NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2018

MATHEMATICS: PAPER I

Time: 3 hours | 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2. Read the questions carefully.

3. Answer all the questions.

4. Number your answers exactly as the questions are numbered.

5. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.

6. Clearly show **ALL** calculations, diagrams, graphs et cetera that you have used in determining your answers.

   Answers only will NOT necessarily be awarded full marks.

7. Diagrams are not necessarily drawn to scale.

8. If necessary, round off answers to **ONE** decimal place, unless stated otherwise.

9. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a) The 100th term of an arithmetic sequence is 512 and its common difference is 7, determine the first term of the sequence. (3)

(b) The general term of a sequence is $T_n = 2n + 3$.

(1) Show that the sequence is arithmetic. (3)

(2) Determine, in terms of n, a simplified expression for S_n, the sum of the first n terms. (3)

(c) Consider the given quadratic sequence: $4\,;7\,;14\,;25\,;...$
Determine a simplified expression for the nth term of the sequence. (4)

[13]

QUESTION 2

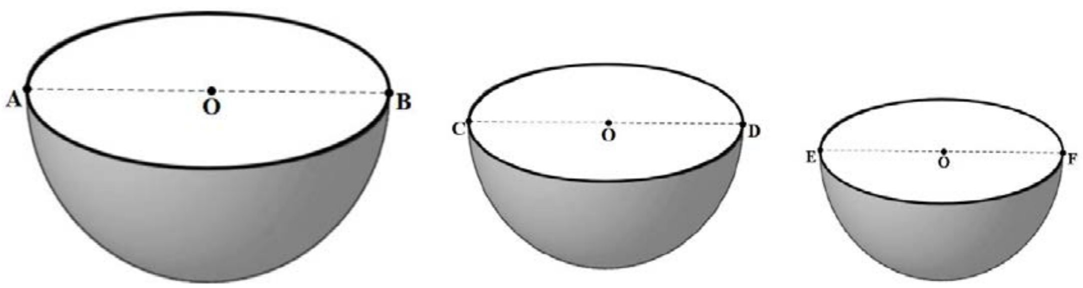
(a) Given: $\sum_{n=1}^{x} 108 \times \left(\frac{2}{3}\right)^n$

(1) Determine the first two terms. (2)

(2) If $\sum_{n=1}^{x} 108 \times \left(\frac{2}{3}\right)^n = \frac{520}{3}$, determine the value of x . (4)

(b) Hollow plastic hemispheres are created such that each successive one fits into the previous one.

The radii $\text{OB} = 21$ cm, $\text{OD} = 3$ cm and $\text{OF} = \frac{3}{7}$ cm.



Determine the sum of the outer surface areas, as shaded in the diagram, of all such hemispheres created by continuing the pattern indefinitely.

Useful formula: Surface Area of a hemisphere $= 2\pi r^2$ (5)

[11]

QUESTION 3

(a) Given: $f(x) = x^2 - 3x - 4$ and $g(x) = x + 1$

Calculate the following:

(1) x, if $\dfrac{1}{f(x).g(x)}$ is undefined (4)

(2) x, if $f(x) \leq 0$ (4)

(b) Consider the equation: $\sqrt{x+4} - 3 = x$

(1) Show, without solving the equation, that $x \geq -4$ (2)

(2) Solve for x correct to one decimal place. (6)

[16]

QUESTION 4

(a) Given: $f(x) = 2x^3$

(1) Determine the average gradient of f between the points $x = 1$ and $x = 1 + h$. (4)

(2) Hence, or otherwise, determine $f'(1)$. (2)

(b) Determine $\dfrac{dy}{dx}$: $y = \dfrac{3}{x^2} - 10\sqrt[5]{x}$. (4)

[10]

QUESTION 5

(a) Riyan opened a bank account 15 years ago, with the intention of saving money for when he retires.

The bank offered him an interest rate of 16% per annum compounded monthly for the first 5 years and thereafter changed the interest rate to 11% per annum (compounded annually).

Riyan made an immediate deposit of R300 000 upon opening the account and withdrew R500 000 at the end of 13 years.

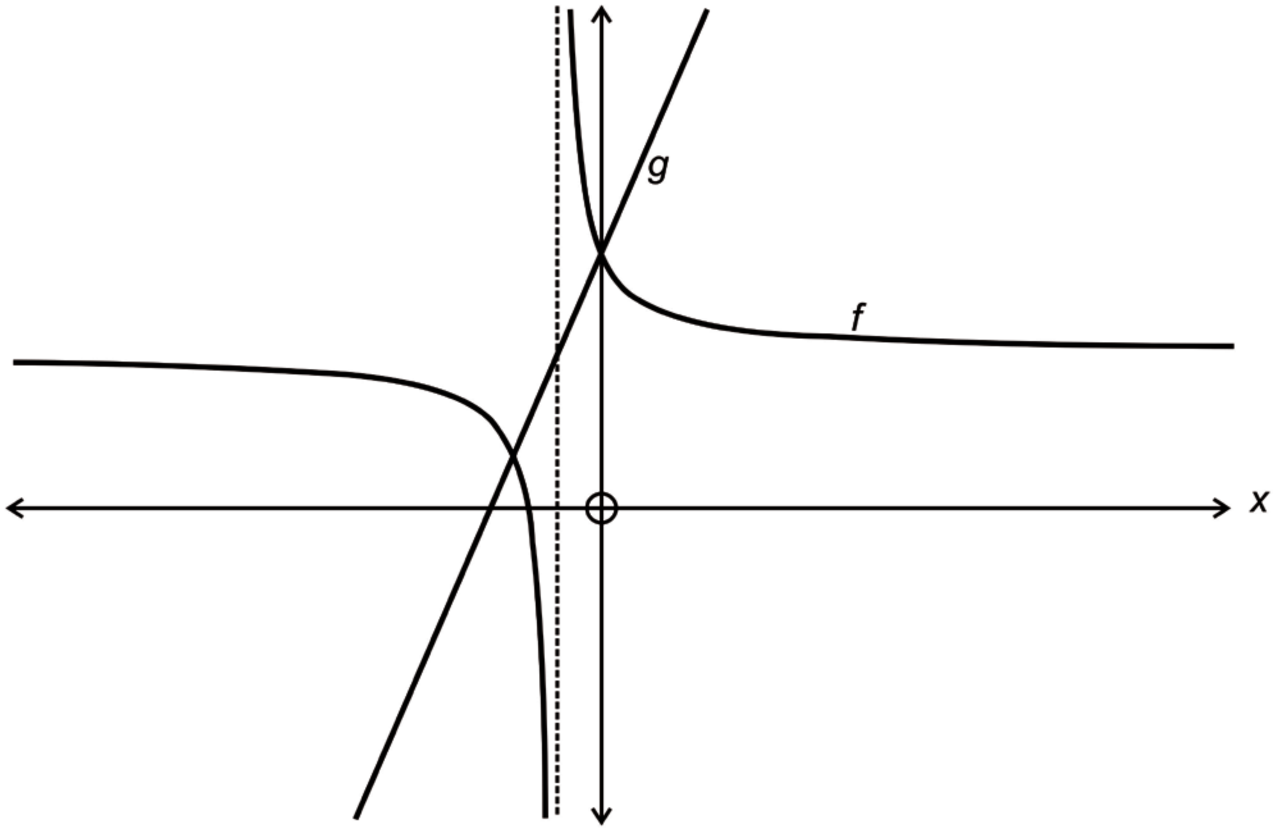
Calculate how much money he would have in this account at the end of the 15th year. (5)

(b) If instead, Riyan had taken a retirement annuity over the same period of 15 years, and the insurance company had offered him 8% per annum compounded monthly, what would his **monthly payments** have been if he were to save an amount of R1 270 000 at the end of the 15th year. (4)

[9]

QUESTION 6

In the diagram below, the graphs of $f(x)=\dfrac{a}{x+b}+c$ and $g(x)=2x+5$ are given.



The graph of f has a vertical asymptote at $x=-1$, both graphs intersect on the y-axis and the graph of g intersects the horizontal asymptote of f at the point $(-1\,;\,y)$.

(a) Determine a, b and c. Show all working. (6)

(b) If $f(x)=\dfrac{2}{x+1}+3$ and $g(x)=2x+5$:

(1) Determine the x-intercepts of f and g. (3)

(2) Hence, or otherwise, solve for x if $f(x).g(x)\leq 0$. (3)

(c) (1) Determine g^{-1}, the inverse of g in the form $y=....$ (3)

(2) Hence, or otherwise, determine the values of x for which $g^{-1}(x)>g(x)$. (3)

[18]

77 marks

SECTION B

QUESTION 7

Answers only will not be awarded full marks.

(a) The roots of a quadratic equation are given as $5-\sqrt{2}$ and $5+\sqrt{2}$. Determine the equation in the form $ax^2+bx+c=0$. (4)

(b) The equations $x^2+ax+b=0$ and $x^2+bx+a=0$ both have real and equal roots. Solve for a and b, where $a>0$ and $b>0$. (7)

[11]

QUESTION 8

Katy invested in Bitcoins (a digital currency) which increased in value at a rate of 200% per annum over a period of time.

Her original investment of (y) rands was squared in value after (x) years when she sold her investment.

(a) Write down an equation representing the relationship between y and x in the form: $y=....$ (3)

(b) Sketch the graph of (a) showing any intercepts and asymptotes if they exist. (3)

(c) If Katy's original investment was R750:

(1) Determine correct to the nearest month, the number of years it took to square in value. (3)

(2) Determine a restriction on the domain of the graph sketched in (b) that could represent Katy's investment. (1)

[10]

QUESTION 9

Consider the graphs of $g(x) = x^3 - 3x^2$ and $h(x) = -\frac{2}{3}x - \frac{4}{3}$.

(a) Determine whether the graph of h intersects the graph of g at its point of inflection. Show all working. (6)

(b) (1) Determine the stationary point(s) of $y = g'(x)$. Classify your stationary point(s). (4)

(2) Hence, or otherwise, determine:

(i) the value(s) of x for which g is concave down. (1)

(ii) the gradient of the tangent to g at its point of inflection. (2)

(3) A student claims that the gradient of g at any point will never be less than -3. Is the student correct? Explain. (2)

(c) Determine the value of k, if the graph of g is shifted so that the values of x for which the new graph $j(x) = (x+k)^3 - 3(x+k)^2$ decreases, is between -3 and -1. (4)

[19]

QUESTION 10

(a) Given: $f(x) = ax^2 + bx + c$ where $b > 2a > 0$ and $a > c > 0$

(1) Show that $b^2 > 4ac$. (2)

(2) Draw a sketch graph of f. (4)

(b) Given: $g(x) = \frac{1}{x+2} - \frac{1}{2}$ and $h(x) = 2^x + p$

(1) Sketch the graph of g. (3)

(2) Determine the value(s) of p for which $g(x) = h(x)$ has only one root. (2)

[11]

QUESTION 11

Consider the word: C I R C L E

Note: The repeated letters are treated as identical.

(a) If two letters are selected at random, without replacement, determine the probability that:

(1) Both letters are "C". (2)

(2) Only one letter is "C". (3)

(b) Determine the number of different 6-letter arrangements that can be made with the letters. (2)

(c) How many word arrangements can be made if the word starts and ends with the same letter? (2)

[9]

QUESTION 12

Lulu and Riempie have been invited to observe a missile testing experiment.

[<https://commons.wikimedia.org/wiki/File:Roland_(missile).jpg>]

The missile engineer informed them that the probability that a missile will hit its target is 0,9.

He then asks Lulu and Riempie to work out the **minimum** number of missiles that would need to be fired at the target to ensure a 0,97 chance of hitting the target.

Lulu calculated that at least 2 missiles needed to be fired at the target.

Riempie calculated that at least 3 missiles needed to be fired.

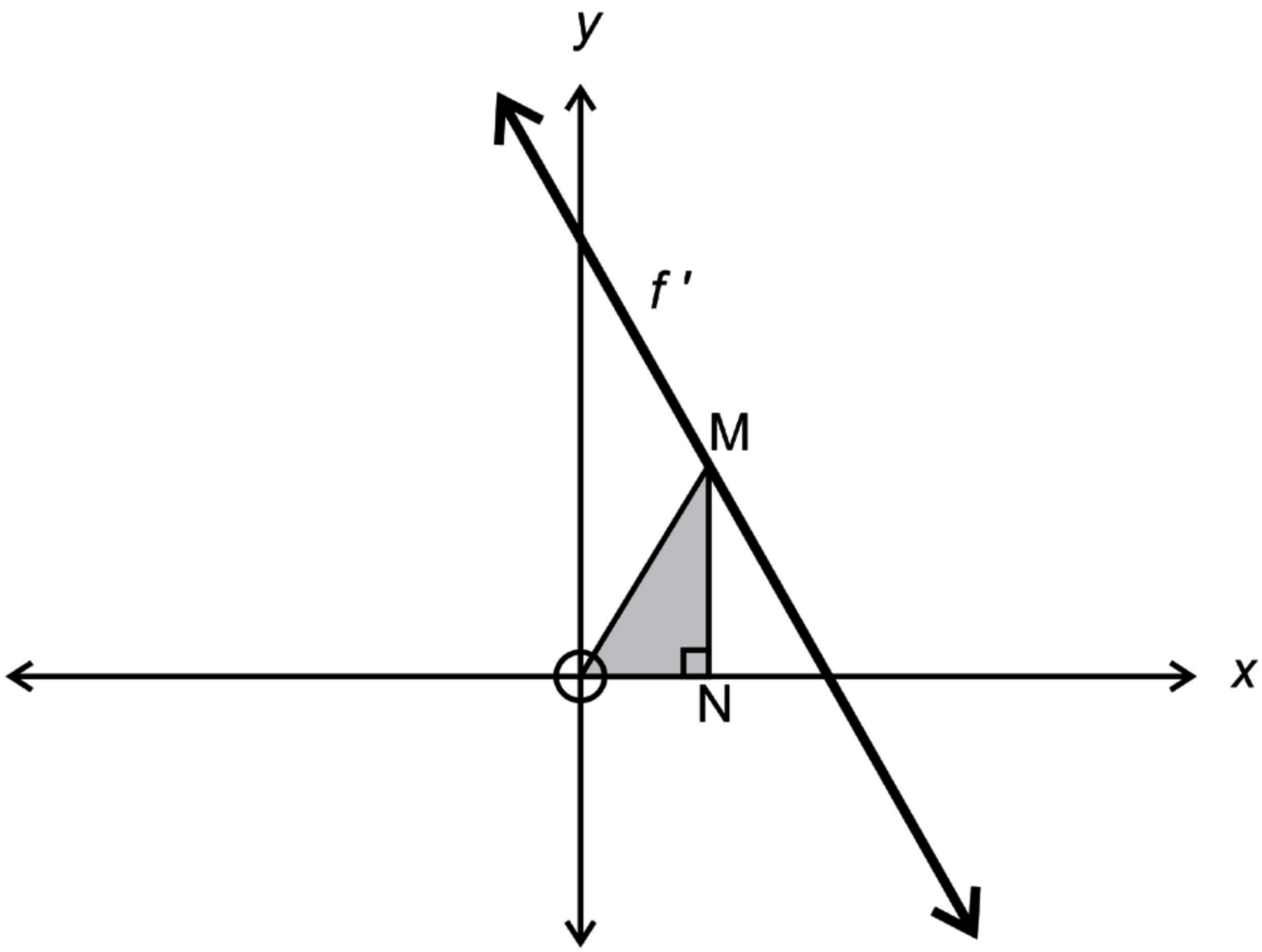
Determine who was correct. Show all calculations.

[6]

QUESTION 13

In the diagram, the vertices of the shaded right-angled triangle OMN are O $(0;0)$, the variable point N $(x_1;0)$ which is on the x-axis where $0 \le x_1 \le 3$ and point M which lies on the line $2y + 3x - 6 = 0$.

The line represents the graph of the first derivative function of the function of *f*.



The area of the shaded region is given as $A = rx^2 + tx$ and $f(x) = rx^2 + bx + c$ has a stationary point at $(x;5)$.

Determine whether the value of x_1 that yields the maximum area of Δ OMN is also the value of x_2 that yields the maximum distance between the graphs of *f* and its derivative f'.

Show all working.

[7]

73 marks

Total: 150 marks