



NATIONAL SENIOR CERTIFICATE EXAMINATION  
NOVEMBER 2019

**MATHEMATICS: PAPER I**  
**MARKING GUIDELINES**

Time: 3 hours

150 marks

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**These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.**

**The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.**

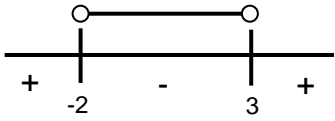
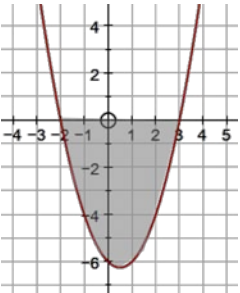
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**NOTE:**

- If a student answers a question more than once, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking memorandum.

**SECTION A**

**QUESTION 1**

(a)(1)	$2(-2)^2 + (-2) + k = 0$ $8 - 2 + k = 0$ $k = -6$	correct subst. of $-2$  $k = -6$
(a)(2)	$\therefore 2x^2 + x - 6 = 0$ $(2x - 3)(x + 2) = 0$ $\therefore \text{Other root is } \frac{3}{2}$	Factors/correct subst. in formula  $\frac{3}{2}$
(b)(1)	$x - 2 = 3\sqrt{x + 2}$ $(x - 2)^2 = (3\sqrt{x + 2})^2$ $x^2 - 4x + 4 = 9(x + 2)$ $x^2 - 13x - 14 = 0$ $(x - 14)(x + 1) = 0$ $x = 14 \text{ or } x = -1$ Check: $x = -1$ is not valid	Isolate surd $x^2 - 4x + 4$ $9(x + 2)$ $x^2 - 13x - 14$ factors answer with selection
(b)(2)	$x^2 - x - 6 \leq 0$ $(x - 3)(x + 2) \leq 0$ Crit. values: $3 ; -2$   Solution: $-2 \leq x \leq 3$	Factors/critical values  Number line/graph  $x \geq -2$  $x \leq 3$ 



**QUESTION 2**

(a)	$A = P(1 + i)^n$ $A = 12\,349 \left(1 + \frac{0,123}{52}\right)^1$ $A = R12\,378,21$	Sub. P into correct formula $\frac{0,123}{52}$ $n = 1$
(b)	$P = x \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$ $12\,349 = 94,75 \left[ \frac{1 - \left(1 + \frac{0,123}{52}\right)^{-52n}}{\frac{0,123}{52}} \right]$ $0,6917\dots = (1,00236\dots)^{-52n}$ $\log_{1,00236\dots} 0,6917\dots = -52n$ $n \approx 3 \text{ years}$	$\frac{0,123}{52}$  Correct P & x in correct formula  conversion to logs  answer
(c)	$A = P(1 - in)$ $A = 12\,349(1 - 0,2 \times 2)$ $A = 7\,409,40$	$P = 12\,349$ $0,2 \times 2$ answer
(d)	Balance outstanding = A – F $= 12\,349 \left(1 + \frac{0,123}{52}\right)^{2 \times 52} - \frac{94,75 \left[ \left(1 + \frac{0,123}{52}\right)^{2 \times 52} - 1 \right]}{\frac{0,123}{52}}$ $= 15\,788,54384 - 11\,156,97628$ $= R4\,631,57$ Depreciated amount = R7 409,40 Therefore it would be sufficient.  <p style="text-align: center;"><b>OR</b></p> $P = \frac{x \left[ 1 - (1 + i)^{-n} \right]}{i}$ $P = \frac{94,75 \left[ 1 - \left(1 + \frac{0,123}{52}\right)^{-52} \right]}{\frac{0,123}{52}}$ $P = R4\,630,90$ Depreciated amount = R7 409,40 Therefore it would be sufficient.	Use of correct formula $n = 104$ in A-F formula $94,75$ rate $\frac{123}{5200}$  Answer  Conclusion   Correct Pv formula $94,75$ in P formula $n \approx 52$ into formula rate $\frac{123}{5200}$  Answer  Conclusion

**QUESTION 3**

<p>(a)(1)</p>	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-5(x+h)^2 + (x+h) - (-5x^2 + x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-5(x^2 + 2xh + h^2) + x + h + 5x^2 - x}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-5x^2 - 10xh - 5h^2 + x + h + 5x^2 - x}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-10x - 5h + 1)}{h}$ $f'(x) = \lim_{h \rightarrow 0} (-10x - 5h + 1)$ $= -10x + 1$ <p style="text-align: center;"><b>OR</b></p> $f(x+h) = -5(x+h)^2 + (x+h)$ $f(x+h) = -5(x^2 + 2xh + h^2) + x + h$ $f(x+h) = -5x^2 - 10xh - 5h^2 + x + h$ $f(x+h) - f(x) = -5x^2 - 10xh - 5h^2 + x + h - (-5x^2 + x)$ $f(x+h) - f(x) = -10xh - 5h^2 + h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-10x - 5h + 1)}{h}$ $f'(x) = \lim_{h \rightarrow 0} (-10x - 5h + 1)$ $= -10x + 1$	$-5(x+h)^2 + (x+h)$ <p>Squaring &amp; distributing</p> <p>Factorisation</p> <p>notation</p> <p>sub. in 0 to get <math>-10x + 1</math></p> $-5(x+h)^2 + (x+h)$ <p>Squaring &amp; distributing</p> <p>Factorisation</p> <p>notation</p> <p>sub. in 0 to get <math>-10x + 1</math></p>
<p>(a)(2)</p>	<p>At: <math>x = 1, f'(1) = -10(1) + 1</math></p> $f'(1) = -9$ <p><math>\therefore</math> Eq. of tangent: <math>y = -9x + c</math></p> <p>Substitute: <math>(1; -4)</math></p> $-4 = -9(1) + c$ $c = 5$ $\therefore y = -9x + 5$ <p style="text-align: center;"><b>OR</b></p> <p>At: <math>x = 1, f'(1) = -10(1) + 1</math></p> $f'(1) = -9$ <p>Substitute: <math>(1; -4)</math></p> $y - (-4) = -9(x - 1)$ $\therefore y = -9x + 5$	$f'(1) = -9$ <p>Calculating y-coord of <math>-4</math></p> <p>Answer</p> $f'(1) = -9$ <p>Calculating y-coord of <math>-4</math></p> <p>Answer</p>

(b)(1)	$y = \frac{x^3 + x^{\frac{3}{2}}}{x}$ $y = \frac{x^3}{x} + \frac{x^{\frac{3}{2}}}{x}$ $y = x^2 + x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x + \frac{1}{2}x^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2x + \frac{1}{2\sqrt{x}}$	$x^2$  $x^{\frac{1}{2}}$  $2x$  $\frac{1}{2}x^{-\frac{1}{2}}$
(b)(2)	$D_x \left[ \frac{(2x-3)(4x^2+6x+9)}{(4x^2+6x+9)} \right]$ $D_x(2x-3)$ $= 2$	$(2x-3)(4x^2+6x+9)$  $D_x(2x-3)$ $= 2$



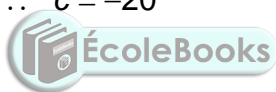
**QUESTION 4**

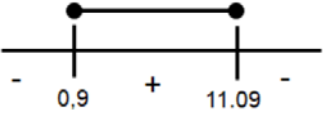
<p>(4)(a)</p>	$T_n = a + (n-1)d$ $T_n = 5 + (n-1)(4)$ $T_n = 4n + 1$ $100 = 4n + 1$ $4n = 99$ $n = 24\frac{3}{4}$ <p>24 pentagons</p>	$d = 4$ $T_n = 4n + 1$ $T_n = 100$ <p>Answer</p>
<p>(b)(1)</p>	$T_1 = 3 \text{ and } T_n = 47$ $S_n = 300$ $S_n = \frac{n}{2}(a + l)$ $300 = \frac{n}{2}(3 + 47)$ $n = 12$	<p>Correct formula</p> $300 = \frac{n}{2}(3 + 47)$ <p>Answer</p>
<p>(b)(2)</p>	$T_n = a + (n-1)d$ $47 = 3 + 11d$ $d = 4$	<p>Correct formula</p> $47 = 3 + 11d$ <p>Answer</p>
<p>(c)</p>	<p>Series: <math>\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots</math></p> <p><math>r = \frac{1}{2}</math>, converging series</p> $S_\infty = \frac{a}{1-r} ; -1 < r < 1$ $S_\infty = \frac{\frac{1}{4}}{1 - \frac{1}{2}}$ $S_\infty = \frac{1}{2}$	<p>Expansion</p> $r = \frac{1}{2}$ <p>Correct formula</p> <p>Answer</p>
<p>(d)</p>	$\frac{T_5}{T_3} = \frac{T_7}{T_5}$ $\frac{4}{5p+1} = \frac{1}{4}$ $p = 3$	<p>Equating ratios</p> <p>Correct substitution</p> <p>Answer</p>



**QUESTION 5**

(a)	50	50
(b)	$2a = -4 \quad 3a + b = 18 \quad a + b + c = 2$ $a = -2 \quad 3(-2) + b = 18 \quad (-2) + 24 + c = 2$ $b = 24 \quad c = -20$ $T_n = -2n^2 + 24n - 20$ <p style="text-align: center;"><b>OR</b></p> $2a = -4$ $a = -2$ $T_n = -2n^2 + bn + c$ $T_1: -2 + b + c = 2 \quad \therefore b + c = 4 \quad \dots \text{eq 1}$ $T_2: -8 + 2b + c = 20 \quad \therefore 2b + c = 28 \quad \dots \text{eq 2}$ $T_2 - T_1: b = 24$ <p>Subst. in eq. 1: <math>24 + c = 4 \quad \therefore c = -20</math></p> $T_n = -2n^2 + 24n - 20$ <p style="text-align: center;"><b>OR</b></p> $T_n = T_2(n-1) - T_1(n-2) + \frac{(n-1)(n-2)}{2} \times (2^{\text{nd}} \text{ diff.})$ $T_n = 20(n-1) - 2(n-2) + \frac{(n-1)(n-2)}{2} \times (-4)$ $T_n = 20n - 20 - 2n + 4 - 2(n^2 - 3n + 2)$ $T_n = 20n - 20 - 2n + 4 - 2n^2 + 6n - 4$ $T_n = -2n^2 + 24n - 20$	<p>Determining differences</p> $a = -2$ $b = 24$ $c = -20$ <p>Determining differences</p> $a = -2$ $b = 24$ $c = -20$ <p>Determining differences</p> $a = -2$ $b = 24$ $c = -20$



<p>(c)</p>	$T_n = -2n^2 + 24n - 20$ $T_n = -2(n^2 - 12n + 10)$ $T_n = -2[(n-6)^2 - 26]$ $T_n = -2(n-6)^2 + 52$ <p>Max. passengers is 52</p> <p style="text-align: center;"><b>OR</b></p> $T_n' = -4n + 24$ $-4n + 24 = 0$ $n = 6$ <p>Subst. <math>n = 6</math></p> $T_n = -2(6)^2 + 24(6) - 20$ $T_6 = 52$ <p>Max. passengers is 52</p>	<p>Determining n</p> <p><math>n = 6</math></p> <p>Answer</p> <p>Determining n</p> <p><math>n = 6</math></p> <p>Answer</p>
<p>(d)</p>	<p>Let <math>n</math> be 12 stops</p> $T_{12} = -2(12)^2 + 24(12) - 20$ $\therefore T_{12} = -20$ <p>Invalid due to negative answer</p> <p style="text-align: center;"><b>OR</b></p> $-2n^2 + 24n - 20 \geq 0$ <p>Passengers must be <math>\geq 0</math></p> <p>Crit. Values: <math>6 \pm \sqrt{26}</math></p>  <p>Hence <math>0,9 \leq n \leq 11,09</math></p>	<p>Substitution Explanation</p> <p>Substitution Explanation</p>





**SECTION B**

**QUESTION 6**

(a)	$f(x) > g(x)$ for $0 < x < 3$	$x > 0$ $x < 3$
(b)	$g(x) = \log_a x$ subst. (3;1) $1 = \log_a 3$ $a = 3$  $f(x) = \sqrt{kx}$ subst. (3;1) $1 = \sqrt{3k}$ $(1)^2 = (\sqrt{3k})^2$ $k = \frac{1}{3}$	Substitution  Answer  Substitution  Answer
(c)	$f : y = \sqrt{\frac{1}{3}x}$ $f^{-1} : x = \sqrt{\frac{1}{3}y}$ $x^2 = \frac{1}{3}y$ $y = 3x^2$ for $x \geq 0$	Changing x and y    $y = 3x^2$ Domain: $x \geq 0$

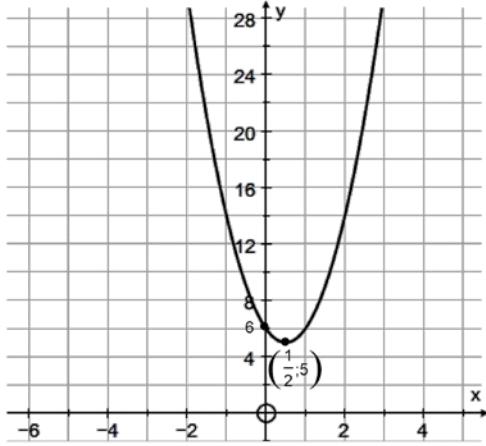


**QUESTION 7**

<p>(a)</p>		<p>Shape of f and g</p> <p>Intersection: (1;9)</p> <p><b>Graph g:</b> Horizontal asymptote (0;1)</p> <p><b>Graph f:</b> Horizontal asymptote (0;3)</p>
<p>(b)</p>	$a^{2x} = 3^{x+1}$ $\frac{a^{2x}}{3^x} = 3$ $\left(\frac{a^2}{3}\right)^x = 3$ $\log_3\left(\frac{1}{3}a^2\right) = x$ <p style="text-align: center;"><b>OR</b></p> $x = \log_{\frac{a^2}{3}} 3$ <p style="text-align: center;"><b>OR</b></p> $x = \frac{\log 3}{\log a^2 - \log 3}$	<p>Isolating x : <math>\left(\frac{a^2}{3}\right)^x</math></p> <p>Conversion to logs</p> <p>Answer</p>



**QUESTION 8**

(a)(1)	$(2x-1)^2 \geq 0$	Answer
(a)(2)		Shape T.P. $\left(\frac{1}{2}; 5\right)$ y-int.: (0;6)
(a)(3)	Shift 5 units down	Answer
(a)(4)	$(2x-1)^2 = k$ $4x^2 - 4x + (1-k) = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1-k)}}{2(4)}$ $x = \frac{4 \pm \sqrt{16k}}{8}$ $x = \frac{4 \pm 4\sqrt{k}}{8}$ $x = \frac{1 \pm \sqrt{k}}{2}$ Roots are real for $k \geq 0$ <p style="text-align: center;"><b>OR</b></p> $(2x-1)^2 = k$ $4x^2 - 4x + (1-k) = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1-k)}}{2(4)}$ $x = \frac{4 \pm \sqrt{16k}}{8}$ Roots are real for $16k \geq 0 \therefore k \geq 0$ <p style="text-align: center;"><b>OR</b></p> $(2x-1)^2 = k$ $2x-1 = \pm\sqrt{k}$ $x = \frac{1 \pm \sqrt{k}}{2}$ Roots are real for $k \geq 0$	Sub in formula $x = \frac{1 \pm \sqrt{k}}{2}$ $k \geq 0$ Sub in formula $x = \frac{4 \pm \sqrt{16k}}{8}$ $16k \geq 0 \therefore k \geq 0$ $2x-1 = \pm\sqrt{k}$ $x = \frac{1 \pm \sqrt{k}}{2}$ $k \geq 0$



<p>(a)(5)</p>	<p><math>y = 4x^2 - 4x + (1+k)</math>                  For real, unequal and rational, <math>\Delta &gt; 0</math> and perfect square</p> <p><math>\Delta = (-4)^2 - 4(4)(1+k)</math>  <math>\Delta = -16k</math></p> <p><math>\therefore k = -1, k = -\frac{1}{4}, k = -\frac{1}{16}</math> etc.</p> <p style="text-align: center;"><b>OR</b></p> <p><math>y = 4x^2 - 4x + (1+k)</math>                  Solve for <math>4x^2 - 4x + (1+k) = 0</math> through trial &amp; improvement:                  When <math>k = -1</math> roots are real, rational &amp; unequal                  When <math>k = -4</math> roots are real, rational &amp; unequal, etc.</p>	<p><math>\Delta = -16k</math></p> <p>accurate value of k                  accurate value of k                  accurate value of k</p> <p><math>4x^2 - 4x + (1+k) = 0</math></p> <p>accurate value of k                  accurate value of k                  accurate value of k</p>
<p>(b)</p>	<p><math>px^2 + qx + r = 0</math>  <math>x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}</math>  <math>\therefore P = \frac{-q + \sqrt{q^2 - 4pr}}{2p}</math>  <math>x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2}</math>  <math>\therefore Q = \frac{-q + \sqrt{q^2 - 4pr}}{2}</math></p> <p>For: <math>P : Q</math>,</p> <p><math>\frac{1}{p} \left[ \frac{-q + \sqrt{q^2 - 4pr}}{2} \right] : 1 \left[ \frac{-q + \sqrt{q^2 - 4pr}}{2} \right]</math></p> <p>Ratio: <math>\frac{1}{p} : 1</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Ratio: <math>1 : p</math></p>	<p><math>P = \frac{-q + \sqrt{q^2 - 4pr}}{2p}</math></p> <p><math>Q = \frac{-q + \sqrt{q^2 - 4pr}}{2}</math></p> <p>Answer</p>



**QUESTION 9**

<p>(a)(1)</p>	$y = a(x - x_1)(x - x_2)(x - x_3)$ $y = a(x + 3)(x + 3)\left(x - \frac{1}{2}\right)$ <p>Subst.: (0;9)  <math>a = -2</math></p> $y = -2(x + 3)^2\left(x - \frac{1}{2}\right)$ $y = -2\left(x - \frac{1}{2}\right)(x^2 + 6x + 9)$ $y = -2\left(x^3 + 6x^2 + 9x - \frac{1}{2}x^2 - 3x - 4\frac{1}{2}\right)$ $y = -2\left(x^3 + 5\frac{1}{2}x^2 + 6x - 4\frac{1}{2}\right)$ $y = -2x^3 - 11x^2 - 12x + 9$ <p style="text-align: center;"><b>OR</b></p> $y = a(x + 3)(x + 3)(2x - 1)$ <p>Subst.: (0;9)  <math>a = -1</math></p> $y = -1(x + 3)(x + 3)(2x - 1)$ $y = -1(x^2 + 6x + 9)(2x - 1)$ $y = -1(2x^3 + 12x^2 + 18x - x^2 - 6x - 9)$ $y = -1(2x^3 + 11x^2 + 12x - 9)$ $y = -2x^3 - 11x^2 - 12x + 9$	<p>formula</p> <p>Substitution of intercepts</p> <p>Subst. of (0;9)</p> <p><math>a = -2</math></p> <p>Answer showing  <math>a, b, c</math> and <math>d</math></p> <p>formula</p> <p>Substitution of intercepts</p> <p>Subst. of (0;9)</p> <p><math>a = -1</math></p> <p>Answer showing  <math>a, b, c</math> and <math>d</math></p>
<p>(a)(2)</p>	$f(x) = -2x^3 - 11x^2 - 12x + 9$ $f'(x) = -6x^2 - 22x - 12$ $f''(x) = -12x - 22$ $-12x - 22 = 0$ $x = -\frac{11}{6}$	$f'(x) = -6x^2 - 22x - 12$ $f''(x) = -12x - 22$ $x = -\frac{11}{6}$
<p>(b)</p>	$f'(x) = 8$ $-6x^2 - 22x - 12 = 8$ $-6x^2 - 22x - 20 = 0$ $x = -\frac{5}{3} \text{ or } x = -2$ <p><math>E(-2;5)</math></p>	$f'(x) = 8$ $x = -2$ $y = 5$

<p>(c)</p>	$y = \frac{2}{x+p} + q$ $p = 3$ $y = \frac{2}{x+3} + q \text{ subst. } (-2;5)$ $5 = \frac{2}{-2+3} + q$ $q = 3$ $\therefore y = \frac{2}{x+3} + 3$	<p><math>x + 3</math></p> <p>Substitution</p> <p><math>q = 3</math></p>
<p>(d)</p>	$y = x + 6$ <p style="text-align: center;"><b>OR</b></p> <p>Line goes through <math>(-3;3)</math></p> $y = x + c \quad \dots \text{ substitute } (-3;3)$ $\therefore y = x + 6$	<p><math>y = x</math></p> <p><math>y = x + 6</math></p> <p><math>(-3;3)</math></p> <p><math>y = x + 6</math></p>
<p>(e)</p>	$(-\infty; -3) \cup [-2; 0]$	<p><math>(-\infty; -3)</math></p> <p>( ) due to asymptote</p> <p><math>[-2; 0]</math></p>
<p>(f)</p>	$h(x) - k = -2x^3 - 11x^2 - 12x + 9$ <p>For h: y-int. <math>(0;9)</math></p> <p>For g: y-int. <math>\left(0; \frac{11}{3}\right)</math></p> $k < -\frac{16}{3}$	<p>y-int. <math>(0;9)</math></p> <p>y-int. <math>\left(0; \frac{11}{3}\right)</math></p> $k < -\frac{16}{3}$





**QUESTION 11**

(a)	$\frac{10!}{5! \times 2! \times 3!}$ $= 2\,520$	$\frac{10!}{[ ] [ ]}$ $\frac{[ ]}{5! \times 2! \times 3!}$ Answer
(b)(1)	$P(\text{not picked}) = 0,3 \times 0,65$ $= 0,195$	0,3 and 0,65 multiplication of above Answer
(b)(2)	$P(\text{made into juice})$ $= (0,7 \times 0,6) + (0,3 \times 0,35 \times 0,6)$ $= 0,483$ $\therefore \text{Approx. } 48,3\% \text{ will be made into juice}$ <p style="text-align: center;"><b>OR</b></p> $P(\text{made into juice})$ $= (1 - 0,195) \times 0,6$ $= 0,483$ $\therefore \text{Approx. } 48,3\% \text{ will be made into juice}$	indicating 0,6 or 60% $(0,7 \times 0,6)$ $(0,3 \times 0,35 \times 0,6)$ 0,483  indicating 0,6 or 60% $(1 - \dots)$ $(1 - 0,195) \times 0,6$ 0,483





<p>(b)(3) Total oranges exported  <math>= 120 \times 172</math>  <math>= 20\ 640</math></p> <p><math>P(\text{exported})</math>  <math>= (0,7 \times 0,09) + (0,3 \times 0,35 \times 0,09)</math>  <math>= 0,07245</math>  <math>\therefore 7,245\% \text{ exported}</math></p> <p>Let total number of oranges = <math>x</math>  <math>\frac{20\ 640}{x} \times 100 = 7,245</math>  <math>\therefore x = 284\ 886 \text{ oranges in total}</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Total oranges exported  <math>= 120 \times 172</math>  <math>= 20\ 640</math></p> <p><math>P(\text{exported})</math>  <math>= (1 - 0,195) \times 0,09</math>  <math>= 0,07245</math>  <math>\therefore 7,245\% \text{ exported}</math></p> <p>Let total number of oranges = <math>x</math>  <math>\frac{20\ 640}{x} \times 100 = 7,245</math>  <math>\therefore x = 284\ 886 \text{ oranges in total}</math></p>	<p>20 640</p> <p><math>(0,7 \times 0,09)</math>  <math>(0,3 \times 0,35 \times 0,09)</math></p> <p>Answer</p> <p>20 640</p> <p><math>(1 - 0,195)</math>  <math>(1 - 0,195) \times 0,09</math></p> <p>Answer</p>
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**Total: 150 marks**