



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2018

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 24pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

QUESTION/VRAAG 1

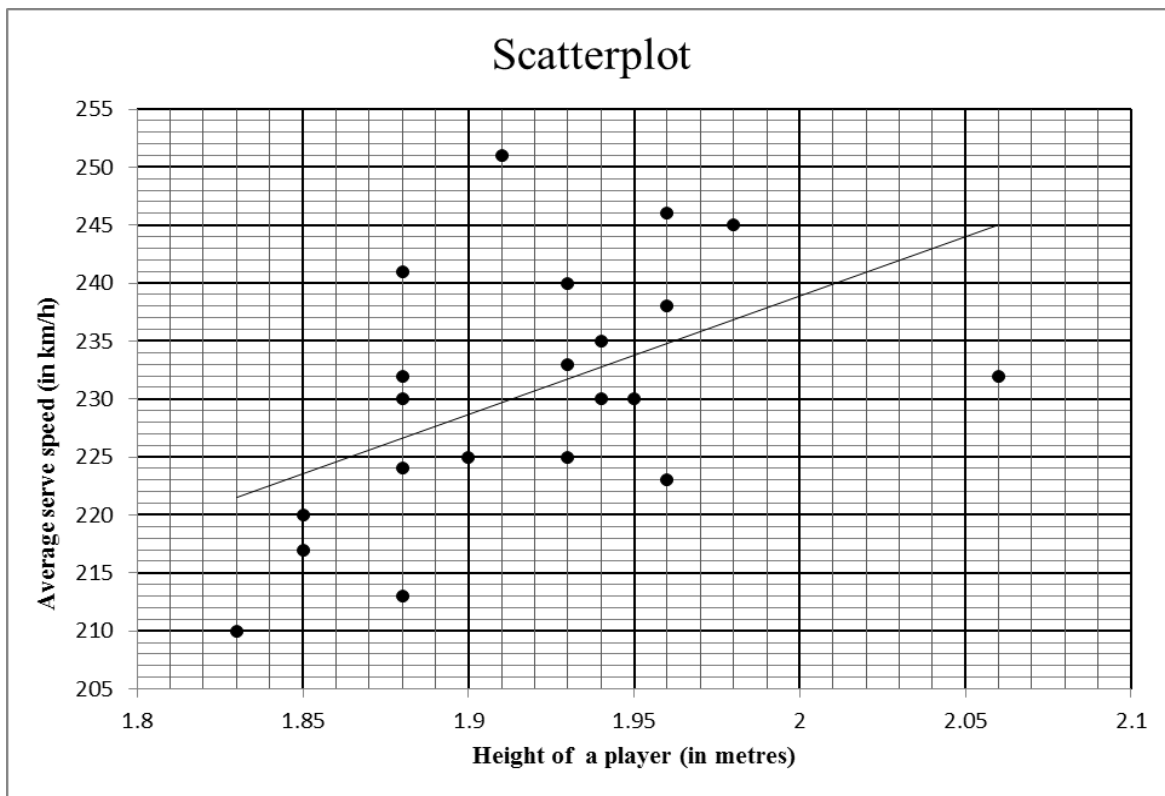
1.1.1	140 items	✓ answer	(1)
1.1.2	Modal class/modale klas: $20 < x \leq 30$ minutes OR/OF $20 \leq x < 30$ minutes	✓ answer ✓ answer	(1) (1)
1.1.3	Number of minutes taken = 20 minutes	✓ answer	(1)
1.1.4	140 – 126 [Accept: 124 to 128] 14 orders (12 to 16) <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Answer only: Full marks</div>	✓ 126 ✓ answer	(2)
1.1.5	75 th percentile is at 105 items =37 minutes [accept 36 – 38 minutes] <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Answer only: Full marks</div>	✓ 105 ✓ answer	(2)
1.1.6	Lower quartile is at 35 items =21,5 min [accept 21 – 23 min] IQR = 37 – 21,5 = 15,5 min [accept 13 – 17 min] <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Answer only: Full marks</div>	✓ lower quartile (Q ₁) ✓ answer	(2)


35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1(a)	$\bar{x} = \frac{1420}{15}$ = R94,666... = R94,67 <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Answer only: Full marks</div>	✓ 1420 ✓ answer	(2)
1.2.1(b)	$\sigma = R22,691... = R22,69$	✓✓ answer	(2)
1.2.2(a)	They both collected the same (equal) amount in tips, i.e. R1 420 over the 15-day period. <i>Hulle albei het dieselfde bedrag met footjies ontvang, nl. R1 420 oor die 15 dae-tydperk</i>	✓ answer	(1)
1.2.2(b)	Mary's standard deviation is smaller than Reggie's which suggests that there was greater variation in the amount of tips that Reggie collected each day compared to the number of tips that Mary collected each day. <i>Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar groter variasie/verspreiding in die footjies was wat Reggie elke dag ontvang het in vergelyking met die getal footjies wat Marie elke dag ontvang het.</i>	✓ explanation	(1)

[15]

QUESTION/VRAAG 2

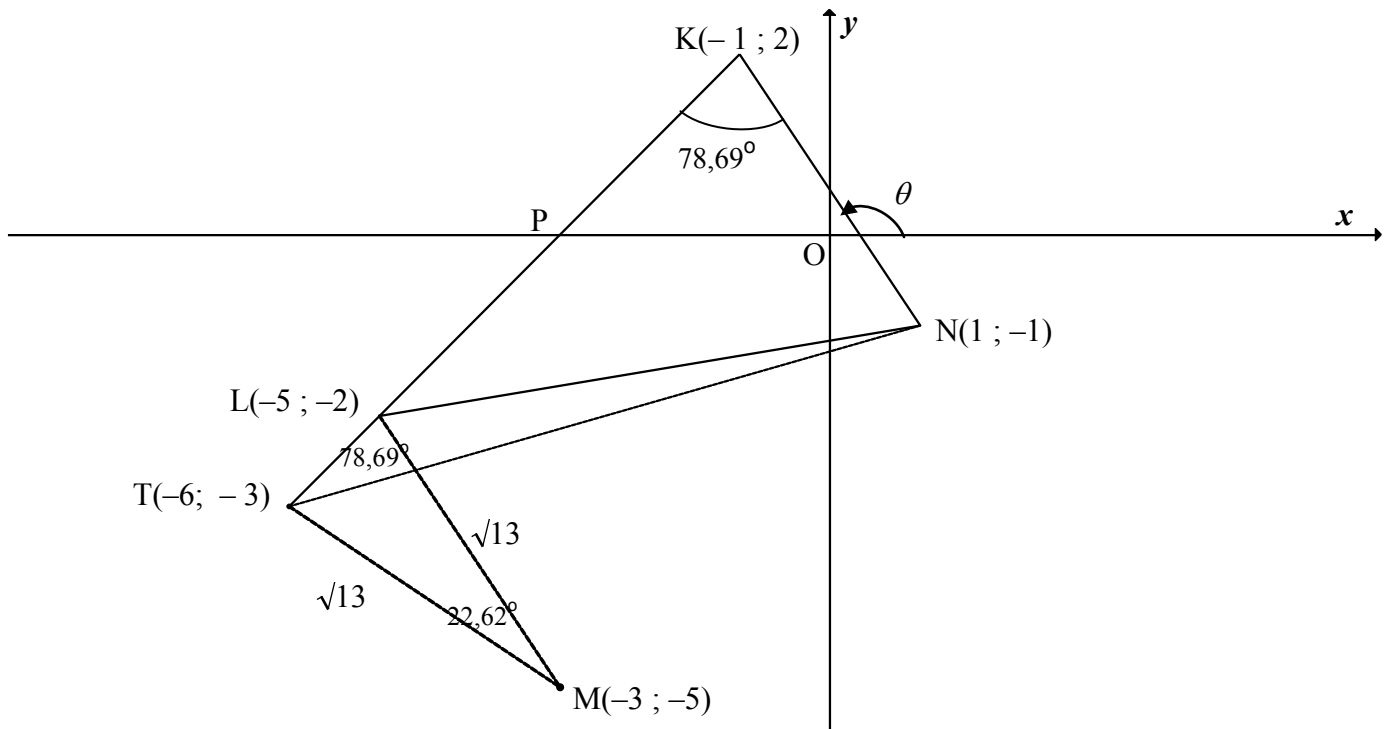



2.1	251 km/h		✓ answer	(1)
2.2.1	$r = 0,52$ OR C		✓ answer	(1)
2.2.2	The points are fairly scattered and the least squares regression line is increasing. <i>Die punte is redelik verspreid en die kleinstekwadrate-regressielyn neem toe.</i>		✓ reason	(1)
2.3	There is a weak positive relation hence the height could have an influence <i>Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.</i>		✓ answer	(1)
	OR/OF There is no conclusive evidence that the height of a player will influence his/her tennis serve speed. <i>Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.</i>		✓ answer	(1)
	OR/OF There is no conclusive evidence that a taller person will serve faster than a shorter person. <i>Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.</i>		✓ answer	(1)

2.4	<p>For $(0 ; 27,07)$, it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h. It is impossible for a person to have a height of 0 m.</p> <p><i>(0 ; 27,07) beteken dat 'n speler 'n lengte van 0 m kan hê en teen 'n spoed van 27,07 km/h kan afslaan. Dit is onmoontlik om 'n lengte van 0 m te hê.</i></p> <p>OR/OF</p> <p>This means that the player does not exist and therefore cannot serve and have a serve speed.</p> <p><i>Dit beteken dat die speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.</i></p>	<p>✓ explanation (1)</p> <p>✓ explanation (1)</p>
[5]		



QUESTION/VRAAG 3



3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2 - (-1)}{-1 - 1}$ $= -\frac{3}{2}$ <div style="text-align: center; border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Answer only: Full marks </div>	<div style="text-align: center;">  </div> ✓ correct substitution ✓ answer (2)
3.1.2	$\tan \theta = m_{KN} = -\frac{3}{2}$ $\theta = 180^\circ - 56,31^\circ$ $\theta = 123,69^\circ$ <div style="text-align: center; border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> Answer only: Full marks </div>	✓ $\tan \theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)
3.2	Inclination $KL = 123,69^\circ - 78,69^\circ = 45^\circ$ [ext $\angle \Delta$] $\tan 45^\circ = m_{KL} = 1$	✓ S ✓ $\tan 45^\circ = m_{KL} = 1$ (2)
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ <p>OR/OF</p> $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$	✓ substitute $(-1; 2)$ and m ✓ equation (2) ✓ substitute $(-1; 2)$ and m ✓ equation (2)

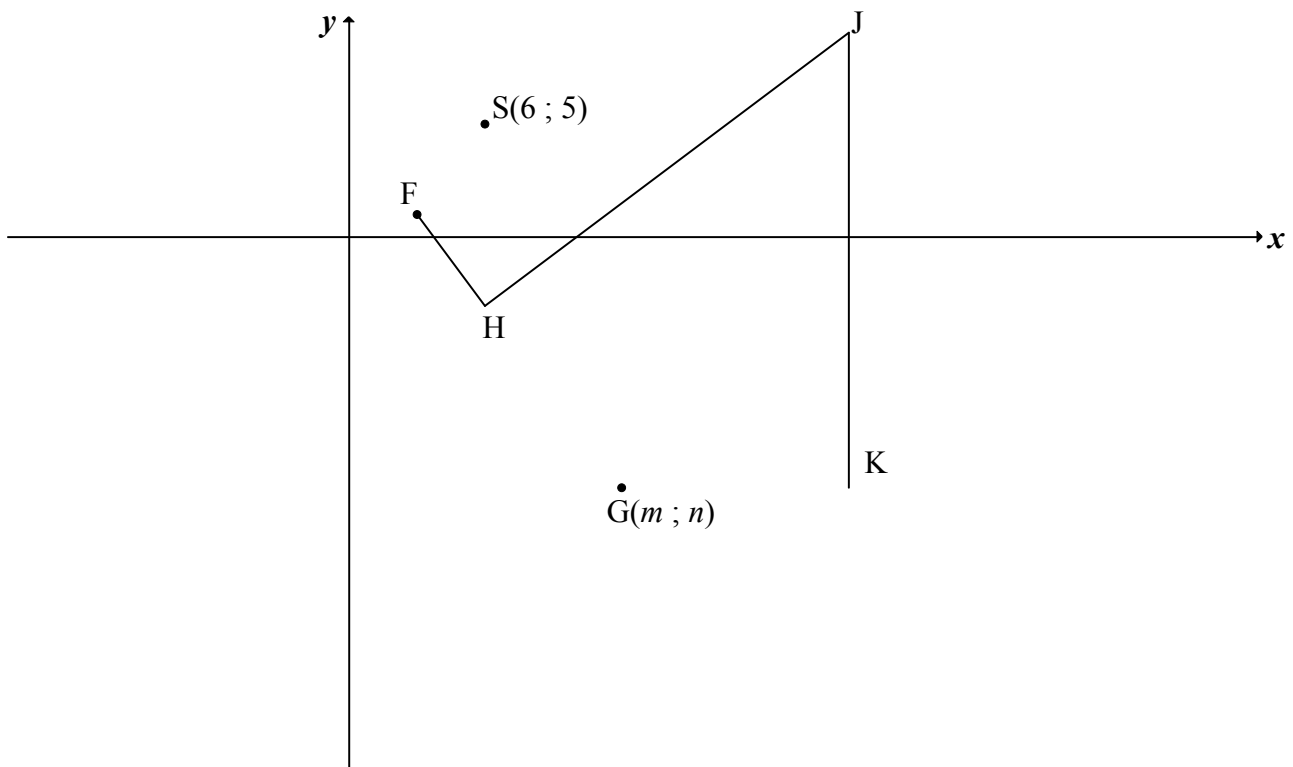
3.4	$KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13} \text{ or } 3,61$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: Full marks</div>	✓ substitute K and N into distance formula ✓ answer (2)
3.5.1	$(x+3)^2 + (y+5)^2 = 13 \quad \dots(1)$ L is a point on KL $y = x + 3 \quad \dots(2)$ (2) in (1): $(x+3)^2 + (x+3+5)^2 = 13$ $x^2 + 6x + 9 + x^2 + 16x + 64 = 13$ $2x^2 + 22x + 60 = 0$ $x^2 + 11x + 30 = 0$ $(x+5)(x+6) = 0$ $x = -5 \text{ or } x = -6$ $y = -2 \text{ or } y = -3$ L(-5 ; -2) or (-6 ; -3) OR/OF $(x+3)^2 + (y+5)^2 = 13 \quad \dots(1)$ L is a point on KL $y = x + 3 \quad \therefore x = y - 3 \quad \dots(2)$ (2) in (1): $(y-3+3)^2 + (y+5)^2 = 13$ $y^2 + y^2 + 10y + 25 = 13$ $2y^2 + 10y + 12 = 0$ $y^2 + 5y + 6 = 0$ $(y+2)(y+3) = 0$ $y = -2 \text{ or } y = -3$ $x = -5 \text{ or } x = -6$ L(-5 ; -2) or (-6 ; -3)	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ x-values ✓ y-values (5) ✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ y-values (both) ✓ x-values (both) (5)
3.5.2	Midpoint of KM: (-2 ; -1,5) $\therefore \frac{x_L + 1}{2} = -2 \text{ and } \frac{y_L - 1}{2} = -\frac{3}{2}$ $\therefore L(-5 ; -2)$ OR/OF $m_{KN} = m_{LM}$ $\frac{y - (-5)}{x - (-3)} = -\frac{3}{2}$ $2(x+3+5) = -3(x+3)$ $2x+16 = -3x-9$ $5x = -25$ $x = -5$ $\therefore L(-5 ; -2)$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: Full marks</div>	✓ midpoint of KM ✓ x value ✓ y value (3) ✓ $m_{LM} = m_{KN}$ ✓ x value ✓ y value (3)

	<p>OR/OF $N \rightarrow M:$ $(x; y) \rightarrow (x - 4; y - 4)$ $\therefore L(-1 - 4; 2 - 4)$ $\therefore L(-5; -2)$</p> <p>$N \rightarrow K:$ $(x; y) \rightarrow (x - 2; y + 3)$ $\therefore L(-3 - 2; -5 + 3)$ $\therefore L(-5; -2)$</p>	
	<p>OR/OF $N \rightarrow M:$ $(x; y) \rightarrow (x - 4; y - 4)$ $\therefore L(-1 - 4; 2 - 4)$ $\therefore L(-5; -2)$</p> <p>$N \rightarrow K:$ $(x; y) \rightarrow (x - 2; y + 3)$ $\therefore L(-3 - 2; -5 + 3)$ $\therefore L(-5; -2)$</p>	
	<p>OR/OF $N \rightarrow M:$ $(x; y) \rightarrow (x - 4; y - 4)$ $\therefore L(-1 - 4; 2 - 4)$ $\therefore L(-5; -2)$</p> <p>$N \rightarrow K:$ $(x; y) \rightarrow (x - 2; y + 3)$ $\therefore L(-3 - 2; -5 + 3)$ $\therefore L(-5; -2)$</p>	
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	<p>OR/OF $N \rightarrow M:$ $(x; y) \rightarrow (x - 4; y - 4)$ $\therefore L(-1 - 4; 2 - 4)$ $\therefore L(-5; -2)$</p> <p>$N \rightarrow K:$ $(x; y) \rightarrow (x - 2; y + 3)$ $\therefore L(-3 - 2; -5 + 3)$ $\therefore L(-5; -2)$</p>	<p>✓ transformation</p> <p>✓ x value ✓ y value</p> <p>(3)</p>
3.6	<p>$T(-6; -3)$ (from Question 3.5.1)</p> $KT = \sqrt{(-1 - (-6))^2 + (2 - (-3))^2}$ $= \sqrt{50}$ <p>$KN = \sqrt{13}$ (CA from 3.4)</p> $\text{Area of } \Delta KTN = \frac{1}{2} KT \cdot KN \sin \hat{LKN}$ $= \frac{1}{2} \sqrt{50} \cdot \sqrt{13} \sin 78,69^\circ$ $= 12,50 \text{ square units}$	<p>✓ coordinates of T</p> <p>✓ length of KT</p> <p>✓ substitution into area rule</p> <p>✓ answer</p> <p>(4)</p>

	<p>OR/OF</p> <p>In ΔKLM:</p> $\frac{TL}{\sin 22,62^\circ} = \frac{\sqrt{13}}{\sin 78,69^\circ}$ $TL = 1,414..$ $KL = \sqrt{(-1 - (-5))^2 + (2 - (-2))^2}$ $= \sqrt{32}$ $\therefore KT = 7,0708...$ <p>Area of $\Delta KTN = \frac{1}{2} KT \cdot KN \sin \hat{LKN}$</p> $= \frac{1}{2} (7,0708) \cdot \sqrt{13} \sin 78,69^\circ$ $= 12,50 \text{ square units}$	<p>✓ length of TL</p> <p>✓ length of KT</p> <p>✓ substitution into area rule</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
[22]		



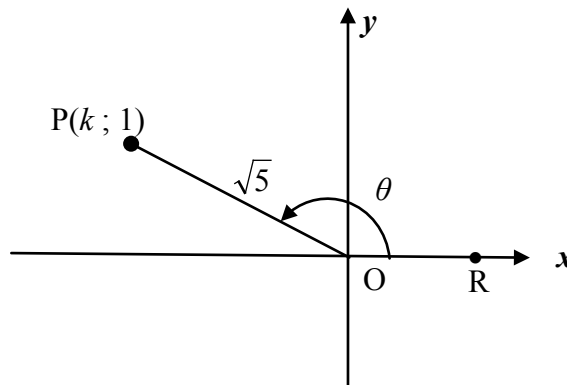
QUESTION/VRAAG 4



4.1	F(3;1)	✓ x value ✓ y value (2)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ $FS = 5$	✓ substitution of F & S ✓ answer (2)
4.3	$FH(FS) : HG = 1 : 2$ $\therefore HG = 2 FH$ $= 10$	✓ $HG = 10$ (1)
4.4	Tangents from common/same point / <i>Raaklyne vanaf gemeenskaplike of dieselfde punt</i>	✓ answer (1)
4.5.1	$\hat{F}HJ = 90^\circ$ [tan \perp radius / rkl \perp radius] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/stelling] $FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x - m)^2 + (y - n)^2 = 100$	✓ answer (1)

<p>4.5.3</p>	<p>K(22; n) GK = HG = 10 FH = FS = 5 $m = 22 - 10$ $m = 12$ F, H and G are collinear <i>F, H en G is saamlynig</i> $FG^2 = (12 - 3)^2 + (n - 1)^2$ $15^2 = 81 + (n - 1)^2$ $(n - 1)^2 = 144$ $n - 1 = \pm 12$ $n \neq 13$ or $n = -11$ $\therefore G(12; -11)$</p> <p>OR/OF</p> <p>K(22; n) GK = HG = 10 FH = FS = 5 $m = 22 - 10$ $m = 12$ Let J(22 ; y): $FJ^2 = (22 - 3)^2 + (y - 1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y - 9)(y + 7)$ $\therefore y = 9$ or/of $y \neq -7$ $\therefore n = 9 - 20 = -11$ $\therefore G(12; -11)$</p>	<p>[radius \perp tangent] [radii] [radii]</p> <p>[HJ is a common tangent] <i>[HJ is 'n gemeenskaplike raaklyn]</i></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $n^2 - 2n - 143 = 0$ $(n + 11)(n - 13) = 0$ $n = -11$ or $n \neq 13$ </div> <p>[radius \perp tangent] [radii] [radii]</p>	<p>✓ K(22; n)</p> <p>✓ value of m</p> <p>✓ subst. of F and G in distance formula ✓ $FG = 15$</p> <p>✓ simplification/ standard form ✓ value of n ✓ coordinates of G (7)</p> <p>✓ K(22; n)</p> <p>✓ value of m</p> <p>✓ subst. of F and J in distance formula ✓ $FJ = \sqrt{425}$ ✓ standard form</p> <p>✓ value of n ✓ coordinates of G (7)</p>
[18]			


QUESTION/VRAAG 5



5.1.1	$k^2 = (\sqrt{5})^2 - 1^2$ $= 4$ $k = -2$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>	✓ substitution into theorem of Pythagoras ✓ answer (2)
5.1.2(a)	$\tan \theta = -\frac{1}{2}$	✓ answer (1)
5.1.2(b)	$\cos(180^\circ + \theta) = -\cos \theta$ $= \frac{2}{\sqrt{5}}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>	✓ reduction ✓ answer (2)
5.1.2(c)	$\sin(\theta + 60^\circ) = \frac{a+b}{\sqrt{20}}$ $\text{LHS} = \sin \theta \cos 60^\circ + \cos \theta \sin 60^\circ$ $= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-2\sqrt{3}}{2\sqrt{5}}$ $= \frac{1-2\sqrt{3}}{\sqrt{20}}$	✓ expansion ✓ subst of $\sin \theta$ ✓ subst of $\cos \theta$ ✓ both special \angle s ✓ $\frac{1-2\sqrt{3}}{2\sqrt{5}}$ (5)
5.1.3	$\tan \theta = -\frac{1}{2}$ $\therefore \theta = 180^\circ - 26,57^\circ$ $\therefore \theta = 153,43^\circ$ $\tan(2\theta - 40^\circ) = \tan[(2 \times 153,43^\circ) - 40^\circ]$ $= \tan 266,87^\circ$ $= 18,3$	✓ θ ✓ substitution ✓ answer (3)

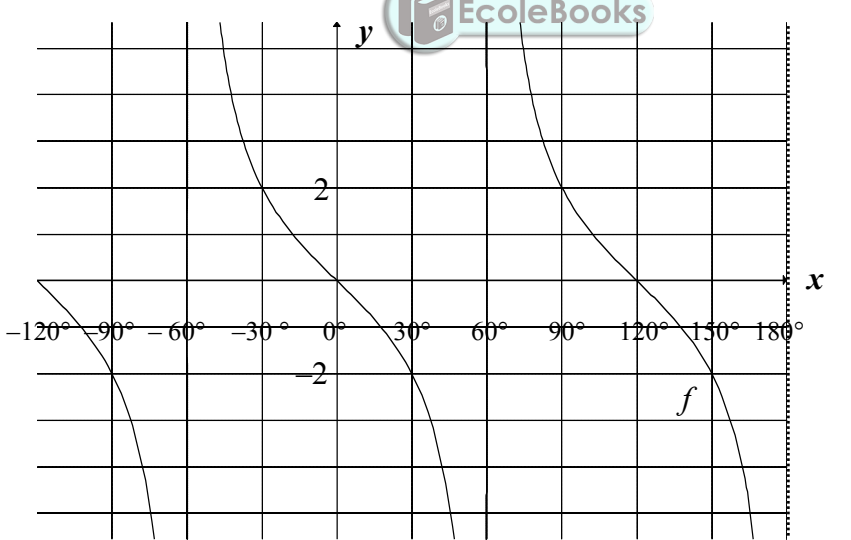
<p>5.2</p>	<p>LHS = $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$ RHS = $2 \tan 2x$</p> <p>= $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$</p> <p>= $\frac{\cos^2 x + 2 \sin x \cos x + \sin^2 x - \cos^2 x + 2 \sin x \cos x - \sin^2 x}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{2(2 \sin x \cos x)}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{2 \sin 2x}{\cos 2x}$</p> <p>= $2 \tan 2x$</p> <p>= RHS</p> <p>OR/OF</p> <p>LHS = $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$ RHS = $2 \tan 2x$</p> <p>= $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$</p> <p>= $\frac{(\cos x + \sin x + \cos x - \sin x)(\cos x + \sin x - \cos x + \sin x)}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{(2 \cos x)(2 \sin x)}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{2(2 \sin x \cos x)}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{2 \sin 2x}{\cos 2x}$</p> <p>= $2 \tan 2x$</p> <p>= RHS</p> <p>OR/OF</p> <p>RHS = $2 \tan 2x$</p> <p>= $\frac{2 \sin 2x}{\cos 2x}$</p> <p>= $\frac{2(2 \sin x \cdot \cos x)}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{4 \sin x \cdot \cos x}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{1 + 2 \sin x \cdot \cos x - (1 - 2 \sin x \cdot \cos x)}{\cos^2 x - \sin^2 x}$</p> <p>= $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$</p> <p>= $\frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} - \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$</p> <p>= $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = \text{LHS}$</p>	<p>✓ single fraction</p> <p>✓ expansion</p> <p>✓ simplification (both)</p> <p>✓ double ∠ identity</p> <p>✓ double ∠ identity</p> <p>(5)</p> <p>✓ single fraction</p> <p>✓ difference of two squares</p> <p>✓ simplification (both)</p> <p>✓ double ∠ identity</p> <p>✓ double ∠ identity</p> <p>(5)</p> <p>✓ double ∠ identity</p> <p>✓ double ∠ identity</p> <p>✓ identity & method</p> <p>✓ factorising numerator and denominator</p> <p>✓ writing as 2 terms</p> <p>(5)</p>
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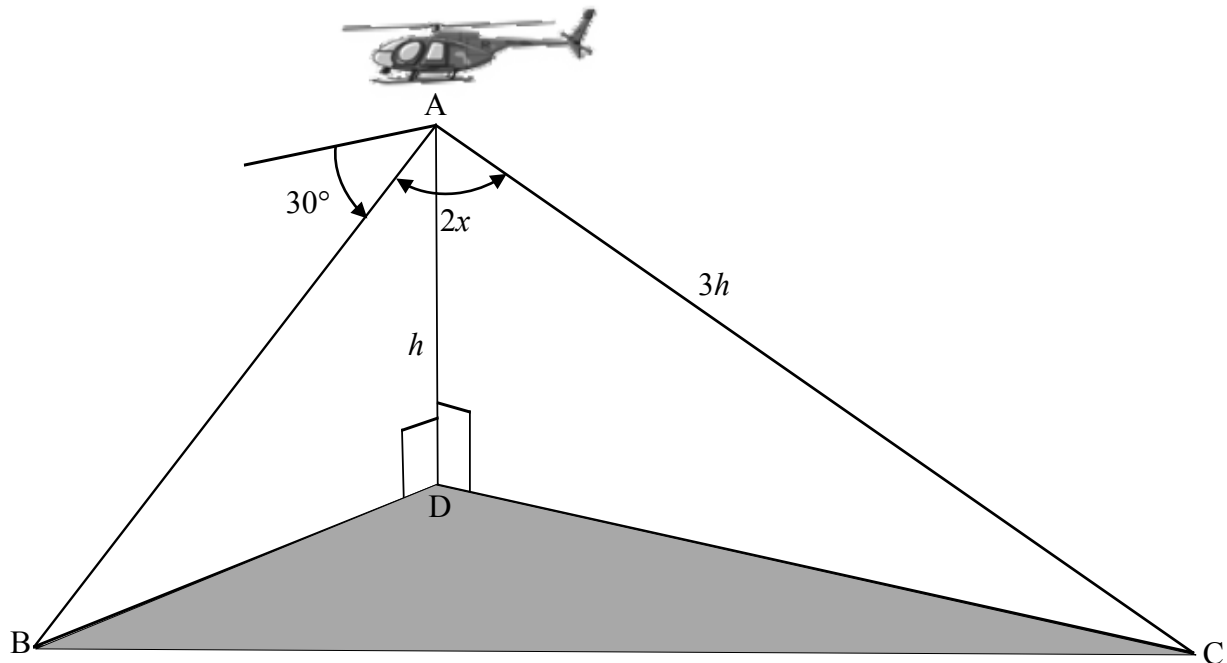
<p>5.3</p>	$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$ $= \cos^2 38^\circ + \cos^2 39^\circ + \cos^2 40^\circ + \dots + \cos^2 51^\circ + \cos^2 52^\circ$ $= \sin^2 52^\circ + \sin^2 51^\circ + \sin^2 50^\circ + \dots + \cos^2 51^\circ + \cos^2 52^\circ$ $= 7(1) + \cos^2 45^\circ$ $= 7 + \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{or} \quad = 7 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= 7\frac{1}{2}$ <p>OR/OF</p> $\sum_{A=38^\circ}^{52^\circ} \cos^2 A$ $= \cos^2 38^\circ + \cos^2 39^\circ + \cos^2 40^\circ + \dots + \cos^2 51^\circ + \cos^2 52^\circ$ $= (\cos^2 38^\circ + \sin^2 52^\circ) + (\cos^2 39^\circ + \sin^2 51^\circ) \dots + \cos^2 45^\circ$ $= 7(1) + \cos^2 45^\circ$ $= 7 + \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{or} \quad = 7 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= 7\frac{1}{2}$ 	<ul style="list-style-type: none"> ✓ expansion ✓ co ratio ✓ $\cos^2 45^\circ$ ✓ $7 \times$ identity ✓ answer <p style="text-align: right;">(5)</p> <ul style="list-style-type: none"> ✓ expansion ✓ pairing ✓ $\cos^2 45^\circ$ ✓ $7 \times$ identity ✓ answer <p style="text-align: right;">(5)</p>
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[23]

QUESTION/VRAAG 6

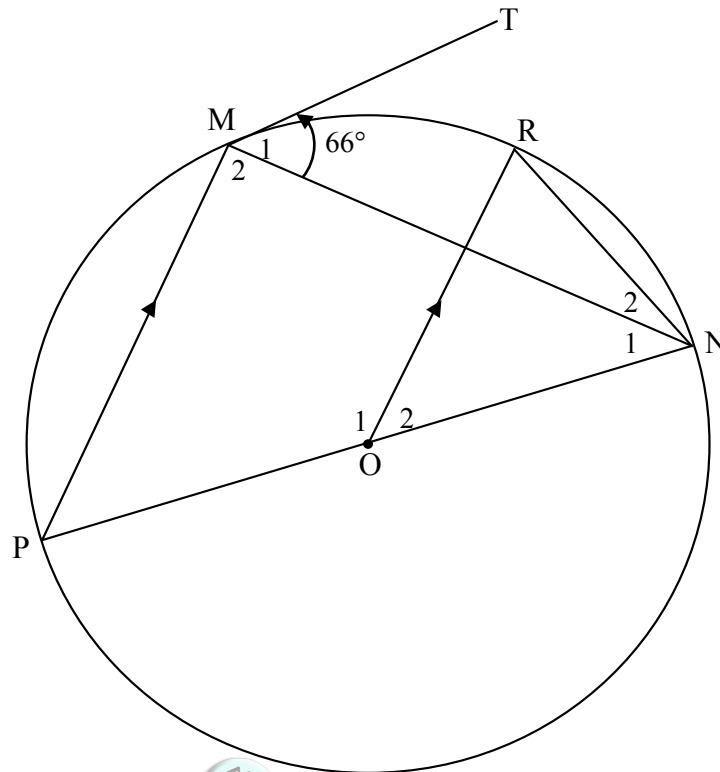
6.1	Period = 120°	✓ answer (1)
6.2	$2 = -2 \tan \frac{3}{2}x$ $\tan \left(\frac{3}{2}t \right) = -1$ $\frac{3}{2}t = 135^\circ + k.180^\circ \quad \text{OR/OF} \quad \frac{3}{2}t = -45^\circ + k.180^\circ$ $t = 90^\circ + k.120^\circ ; k \in Z \quad \quad \quad t = -30^\circ + k.120^\circ ; k \in Z$ <p>OR/OF</p> $2 = -2 \tan \frac{3}{2}x$ $\tan \left(\frac{3}{2}t \right) = -1$ $\frac{3}{2}t = 135^\circ + k.360^\circ \quad \text{or/of} \quad \frac{3}{2}t = 315^\circ + k.360^\circ$ $t = 90^\circ + k.240^\circ \quad \text{or/of} \quad t = 210^\circ + k.240^\circ ; k \in Z$	✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3) ✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3)
6.3		✓ asymptotes: $x = \pm 60^\circ ; x = 180^\circ$ ✓ x-intercepts $0^\circ ; \pm 120^\circ$ ✓ negative shape ✓ $(90^\circ ; 2)$ or $(-30^\circ ; 2)$ or $(30^\circ ; -2)$ or $(-90^\circ ; -2)$ (4)
6.4	$x \in (-60^\circ ; -30^\circ] \text{ or } (60^\circ ; 90^\circ]$ <p>OR/OF</p> $-60^\circ < x \leq -30^\circ \text{ or } 60^\circ < x \leq 90^\circ$	✓ interval ✓ interval ✓ notation (3) ✓ interval ✓ interval ✓ notation (3)
6.5	$g(x) = -2 \tan \left[\frac{3}{2}(x + 40^\circ) \right] = f(x + 40^\circ)$ Translation of 40° to the left / skuif met 40° links	✓ Translation of 40° ✓ to the left (2)
[13]		

QUESTION/VRAAG 7



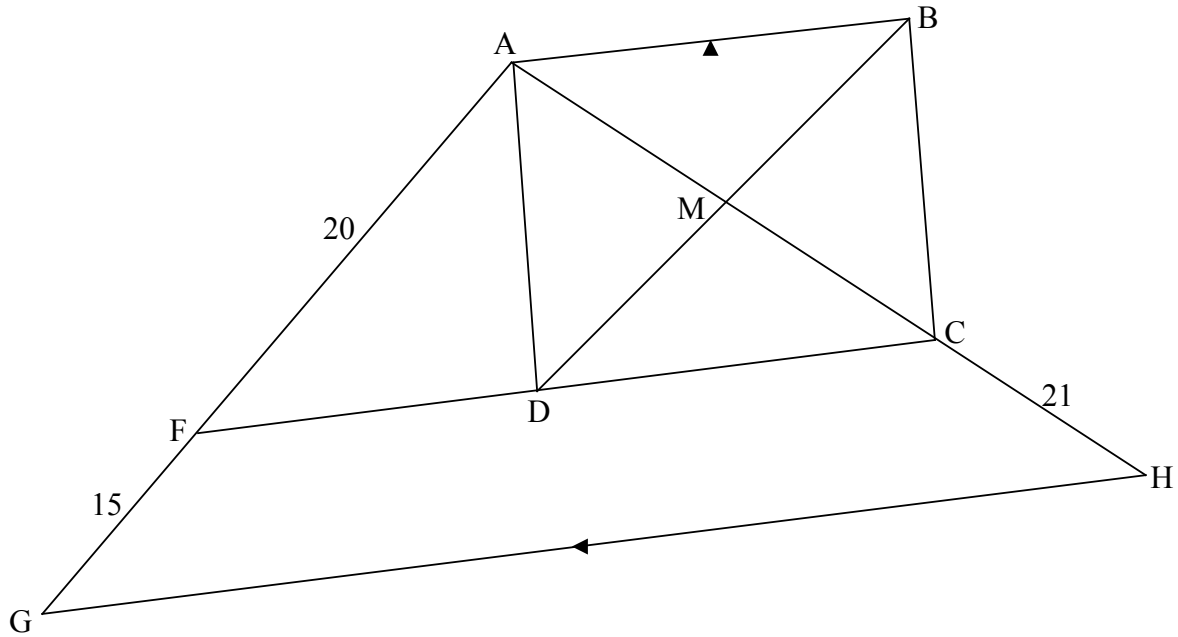
<p>7.1</p>	<p>$\hat{A}BD = 30^\circ$ $\sin 30^\circ = \frac{h}{AB}$ $AB = \frac{h}{\sin 30^\circ}$ OR $AB = \frac{h}{\frac{1}{2}}$ OR $AB = 2h$</p> <p>OR/OF</p> <p>$\hat{B}AD = 60^\circ$ $\cos 60^\circ = \frac{h}{AB}$ $AB = \frac{h}{\cos 60^\circ}$ OR $AB = \frac{h}{\frac{1}{2}}$ OR $AB = 2h$</p>	<p>✓ $\hat{A}BD = 30^\circ$</p> <p>✓ answer (2)</p> <p>✓ $\hat{B}AD = 60^\circ$</p> <p>✓ answer (2)</p>
<p>7.2</p>	<p>$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos \hat{B}AC$ $= (2h)^2 + (3h)^2 - 2(2h)(3h) \cos 2x$ $= 13h^2 - 12h^2 (2 \cos^2 x - 1)$ $= 13h^2 - 24h^2 \cos^2 x + 12h^2$ $= 25h^2 - 24h^2 \cos^2 x$ $BC = h\sqrt{25 - 24 \cos^2 x}$</p>	<p>✓ use of cosine rule in $\triangle ABC$</p> <p>✓ substitution</p> <p>✓ double angle identity</p> <p>✓ $25h^2 - 24h^2 \cos^2 x$</p> <p>(4)</p>
<p>[6]</p>		

QUESTION/VRAAG 8



8.1.1	$\hat{P} = \hat{M}_1 = 66^\circ$	[tan chord theorem/raaklyn koordst]	✓S ✓R	(2)
8.1.2	$\hat{M}_2 = 90^\circ$	[∠ in semi circle/∠ in halfsirkel]	✓S ✓R	(2)
8.1.3	$\hat{N}_1 = 180^\circ - (90^\circ + 66^\circ)$ $= 24^\circ$	[sum of ∠s of /som van ∠e ΔMNP]	✓S	(1)
8.1.4	$\hat{O}_2 = \hat{P} = 66^\circ$	[corres. ∠s;/ooreenk ∠e, PM OR]	✓S ✓R	(2)
8.1.5	$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180^\circ - 66^\circ$ $= 114^\circ$ $\hat{R} = \hat{N}_1 + \hat{N}_2 = 57^\circ$ $\therefore \hat{N}_2 = 33^\circ$	[sum of ∠s of /som van ∠e ΔRNO]	✓S ✓S/R ✓S	(3)
	OR/OF $\hat{P}\hat{O}\hat{R} = 114^\circ$ $\hat{P}\hat{N}\hat{R} = 57^\circ$ $\therefore \hat{N}_2 = 33^\circ$	[∠s on straight line/∠e op reguitlyn] [∠ at centre = twice ∠ at circumference/ midpts∠ = 2 × omtreks∠]	✓S ✓S/R ✓S	(3)

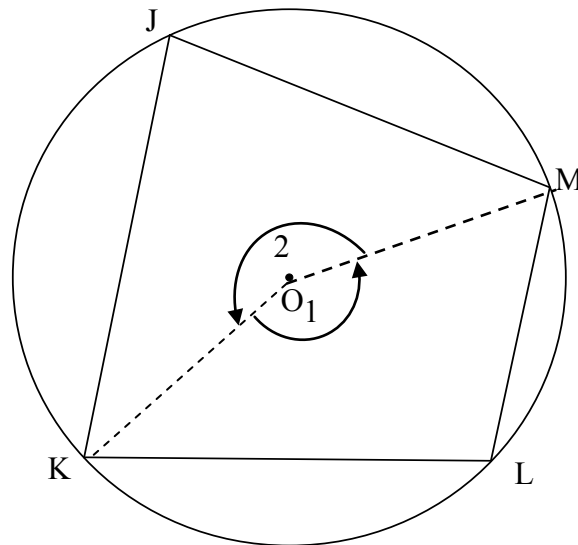
8.2



8.2.1	$FC \parallel AB \parallel GH$ [opp sides of rectangle / <i>teenoorst sye v reghoek</i>]	✓ R (1)
8.2.2	$\frac{AC}{CH} = \frac{AF}{FG}$ [line \parallel one side of Δ] OR [prop theorem; $FC \parallel GH$] <i>[lyn \parallel een sy van Δ] OF [eweredighst; $FC \parallel GH$]</i> $\frac{AC}{21} = \frac{20}{15}$ $AC = \frac{20 \times 21}{15}$ $= 28$ $DB = AC = 28$ [diags of rectangle =/ <i>hoeklyne v reghoek</i> =] $DM = \frac{1}{2}DB = 14$ [diags of rectangle bisect/ <i>hoekl v reghoek halveer</i>]	✓ S ✓ R ✓ AC ✓ S ✓ S (5)
[16]		

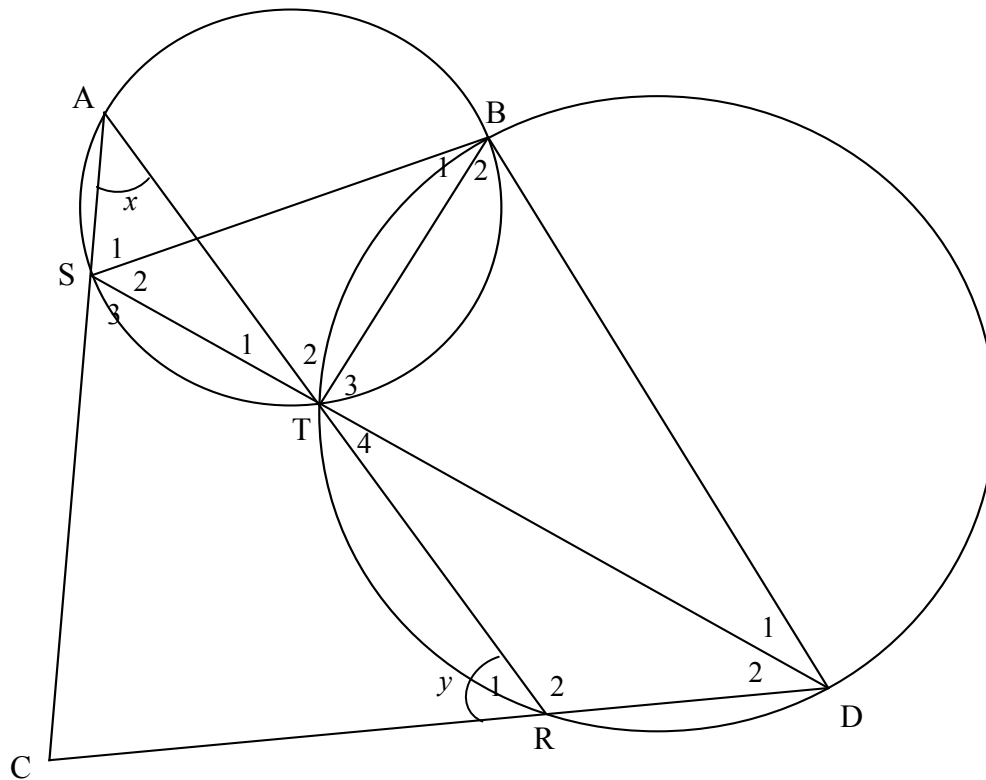
QUESTION/VRAAG 9

9.1



<p>9.1</p>	<p>Constr/Konstr.: Draw KO and MO/Trek KO en MO</p> <p>Proof:</p> <p>$\hat{O}_1 = 2\hat{J}$ [∠ at centre = twice ∠ at circumference] <i>[midpts ∠ = 2 × omtreks ∠]</i></p> <p>$\hat{O}_2 = 2\hat{L}$ [∠ at centre = twice ∠ at circumference]</p> <p>$\hat{O}_1 + \hat{O}_2 = 360^\circ$ [∠s around a point / ∠e om 'n punt]</p> <p>$\therefore 2\hat{J} + 2\hat{L} = 360^\circ$</p> <p>$\therefore 2(\hat{J} + \hat{L}) = 360^\circ$</p> <p>$\therefore \hat{J} + \hat{L} = 180^\circ$</p> <p>OR/OF</p> <p>Constr/Konstr.: Draw KO and MO/Trek KO en MO</p> <p>Proof:</p> <p>Let $\hat{J} = x$</p> <p>$\hat{O}_1 = 2x$ [∠ at centre = twice ∠ at circumference] <i>[midpts ∠ = 2 × omtreks ∠]</i></p> <p>$\hat{O}_2 = 360^\circ - 2x$ [∠s around a point / ∠e om 'n punt]</p> <p>$\therefore \hat{L} = 180^\circ - x$ [∠ at centre = twice ∠ at circumference]</p> <p>$\therefore \hat{J} + \hat{L} = 180^\circ$</p>	<p>✓ construction</p> <p>✓ S/R</p> <p>✓ S</p> <p>✓ S/R</p> <p>✓ S</p> <p>(5)</p> <p>✓ construction</p> <p>✓ S ✓ R</p> <p>✓ S/R</p> <p>✓ S</p> <p>(5)</p>
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9.2

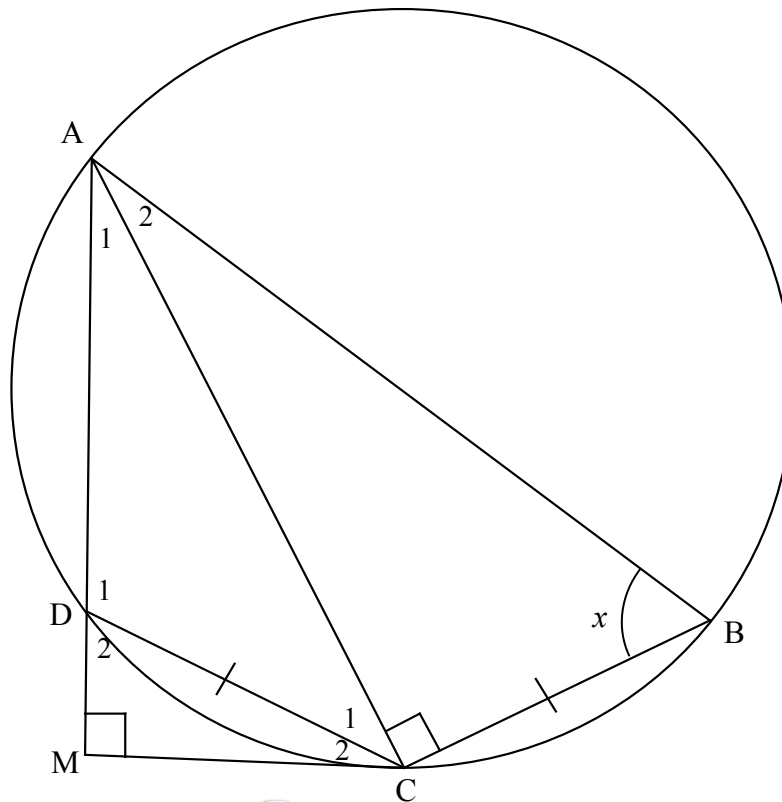


9.2.1(a)	$\hat{B}_1 = x$ [∠s in same seg/∠e in dieselfde segm]	✓ S ✓ R	(2)
9.2.1(b)	$\hat{B}_2 = y$ [ext ∠ of cyclic quad/buite∠ koordevh]	✓ S ✓ R	(2)
9.2.2	$\hat{C} = 180^\circ - (x + y)$ [sum of ∠s of/som v ∠e, ΔACR] $\hat{SBD} + \hat{C} = x + y + 180^\circ - (x + y)$ $\hat{SBD} + \hat{C} = 180^\circ$ SCDB is a cyclic quad [converse opp angles of cyclic quad] [omgekeerde teenoorst ∠e koordevh]	✓ S ✓ S ✓ R	(3)
	OR/OF $\hat{S}_1 = \hat{T}_2$ [∠s in same segment/∠e in dies. segment] $\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = \hat{BDR}$ [ext ∠ of cyc quad/buite∠ koordevh] $\therefore \hat{S}_1 = \hat{BDR}$ \therefore SCDB is cyc quad [ext ∠ of quad = opp ∠/ buite∠ = tos ∠]	✓ S ✓ S ✓ R	(3)

9.2.3	$\hat{T}_4 = y - 30^\circ$ [ext \angle of/buite \angle Δ TDR] $\hat{T}_1 = y - 30^\circ$ [vert opp \angle s =/regoorst \angle e =] $y - 30^\circ + x + 100^\circ = 180^\circ$ [sum of \angle s of/som v \angle e, Δ AST] $\therefore x + y = 110^\circ$ $\hat{S}\hat{B}\hat{D} = 110^\circ$ \therefore SD not diameter [line does not subtend $90^\circ \angle$] <i>SD nie 'n middellyn [lyn onderspan nie $90^\circ \angle$]</i>	✓ S ✓ S ✓ S ✓ R ✓ S ✓ S ✓ S ✓ R	(4)
	<p>OR/OF</p> $A\hat{S}\hat{T} = \hat{C} + \hat{D}_2$ [ext \angle of/buite \angle Δ SCD] $\hat{C} = 100^\circ - 30^\circ = 70^\circ$ $\hat{S}\hat{B}\hat{D} = 180^\circ - 70^\circ$ [opp \angle s cyclic quad/ teenoorst \angle e kdvh] $= 110^\circ$ \therefore SD not diameter [line does not subtend $90^\circ \angle$] <i>SD nie 'n middellyn [lyn onderspan nie $90^\circ \angle$]</i>	✓ S ✓ S ✓ S ✓ R	(4)
			[16]



QUESTION/VRAAG 10



<p>10.1.1</p>	<p>$\hat{A}_2 = \hat{A}_1 = 90^\circ - x$ [= chords subtend = $\angle s$ = <i>kde onderspan</i> = $\angle e$]</p> <p>$\hat{D}_2 = x$ [exterior angle of cyclic quad/<i>buite</i> \angle <i>koordevh.</i>]</p> <p>$\therefore \hat{C}_2 = 90^\circ - x$ [sum of $\angle s$ of/<i>som v</i> $\angle e$, $\triangle DCM$]</p> <p>$\therefore \hat{C}_2 = \hat{A}_1 = 90^\circ - x$</p> <p>$\therefore MC$ is a tangent to the circle at C [converse: tan chord th] <i>MC is 'n raaklyn by C [omgekeerde raakl koordst]</i></p> <p>OR/OF</p> <p>$\hat{A}_2 = \hat{A}_1 = 90^\circ - x$ [= chords subtend = $\angle s$/ = <i>kde onderspan</i> = $\angle e$]</p> <p>$\hat{C}_1 + \hat{C}_2 = x$ [sum of $\angle s$ of/<i>som v</i> $\angle e$, $\triangle ACM$]</p> <p>$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$</p> <p>$\therefore MC$ is a tangent to the circle at C [converse : tan chord th] <i>MC is 'n raaklyn by C [omgekeerde raakl koordst]</i></p> <p>OR/OF</p> <p>In $\triangle AMC$ and $\triangle ACB$:</p> <p>$\hat{A}_2 = \hat{A}_1 = 90^\circ - x$ [= chords subtend = $\angle s$/ = <i>kde onderspan</i> = $\angle e$]</p> <p>$\hat{A}MC = \hat{A}CB = 90^\circ$ [given]</p> <p>$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$</p>	<p>✓ S ✓ R</p> <p>✓ S/R</p> <p>✓ $\hat{C}_2 = 90^\circ - x$</p> <p>✓ R</p> <p>(5)</p> <p>✓ S ✓ R</p> <p>✓✓ $\hat{C}_1 + \hat{C}_2 = x$</p> <p>✓ R</p> <p>(5)</p> <p>✓ S ✓ R</p> <p>✓✓ $\hat{C}_1 + \hat{C}_2 = x$</p>
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	<p>$\therefore MC$ is a tangent to the circle at C [converse : tan chord th] <i>MC is 'n raaklyn by C [omgekeerde raakl koordst]</i></p>	<p>✓ R (5)</p>
	<p>In $\triangle ACB$ and/en $\triangle CMD$ $\hat{B} = \hat{D}_2 = x$ [proved OR exterior \angle of cyclic quad.] <i>[bewys OF buite \angle v koordevh]</i> $\hat{A}_2 = \hat{C}_2 = 90^\circ - x$ [proved OR sum of \angles in \triangle] <i>[Bewys OF som v \anglee in \triangle]</i> $\triangle ACB \parallel \triangle CMD$ [\angle, \angle, \angle]</p> <p>OR/OF In $\triangle ACB$ and/en $\triangle CMD$ $\hat{B} = \hat{D}_2 = x$ [proved OR exterior \angle of cyclic quad.] <i>[bewys OF buite \angle v koordevh]</i> $\hat{A}\hat{C}B = \hat{A}\hat{M}C = 90^\circ$ [given/gegee] $\triangle ACB \parallel \triangle CMD$ [\angle, \angle, \angle]</p> <p>OR/OF In $\triangle ACB$ and/en $\triangle CMD$ $\hat{B} = \hat{D}_2 = x$ [proved OR exterior \angle of cyclic quad] <i>[bewys OF buite \angle v koordevh]</i> $\hat{A}_2 = \hat{C}_2 = 90^\circ - x$ [proved OR sum of \angles in \triangle] <i>[Bewys OF som v \anglee in \triangle]</i> $\hat{A}\hat{C}B = \hat{A}\hat{M}C = 90^\circ$ [given OR sum of \angles in \triangle] <i>[gegee OF som v \anglee in \triangle]</i> $\triangle ACB \parallel \triangle CMD$</p>	<p>✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)</p>
10.2.1	<p>$\frac{BC}{MD} = \frac{AB}{DC}$ [$\triangle ACB \parallel \triangle CMD$] $\frac{DC}{MD} = \frac{AB}{DC}$ [$BC = DC$] $\therefore DC^2 = AB \times MD$</p> <p>In $\triangle AMC$ and/en $\triangle CMD$ \hat{M} is common/<i>gemeen</i> $\hat{A}_1 = \hat{C}_2$ [tan chord th /<i>raaklyn koordst</i>]</p> <p>OR/OF $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x$ [tan chord th /<i>raaklyn koordst OR/OF</i> exterior \angle of cyclic quad/ <i>buite \angle v kdvh</i>]</p> <p>$\triangle AMC \parallel \triangle CMD$ [\angle, \angle, \angle] $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$</p>	<p>✓ $\frac{BC}{MD} = \frac{AB}{DC}$ ✓ $DC^2 = AB \times MD$ ✓ S ✓ S ✓ $CM^2 = AM \times MD$ ✓ $\frac{AM \times MD}{AB \times MD}$ (6)</p>

	<p>OR/OF</p> $\frac{AC}{MC} = \frac{AB}{DC} \quad [\Delta ACB \parallel \Delta CMD]$ $\therefore CM \times AB = AC \times DC$ <p>In ΔAMC and/en ΔACB $\hat{C} = \hat{M} = 90^\circ$ [given] $\hat{A}_1 = \hat{A}_2$ [proven]</p> <p>OR/OF</p> $\hat{A}\hat{C}\hat{M} = \hat{B} = x$ [proven] $\Delta AMC \parallel \Delta ACB$ [\angle, \angle, \angle] $\frac{AC}{AM} = \frac{BC}{MC}$ $\therefore AC \times MC = AM \times BC$ $\therefore AC = \frac{BC \cdot AM}{MC}$ $CM \times AB = \frac{BC \cdot AM}{MC} \times DC$ $CM^2 = \frac{DC \cdot AM}{AB} \times DC \quad [BC = DC]$ $\frac{CM^2}{DC^2} = \frac{AM}{AB}$	<p>✓ $\frac{AC}{MC} = \frac{AB}{DC}$</p> <p>✓ S</p> <p>✓ S</p> <p>✓ $AC \cdot MC = AM \cdot BC$</p> <p>✓ equating</p> <p>✓ S</p> <p>(6)</p>
<p>10.2.2</p>	<p>In ΔDMC:</p> $\frac{CM}{DC} = \sin x$ $\frac{CM^2}{DC^2} = \sin^2 x \quad \frac{AC}{AB} = \frac{CM}{DC}$ $\therefore \frac{AM}{AB} = \sin^2 x$ <p>OR/OF</p> <p>In ΔABC:</p> $\sin x = \frac{AC}{AB}$ <p>In ΔAMC:</p> $\sin x = \frac{AM}{AC}$ $\sin x \cdot \sin x = \frac{AC}{AB} \times \frac{AM}{AC} = \frac{AM}{AB}$	<p>✓ trig ratio</p> <p>✓ square both sides</p> <p>(2)</p> <p>✓ 2 equations for $\sin x$</p> <p>✓ product</p> <p>(2)</p>
<p>[16]</p>		

TOTAL/TOTAAL: 150