



NATIONAL SENIOR
CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT

GRADE/GRAAD 12

SEPTEMBER 2020

**TECHNICAL MATHEMATICS P2/TEGNIESE WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 16 pages.
Hierdie nasienriglyn bestaan uit 16 bladsye.

NOTE:

- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

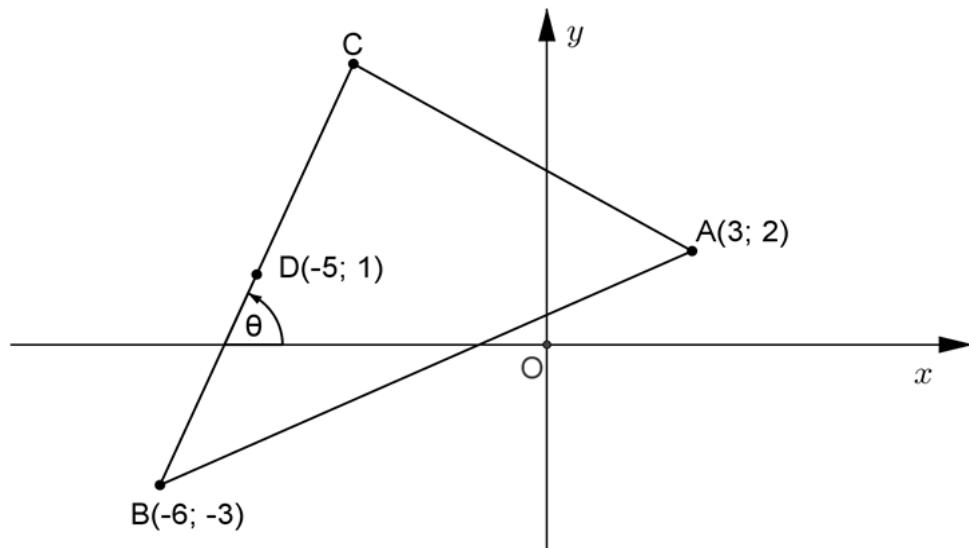
LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos in hierdie nasienriglyn aangedui.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method/Metode
A	Accuracy/Akkuraatheid
AO	Answer only/Slegs antwoord
CA	Consistent accuracy/Deurlopende akkuraatheid
F	Formula/Formule
I	Identity/Identiteit
R	Rounding/Afronding
S	Simplification/Vereenvoudiging
ST	Statement/Bewering
RE	Reason/Rede
ST RE	Statement and correct reason/Bewering en korrekte rede
SF	Substitution correctly in correct formula/Korrekte vervanging in die korrekte formule
NPU	No penalty for omitting units/Geen penalisering vir eenhede uitgelaat



QUESTION/VRAAG 1

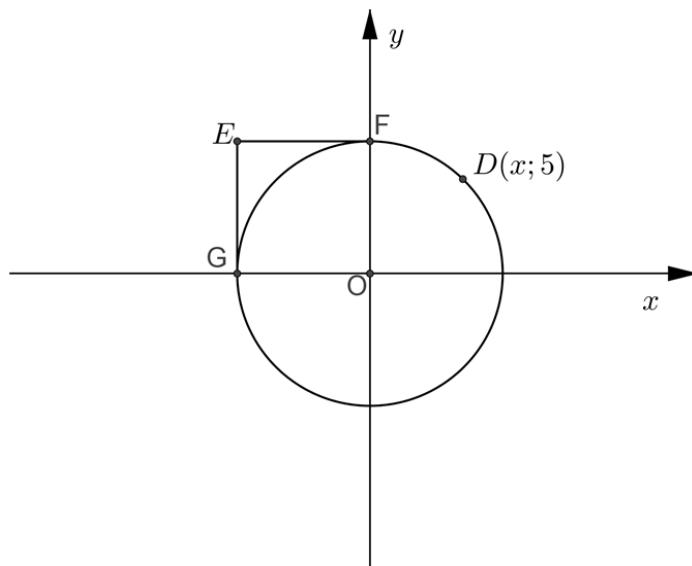


1.1	$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3+6)^2 + (2+3)^2} = \sqrt{(-6-3)^2 + (-3-2)^2} \\ &= \sqrt{106} \end{aligned}$	✓SF ✓CA A (2)
1.2.1	$\begin{aligned} m_{BC} &= m_{BD} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1+3}{-5+6} = \frac{-3-1}{-6+5} \\ &= 4 \end{aligned}$	✓SF A ✓ gradient / gradiënt CA (2)
1.2.2	$\begin{aligned} \tan \theta &= m_{BC} = 4 \\ \therefore \theta &= \tan^{-1}(4) \\ \therefore \theta &\approx 76^\circ \end{aligned}$	✓M ✓CA Rounded answer / afgeronde antwoord (2)
1.2.3	$\begin{aligned} (-5; 1) &= \left(\frac{x_C - 6}{2}; \frac{y_C - 3}{2} \right) \\ \therefore x_C - 6 &= -10 & y_C - 3 &= 2 \\ \therefore x_C &= -4 & y_C &= 5 \end{aligned}$	✓M ✓S ✓S ✓CA both answers / beide antwoorde (4)

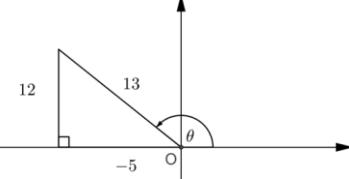
<p>1.2.4</p> $m_{\text{new line}} = 4$ nuwe lyn $y - y_1 = m(x - x_1)$ $\therefore y - 2 = 4(x - 3)$ $\therefore y - 2 = 4x - 12$ $\therefore y = 4x - 10$ <p style="text-align: center;">OR/OF</p> $m_{\text{new line}} = 4$ nuwe lyn $y = mx + c$ $2 = -4(-3) + c$ $\therefore c = -10$ $\therefore y = 4x - 10$	<p>✓CA gradient of new line / gradiënt van nuwe lyn</p> <p>✓SF CA gradient and point A / gradiënt en punt A</p> <p>✓S CA</p> <p>OR/OF</p> <p>✓CA gradient of new line / gradiënt van nuwe lyn</p> <p>✓SF CA gradient and point A / gradiënt en punt A</p> <p>✓S CA</p>	<p>(3)</p>
		[13]



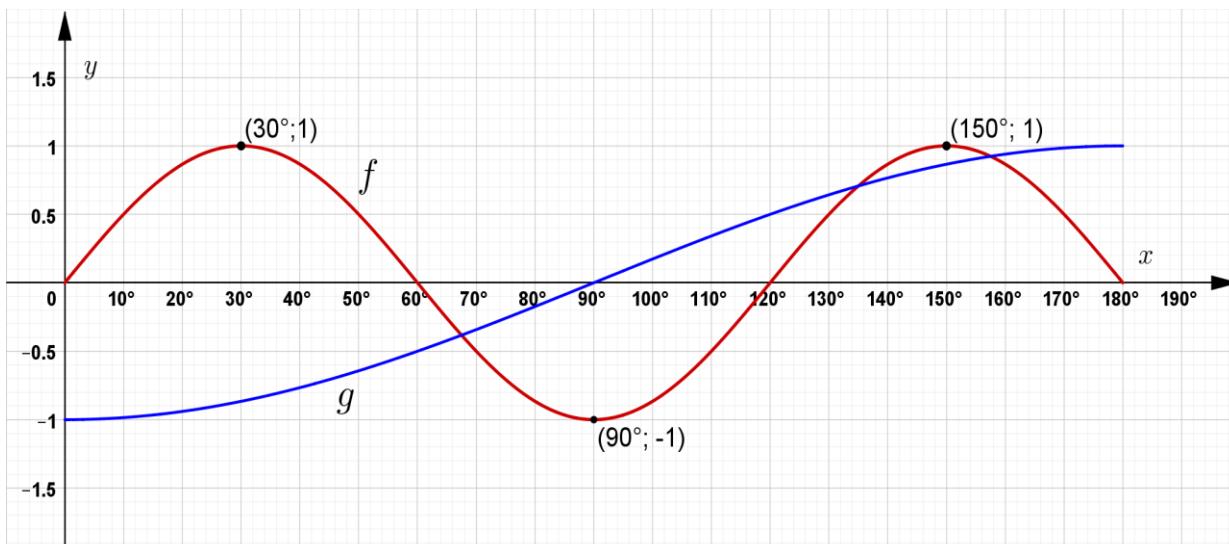
QUESTION/VRAAG 2



2.1.1	$x^2 + y^2 = 49$ $\therefore x^2 + 25 = 49$ $\therefore x^2 = 24$ $\therefore x = \sqrt{24} = 2\sqrt{6}$	✓SF A ✓S CA ✓CA surd form / wortelvorm	(3)
2.1.2 (a)	EF: $y = 7$ EG: $x = -7$	✓A y ✓A 7 ✓A x ✓A -7	(4)
2.1.2 (b)	E(-7 ; 7)	✓CA	(1)
2.2.1	$16x^2 + 49y^2 = 784$ $\frac{x^2}{49} + \frac{y^2}{16} = 1$	✓A LHS / LK ✓A RHS / RK	(2)
2.2.2		✓CA x-intercepts/ afsnitte ✓CA y-intercepts/ afsnitte ✓CA elliptical shape/ elliptiese vorm	(3)
			[13]

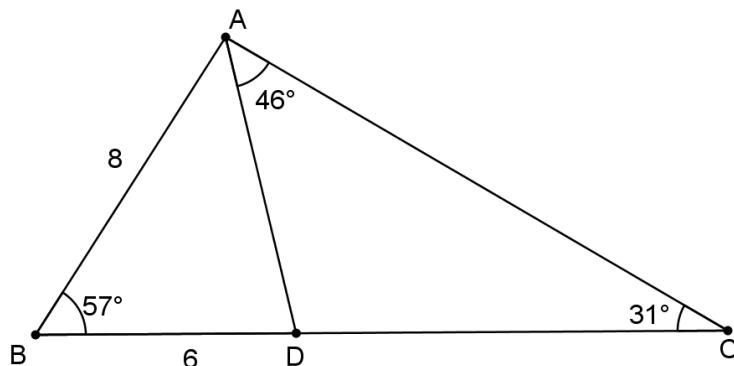
QUESTION/VRAAG 3			
3.1.1	$\begin{aligned} \text{cosec } A - \tan B \\ = \text{cosec } 123^\circ - \tan 65^\circ \\ = \frac{1}{\sin 123^\circ} - \tan 65^\circ \\ \approx -0,95 \end{aligned}$	$\checkmark \mathbf{A} \frac{1}{\sin 123^\circ}$ $\checkmark \mathbf{A} -0,95$	AO: Full marks / Volpunte (2)
3.1.2	$\begin{aligned} \cot^2(A + 2B) \\ = \left[\frac{1}{\tan(123^\circ + 2 \times 65^\circ)} \right]^2 \\ \approx 0,09 \end{aligned}$	$\checkmark \mathbf{A} \left[\frac{1}{\tan(123^\circ + 2 \times 65^\circ)} \right]^2$ $\checkmark \mathbf{A} 0,09$	AO: Full marks / Volpunte (2)
3.2	$\begin{aligned} \sin \frac{\pi}{6} + \sec^2 \frac{\pi}{4} &= \sin 30^\circ + \frac{1}{\cos^2 45^\circ} \\ &= 0,5 + 2 \\ &= 2,5 \end{aligned}$	$\checkmark \mathbf{A} 0,5$ $\checkmark \mathbf{A} 2$ $\checkmark \mathbf{CA} 2,5$ AO: Full marks / Volpunte	(3)
3.3	 $\cot \theta - \sec \theta = \frac{-5}{12} - \frac{13}{-5} = \frac{131}{60} \approx 2,18$	$\checkmark \mathbf{A}$ diagram in correct quadrant / diagram in korrekte kwadrant $\checkmark \mathbf{A}$ values of sides / waardes van sye $\checkmark \mathbf{CA}$  $\cot \theta = \frac{-5}{12}$ $\checkmark \mathbf{CA} \sec \theta = \frac{13}{-5}$ $\checkmark 2,18$	(5)
3.4	$\begin{aligned} (\tan^2 \theta + 1)(1 - \cos^2 \theta) \\ = \sec^2 \theta \times \sin^2 \theta \\ = \frac{1}{\cos^2 \theta} \times \sin^2 \theta \\ = \tan^2 \theta \end{aligned}$	$\checkmark \mathbf{A} \sec^2 \theta$ $\checkmark \mathbf{A} \sin^2 \theta$ $\checkmark \mathbf{A} \frac{1}{\cos^2 \theta}$ $\checkmark \mathbf{A} \tan^2 \theta$	(4)
3.5	$\begin{aligned} \frac{\sin(180^\circ + x) \cdot \tan 135^\circ}{\sec(180^\circ - x) \cdot \cos(360^\circ - x)} \\ = \frac{(-\sin x)(-1)}{(-\sec x)(\cos x)} \\ = \frac{\sin x}{-1} \\ = -\sin x \end{aligned}$	$\checkmark \mathbf{A} \sin x$ $\checkmark \mathbf{A} -1$ $\checkmark \mathbf{A} -\sec x$ $\checkmark \mathbf{A} \cos x$ $\checkmark \mathbf{A} -1$ $\checkmark \mathbf{CA} -\sin x$	(6)
			[22]

QUESTION/VRAAG 4



4.1		f: ✓ A x-intercepts / afsnitte ✓ A turning points / draaipunte ✓ A shape – starting at origin / vorm – getrek vanuit oorsprong g: ✓ A start and endpoint / begin en eindpunt ✓ A x-intercept / afsnit	(5)
4.2.1	120° OR/OF Period = $\frac{360^\circ}{3} = 120^\circ$	✓ A	(1)
4.2.2 (a)	$90^\circ \leq x \leq 180^\circ$	✓ CA interval ✓ CA notation / notasie	(2)
4.2.2 (b)	$0^\circ \leq x \leq 60^\circ$ OR/OF $90^\circ \leq x \leq 120^\circ$	✓ CA interval 1 ✓ CA notation / notasie ✓ CA interval 2 ✓ CA notation / notasie	(4)
4.2.2 (c)	$x = 90^\circ$ and/ en $x = 180^\circ$	✓ CA 90° ✓ CA 180°	(2)
			[14]

QUESTION/VRAAG 5



5.1	$\text{Area } \Delta ABD = \frac{1}{2} ad \sin B$ $= \frac{1}{2} \times 6 \times 8 \sin 57^\circ$ $= 20,13 \text{ sq units /vk eenhede}$	✓F ✓SF A ✓CA (3)
5.2	$AD^2 = AB^2 + BD^2 - 2AB \cdot BD \cos B$ $= 8^2 + 6^2 - 2 \times 8 \times 6 \cos 57^\circ$ $= 47,714\dots$ $AD \approx 6,91 \text{ units /eenhede}$	✓F ✓SF A ✓CA ✓R (4)
5.3	$\frac{CD}{\sin CAD} = \frac{AD}{\sin C}$ $\frac{CD}{\sin 46^\circ} = \frac{6,91}{\sin 31^\circ}$ $CD = \frac{6,91 \sin 46^\circ}{\sin 31^\circ}$ $\approx 9,65 \text{ units /eenhede}$	✓F ✓SF A ✓S CA ✓CA (4)
		[11]



QUESTION/VRAAG 6

6.1	Bisect the chord / halveer die koord	✓A	(1)
6.2			
6.2.1	$AO = x + 7$	✓A	(1)
6.2.2	$AF = 24 \text{ units}$ $AO^2 = AF^2 + OF^2$ (line from centre \perp to chord) $(\text{lyn vanaf midpt } \perp \text{ op koord})$ (Pyth) $\therefore (x+7)^2 = 24^2 + 7^2$ $\therefore x^2 + 14x + 49 = 625$ $\therefore x^2 + 14x - 576 = 0$ $\therefore (x+32)(x-18) = 0$ $\therefore x \neq -32 \text{ or/of } x = 18$	✓ST ✓RE ✓M ✓ST CA ✓ST CA	(5)
6.3			
6.3.1	$\hat{E} = 66^\circ$ (Int \angle s of Δ) $\angle e \text{ van } \Delta$ $\therefore \hat{C}_2 = 66^\circ$ (given / gegee) $\hat{D} = 66^\circ$ (\angle s in same seg) $\angle e \text{ in dies } \square \text{ segm}$	✓ST RE ✓ST ✓ST ✓RE	(4)

6.3.2	$H\hat{G}C = 114^\circ$ $\left(\text{opp } \angle s \text{ of cyclic quad} \right)$ <i>(teenoorste } \angle e \text{ van kdvh}</i>	✓ST ✓RE	(2)
6.3.3	$\hat{C}_1 = 24^\circ$ $\left(\angle s \text{ in same seg} \right)$ $\therefore \hat{C} = 90^\circ$ $\therefore \text{DG is a diameter / is 'n middellyn}$ $\text{(chord subtends } 90^\circ / \text{ koord onderspan } 90^\circ)$ OR/OF $\therefore \text{DG is a diameter / is 'n middellyn}$ $\text{(converse } \angle \text{ in semi-circle / omgekeerd } \angle \text{ in semi-sirkel)}$	✓ST ✓RE ✓RE	(3)
			[16]

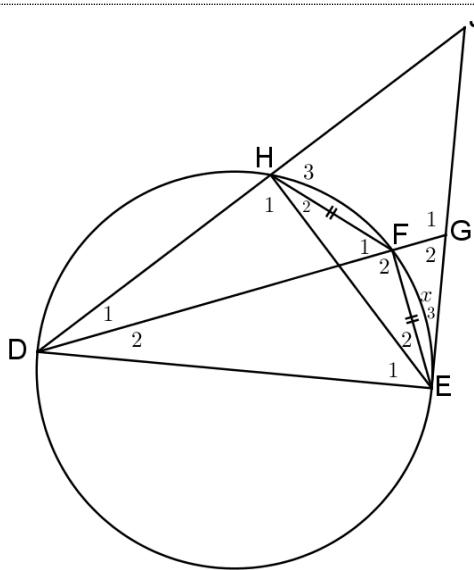


QUESTION/VRAAG 7

7.1	tangent to the circle / raaklyn tot die sirkel	✓A	(1)
7.2			
7.2.1	$\hat{H}_2 = 37^\circ$ (tan-chord raaklyn koord)	✓ST ✓RE	(2)
7.2.2	$\hat{F}_2 = 53^\circ$ (rad \perp tangent rad \perp raaklyn)	✓ST ✓RE	(2)
7.2.3	$\hat{D} = 53^\circ$ (\angle s opp equal sides: DO and FO radii \angle e teenoor gelyke sye: DO en FO radiusse) $\therefore \hat{O}_2 = 106^\circ$ (ext \angle of Δ buite \angle van Δ) OR / OF $\hat{D} = 53^\circ$ (\angle s opp equal sides: DO and FO radii \angle e teenoor gelyke sye: DO en FO radiusse) $\therefore \hat{O}_2 = 106^\circ$ (\angle at centre = $2 \times \angle$ at circumf middelpnts \angle = $2 \times \angle$ by omtrek)	✓ST RE ✓ST RE OR/OF ✓ST RE ✓ST RE	(2)
7.2.4	$\hat{E}_2 = \hat{D}_2$ (\angle s in same segm \angle e in dies. segm) $\hat{E}_2 = 53^\circ - 16^\circ$ $= 37^\circ$	✓ST ✓RE ✓ST	(3)
			[10]

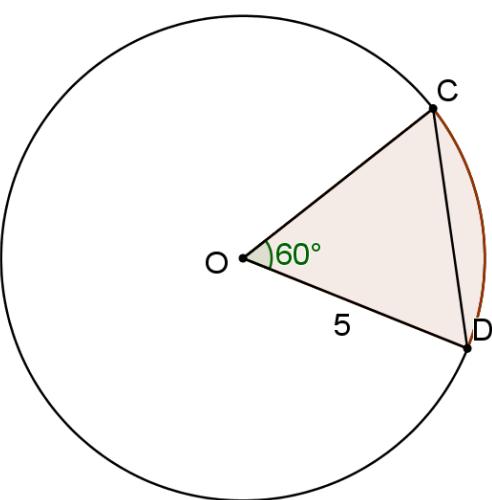
QUESTION/VRAAG 8			
8.1	Parallel / ewewydig	✓A	(1)
8.2			
	<p>A diagram showing triangle ABC. Point G is on the base BC. Line AG connects A to G. Line HG connects H (on AB) to G. Line FD connects F (on BC) to G. Arrows on HG and FD indicate they are parallel to the base AC.</p>		
8.2.1	$\frac{BF}{FD} = \frac{AG}{GD}$ $= \frac{1}{2}$ <p>(line \parallel to one side of Δ) (lyn \parallel aan een sy van Δ)</p>	✓ST ✓RE ✓ST	(3)
8.2.2	$\frac{CG}{CH} = \frac{CF}{CB}$ $= \frac{3+2}{3+3}$ $= \frac{5}{6}$ <p>(line \parallel to one side of Δ) (lyn \parallel aan een sy van Δ)</p> <p></p> <p>(D midpoint / middelpunt)</p>	✓ST RE ✓ST ✓ST	(3)

8.3



8.3.1	$\hat{D}_2 = x$ (tan-chord <i>raaklyn - krd</i>) $\hat{H}_2 = x$ (\angle s in same seg <i>hoek in dies segm</i>) $\hat{E}_2 = x$ (\angle s opp equal sides <i>\anglee teenoor gelyke sye</i>)	✓ST ✓ RE ✓ST ✓ RE ✓ST RE	
	OR/OF $\hat{D}_2 = x$ (tan-chord <i>raaklyn - krd</i>) $\hat{H}_2 = x$ (tan-chord <i>raaklyn - krd</i>) $\hat{E}_2 = x$ (\angle s opp equal sides <i>\anglee teenoor gelyke sye</i>)	OR/OF ✓ST ✓ RE ✓ST ✓ RE ✓ST RE	
8.3.2	In ΔDFE and / en ΔDEG : \hat{D}_2 (common/gemeen) $\hat{F}_2 = 90^\circ$ (\angle in semi-circle <i>\angle in semi-sirkel</i>) $\hat{E} = 90^\circ$ (tan/ <i>raaklyn</i> \perp radius) $\therefore \Delta DFE // \Delta DEG$ (AAA)	✓ST RE ✓ST ✓ RE ✓ST ✓ RE ✓RE	(5)

QUESTION/VRAAG 9



9.1	$s = r\theta$ $= (5) \left(60^\circ \times \frac{\pi}{180^\circ} \right)$ $= \frac{5\pi}{3}$ $\approx 5,24 \text{ cm}$	✓F ✓SF A $\times \frac{\pi}{180^\circ}$ ✓M ✓CA (4)
9.2	Area of sector = $\frac{rs}{2}$ Opp van sektor = $\frac{5 \times \frac{5\pi}{3}}{2}$ $= \frac{25}{6}\pi$ $\approx 13 \text{ cm}^2$ OR / OF Area of sector = $\frac{r^2\theta}{2}$ Opp van sektor = $\frac{5^2 \times \frac{\pi}{3}}{2}$ $= \frac{25}{6}\pi$ $\approx 13 \text{ cm}^2$	✓F ✓SF A ✓CA ✓R OR / OF ✓F ✓SF A ✓CA ✓R (4)
9.3 (a)	$O\hat{C}D = O\hat{D}C$ $\begin{pmatrix} \angle s \text{ opp } = \text{sides} \\ \angle e \text{ teenoor } = \text{sye} \end{pmatrix}$ $CD = 5 \text{ cm}$ $\begin{pmatrix} \text{equiangular } \Delta \\ \text{gelyksydige } \Delta \end{pmatrix}$	✓ST ✓ST AO: Full marks (2)

9.3 (b)	$4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (5)^2 = 0$ $4h^2 - 40h + 25 = 0$ $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{40 \pm \sqrt{40^2 - 4 \times 4 \times 25}}{8}$ $= \frac{40 \pm \sqrt{1200}}{8}$ $= 0,67 \text{ or/of } 9,33$ <p>\therefore required height / vereiste hoogte = 0,67 cm</p>	✓F ✓SF A ✓S CA ✓CA ✓C	(5)
			[15]

QUESTION/VRAAG 10

10.1.1	$180 \text{ rpm} = \frac{180 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec/sek}} = 3 \text{ rps}$	✓ conversion / herleiding ✓S A AO: Full marks	(2)
10.1.2	$n = 3 \text{ rps}$ $v = \pi Dn$ $r = 6 \Rightarrow d = 12$ $v = \pi \times 12 \times 3$ $= 36\pi \text{ cm/s}$	✓F ✓SF A ✓CA	(3)
10.1.3	$\omega = \frac{v}{r}$ $= \frac{36\pi}{6}$ $= 6\pi \text{ rad/s}$	✓F ✓SF A ✓CA	(3)
10.2	$V_{\text{cylinder}} = \pi r^2 h$ $1l = \pi (12 \text{ cm})^2 h$ $1000 \text{ cm}^3 = 144\pi h \text{ cm}^2$ $h \approx 2,2 \text{ cm}$	✓F ✓A $1\ell = 1000 \text{ cm}^3$ ✓SF A ✓CA	(4) [12]



QUESTION/VRAAG 11			
11.1	$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$ $256 = a \left(\frac{2,2+2,1}{2} + 2,8 + 3,1 + 3,2 + 2,9 + 2,6 \right)$ $= 16,75a$ $a = 15,28$ <p style="text-align: center;">OR / OF</p> $A_T = a(m_1 + m_2 + m_3 + \dots + m_{n-1})$ $256 = a \left(\frac{2,2+2,8}{2} + \frac{2,8+3,1}{2} + \frac{3,1+3,2}{2} + \frac{3,2+2,9}{2} + \frac{2,9+2,6}{2} + \frac{2,6+2,1}{2} \right)$ $= 16,75a$ $a = 15,28$	✓F ✓SF A ✓CA ✓ value of a / waarde van a OR / OF ✓F ✓SF A ✓CA ✓ value of a / waarde van a	(4)
11.2	Length of straight edge / Lengte van reguitsy $= 15,28 \times 6 = 91,68$ m	✓M ✓CA	(2)
			[6]

TOTAL/TOTAAL: 150

