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# Revision of Grade 10 and 11 Probability

## Grade 12 CAPS Mathematics Series

Govan Mbeki Mathematics Development Centre
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# Outcomes for this Topic

In this topic we will revise :

- **Probability terminology.**

Unit 1

- **Probability identities.**

Unit 2

- **Probability for dependent and independent events.**

Unit 3

- **Conditional probability.**

**Not in the CAPS Curriculum anymore**

Unit 4

- **Use of Venn Diagrams and Contingency Tables.**

Unit 5

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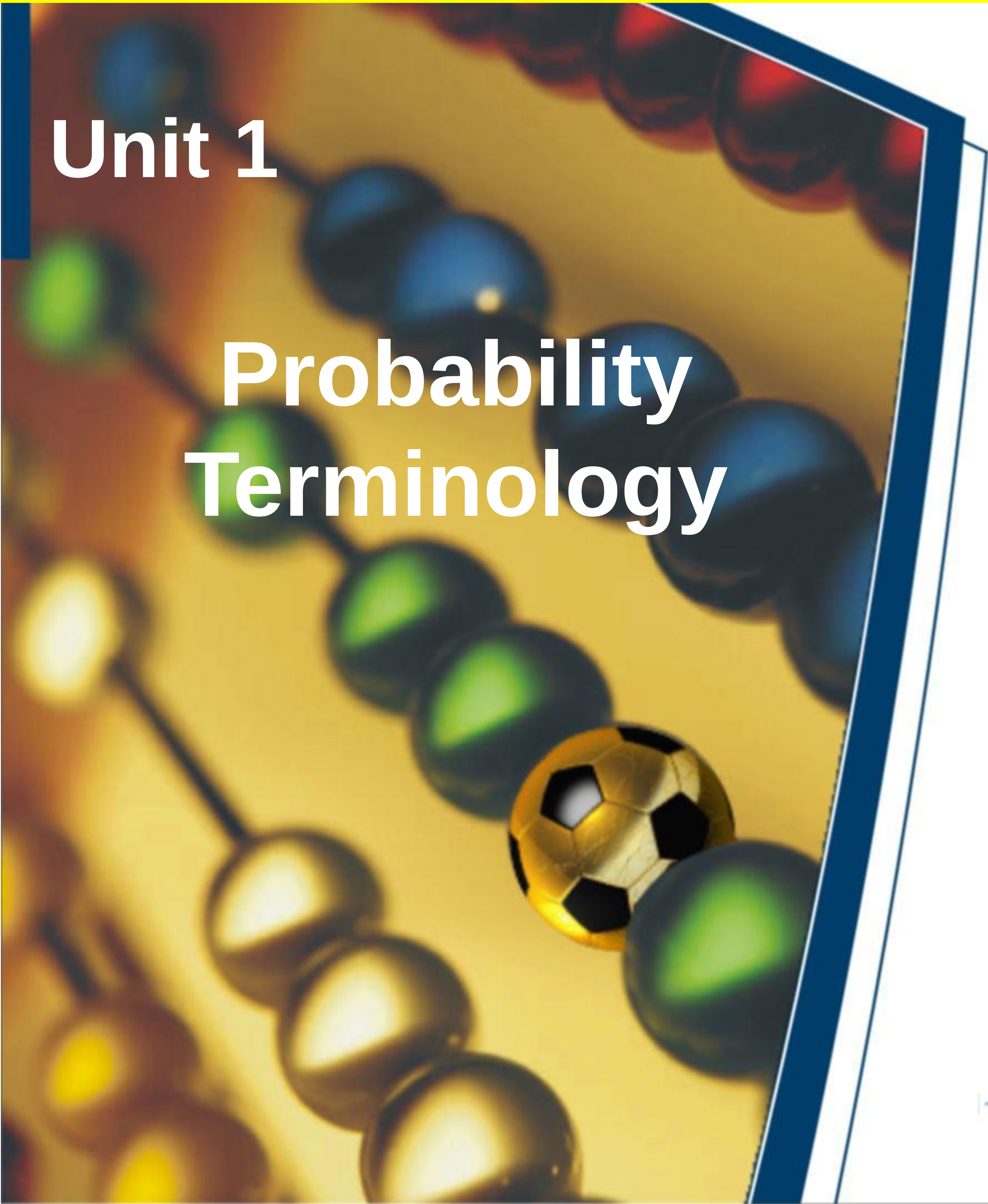

# Unit 1

# Probability Terminology

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# Outcomes for Unit 1

In this Unit section our revision exercise will include discussing :

- ✓ what **probability** is
- ✓ what a **statistical experiment** is
- ✓ the concept **outcome of an event**
- ✓ the concept **sample space**
- ✓ the concepts **event** and **number of events** in a sample space
- ✓ the concept of **theoretical probability**
- ✓ concepts **union** and **intersection** of events

# What is Probability?

- **Probability** is the mathematics of chance that enables us to predict the likelihood of uncertain occurrences and tells us the relative frequency with which we expect an event to occur.

- **Examples :**

1. The probability that it will rain in PE today is $0{,}3 = 30\%$.
2. The probability of contracting the flu virus is $70{:}100 = 70\%$.
3. The probability of winning the Lotto is $\dfrac{1}{13\ 983\ 816} \approx 0{,}000\ 007\%$.

- Probability can be reported as a **fraction, decimal or percentage**.

- The greater the probability, the more likely the event is to occur. Conversely, the smaller the probability, the less likely the event is to occur. In the examples there is a **strong probability** that you may contract the virus, a **reasonable chance** of rain in PE today, but almost **no chance** of you winning the Lotto.

# Experiment, Outcome, Sample Space and Event

- To study probability in a mathematically precise way, we need special **terminology** and **notation**.

- **An experiment** is the act of making an observation or taking a measurement e.g. toss a fair coin and record whether the side up is heads or tails.

- An **outcome** is one of the possible things that can occur as a result of an experiment e.g. H (heads) and T (tails) are the two possible outcomes in the coin-tossing experiment.

- The set of all possible outcomes is called the **sample space**. In the coin-tossing experiment $S = \{H, T\}$ is the sample space. An **event** is any subset of the sample space.

# Number of Events in Sample Space : $2^{n(S)}$

| Sample Space | Events | Number of Events |
|---|---|---|
| $S=\{\ \}$ | $\{\ \}$ | $1=2^0=2^{n(S)}$ |
| $S=\{a\}$ | $\{\ \},\{a\}$ | $1+1=2=2^1=2^{n(S)}$ |
| $S=\{a;b\}$ | $\{\ \},\{a\},\{b\},\{a;b\}$ | $1+2+1=4=2^2=2^{n(S)}$ |
| $S=\{a;b;c\}$ | $\{\ \}$<br>$\{a\},\{b\},\{c\}$<br>$\{a;b\},\{a;c\},\{b;c\}$<br>$\{a;b;c\}$ | $1+3+3+1=8=2^3=2^{n(S)}$ |
| $S=\{a;b;c;d\}$ | $\varnothing$<br>$\{a\},\{b\},\{c\},\{d\}$<br>$\{a;b\},\{a;c\},\{a;d\},\{b;c\},\{b;d\},\{c;d\}$<br>$\{a;b;c\},\{a;b;d\},\{a;c;d\},\{b;c;d\}$<br>$S$ | $1+4+6+4+1=16=2^{n(S)}$ |

# Tossing a single coin in an experiment

**Experiment 1 :** Toss a fair coin, and record after each trial whether the side up is heads or tails.


- Two possible outcomes $H$ or $T$.
  Hence the sample space $S = \{H, T\}$ and $n(S) = 2$.

- There are $2^2 = 4$ events and since an event is simply a subset of the sample space the events in this experiment are $\{\ \}$, $\{H\}$, $\{T\}$ and $\{H, T\}$.

- Events in this experiment can be described in the following way:

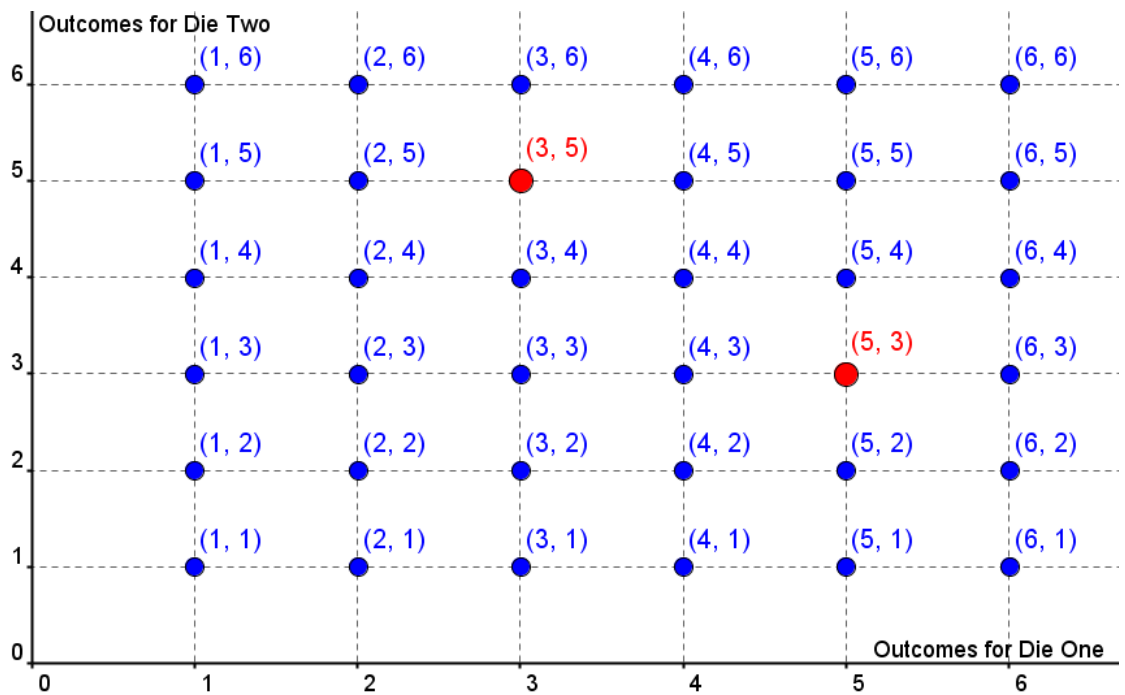
| **Event** | **Description of event** |
|---|---|
| $\{H\}$ | Getting heads |
| $\{T\}$ | Getting tails |
| $S = \{H, T\}$ | Getting either heads or tails |
| $\{\ \} = \varnothing$ | Getting neither heads nor tails<br>How is this possible? |

It is not always possible to give descriptions of the different events.

# Rolling a pair of dice in an experiment

**Experiment 2 :** Roll a pair of six-sided fair dice and record, respectively, the number value and number of dots showing on the top faces of the die after each trial.

- For this experiment, the sample space consist of 36 ordered pairs of outcomes and $2^{36}$ events. The 36 ordered pairs can be illustrated graphically.

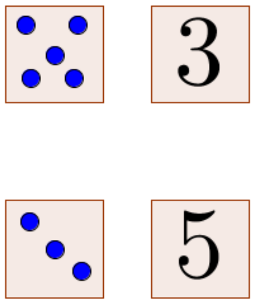



**Note :**

$(5;3)$ & $(3;5)$

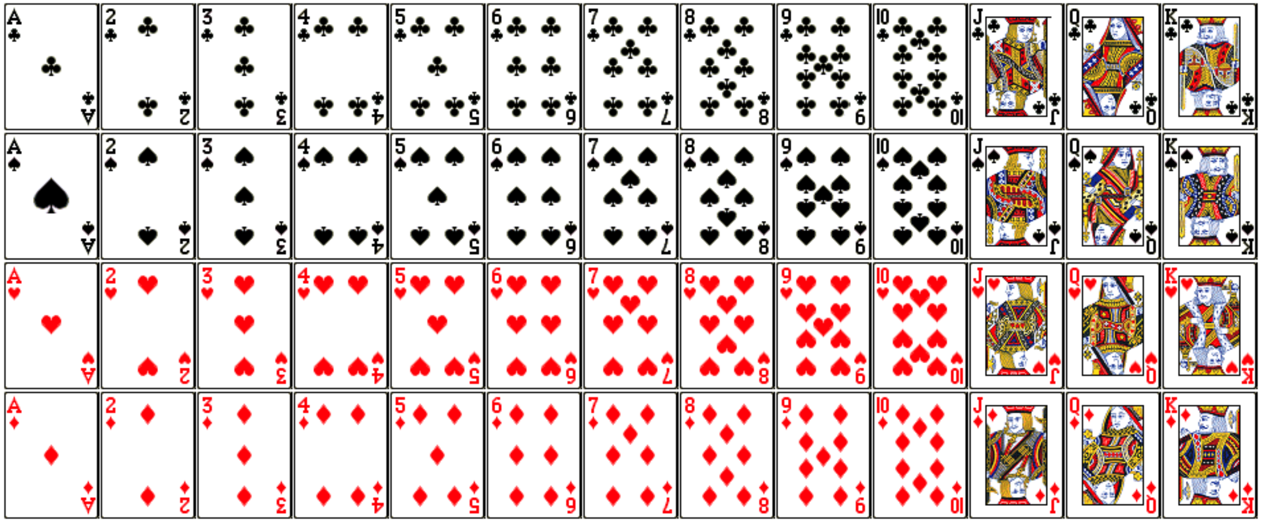
Are different outcomes

Many more such pairs!



# Experiment : Draw a card from a standard pack of cards

**Experiment 3 :** A card is randomly drawn from a standard deck of 52 playing cards.



**See illustration of :**
Drawing a face card,
as one of these events.

- For this experiment, the sample space consist of 52 outcomes and $2^{52} = 4\ 503\ 599\ 627\ 370\ 496$ events.

# Theoretical Probability

- **Probability** is a real number in the closed interval $[0;1]$ that describes how likely it is that an event will occur.

- $\begin{cases} \text{A probability of 0 means that an event will never occur (impossible).} \\ \text{A probability of 1 means that an event will always occur (certain).} \\ \text{A probability of 0,5 means that an event will occur half the time.} \end{cases}$

- Define theoretical probability of an event as:

$$P(E) = \frac{\text{Number of outcomes in } E}{\text{Number of outcomes in } S} = \frac{n(E)}{n(S)}$$

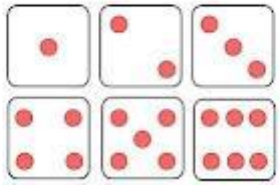
- $\begin{cases} P(\varnothing) = 0 \text{ and } P(S) = 1 \\ P(\varnothing) \le P(E) \le P(S) \\ 0 \le P(E) \le 1 \end{cases}$

# Theoretical Probability from single coin - tossing experiment

| Event | Description of event |
|---|---|
| $E_1 = \{H\}$ | Getting heads |
| $E_2 = \{T\}$ | Getting tails |
| $E_3 = S = \{H, T\}$ | Getting either heads or tails |
| $E_4 = \{\ \} = \varnothing$ | Getting neither heads nor tails |

$$\begin{cases} P(\text{getting heads}) = P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{1}{2} = 50\% \\ P(\text{getting tails}) = P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{1}{2} = 50\% \\ P(\text{getting either heads or tails}) = P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{2}{2} = 1 = 100\% \\ P(\text{getting neither heads nor tails}) = P(E_4) = \dfrac{n(E_4)}{n(S)} = \dfrac{0}{2} = 0 = 0\% \end{cases}$$

# Theoretical Probability from rolling a single die experiment



$$S=\{1;\ 2;\ 3;\ 4;\ 5;\ 6\}\Rightarrow n(S)=6$$

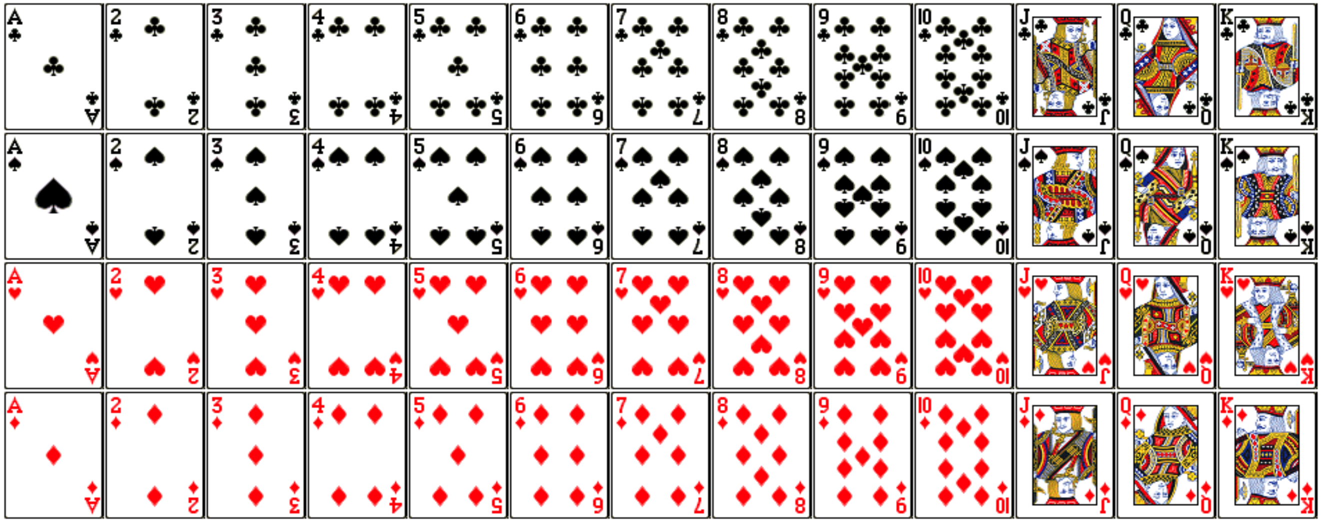
$$P(\text{getting a number smaller than 3})$$
$$=P(E)\qquad \text{where } E=\{1;2\}$$
$$=\frac{n(E)}{n(S)}=\frac{2}{6}=\frac{1}{3}$$

$$P(\text{getting a six})$$
$$=P(E)\qquad \text{where } E=\{6\}$$
$$=\frac{n(E)}{n(S)}=\frac{1}{6}$$

$$P(\text{getting an even number})$$
$$=P(E)\qquad \text{where } E=\{2;4;6\}$$
$$=\frac{n(E)}{n(S)}=\frac{3}{6}=\frac{1}{2}$$

$$P(\text{getting a number greater than six})$$
$$=P(E)\qquad \text{where } E=\{\ \}=\varnothing$$
$$=\frac{n(E)}{n(S)}=\frac{0}{6}=0$$

# Theoretical Probability from drawing a card experiment



$$\therefore n(S) = 52$$

$$P(\text{drawing a diamond}) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$P(\text{drawing either an ace or a king}) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

$$P(\text{drawing a diamond face card}) = \frac{n(E)}{n(S)} = \frac{3}{52}$$

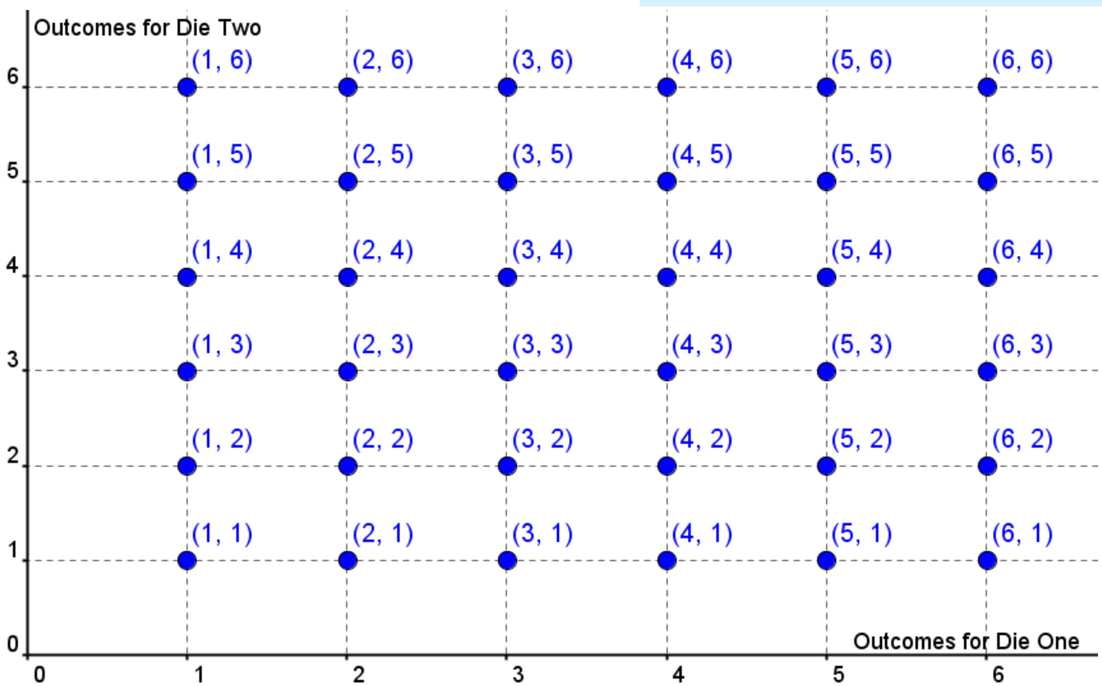
$$P(\text{drawing a face card}) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

# Tutorial 1: Theoretical Probabilities

Roll a pair of six-sided fair dice and record, respectively, the number and number of dots showing on the top faces of the die after each trial.

Determine the theoretical probability for each of the following events:

1. Event $A$: The sum of the dots on the dice is equal to 7
2. Event $B$: The sum of the dice is greater than 5
3. Event $C$: At least one die must show a 4 faceup.




**PAUSE**

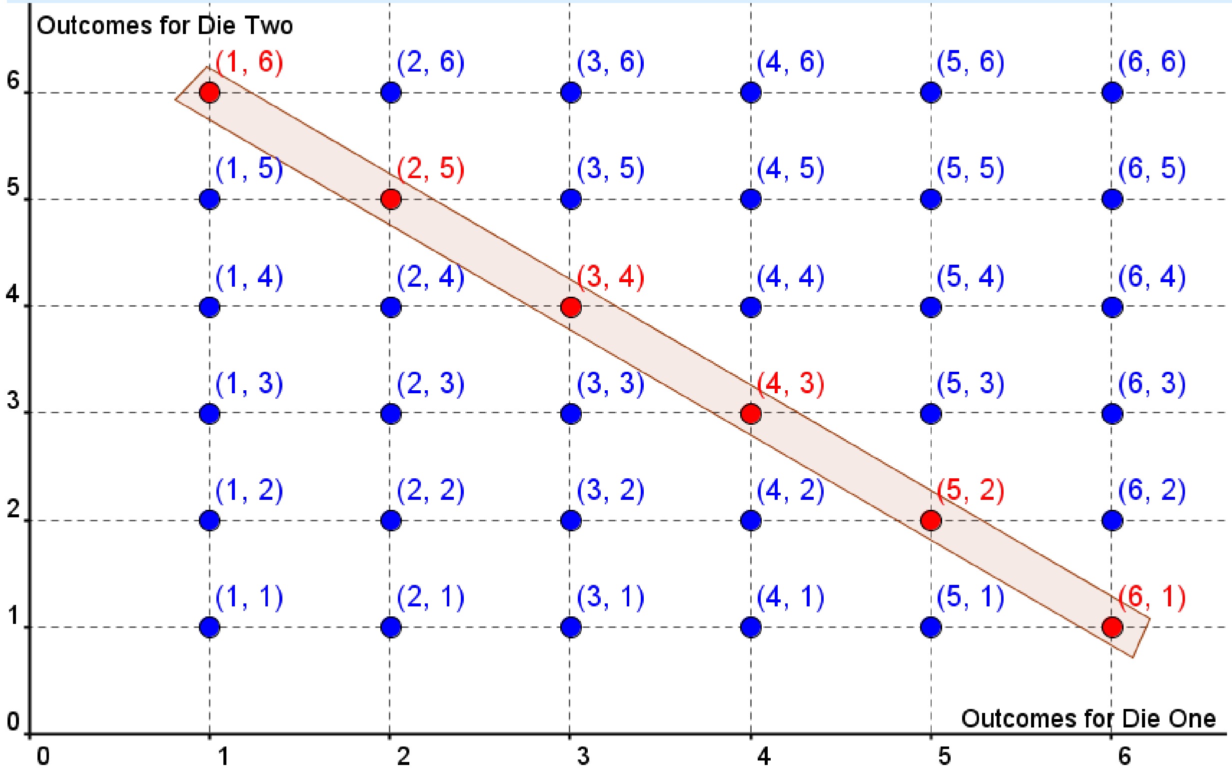
- **Do Tutorial 1**
- **Then View Solutions**

# Tutorial 1: Problem 1: Suggested Solution

1. Determine the theoretical probability for event *A*.

Event $A$: The sum of the dots on the dice is equal to 7.

$$A=\{(1;6);(2;5);(3;4);(4;3);(5;2);(6;1)\}\Rightarrow n(A)=6$$

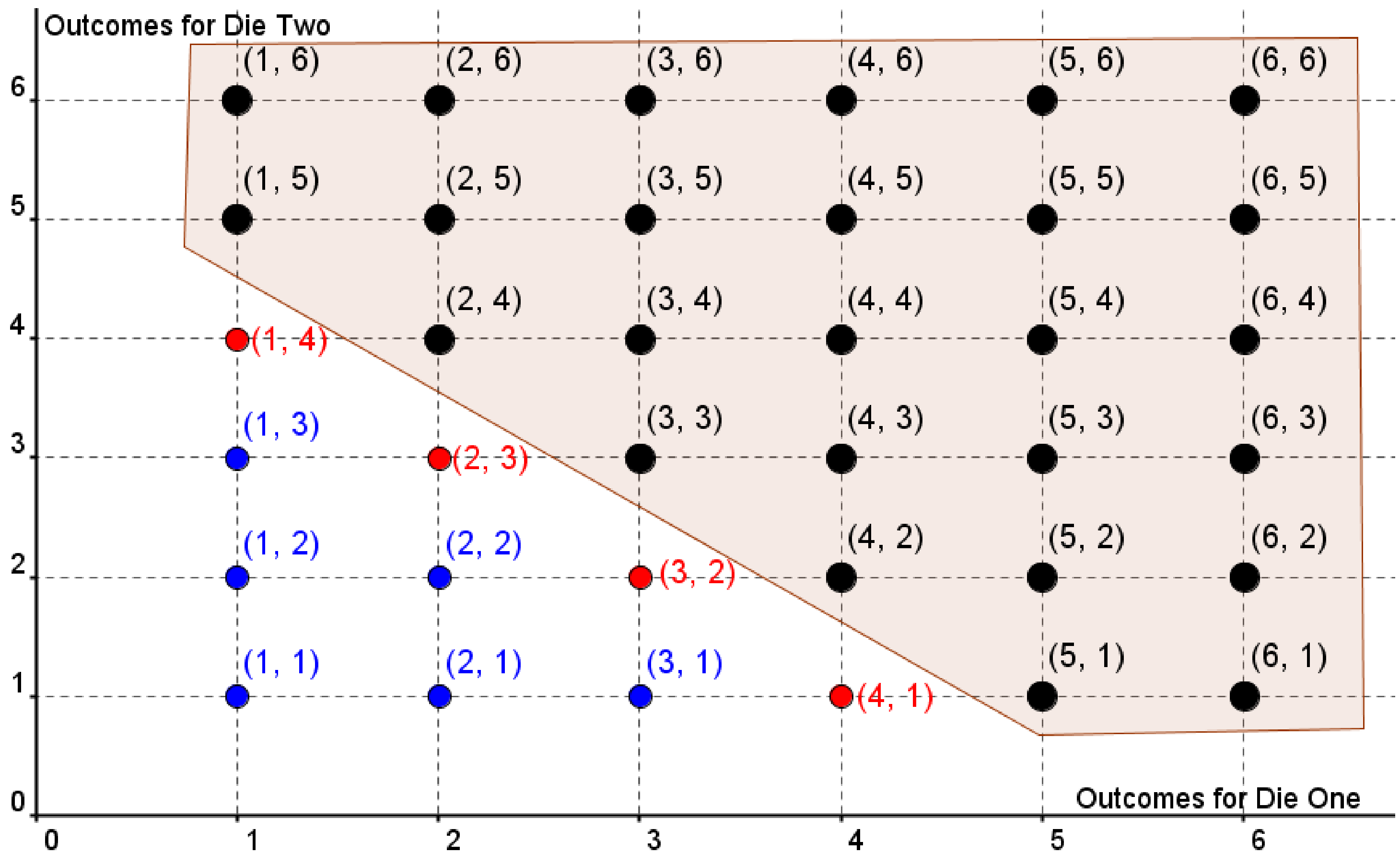



$$P(A)=\frac{n(A)}{n(S)}=\frac{6}{36}=\frac{1}{6}$$

# Tutorial 1: Problem 2: Suggested Solution

2. Determine the theoretical probability for event *B*.

Event *B*: The sum of the dots on the dice is greater than 5



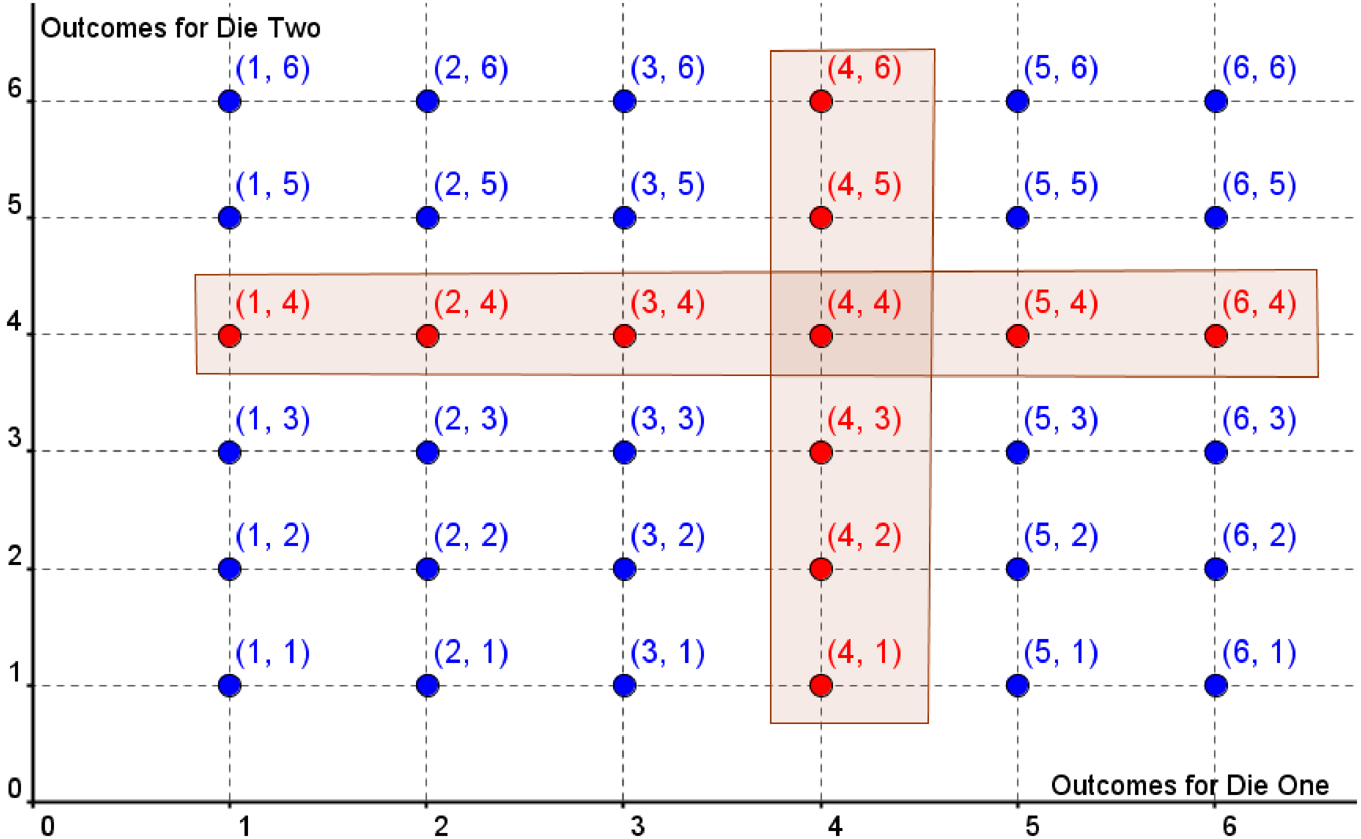

$$n(B) = 26$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{26}{36} = \frac{13}{18}$$

# Tutorial 1: Problem 3: Suggested Solution

3. Determine the theoretical probability for event $C$.

Event $C$ : At least one die must show a 4 face-up.




$$n(C) = 11$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{11}{36}$$

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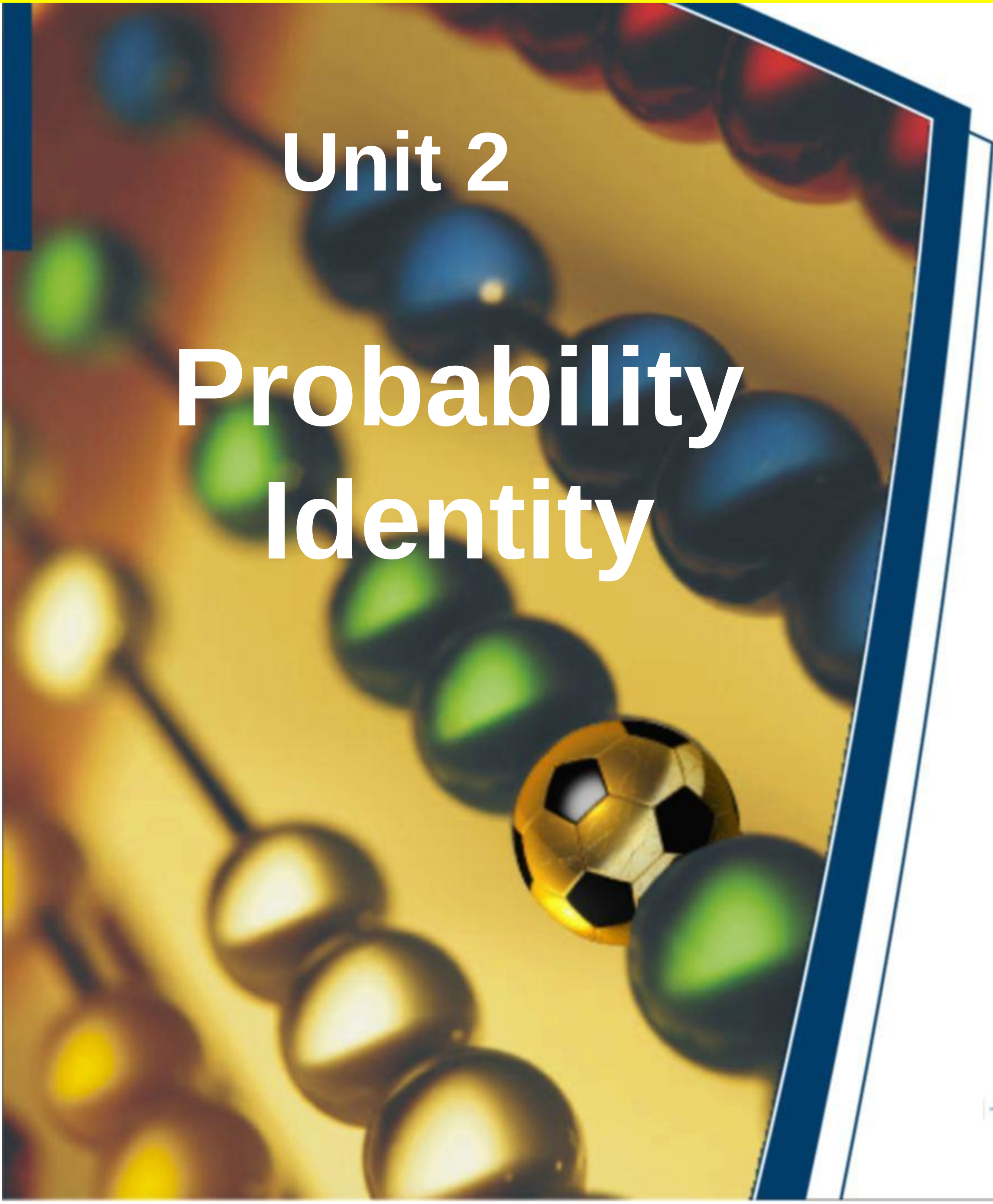

# Unit 2

# Probability Identity

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# Outcomes for Unit 2

## In this Unit section we will revise :

- ✓ The probability identity

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

- ✓ Concept of **mutually exclusive** events when :

$$A\cap B=\varnothing$$

- ✓ Concept of **complementary** events when :

$$A\cap B=\varnothing \textbf{ and } A\cup B=S$$

# Venn diagram representation of events and sample space

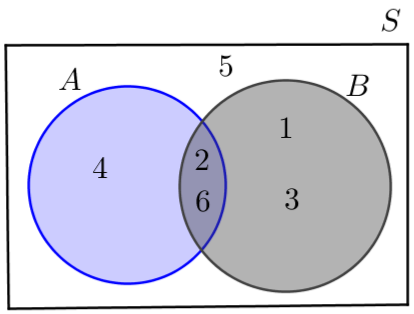
Consider the following three sets of events linked to the sample space $S=\{\text{outcomes when rolling a single die}\}=\{1;2;3;4;5;6\}$.

1. $A=\{\text{getting an even number}\}=\{2;4;6\}$

   $B=\{\text{getting a factor of 6}\}=\{1;2;3;6\}$

   $A\cup B=\{\text{getting an even number } \textbf{or} \text{ a factor of 6}\}=\{1;2;3;4;6\}$

   $A\cap B=\{\text{getting an even number } \textbf{and} \text{ a factor of 6}\}=\{2;6\}$

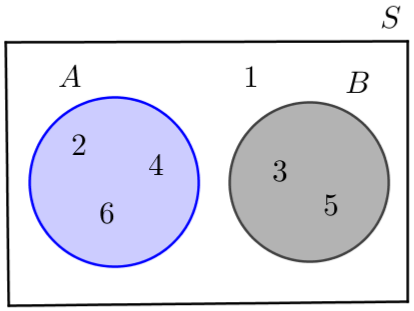



2. $A=\{\text{getting an even number}\}=\{2;4;6\}$

   $B=\{\text{getting a odd prime number}\}=\{3;5\}$

   $A\cup B=\{\text{getting an even number } \textbf{or} \text{ odd prime number}\}=\{2;3;4;5;6\}$

   $A\cap B=\{\text{getting an even number } \textbf{and} \text{ odd prime number}\}=\{\ \}=\varnothing$

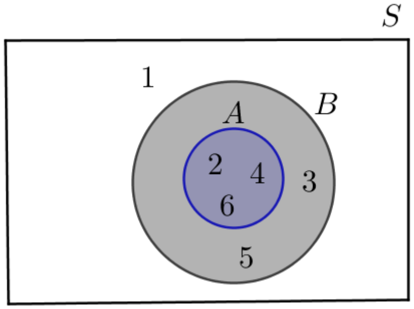



3. $A=\{\text{getting an even number}\}=\{2;4;6\}$

   $B=\{\text{getting a number } >1\}=\{2;3;4;5;6\}$

   $A\cup B=\{\text{getting an even number } \textbf{or} \text{ a number } >1\}=\{2;3;4;5;6\}=B$

   $A\cap B=\{\text{getting an even number } \textbf{and} \text{ a number } >1\}=\{2;4;6\}=A$



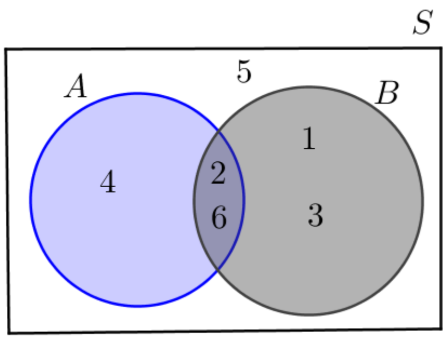
# Example 1: Identity $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

1. Consider the example where:

$S=\{\text{outcomes when rolling a single die}\}$ and the events

$A=\{\text{getting even number}\}=\{2;4;6\}$ and

$B=\{\text{getting a factor of 6}\}=\{1;2;3;6\}$




Show that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ holds for this example:

$S=\{1;2;3;4;5;6\}\Rightarrow n(S)=6$

- $A=\{2;4;6\}\Rightarrow n(A)=3$
- $B=\{1;2;3;6\}\Rightarrow n(B)=4$
- $A\cup B=\{1;2;3;4;6\}\Rightarrow n(A\cup B)=5$
- $A\cap B=\{2;6\}\Rightarrow n(A\cap B)=2$

- $P(A\cup B)=\dfrac{n(A\cup B)}{n(S)}=\dfrac{5}{6}$
- $P(A)=\dfrac{n(A)}{n(S)}=\dfrac{3}{6}$
- $P(B)=\dfrac{n(B)}{n(S)}=\dfrac{4}{6}$
- $P(A\cap B)=\dfrac{n(A\cap B)}{n(S)}=\dfrac{2}{6}$

$$\therefore P(A)+P(B)-P(A\cap B)=\frac{3}{6}+\frac{4}{6}-\frac{2}{6}=\frac{5}{6}=P(A\cup B)$$

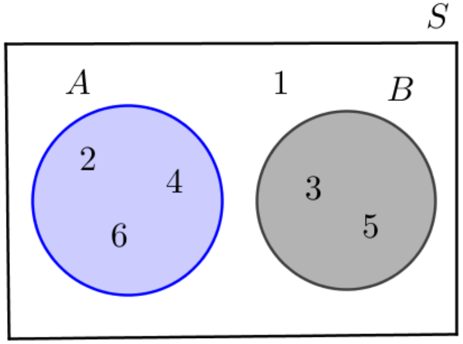
# Example 2: Identity $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. Consider the example where:

$S = \{\text{outcomes when rolling a single die}\}$ and the events

$A = \{\text{getting even number}\} = \{2;4;6\}$ and

$B = \{\text{getting an odd prime number}\} = \{3;5\}$




Show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ holds for this example:

$S = \{1;2;3;4;5;6\} \Rightarrow n(S) = 6$

- $A = \{2;4;6\} \Rightarrow n(A) = 3$
- $B = \{3;5\} \Rightarrow n(B) = 2$
- $A \cup B = \{2;3;4;5;6\} \Rightarrow n(A \cup B) = 5$
- $A \cap B = \{\ \} = \varnothing \Rightarrow n(A \cap B) = 0$

- $P(A \cup B) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{5}{6}$
- $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6}$
- $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{6}$
- $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{0}{6}$

$$\therefore P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{2}{6} - \frac{0}{6} = \frac{5}{6} = P(A \cup B)$$

# Tutorial 2: Identity $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Consider the example where:

$S = \{\text{outcomes when rolling a single die}\}$ and the events

$A = \{\text{getting an even number}\} = \{2;4;6\}$ and

$B = \{\text{getting a number } > 1\} = \{2;3;4;5;6\}$

1. Illustrate the events and sample space by means of an appropriate Venn diagram.
2. Confirm that the Identity $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ holds for this example.

**PAUSE**

- **Do Tutorial 2**
- **Then View Solutions**

# Tutorial 2: Problems 1 and 2: Suggested solutions

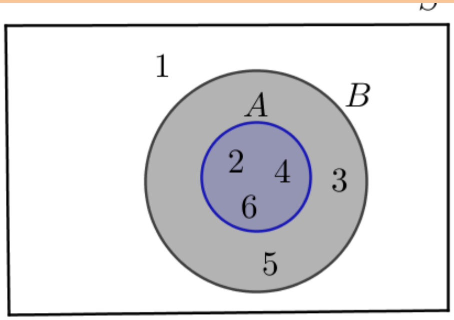
Given:

$S = \{\text{outcomes when rolling a single die}\}$ and the events

$A = \{\text{getting an even number}\} = \{2;4;6\}$ and

$B = \{\text{getting a number } > 1\} = \{2;3;4;5;6\}$

1. Appropriate Venn Diagram:




2. Show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ holds for this example:

$$S = \{1;2;3;4;5;6\} \Rightarrow n(S) = 6$$

- $A = \{2;4;6\} \Rightarrow n(A) = 3$
- $B = \{2;3;4;5;6\} \Rightarrow n(B) = 5$
- $A \cup B = B = \{2;3;4;5;6\} \Rightarrow n(A \cup B) = 5$
- $A \cap B = A = \{2;4;6\} \Rightarrow n(A \cap B) = 3$

- $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6}$
- $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{5}{6}$
- $P(A \cup B) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{5}{6} = P(B)$
- $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{3}{6} = P(A)$

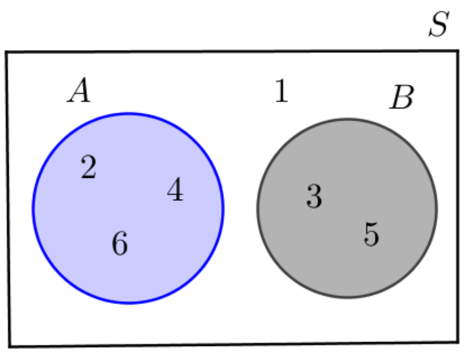
$$\therefore P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{5}{6} - \frac{3}{6} = \frac{5}{6} = P(A \cup B)$$

# Mutually exclusive events: $A \cap B = \varnothing$

**Definition :**

- Two events are called mutually exclusive if they cannot occur at the same time.
- Alternatively two event sets, $A$ and $B$, are mutually exclusive if they do not have any elements in common.

  This implies that $A \cap B = \varnothing$ or $P(A \cap B) = 0$.
- In addition: $n(A \cup B) = n(A) + n(B) \Rightarrow P(A \cup B) = P(A) + P(B)$




$A \cap B = \varnothing \Leftrightarrow n(A \cap B) = 0$

**Example of two mutually exclusive events :**

- Sample space $S = \{\text{outcomes when rolling a single die}\} = \{1;2;3;4;5;6\}$
- Events: $\begin{cases} A = \{\text{getting an even number}\} = \{2;4;6\} \\ B = \{\text{getting a odd prime number}\} = \{3;5\} \end{cases}$

In addition: $P(A) + P(B) = \dfrac{3}{6} + \dfrac{2}{6} = \dfrac{5}{6} = P(A \cup B)$

# Complementary events: $A \cap B = \varnothing$ and $A \cup B = S$

**Note :**

(1) $A \cap A' = \varnothing$

(2) $A \cup A' = S$

(3) $1 = P(S) = P(A \cup A')$
$= P(A) + P(A') - P(A \cap A')$
$= P(A) + P(A')$

(4) $P(A) = 1 - P(A')$ or $P(A') = 1 - P(A)$

**Definition :**

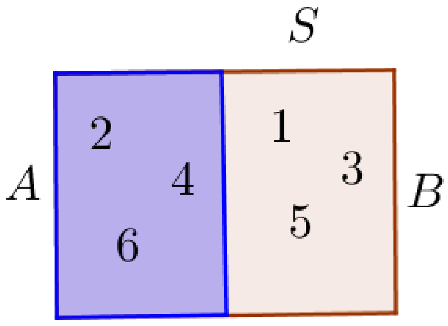
Two events, $A$ and $B$, are complementary events if and only if

1. $A \cap B = \varnothing$ and
2. $A \cup B = S$

In addition:
- $A' = B = S - A$
- $B' = A = S - B$
- $A$ and $B$ forms a **partitioning** of $S$

**Example of two complementary events :**

- Sample space $S = \{\text{rolling a die}\} = \{1, 2, 3, 4, 5, 6\}$
- Events: $A = B' = \{\text{getting an even number}\} = \{2, 4, 6\}$; $B = A' = \{\text{getting a odd number}\} = \{1, 3, 5\}$



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# Unit 3

# Dependent and Independent Events

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# Outcomes for Unit 3

In this Unit section we will revise :

- Identification of independent and dependent events.

- Calculation of the probability for dependent and independent events by means of Contingency Tables.

- Calculation of the probability for dependent and independent events by means of Tree Diagrams.

- Product Rule for independent events : $P(A \cap B) = P(A) \times P(B)$

# Independent Events and Product Rule

Two events are said to be independent if the result of the second event is not affected by the result of the first event.

**Example 1:** A white ball, red ball and yellow ball are placed in a box. In an experiment a ball is randomly drawn, its color recorded and it is **returned** to the box. A second ball is drawn and its color recorded. What are the probability of drawing a red ball and then a white ball?

Use a contingency table:

- $n(S) = 3 \times 3 = 9$

| | | First Draw | | |
|---|---|---|---|---|
| | | W | R | Y |
| Second Draw | W | WW | **RW** | YW |
| | R | WR | RR | YR |
| | Y | WY | RY | YY |

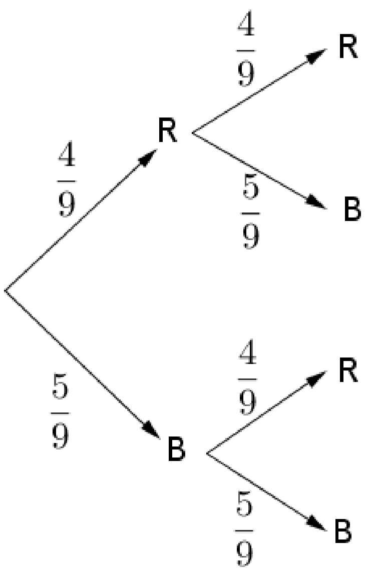
$\therefore$ If $A$ and $B$ are independent events then $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$

- $A = \{\text{red first}\} = \{RW; RR; RY\} \Rightarrow P(A) = \frac{3}{9} = \frac{1}{3}$
- $B = \{\text{white second}\} = \{WW; RW; YW\} \Rightarrow P(B) = \frac{3}{9} = \frac{1}{3}$
- $A \cap B = \{\text{first red then white}\} = \{RW\} \Rightarrow P(A \cap B) = \frac{1}{9} \Rightarrow P(A) \times P(B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} = P(A \cap B)$

# Independent Events : Use Tree Diagram

**Example 2 :** A packet contains 4 red paperclips and 5 blue paperclips. One clip is taken from the packet, the color recorded and then **placed back** in packet. A second clip is taken from the packet and the color recorded. What is the probability that the first clip is red and the second blue?

**Tree Diagram**




**Solution :**

$$P(A) = P(\{\text{First Red}\}) = P(RR) + P(RB)$$

$$= \frac{4}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9} = \frac{16+20}{81} = \frac{36}{81} = \frac{4}{9}$$

$$P(B) = P(\{\text{Second Blue}\}) = P(RB) + P(BB)$$

$$= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{5}{9} = \frac{20+25}{81} = \frac{45}{81} = \frac{5}{9}$$

$$P(A \cap B) = P(\{\text{First Red and Second Blue}\})$$

$$= \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$$

$$\therefore P(A) \times P(B)$$

$$= \frac{4}{9} \times \frac{5}{9} = \frac{20}{81} = P(A \cap B)$$

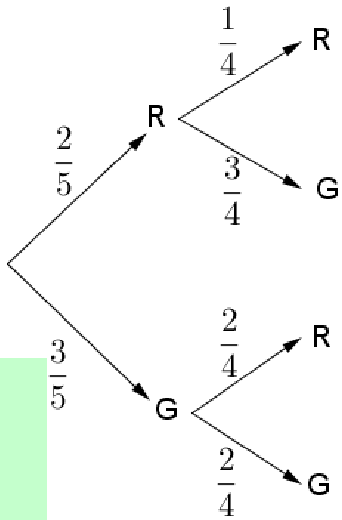
$\therefore$ $A$ and $B$ are independent events.

Thus draws **with replacement** lead to **independent events**.

# Probability for Dependent Events : Tree Diagram

**Example 3 :** A bag contains five marbles : two red and three green. Two marbles are drawn from the bag, one after the other, **without replacing** the first one drawn, what is the probability that the first marble is red and the second green?

If a result of one event is affected by the result of another event the events are said to be **dependent**.




**Solution :**

$$P(A) = P(\{\text{First Red}\}) = P(RR) + P(RG)$$

$$= \frac{2}{5}\times\frac{1}{4} + \frac{2}{5}\times\frac{3}{4} = \frac{2+6}{20} = \frac{8}{20} = \frac{2}{5}$$

$$P(B) = P(\{\text{Second Green}\}) = P(RG) + P(GG)$$

$$= \frac{2}{5}\times\frac{3}{4} + \frac{3}{5}\times\frac{2}{4} = \frac{6+6}{20} = \frac{12}{20} = \frac{3}{5}$$

$$P(A\cap B) = P(\{\text{First Red and Second Green}\})$$

$$= \frac{2}{5}\times\frac{3}{4} = \frac{6}{20} = \frac{3}{10}$$

$$\therefore P(A)\times P(B)$$

$$= \frac{2}{5}\times\frac{3}{5} = \frac{6}{25}$$

$$\neq P(A\cap B) \quad \left(\because \frac{6}{25} \neq \frac{3}{10}\right)$$

$\therefore$ If $P(A)\times P(B) \neq P(A\cap B)$ then $A$ and $B$ are dependent.

# Tutorial 3: Independent and Dependent Events and Identities

**Problem 1:** A bag contains 3 red and 5 blue marbles. One marble is randomly drawn from the bag, its color recorded and then **(a) placed back; (b) not placed back** in the bag. A second marble is then drawn and its color recorded.

1.1 What is the probability that the first marble is red?

1.2 What is the probability that the second marble is blue?

1.3 What is the probability that the first is red **and** the second is blue?

1.4 What is the probability that the first is red **or** the second is blue?

1.5 Are the first marble being red and the second being blue independent events?

**Problem 2:** Given that $P(A) = 0,6,\ P(B) = 0.4$ and $P(A \cap B) = 0,24.$

2.1 Are $A$ and $B$ mutually exclusive? Give a reason for answer.

2.2 Are $A$ and $B$ independent? Give a reason for answer.

2.3 Determine $P(A \cup B)$.

2.4 Determine $P(A \cap B')$.

2.5 Determine $P\left((A \cap B)'\right)$.

PAUSE

- Do Tutorial 3
- Then View Solutions

# Tutorial 3: Problem 1 (a): Suggested Solution

**Problem 1 (a) :** A bag contains 3 red and 5 blue marbles. One marble is randomly drawn from the bag, its color recorded and then **placed back** in the bag. A second marble is drawn and its color recorded.

$$1.1\quad P(A)=P(\{\text{first red}\})=P(RR)+P(RB)=\frac{3}{8}\times\frac{3}{8}+\frac{3}{8}\times\frac{5}{8}=\frac{3}{8}\left(\frac{3}{8}+\frac{5}{8}\right)=\frac{3}{8}\times 1=\frac{3}{8}$$

$$1.2\quad P(B)=P(\{\text{second blue}\})=P(RB)+P(BB)=\frac{3}{8}\times\frac{5}{8}+\frac{5}{8}\times\frac{5}{8}=\left(\frac{3}{8}+\frac{5}{8}\right)\times\frac{5}{8}=1\times\frac{5}{8}=\frac{5}{8}$$

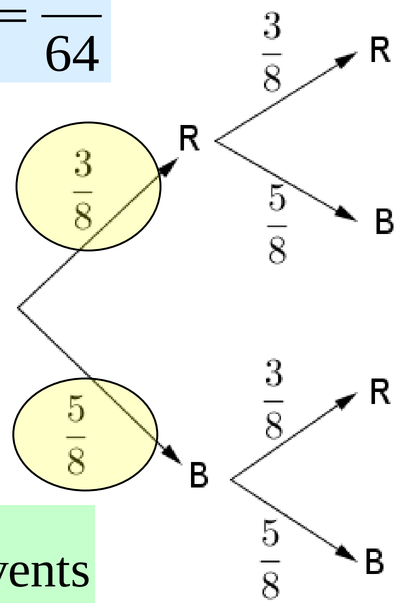
$$1.3\quad P(A\cap B)=P(\{\text{first red and second blue}\})=P(RB)=\frac{3}{8}\times\frac{5}{8}=\frac{15}{64}$$

$$1.4\quad P(A\cup B)=P(A)+P(B)-P(A\cap B)=1-\frac{15}{64}=\frac{49}{64}$$

**or** $P(A\cup B)=P(\{\text{red first or blue second}\})$

$$=P(RR)+P(RB)+P(BB)=\frac{9}{64}+\frac{15}{64}+\frac{25}{64}=\frac{49}{64}$$

$$1.5\quad P(A)\times P(B)=\frac{3}{8}\times\frac{5}{8}=\frac{15}{64}=P(A\cap B)\Rightarrow A \text{ and } B \text{ independent events}$$



# Tutorial 3: Problem 1 (b): Suggested Solution

**Problem 1 (b) :** A bag contains 3 red and 5 blue marbles. One marble is randomly drawn from the bag, its color recorded and then **not placed back** in the bag. A second marble is drawn and its color recorded.

1.1 $P(A) = P(\{\text{first red}\}) = P(RR) + P(RB) = \frac{3}{8}\times\frac{2}{7} + \frac{3}{8}\times\frac{5}{7} = \frac{3}{8}\left(\frac{2}{7}+\frac{5}{7}\right) = \frac{3}{8}\times 1 = \frac{3}{8}$

1.2 $P(B) = P(\{\text{second blue}\}) = P(RB) + P(BB) = \frac{3}{8}\times\frac{5}{7} + \frac{5}{8}\times\frac{4}{7} = \frac{15}{56} + \frac{20}{56} = \frac{35}{56} = \frac{5}{8}$

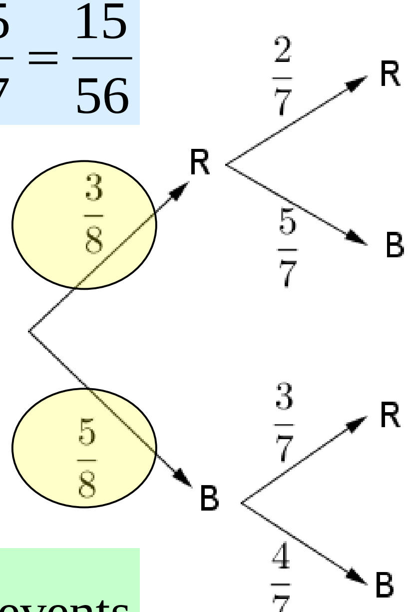
1.3 $P(A\cap B) = P(\{\text{first red and second blue}\}) = P(RB) = \frac{3}{8}\times\frac{5}{7} = \frac{15}{56}$

1.4 $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 1 - \frac{15}{56} = \frac{41}{56}$

**or** $P(A\cup B) = P(\{\text{red first or blue second}\})$

$= P(RR) + P(RB) + P(BB) = \frac{6}{56} + \frac{15}{56} + \frac{20}{56} = \frac{41}{56}$

1.5 $P(A)\times P(B) = \frac{3}{8}\times\frac{5}{8} = \frac{15}{64} \neq P(A\cap B) \Rightarrow A$ and $B$ dependent events



# Tutorial 3: Problem 2: Suggested Solution

**Problem 2 :** Given that $P(A)=0.6,\ P(B)=0,4$ and $P(A\cap B)=0,24.$

2.1 Are $A$ and $B$ mutually exclusive? Give a reason for answer.

2.2 Are $A$ and $B$ independent? Give a reason for answer.

2.3 Determine $P(A\cup B)$.

2.4 Determine $P(A\cap B')$.

2.5 Determine $P\left((A\cap B)'\right)$.

2.1 $A$ and $B$ are not mutually exclusive.

**Reason :** $P(A\cap B)\neq 0 \Rightarrow A\cap B\neq\varnothing$

2.2 $A$ and $B$ are independent.

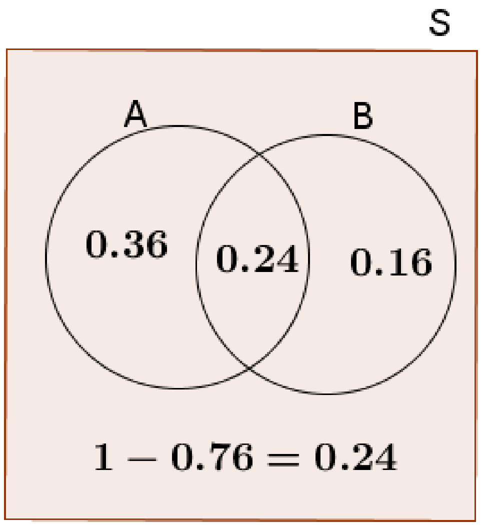
**Reason :** $P(A)\times P(B)=0.6\times0.4=0.24=P(A\cap B)$

2.3 $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0,6+0,4-0,24=0,76$ (**or** $0,36+0,24+0,16$)

2.4 $P(A\cap B')=P(\{A\,\text{only}\})=0,6-0,24=0,36$

2.5 $P\left((A\cap B)'\right)=1-P(A\cap B)=1-0,24=0,76$



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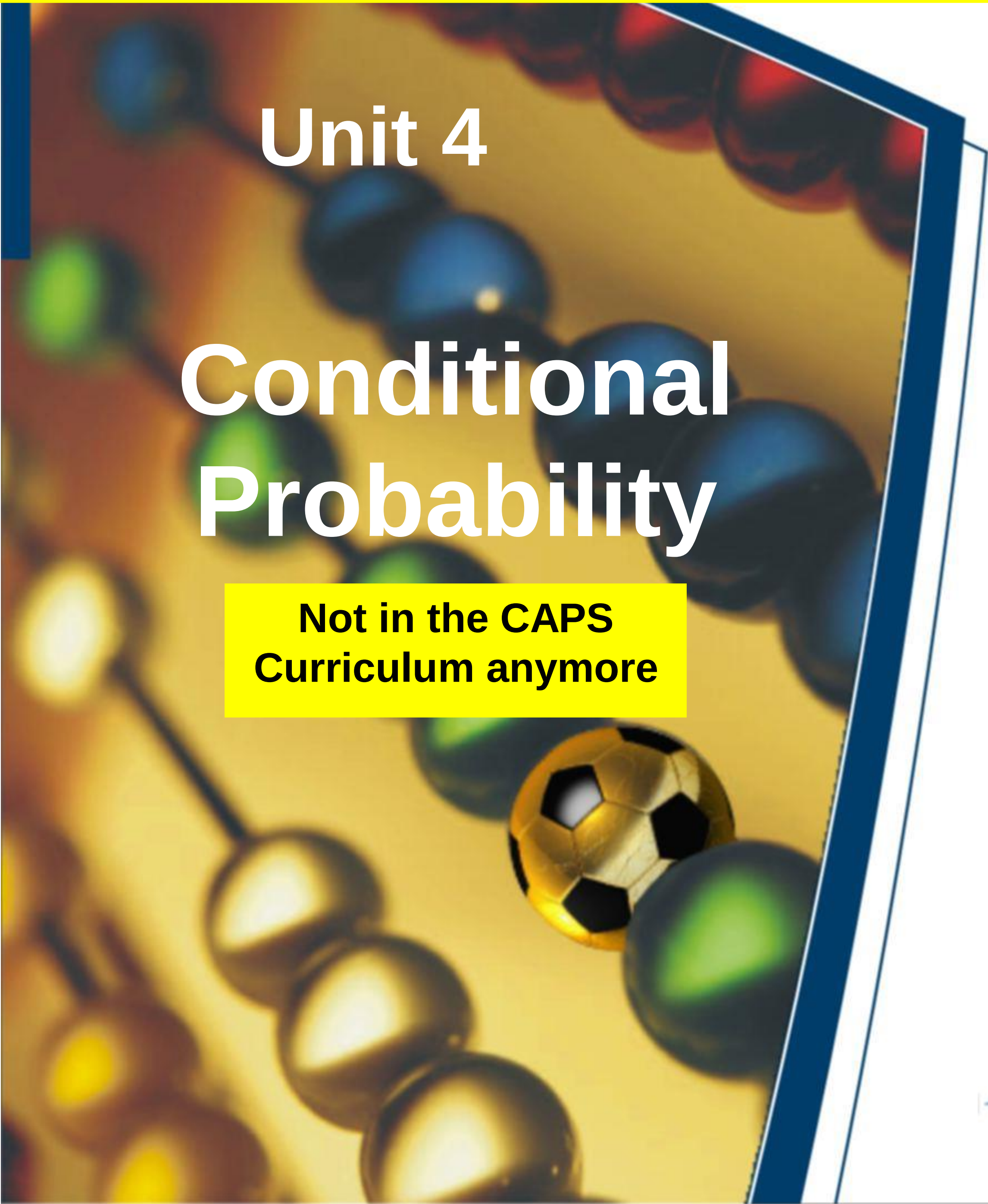

# Unit 4

# Conditional Probability

**Not in the CAPS Curriculum anymore**

## Grade 12 CAPS Mathematics Series

Govan Mbeki Mathematics Development Centre
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# Outcomes for Unit 4

In this Unit section we will revise :

- Definition of conditional probability.
- Solving conditional probability problems by means of tree diagrams.
- Solving conditional probability problems by means of Venn Diagrams.
- Solving conditional probability problems by means of conditional probability formula.

# Conditional Probability : Definition and Formula

Let $A$ and $B$ be events in a sample space $S$ where $P\left(B\right) \neq 0$. The conditional probability that event $A$ occurs, once event $B$ has occured, denoted by $P\left(A|B\right)$, is defined as

$$P\left(A|B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}.$$

If $A$ and $B$ are independent events, then we know that

$$P\left(A \cap B\right) = P\left(A\right) \times P\left(B\right)$$

Conditional probability for two independent events can be redefined using the previous relationship :

$$P\left(B|A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)} = \frac{P\left(A\right) \times P\left(B\right)}{P\left(A\right)} = P\left(B\right)$$

# Conditional Probability of Dependent Events : Using Tree Diagrams

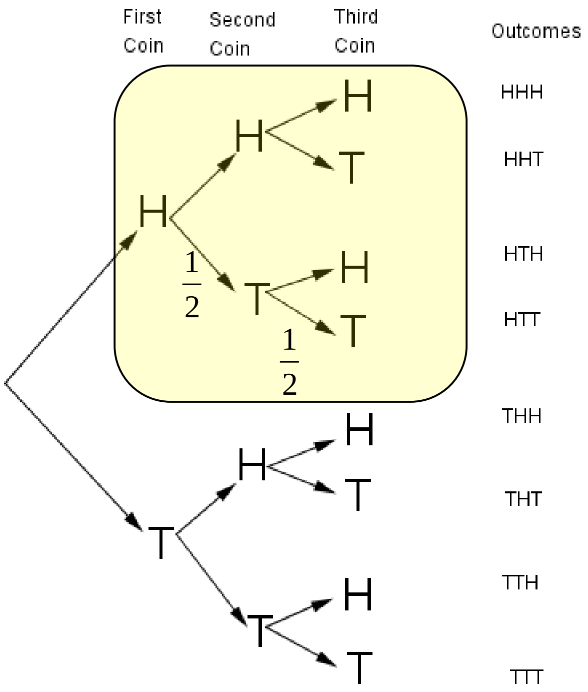
**Example 4 :** When tossing three fair coins, what is the probability of getting two tails once the first coin came up heads?

Tossing three coins implies that sample space is :
$\{HHH;HHT;HTH;HTT;THH;THT;TTH;TTT\}$.

Condition, first coin head, changes sample space to $\{HHH;HHT;HTH;HTT\}$.

Only one outcome, $HTT$, fits the event description.
$\therefore$ Conditional probability of getting two tails given that the first of the three coins is head is $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.

Solve using Tree Diagram



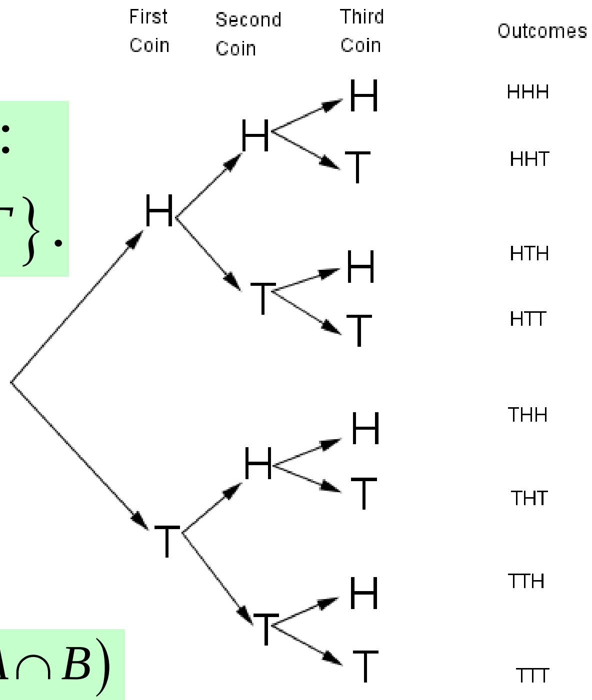

$$P(B|A)=P(\{\text{getting two tails once a head occured}\})=\frac{1}{4}$$

# Probability for Dependent Events : Using Conditional Probability Formula

**Example 4 :** When tossing three fair coins, what is the probability of getting two tails once the first coin came up heads?

**Know:** $P\left(B|A\right)=P\left(\{\text{getting two tails once a head occured}\}\right)=\dfrac{1}{4}$

Tree Diagram




## Alternative Solution :

Tossing three coins implies that sample space is :

$\{HHH;HHT;HTH;HTT;THH;THT;TTH;TTT\}$.

$A=\{\text{first head}\}=\{HHH;HHT;HTH;HTT\}\Rightarrow P(A)=\dfrac{4}{8}$

$B=\{\text{two tails}\}=\{HTT;THT;TTH\}\Rightarrow P(B)=\dfrac{3}{8}$

$\therefore A\cap B=\{HTT\}\Rightarrow P(A\cap B)=\dfrac{1}{8}$

$$P\left(B|A\right)=P\left(\{\text{getting two tails once a head occured}\}\right)=\frac{P(A\cap B)}{P(A)}$$

$$\Rightarrow P\left(B|A\right)=\frac{1}{8}\div\frac{4}{8}=\frac{1}{8}\times\frac{8}{4}=\frac{1}{4}$$

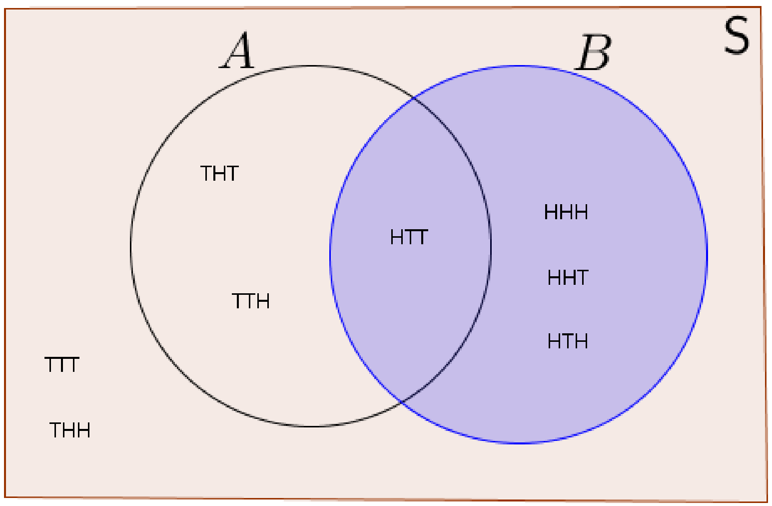
$P(A)\times P(B)=\dfrac{4}{8}\times\dfrac{3}{8}=\dfrac{3}{16}\neq P(A\cap B)\Rightarrow A$ and $B$ are dependent events

# Dependent Events : Using Venn Diagram

**Example 4 :** When tossing three fair coins, what is the probability of getting two tails once the first coin came up heads?

Tossing three coins implies that sample space is :

$S=\{HHH;HHT;HTH;HTT;THH;THT;TTH;TTT\}$.




Let $A$ be the event of getting exactly two tails :

$A=\{HTT;THT;TTH\}$.

Let $B$ be the event of getting head with first coin :

$B=\{HHH;HHT;HTH;HTT\}$

$\therefore A\cap B=\{HTT\}$

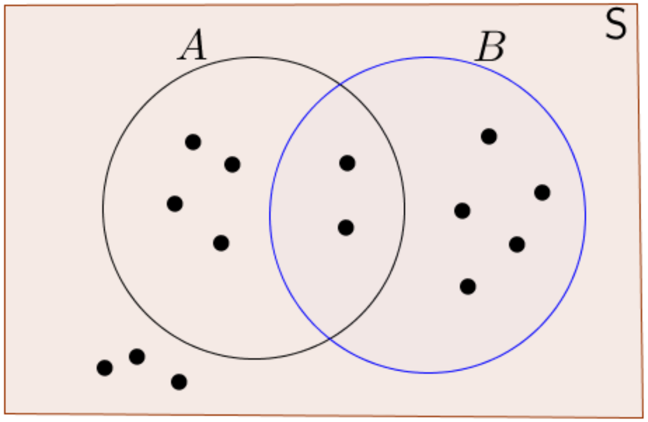
$\therefore P(A|B)=\frac{1}{4}$

Condition of head first limits us to set $B$ (shaded) :

Only one of these four outcomes contains two tails ($HTT$)

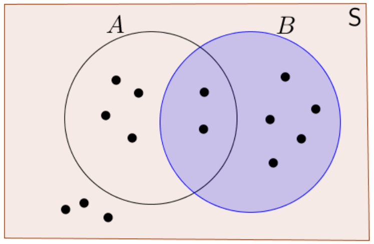
# Venn Diagrams to illustrate definition of Conditional Probability

A sample space S of equally likely outcomes is shown in the next Venn diagram.

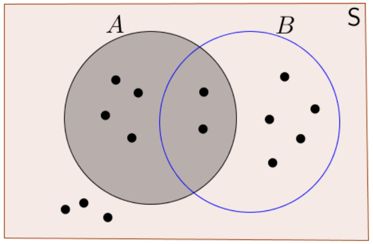



$$\begin{cases} n(S)=14 \\ n(A)=6 \\ n(B)=7 \\ n(A\cap B)=2 \end{cases} \Rightarrow \begin{cases} P(A)=\dfrac{6}{14} \\ P(B)=\dfrac{7}{14} \\ P(A\cap B)=\dfrac{2}{14} \end{cases}$$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{2}{14}\div\frac{7}{14}=\frac{2}{7}$$




Condition $B$ shaded




Condition $A$ shaded

$$P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{2}{14}\div\frac{6}{14}=\frac{2}{6}$$

# Tutorial 4: Conditional Independent and Dependent Events

**Problem 1 :** A packet contains 4 red paperclips and 5 blue paperclips. One clip is taken from the packet, the color recorded and then **placed back** in packet. A second clip is taken from the packet and the color recorded. What is the probability that the second clip is red once the first drawn is blue?

**Problem 2 :** A bag contains two red marbles and three green marbles. Two marbles are drawn from the bag, one after the other, **without replacing** the first one drawn.
What is the probability that the second marble is red once the first drawn is green?

PAUSE

- Do Tutorial 4
- Then View Solutions

# Tutorial 4 : Problem 1 : Suggested Solution

**Problem 1 :** A packet contains 4 red paperclips and 5 blue paperclips.

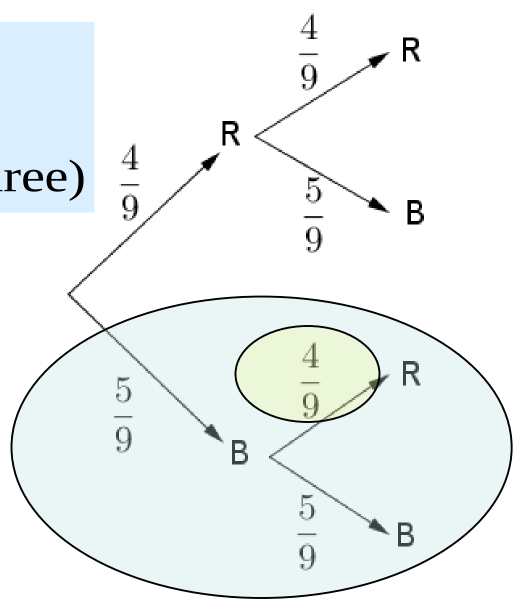
One clip is taken from the packet, the color recorded and then **placed back** in packet.

A second clip is taken from the packet and the color recorded.

What is the probability that the second clip is red once the first drawn is blue?

Let $A = \{\text{first blue}\}$ and $B = \{\text{second red}\}$.

**Tree Diagram**




**Suggested solution:**

Condition limits us to the two bottom branches from the tree diagram.

$$\Rightarrow P(B|A) = \frac{4}{9}$$

(Last part of branch three)

**Second possible solution: Conditional formula**

$$P(A) = P(BR) + P(BB) = \frac{5}{9}\times\frac{4}{9} + \frac{5}{9}\times\frac{5}{9} = \frac{20+25}{81} = \frac{45}{81} = \frac{5}{9}$$

$$P(B) = P(RR) + P(BR) = \frac{4}{9}\times\frac{4}{9} + \frac{5}{9}\times\frac{4}{9} = \frac{16+20}{81} = \frac{36}{81} = \frac{4}{9}$$

$$P(A\cap B) = P(BR) = \frac{5}{9}\times\frac{4}{9} = \frac{20}{81}$$

$$\therefore P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{20}{81} \div \frac{5}{9} = \frac{20}{81}\times\frac{9}{5} = \frac{4}{9}$$

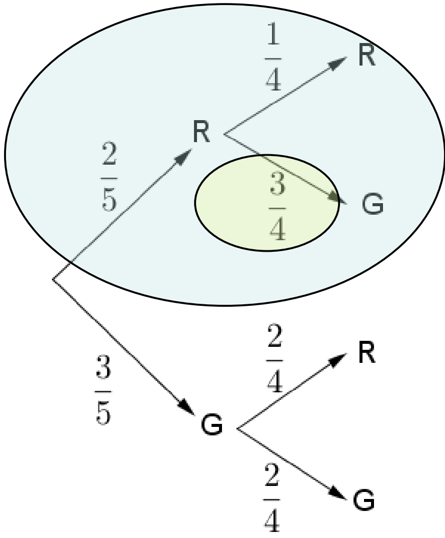
**Third possibility: Independent events**

$$P(B|A) = \frac{P(A)\times P(B)}{P(A)} = P(B) = \frac{4}{9}$$

# Tutorial 4: Problem 2: Suggested Solution

**Problem 2 :** A bag contains two red marbles and three green marbles.
Two marbles are drawn from the bag, one after the other, **without replacing** the first drawn.
What is the probability that the second marble is green once the first drawn is red?

Let $A = \{\text{first red}\}$ and $B = \{\text{second green}\}$.




**Suggested solution:**
Condition limits us to the two top branches from the tree diagram.

$$\Rightarrow P(B|A) = \frac{3}{4}$$

(Last part of branch two)

**Second possible solution: Conditional formula**

$$P(A) = P(RR) + P(RG) = \frac{2}{5}\times\frac{1}{4} + \frac{2}{5}\times\frac{3}{4} = \frac{2+6}{20} = \frac{8}{20} = \frac{2}{5}$$

$$P(B) = P(RG) + P(GG) = \frac{2}{5}\times\frac{3}{4} + \frac{3}{5}\times\frac{2}{4} = \frac{6+6}{20} = \frac{12}{20} = \frac{3}{5}$$

$$P(A\cap B) = P(RG) = \frac{2}{5}\times\frac{3}{4} = \frac{6}{20} = \frac{3}{10}$$

$$\therefore P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{3}{10}\div\frac{2}{5} = \frac{3}{10}\times\frac{5}{2} = \frac{3}{4}$$

**Third option not possible:**

$$P(A)\times P(B) = \frac{2}{5}\times\frac{3}{5} = \frac{6}{25} \neq P(A\cap B)$$

$\therefore$ $A$ and $B$ dependent events.

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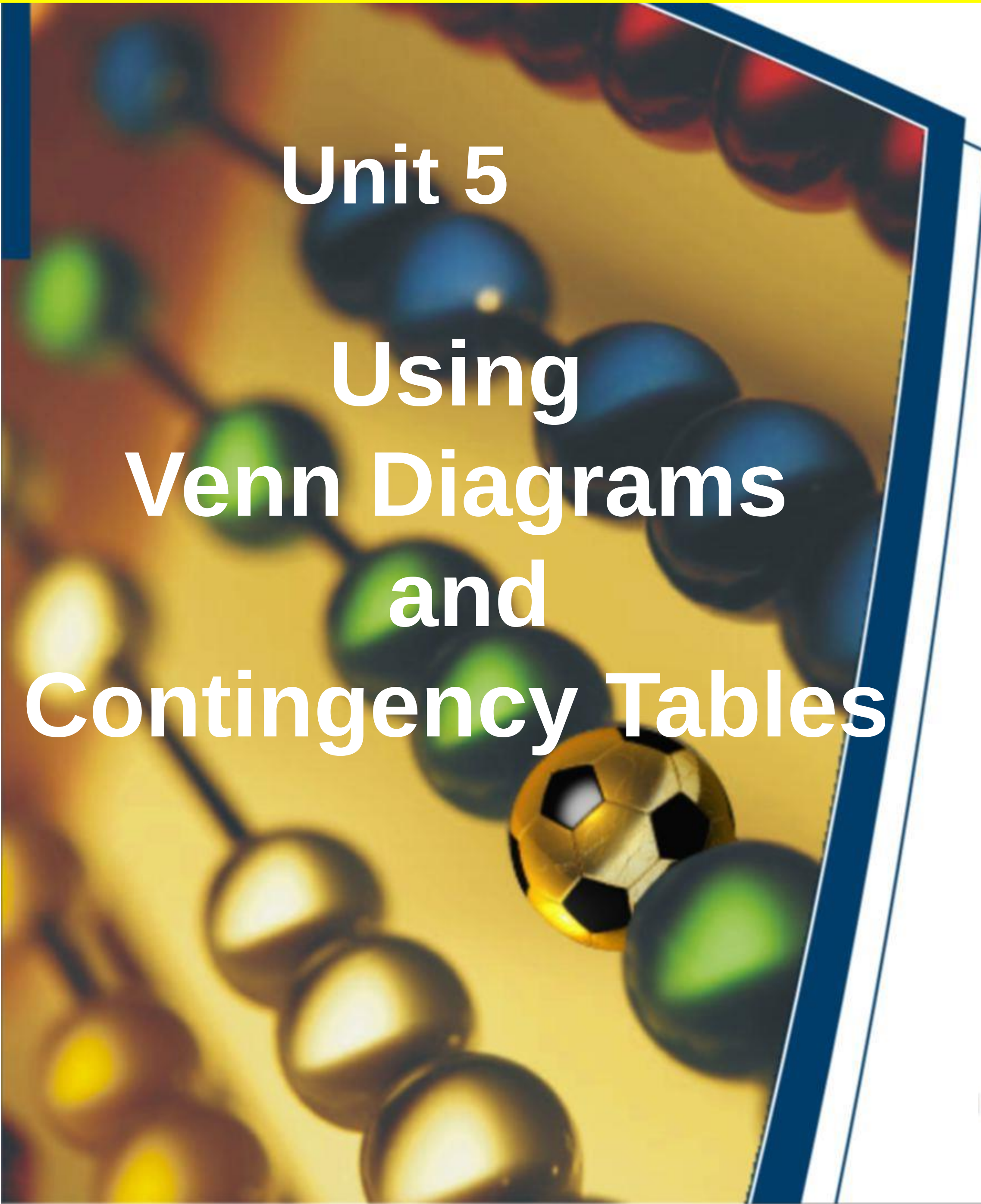

# Unit 5

# Using Venn Diagrams and Contingency Tables

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# Outcomes for Unit 5

In this Unit section we will revise the following :

- Process of solving probability problems by means of Venn diagrams.

- Process of solving probability problems by means of Contingency Tables.

# Solving Probability problems using Venn Diagrams

Consider the die experiment where:

$S = \{1;2;3;4;5;6\}$ and events

$A = \{\text{getting even number}\} = \{2;4;6\}$

$B = \{\text{getting a factor of 6}\} = \{1;2;3;6\}$

1. Venn Diagram can be utilized to:

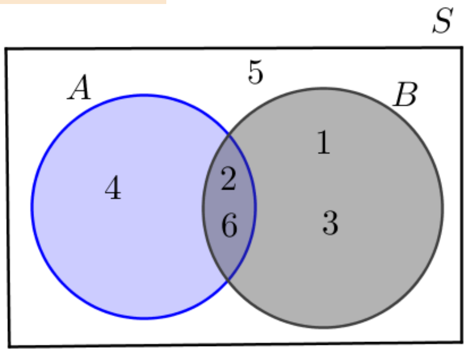
- Display outcomes in sample space
- Display outcomes in events

Venn Diagrams can now be utilized to solve related probability problems:

- $P(A) = \frac{1}{2} \wedge P(B) = \frac{2}{3}$
- $P(A') = \frac{1}{2} \wedge P(B') = \frac{1}{3}$
- $P(A \cup B) = \frac{5}{6} \wedge P(A \cap B) = \frac{1}{3}$
- $P\left((A \cap B)'\right) = P(A' \cup B') = \frac{2}{3}$
- $P(A \cap B') = \frac{1}{6} \wedge P(A' \cap B) = \frac{1}{3}$

Venn Diagram can then be utilized to list related events:

- $A = \{2;4;6\} \wedge B = \{1;2;3;6\}$
- $A' = \{1;3;5\} \wedge B' = \{4;5\}$
- $A \cup B = \{1;2;3;4;6\}$
- $A \cap B = \{2;6\}$
- $(A \cap B)' = \{1;3;4;5\}$
- $A' \cup B' = \{1;3;4;5\}$
- $A \cap B' = \{4\}$
- $A' \cap B = \{1;3\}$



# Solving Probability problems using Venn Diagrams

Consider the die experiment where:

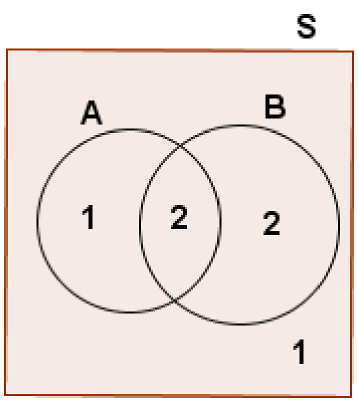
$S = \{1;2;3;4;5;6\}$ and events

$A = \{\text{getting even number}\} = \{2;4;6\}$

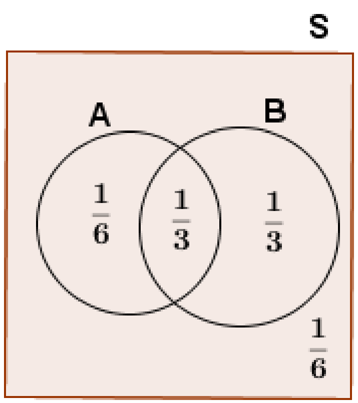
$B = \{\text{getting a factor of } 6\} = \{1;2;3;6\}$

2. Venn Diagrams can be utilized to:

- Display cardinal numbers for regions
- Display probability for regions




Cardinal Numbers




Probabilities

**Note :**

- Sum of cardinal numbers of all the regions is $n(S) = 6$
- Sum of probabilities of all regions is $P(S) = 1$

# Tutorial 5: Using Venn Diagrams to solve Probability Problems

In a survey 100 people were questioned to find out how many read the newspapers *A*, *B* and *C*. The survey revealed that 10 did not read any of the three newspapers; 50 read *A*; 44 read *B*; 20 read *A* and *B*; 9 read *A* and *C*; 6 read *B* and *C* and 5 read all three papers.

**PAUSE**

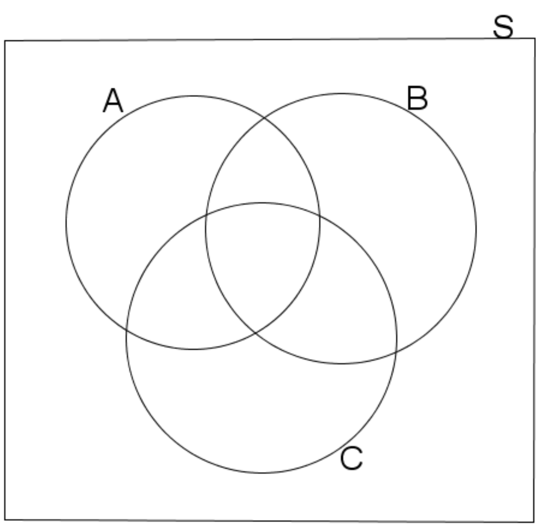
- **Do Tutorial 5**
- **Then View Solutions**

1. Construct an appropriate Venn Diagram.
2. Symbolize given information.
3. Place the appropriate cardinal number in each of the eight regions.
4. Determine how many read newspaper *C* only?
5. Determine the probability represented by each of the eight regions.

# Tutorial 5: Problems 1 and 2: Suggested Solutions

In a survey 100 people were questioned to find out how many read the newspapers *A*, *B* and *C*. The survey revealed that 10 did not read any of the three newspapers; 50 read *A*; 44 read *B*; 20 read *A* and *B*; 9 read *A* and *C*; 6 read *B* and *C* and 5 read all three papers.

1. Construct an appropriate Venn Diagram.
2. Symbolize given information.




$n(S) = 100$

$n(A \cup B \cup C)' = 10$

$n(A) = 50$

$n(B) = 44$

$n(A \cap B) = 20$

$n(A \cap C) = 9$

$n(B \cap C) = 6$

$n(A \cap B \cap C) = 5$

# Tutorial 5: Problems 3 and 4: Suggested Solutions

3. Place the appropriate cardinal number in each of the eight regions.
4. Determine how many read newspaper $C$ only?

$n(S)=100$

$n(A\cup B\cup C)'=10$

$n(A)=50$

$n(B)=44$

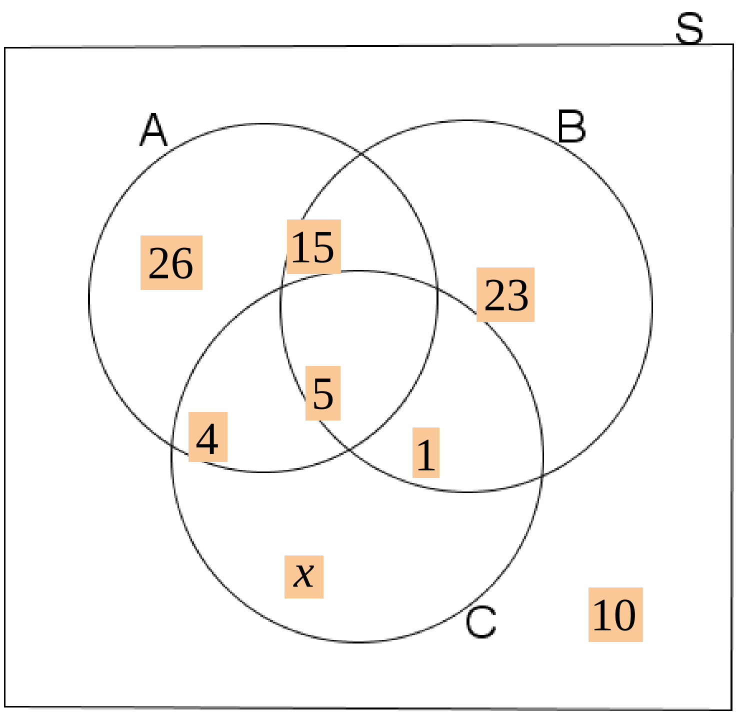
$n(A\cap B)=20$

$n(A\cap C)=9$

$n(B\cap C)=6$

$n(A\cap B\cap C)=5$

Assume that $n(A'\cap B'\cap C)=x$



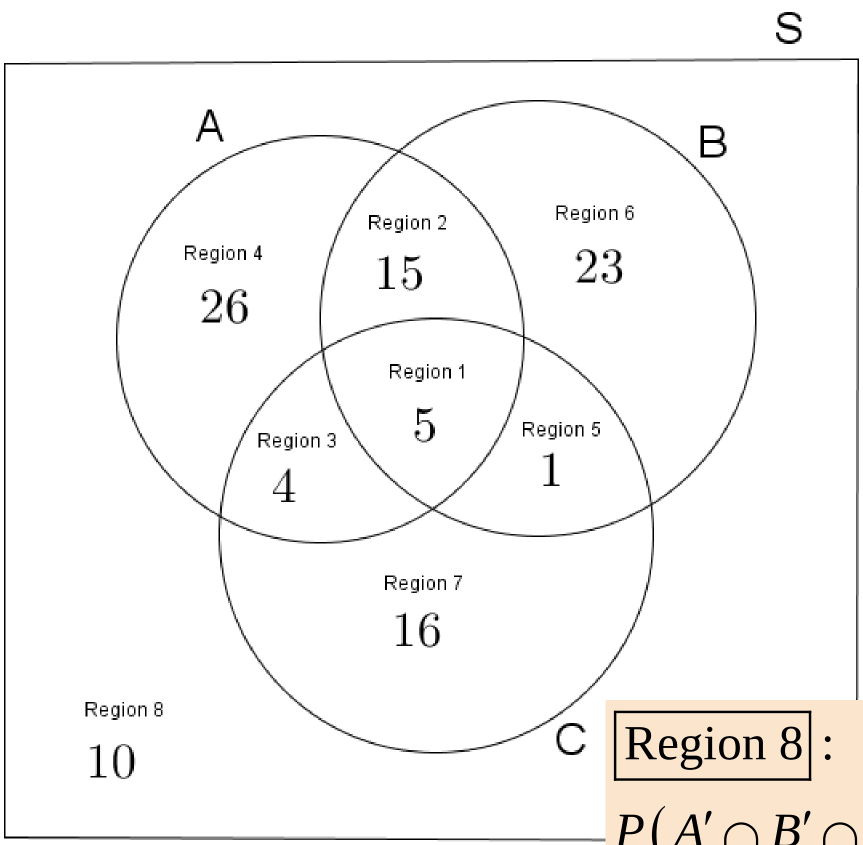

$$26+4+5+15+23+1+x+10=100$$

$$\therefore 84+x=100 \Rightarrow x=16$$

$\therefore$ 16 read $C$ only.

# Tutorial 5: Problem 5: Suggested Solution

5. Determine the probability for each of the eight regions.




Region 1 : $P(A \cap B \cap C) = \frac{5}{100} = \frac{1}{20}$

Region 2 : $P(A \cap B \cap C') = \frac{3}{20}$

Region 3 : $P(A \cap B' \cap C) = \frac{1}{25}$

Region 4 : $P(A \cap B' \cap C') = \frac{13}{50}$

Region 5 : $P(A' \cap B \cap C) = \frac{1}{100}$

Region 6 : $P(A' \cap B \cap C') = \frac{23}{100}$

Region 7 : $P(A' \cap B' \cap C) = \frac{4}{25}$

Region 8 :

$P(A' \cap B' \cap C')$

$= P(A \cup B \cup C)'$

$= \frac{10}{100} = \frac{1}{10}$

# Solving Probability problems using Contingency Tables

A group of people were asked whether they had watched on TV Jesica Fletcher's murder she wrote plays or whether they read her murder stories. Based on the results of the survey, the contingency table was drawn up showing the probabilities of these events.

1. Complete the table.
2. Show that watching her plays and reading her murder stories are independent events.

| | Watch plays | Not watch plays | Total |
|---|---|---|---|
| Read stories | 0,21 | 0.39 | 0,6 |
| Not read stories | 0,14 | 0,26 | 0,4 |
| Total | 0,35 | 0,65 | 1,0 |

$$P(\text{Watch Plays and Read Stories}) = 0,21$$

$$P(\text{Watch Plays}) \times P(\text{Read Stories}) = 0,35 \times 0,6 = 0,21$$

$$\therefore P(\text{Watch Plays}) \times P(\text{Read Stories}) = P(\text{Watch Plays and Read Stories})$$

$\therefore$ Watching the plays and reading the stories are independent events.

# Tutorial 6: Use Contingency Table to solve Probability problem

A study was conducted to investigate the relationship between eye colour and hair colour.

A group of 250 people were observed. The results are:

- 12 people had red hair, 64 had blond hair and 82 had black hair
- 35 of the blondes had blue eyes, and 21 had green eyes
- 68 of the brunettes had brown eyes
- 38 of those with black hair had brown eyes, and 41 had green eyes
- 5 of the redheads had blue eyes
- 116 people had brown eyes and 86 had green eyes

1. Copy and complete the contingency table.
2. Based on these results, determine the probability of a person having:
   - 2.1) blond hair and green eyes
   - 2.2) black hair and brown eyes
   - 2.3) blue eyes, given that are they are brunette — **Not in the CAPS Syllabus**
   - 2.4) red hair, given that they have brown eyes. — **Not in the CAPS Syllabus**

**PAUSE**

- **Do Tutorial 6**
- **Then View Solutions**

| | Blonde hair | Brunette | Black hair | Red hair | Total |
|---|---|---|---|---|---|
| Blue eyes | | | | | |
| Brown eyes | | | | | |
| Green eyes | | | | | |
| Total | | | | | |

# Tutorial 6: Suggested Solution: problem 1

A study was conducted to investigate the relationship between eye colour and hair colour.

A group of 250 people were observed. The results are:

- 12 people had red hair, 64 had blond hair and 82 had black hair
- 35 of the blondes had blue eyes, and 21 had green eyes
- 68 of the brunettes had brown eyes
- 38 of those with black hair had brown eyes, and 41 had green eyes
- 5 of the redheads had blue eyes
- 116 people had brown eyes and 86 had green eyes

1. Complete the contingency table.

Order in which table is completed not unique.

| | Blonde hair | Brunette | Black hair | Red hair | Total |
|---|---|---|---|---|---|
| Blue eyes | 35 | 5 | 3 | 5 | 48 |
| Brown eyes | 8 | 68 | 38 | 2 | 116 |
| Green eyes | 21 | 19 | 41 | 5 | 86 |
| Total | 64 | 92 | 82 | 12 | 250 |

# Tutorial 6: Suggested Solution: problem 2

2. Based on these results, determine the probability of a person having:
   - 2.1) blond hair and green eyes
   - 2.2) black hair and brown eyes
   - 2.3) blue eyes, given that they are brunette **Not in the CAPS Syllabus**
   - 2.4) red hair, given that they have brown eyes. **Not in the CAPS Syllabus**

| | Blonde hair | Brunette | Black hair | Red hair | Total |
|---|---|---|---|---|---|
| Blue eyes | 35 | 5 | 3 | 5 | 48 |
| Brown eyes | 8 | 68 | 38 | 2 | 116 |
| Green eyes | 21 | 19 | 41 | 5 | 86 |
| Total | 64 | 92 | 82 | 12 | 250 |

2.1 $P(\text{Blond and green eyes}) = \frac{21}{250}$

2.2 $P(\text{Black hair and brown eyes}) = \frac{38}{250} = \frac{19}{125}$

2.3 $P(\text{Blue eyes}|\text{Brunette}) = \frac{5}{92}$

2.4 $P(\text{Redhead}|\text{Brown eyes}) = \frac{2}{116} = \frac{1}{58}$

# End of the Topic Slides on revision of Probability for Grades 10 and 11

## REMEMBER!

- Consult text-books and past exam papers & memos for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in these Topic slides.
- Repeat this procedure until you are confident.
- Do not forget: **Practice makes perfect!**