshwantshong se latelang, P, Q (0 ; -2) and R (6 ; -4) ke kgutlo tse tharo tsa kgutlotharo PQR. Ekweishini ya PQ ke $3x - y - 2 = 0$. Ekweishini ya PR ke $y = -x + 2$. RS ke perpendicular hotswa ho R ho y -axis. $\widehat{QPR} = \theta$.



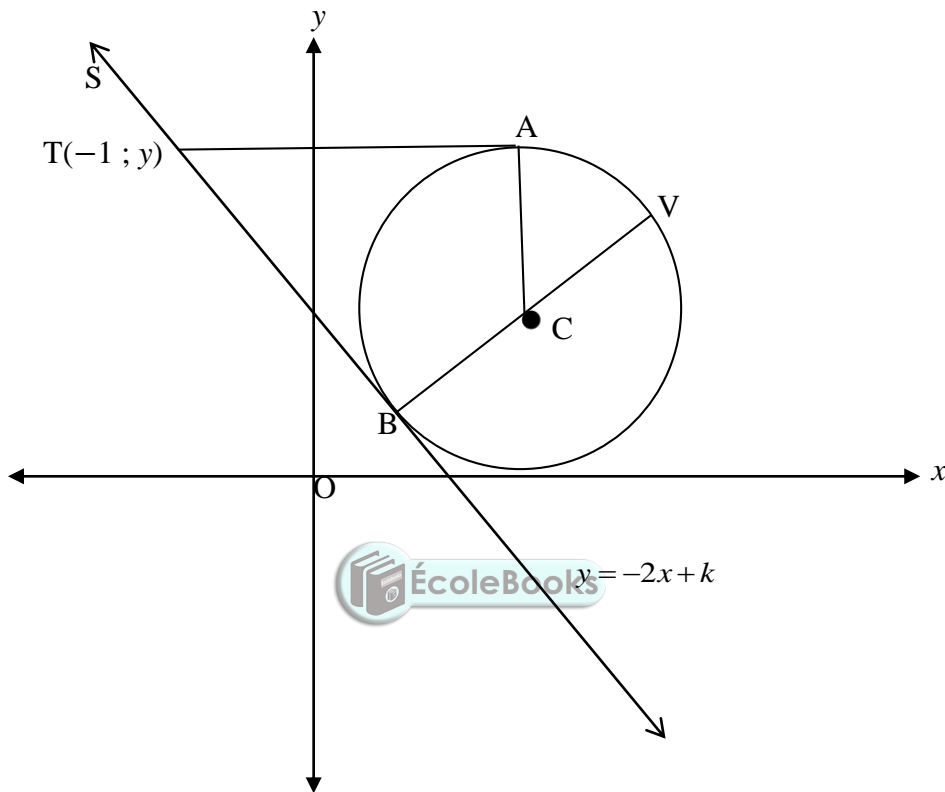
- 3.1 Batla le ho ngola gradient ya QR. (2)
- 3.2 Prova hore $\widehat{PQR} = 90^\circ$. (2)
- 3.3 Fumana coordinates tsa P. (3)
- 3.4 Batla bolelele ba QR. Karabo ha hao ebe ho surd form. (2)
- 3.5 Fumana ekweishini ya sedikadikwe Q, P and R. Karabo ha hao ebe ka tsela e latelang: $(x - a)^2 + (y - b)^2 = r^2$. (5)
- 3.6 Batla size ya angle θ . (5)
- 3.7 Fumana area ya ΔPQR . (3)

[22]

POTSO 4

Ho diagramong e latelang, C ke bohare (centre) ba sedikadikwe se hlalositsweng tjena $x^2 - 6x + y^2 - 4y + 9 = 0$.

T $(-1; y)$ ke point e kantle ho sedikadikwe. Tangents tse 2 di takiwe ho ya ho sedikadikwe hotswa ho T. STB ke tangent ho ya ho sedikadikwe ho B and ekweishini ya yona ke $y = -2x + k$. TA ke tangent ya sedikadikwe ho A hape e parallel ho x -axis. BV ke diameter ya sedikadikwe.



- 4.1 Batla coordinates tsa C. (4)
- 4.2 Fumana ekweishini ya BV. (3)
- 4.3 Fumana ekweishini ya line TA. (1)
- 4.4 Batla bolelele ba line TB. Fana ka mabaka. (4)
- 4.5 Sebetsa le ho ngola value ya k . (2)
- 4.6 Fumana size ya \widehat{ACB} . Ngola lebaka kapa mabaka. (4)

[18]

POTSO 5

5.1 Haeba $\cos 2\theta = p$, fumana tse latelang p e fumaneha karabong (in terms of p):

5.1.1 $\cos 158^\circ$ (2)

5.1.2 $\sin 112^\circ$ (2)

5.1.3 $\sin 38^\circ$ (4)

5.2 Fumana di-value tsa P ho interval $[0^\circ; 360^\circ]$ tsetla kgotsofatsa ekweishini:
 $\sin P = \sin 2P$ (4)

5.3 Haeba $\triangle ABC$ ke scalene triangle, bontsha hore: $\cos(A + B) = -\cos C$ (2)

5.4 Prova identity e latelang:

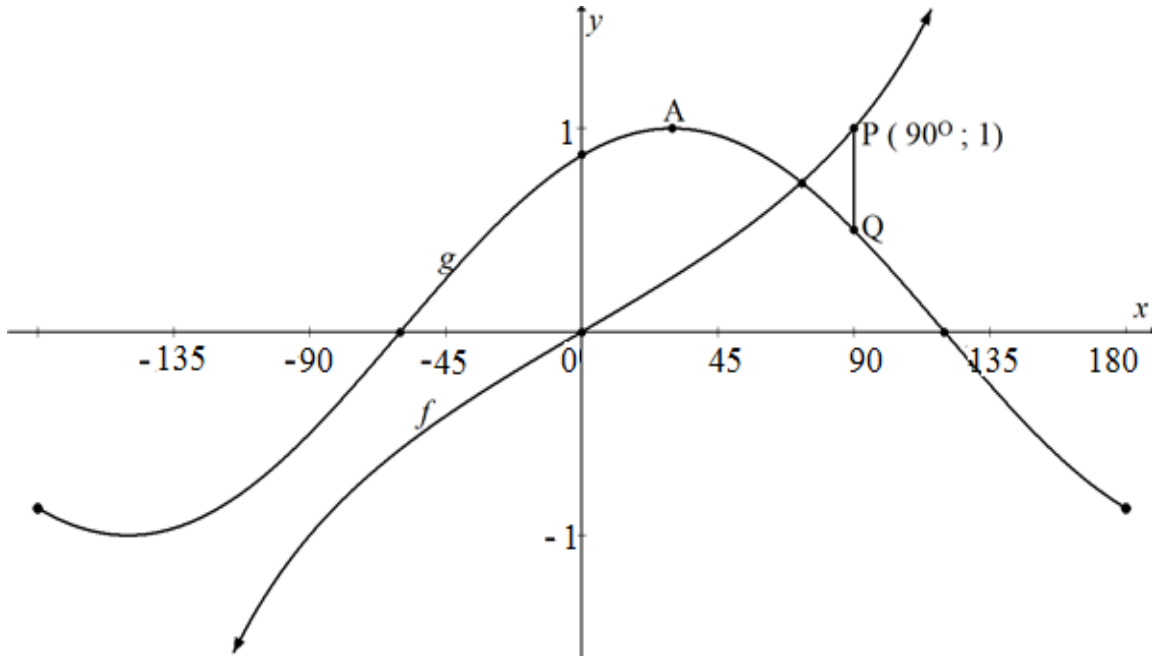
$$\frac{\cos^2 x - \cos x - \sin^2 x}{2 \sin x \cdot \cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x} \quad (5)$$

5.5 Fumana general solution ya: $4 + 7 \cos \theta + \cos 2\theta = 0$. (6)
[25]



POTSO 6

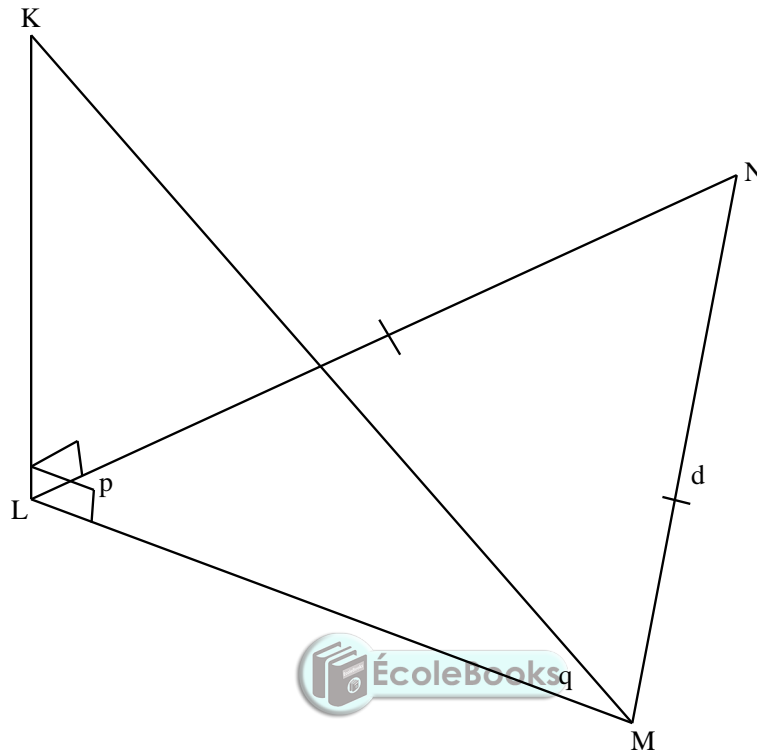
Ho setshwantsho se latelang, digrafo tsa $f(x) = \tan bx$ and $g(x) = \cos(x - 30^\circ)$ di takilwe ho axes tse tshwanang ho $-180^\circ \leq x \leq 180^\circ$. Di-point $P(90^\circ; 1)$ le Q di robetse ho f and g ka mokgwa oo. Sebedisa setshwantsho se latelang ho araba dipotso.



- 6.1 Fumana value ya b . (1)
- 6.2 Ngola fatshe coordinates tsa A , e leng turning point ya g . (2)
- 6.3 Haeba PQ e parallel ho y -axis, fumana coordinates tsa Q . (2)
- 6.4 Ngola fatshe ekweishini ya asymptote(s) ya $y = \tan b(x + 20^\circ)$ ho $x \in [-180^\circ; 180^\circ]$. (1)
- 6.5 Fumana renje ya h if $h(x) = 2g(x) + 1$. (2)
- [8]**

POTSO 7

Points L, M and N di ho horizontal plane ele nngwe. KL ke tora (tower) e tsepameng (vertical). Engele ya elevation ya K hotswa ho M ke q° . $\widehat{NLM} = p^\circ$; $NL = NM = d$ and $KL = h$.



7.1 Fumana size ya \widehat{LNM} moo p e tlabe ele karabong (in terms of p). (2)

7.2 Prova hore $LM = \frac{d \sin 2p}{\sin p}$. (2)

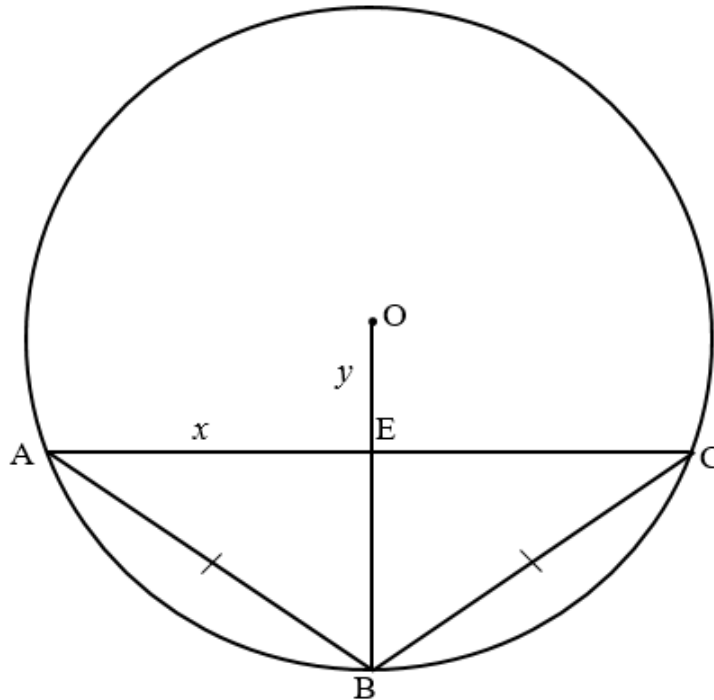
7.3 Tswelapele ho bontsha hore $h = 2d \cos p \tan q$. (3)
[7]

POTSO 8

8.1 Tlatselisa setatemente sa theorem se latelang:

The line drawn from the centre of a circle perpendicular to a chord ... (1)

8.2 Ho setshwantsho se latelang, sedikadikwe ABC le bohare, O, di fuwe. $OB = 8$ units hape $AB = BC = 10$ units. E, ke point e mahareng ya AC. Hare behe $OE = y$ and $AE = x$.

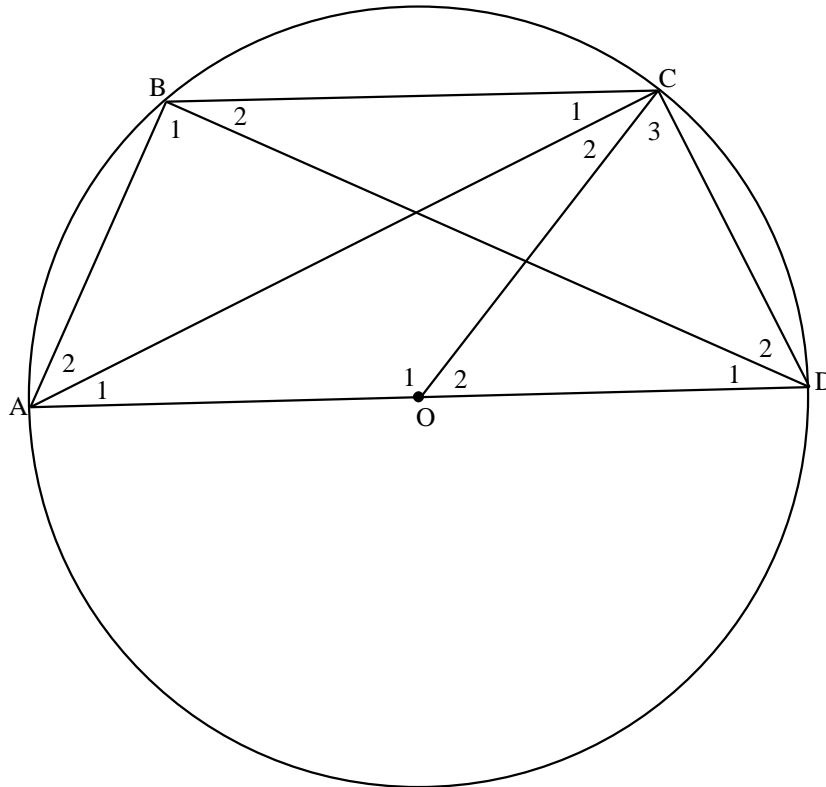


Fumana, ka mabaka, bolelele ba OE. (5)

8.3 Tlatselisa setatemente sa theorem se latelang:

The angle subtended by an arc at the centre of a circle is ... at the circle (on the same side of the chord as the centre). (1)

8.4 Setshwantshong se latelang, O, ke bohare ba sedikadikwe ABCD. AOD ke diameter and OC ke radius. AB, BC, CD, AC and BD ke di straight lines.



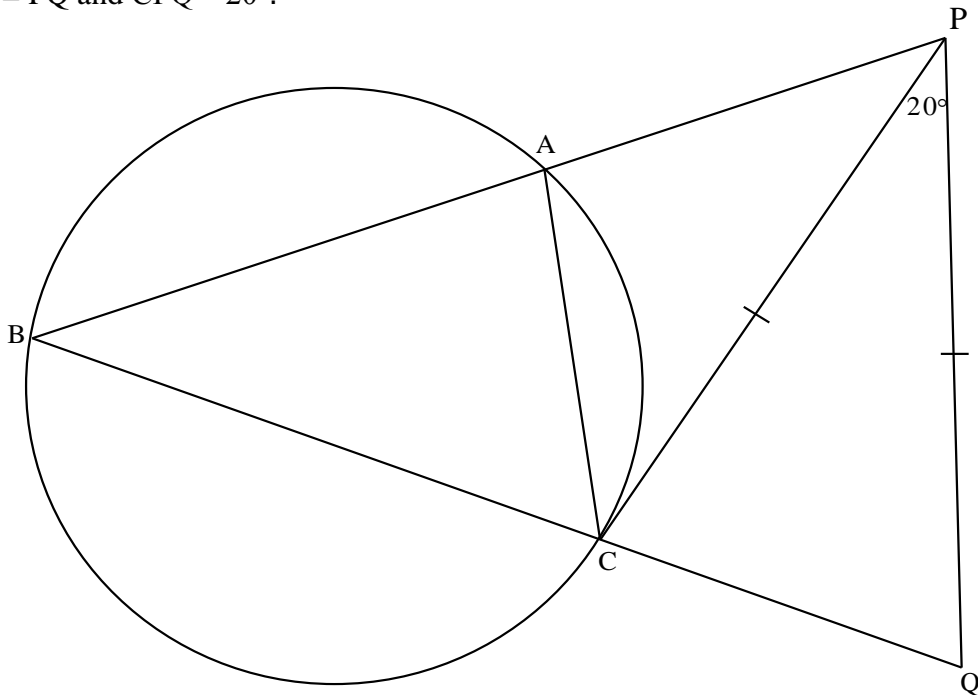
Ngola fatshe, ka mabaka, ekweishini e bontshang kamano pakeng tsa dihlopha tsa di engele.

	DIENGELE	EKWEISHINI/ KAMANO	LEBAKA
e.g.	$\hat{M}_3; \hat{P}$	$\hat{M}_3 = 2 \times \hat{P}$	\angle at centre = $2 \times \angle$ at circum.
8.4.1	$\hat{O}_2; \hat{B}_2$		
8.4.2	$\hat{D}_1; \hat{C}_3; \hat{D}_2$		
8.4.3	$\hat{B}_1; \hat{B}_2; \hat{D}_1; \hat{D}_2$		
8.4.4	$\hat{D}_1; \hat{C}_1$		

(8)
[15]

POTSO 9

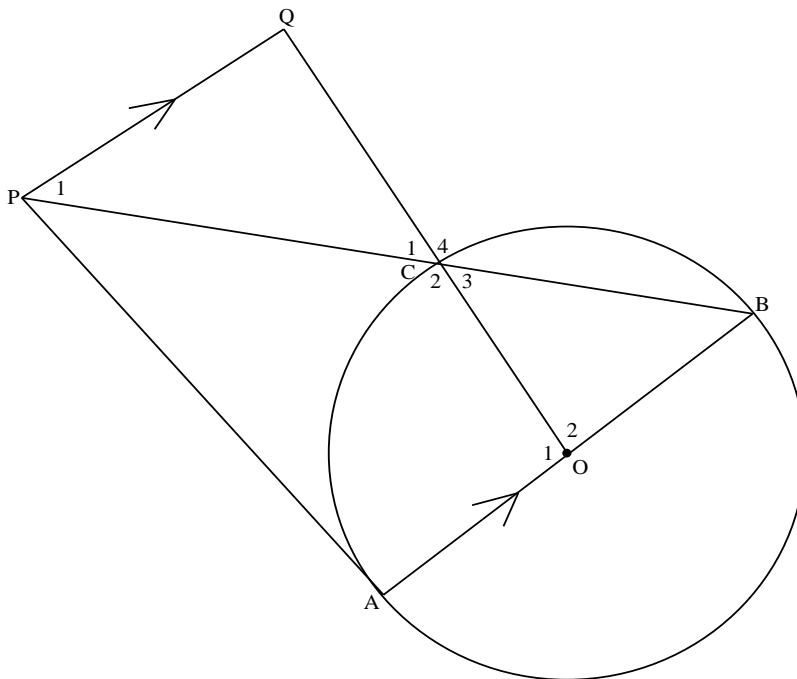
- 9.1 Hare fuwe hore PC ke tangent ya sedikadikwe ACB; BAP and BCQ ke di-straight lines.
 PC = PQ and $\hat{C}PQ = 20^\circ$.



Prova, o fana ka mabaka, hore BC hase diameter.

(5)

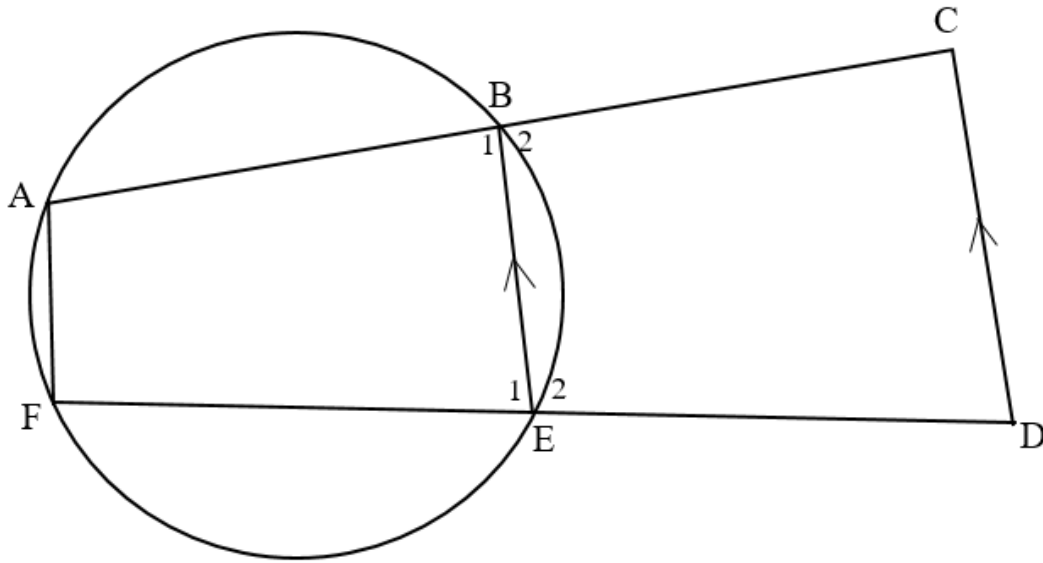
- 9.2 Ho setshwantsho ae latelang, O ke bohare ba sedikadikwe ABC. Tangent PA ho sedikadikwe le diameter AB di kopana ho A. OCQ and BCP ke di-straight lines. $PQ \parallel AB$.



Prova, o fana ka mabaka, hore $PQ = QC$.

(6)

- 9.3 Ho setshwantsho se latelang, chords AB and FE tsa didikadikwe le bohare O, di takilwe ho ya ho points C and D. $BE \parallel CD$.



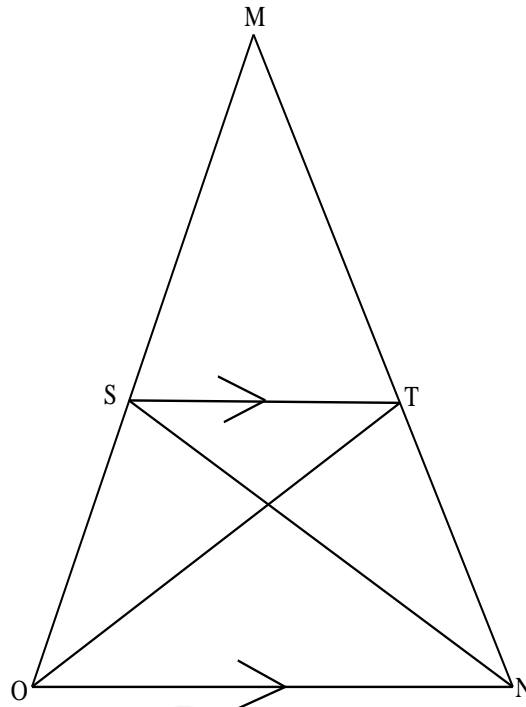
Prova hore ACDF ke cyclic quadrilateral.

(5)
[16]



POTSO 10

- 10.1 Ho setshwantsho, $\triangle MONE$ takilwe. S ke point ho MO and T ke point ho MN hore $ST \parallel ON$, SN and OT di takilwe.

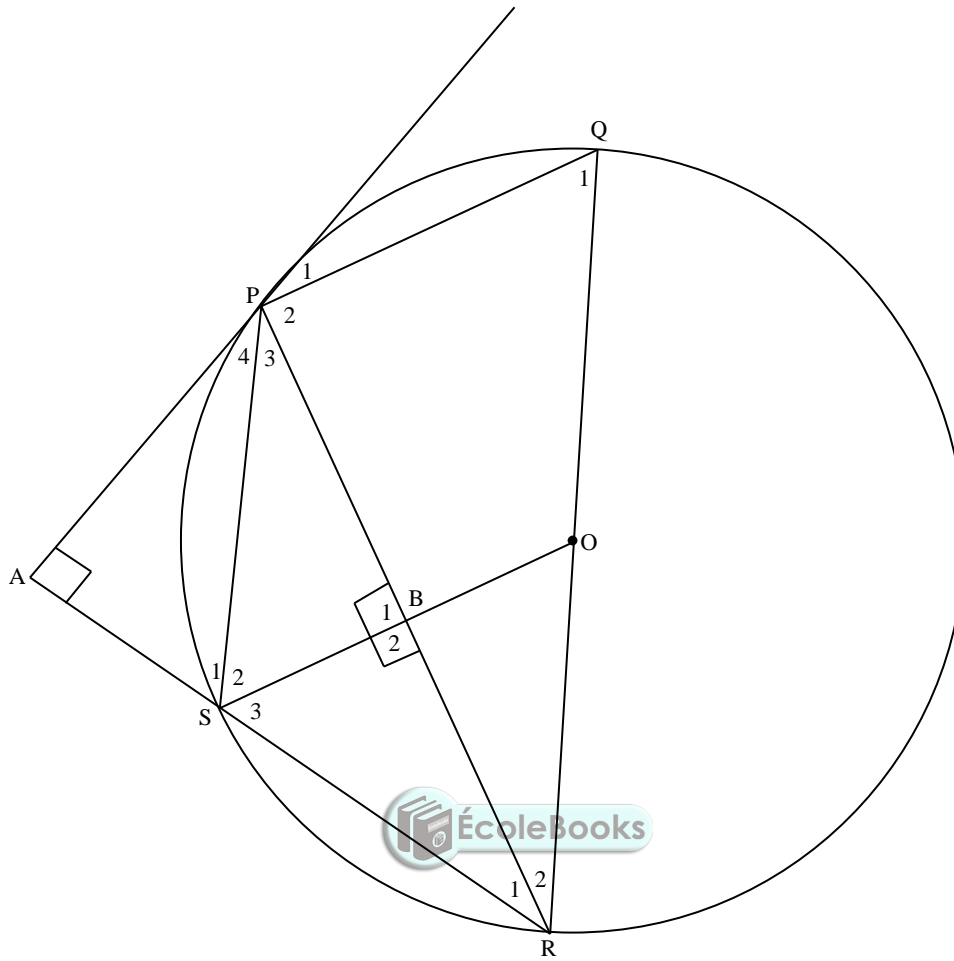


Sebedisa setshwantsho ho prova theorem e bolelang hore a line parallel to one side of a triangle divides the other two sides proportionally. Ka mantswe a mang, prova

hore: $\frac{MS}{SO} = \frac{MT}{TN}$.

(5)

10.2 Ho setshwantsho, O ke bohare ba sedikadikwe. PQRS ke cyclic quadrilateral. Tangent ho feta ho P e kopana le RS le ho eketswa hoja ho A. $OB \perp PR$ and $PA \perp AS$.



Prova hore:

10.2.1 $\Delta APS \parallel \Delta BRS$ (3)

10.2.2 $AP \cdot RS = BR \cdot PS$ (1)

10.2.3 $\hat{P}_4 = \hat{R}_2$ (4)

10.2.4 $BR \cdot RQ = RS \cdot RP$ (6)

[19]

MATSHWOA OHLE A PAMPIRI ENA : 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n \quad \sum_{i=1}^n i = \frac{n(n+1)}{2} \quad T_n = a + (n-1)d \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1 \quad S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \quad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$



$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases} \quad \sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$