MATHEMATICS

MATERIAL FOR GRADE 12

Probability

MEMORANDA

QUESTION 1

<table>
<tr><td>1.1.1</td><td>Three places to fill by 4, 7 or 9 and each time a digit is used, there is an opportunity to use it again. There are 3 possibilities to fill 3 places each time, hence there are $3 \times 3 \times 3 = 27$ ✓possible ways to create the code.</td><td>✓Answer (1)</td></tr>
<tr><td>1.1.2</td><td>$3 \times 2 \times 1$ =3!=6 ways</td><td>✓Answer (1)</td></tr>
<tr><td>1.2</td><td>• 6 Mathematics books can fill six places
• 5 Physical Sciences books can fill 5 places
• Books cannot occupy more than 1 place
• Both Mathematics and Physical Sciences books can be put together hence there are two ways to make the arrangements.
The arrangements of books on the shelve are …
6!×5! ✓×2! ✓=172 800✓</td><td>✓6! ×5!
✓2!
✓Answer (3)</td></tr>
<tr><td>1.3</td><td><table>
<tr><th></th><th>Positive</th><th>Not Positive</th><th></th></tr>
<tr><td>Male</td><td>7</td><td>48</td><td>55</td></tr>
<tr><td>Female</td><td>3</td><td>42</td><td>45</td></tr>
<tr><td></td><td>10</td><td>90</td><td>100</td></tr>
</table></td><td></td></tr>
<tr><td>1.3.1</td><td>$P(M \cap P^{+}) = \frac{7}{100}$✓</td><td>✓Answer (1)</td></tr>
<tr><td>1.3.2</td><td>$n(M \cap P^{+}) = \frac{7}{100} \times 50000000 = 3\,500\,000$ no of possible infected males</td><td>✓Method
$P(M \cap P^{+}) = \frac{7}{100}$✓ (2)</td></tr>
<tr><td>1.3.3</td><td>$P(P^{+} \backslash F) = \frac{3}{45} = \frac{1}{15}$✓✓</td><td>Answer (2)</td></tr>
<tr><td>1.3.4</td><td>$P(M \backslash P^{+}) = \frac{P(M \cap P^{+})}{P(M)}$✓ $= \frac{7}{10}$✓✓</td><td>Answer (3)
[13]</td></tr>
</table>

QUESTION 2

2.1		P (A and B) = P(A) × $P(B)$ = 0,4 × 0,5 = 0,2	✓ ✓	subst. into formula answer	(2)
2.2		P(A or B) = P(A) = P(B) – P(A and B) = 0,4 + 0,5 – 0,2 = 0,7	✓ ✓	subst. into formula answer	(2)
2.3		P (not A and not B) = 1 – P (A or B) = 1 – 0,7 = 0,3	✓ ✓ ✓	formula subst. into formula answer	(3)
					[7]

QUESTION 3

3.1		7! = 5040	✓ ✓	7! answer	(2)
3.2		Girls seated together in 4! ways. With the girls as one unit they can all enter in 4! 4! ways = 576	✓ ✓ ✓	4! ways for girls 4! ways for others with girls as unit answer	(3)
3.3		$\frac{576}{5040}$ = 0,11428 =11,43%	✓	0,11 or 11,43%	(1)
3.4		The girls can sit in $4 \times 3 \times 2 \times 1$ ways. The boys can sit in $3 \times 2 \times 2$ ways the group can sit in $4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 144$ ways. Probability $= \frac{144}{5040} = 0{,}028571 = 2{,}86\%$	✓ ✓ ✓	4! ways 3! ways answer	(3)
					[9]

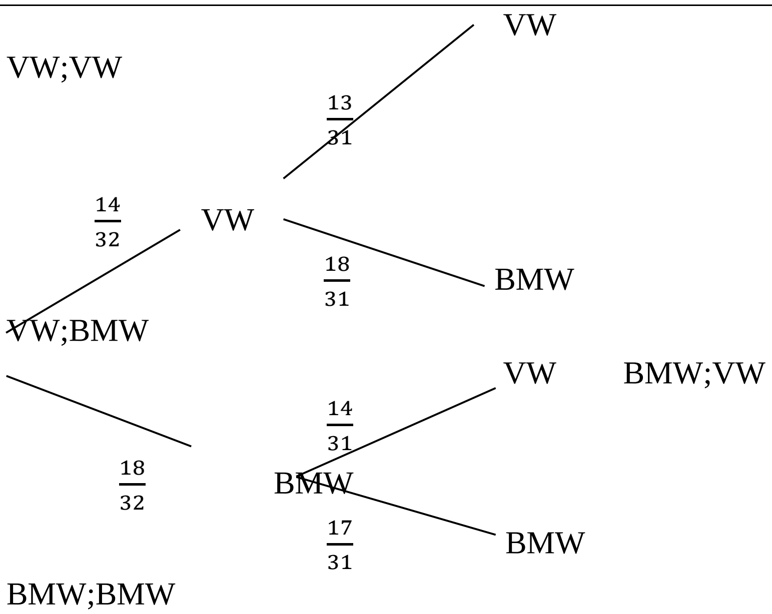
QUESTION 4

4.1.1	$P(boy,\ tennis\ or\ squash)$ $= \frac{18}{120} + \frac{20}{120}$ $= \frac{19}{60}$ $= 0,32$	✓ $\frac{18}{120} + \frac{20}{120}$ ✓ $\frac{19}{60}$ or $0,32$ (2)
4.1.2	$P(learner,\ tennis) = \frac{41}{120}$ $= 0,34$	✓ $\frac{41}{120}$ or $0,34$ (1)
4.1.3	$P(girl) = \frac{50}{120} = \frac{5}{12} = 0,42$	✓ $\frac{50}{120}$ or $\frac{5}{12}$ or $0,42$ (1)
4.2	$P(boy\ and\ golf) = \frac{32}{120} = \frac{4}{15} = 0,27$ $P(boy).\ P(golf) = \frac{70}{120} \cdot \frac{44}{120} = \frac{77}{360} = 0,21$ $\therefore 0,27 \neq 0,21$ $\therefore$ Not independent	✓ $\frac{32}{120}$ or $\frac{4}{15}$ or $0,27$ ✓ $\frac{70}{120} \cdot \frac{44}{120}$ ✓ $\frac{77}{360}$ or $0,21$ ✓ not independent (4) **[8]**

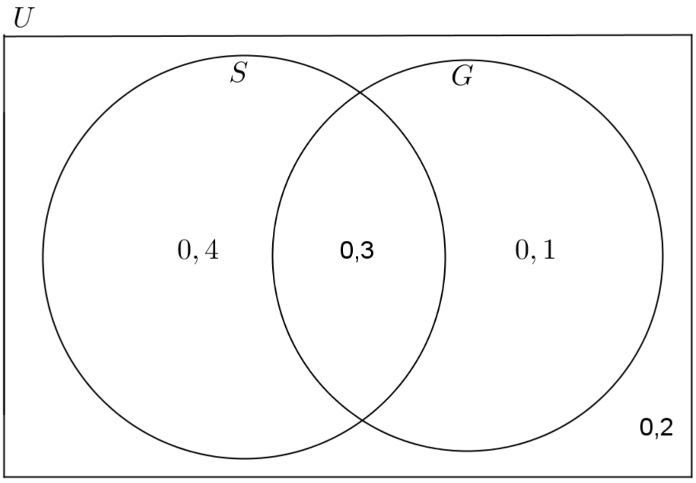
QUESTION 5

5.1.1	$14! = 8,72 \times 10^{10}$	✓ $14!$ or $8,72 \times 10^{10}$ (1)
5.1.2	$4!\ 5!\ 3!\ 4!\ 2! = 829\ 440$	✓ $4!\ 5!\ 3!\ 4!\ 2!$ ✓ $829\ 440$ (2)
5.2.1	PROBABILITY $\frac{11!}{2!\ 2!} = 9\ 979\ 200$	✓ 11! ✓ 2! 2! ✓ answer (3)
5.2.2	$\frac{10!}{2!\ 2!} = 907\ 200$	✓ $\frac{10!}{2!\ 2!}$ ✓ $907\ 200$ (2) **[8]**

QUESTION 6

6.1.1	$P(A \text{ or } B) = \frac{3}{8} + \frac{1}{4} = \frac{5}{8}$	✓ $\frac{5}{8}$	(1)
6.1.2	$P(A \text{ and } B) = \frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{3}{8} + \frac{1}{4} - \frac{3}{32}$ $= \frac{17}{32}$	✓ $P(A \cap B)$ or $P(A \text{ and } B) = \frac{3}{32}$ ✓ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ✓ $\frac{17}{32}$	(3)
6.2.1	 $P(\text{both BMW}) = \frac{18}{32} \times \frac{17}{31} = \frac{153}{496} \approx 0{,}31$	✓✓Tree diagram (branches) ✓ $\frac{18}{32} \times \frac{17}{31}$ ✓ $\frac{153}{496} \approx 0{,}31$	(4)
6.2.1	$P(\text{BMW ... VW}) = \frac{18}{32} \times \frac{14}{31} = \frac{63}{248} \approx 0{,}25$	✓ $\frac{18}{32} \times \frac{14}{31}$ ✓ $\frac{63}{248} \approx 0{,}25$	(2)
6.3.1	$n(E) = 7.13!.6$ $n(S) = 15!$ $P(E) = \frac{n(E)}{n(S)} = \frac{7.13!.6}{15!} = \frac{1}{5}$	✓ $n(E) = 7.13!.6$ ✓ $n(S) = 15!$ ✓ $\frac{1}{5}$	(3)
6.3.2	$n(E) = 8!\,7!$ $n(S) = 15!$ $P(E) = \frac{n(E)}{n(S)} = \frac{8!\,7!}{15!} = \frac{1}{6435}$	✓ $n(E) = 8!\,7!$ ✓ $P(E) = \frac{n(E)}{n(S)} = \frac{8!7!}{15!}$ ✓ $\frac{1}{6435}$	(3)
			[16]

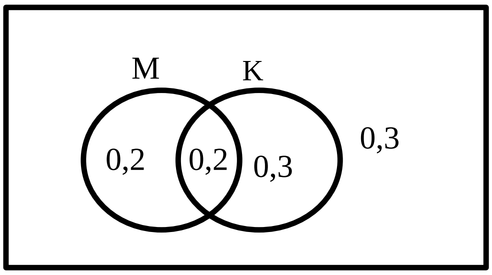
QUESTION 7

7.1.1	$8^8 = 167770216$ ways **MERAFONG**	✓✓ 8^8 (2)
7.1.2	$P(R...N) = \frac{1!\times 6!\times 1!}{8!}$ $= \frac{6!}{8!}$ $= \frac{1}{56}$ $P(R...N) = \frac{1!\times 6^6!\times 1!}{8^8}$ $= \frac{1!\times 6^6 \times 1!}{8^8}$ $= \frac{729}{262144}$ **OR** $P(R....N) = \frac{1}{8}\times\frac{1}{7} = \frac{1}{56}$	✓ $\frac{1!\times 6!\times 1!}{8!}$ ✓ $\frac{1}{56}$ ✓ $\frac{1}{8}\times\frac{1}{7}$ ✓ $\frac{1}{56}$ (2)
7.1.3	$P(a,e,o) = \frac{3!\text{ x }6!}{8!}$ $= \frac{3}{28}$ $P(a,e,o) = \frac{3!\text{ x }6^6}{8^8}$ $= 0{,}02$ **OR** $P(a,e,o) = \frac{3!\text{ x } 5!\text{ x }6}{8!}$ $= \frac{3}{28}$	✓ 3! ; ✓ 6! ✓ $\frac{3}{28}$ ✓ 3! ✓ 6^6 ✓ 0,02 (3)
7.2.1		✓ $P(S\ and\ G) = 0{,}3$ ✓ $P(S\text{ not }G) = 0{,}4$ ✓ $P(G) = 0{,}1$ ✓ $U/(S\ or\ G) = 0{,}2$ (4)
7.2.2 a)	$P(S') = 0{,}3$	✓answer (1)
b)	$P(S\ and\ G)' = 0{,}7$	✓answer (1)
c)	$P(S\ or\ G) = 0{,}8$	✓answer (1)
		[14]

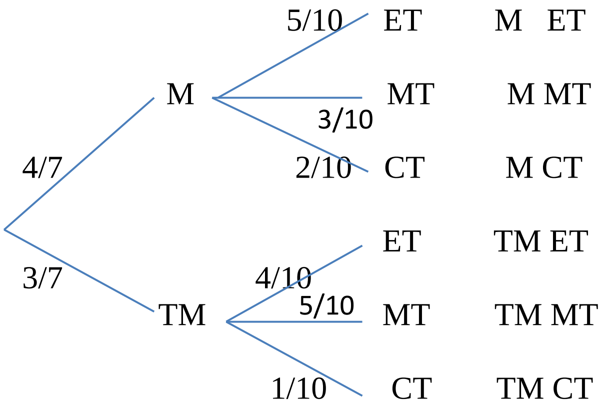
QUESTION 8

8.1.1	150	A✓answer	(1)
8.1.2(a)	$P(\text{Male}) = \frac{70}{150} = \frac{7}{15}$	A✓answer	(1)
8.1.2(b)	$P(\text{Eating Chocolate}) = \frac{80}{150} = \frac{8}{15}$	A✓answer	(1)
8.1.3	$P(\textit{Male eating chocolate}) = \frac{45}{150} = 0{,}3$ $P(\textit{Male}) \times P(\textit{Eating chocolate})$ $= \frac{70}{150} \times \frac{80}{150} = \frac{56}{225} = 0{,}249$ $P(\text{Male and Eating chocolate}) \neq$ $P(\text{Male}) \times P(\text{Eating chocolate})$ Events are not independent	A✓P(male eating chocolate) CA✓ P(Male) x P(eating chocolate) value CA✓conclusion	(3)
8.2.1	6 x 7 x 7 x 7 = 2058	A✓6 x 7 x 7 x 7 A✓2058	(2)
8.2.2	6 x 6 x 5 x 4 = 720	A✓6 x 6 x 5 x 4 A✓720	(2)
8.2.3	Four digit codes divisible by 5:- 6 x 7 x 7 x 1 = 252 Probability of a four-digit code divisible by 5 $= \frac{252}{2058} = \frac{6}{49} = 0{,}1224 = 12{,}24\ \%$	A✓6 x 7 x7 x 1 A✓252 CA✓answer	(3)
			[13]

QUESTION 9

9.1	9.1.1	 P(M) = 0,4	✓ ✓ ✓ ✓	P (M and K) correctly placed/ Placing P (not M and not K) correctly placing 0,3 for P (K only) calculating P(M)	(4)
	9.1.2	P (M) $\times P(K) = 0,4 \times 0,5 \;\; =,2$ $= P$(K and M) so the events are independent	✓ ✓ ✓	P (M) $\times P(K) =$ 0.2 P(K and M) = P (M) $\times P(K)$ Conclusion	(3)
9.2	9.2.1	7! = 5040	✓✓	answer	(2)
	9.2.2	Koketso and Marvin can sit together in 2!6! ways they will not sit together in 7! – 2! 6! ways/ = 3600	✓✓ ✓ ✓	2! 6! 7! – 2! 6! answer	(4)
	9.2.3	$\frac{1}{6}$	✓✓	answer	(2)
					[15]

QUESTION 10

10.1		First event: ✓ M 4/7; TM 3/7 Second event: ✓ M: ET 5/10; MT 3/10; CT 2/10 ✓ TM: ET 4/10; MT 5/10; CT 1/10 ✓ outcomes (4)
10.2.1	$P(\text{TM and MT}) = \frac{3}{7} \cdot \frac{5}{10}$ $= \frac{3}{14}$	✓ $\frac{3}{7} \cdot \frac{5}{10}$ ✓ $\frac{3}{14}$ or 0,21 (2)
10.2.2	$P(\text{ET}) = \frac{4}{7} \cdot \frac{5}{10} + \frac{3}{7} \cdot \frac{4}{10}$ $= \frac{16}{35} = 0,46$	✓ $\frac{4}{7} \cdot \frac{5}{10}$ ✓ $\frac{3}{7} \cdot \frac{4}{10}$ ✓ $\frac{16}{35}$ or 0,46 (3) **[9]**

QUESTION 11

11.1.1	$10^6 = 1\ 000\ 000$	✓ 10^6 or 1 000 000 (1)
11.1.2	(8)(7)(6)(5) = 1680 **OR** ${}_8P_4$ = 1680	✓ (8)(7)(6)(5) ✓ 1680 (2) ✓ ${}_8P_4$ ✓ 1680 (2)
11.2	$10! - (9!)(2!)$ $= 3\ 628\ 800 - 725\ 760$ $= 2\ 903\ 040$	✓ 10! ✓ (9!)(2!) ✓ answer (3) **[6]**

QUESTION 12

#	SUGGESTED ANSWER	DESCRIPTORS	Ma
12.1.1	$P(\text{female and}/\cap\text{ green eyes}) = \frac{147}{540} = \frac{49}{180} = 27{,}22\%$	$\frac{147}{540}$ or $\frac{49}{180}$ or $27{,}22\%$ ✓✓ Answer	(2)
12.1.2	For events to be independent: $P(\text{female and}/\cap\text{ green eyes})$ $= P(\text{green eyes}) \times P(\text{female})$ $P(\text{green eyes}) \times P(\text{female}) = \frac{330}{540} \times \frac{240}{540} = \frac{22}{81}$ $= 0{,}27$ $P(\text{female and}/\cap\text{ green eyes}) = \frac{147}{540} = \frac{49}{180} = 0{,}27$ $\therefore P(\text{female and}/\cap\text{ green eyes})$ $= P(\text{green eyes}) \times P(\text{female})$ **Events are independent and the learner is correct.** **Alternative** For events to be independent: $P(\text{male and}/\cap\text{ green eyes})$ $= P(\text{green eyes}) \times P(\text{male})$ $P(\text{green eyes}) \times P(\text{male}) = \frac{330}{540} \times \frac{300}{540} = \frac{55}{162} = 0{,}34$ $P(\text{male and}/\cap\text{ green eyes}) = \frac{183}{540} = \frac{61}{180} = 0{,}34$ $P(\text{male and}/\cap\text{ green eyes})$ $= P(\text{green eyes}) \times P(\text{male})$ **Events are independent and the learner is correct.**	✓ P(green eyes) = 330/540 ✓ P(female) = 240/540 ✓ 0,27 ✓ P(female and/∩ green eyes) = 0,27 ✓ Deduction ✓ P(green eyes) = 330/540 ✓ P(male) = 300/540 ✓ 0,34 ✓ P(male and/∩ green eyes) = 0,34 ✓ Deduction	(5) (5)

12.2.1	$n(S) = \dfrac{10!}{2! \times 2! \times 3!} = 151200$	✓ Numerator ✓Denominator ✓ Answer	(3)
12.2.2	$n(E) = \dfrac{8!}{2! \times 2!} = 10080$ $\therefore P(E) = \dfrac{10080}{151200} = \dfrac{1}{15}$	✓ $\frac{8!}{2! \times 2!}$ ✓10080 ✓$P(E) = \frac{10080}{151200}$ ✓ Answer	(4)
			[14