MATHEMATICS

Grade 8 - Term 2

CAPS

Learner Book

Revised edition

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8.1 Expanding algebraic expressions

MULTIPLY OFTEN OR MULTIPLY ONCE: IT IS YOUR CHOICE

1. (a) Calculate 5 × 13 and 5 × 87 and add the two answers.
   (b) Add 13 and 87, and then multiply the answer by 5.
   (c) If you do not get the same answer for questions 1(a) and 1(b), you have made a mistake. Redo your work until you get it right.

If you work correctly, you get the same answer for questions 1(a) and 1(b); this is an example of a certain property of addition and multiplication called the **distributive property**. You use this property each time you multiply a number in parts. For example, you may calculate 3 × 24 by calculating 3 × 20 and 3 × 4, and then add the two answers:

$$3 \times 24 = 3 \times 20 + 3 \times 4$$

What you saw in question 1 was that 5 × 100 = 5 × 13 + 5 × 87. This can also be expressed by writing 5(13 + 87).

The word *distribute* means "to spread out". The distributive properties may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

2. (a) Calculate 10 × 56. (b) Calculate 10 × 16 + 10 × 40.

3. Write down any two numbers smaller than 100. Let us call them x and y.
   (a) Add your two numbers and multiply the answer by 6.
   (b) Calculate $6 \times x$ and $6 \times y$ and add the two answers.
   (c) If you do not get the same answers for (a) and (b), you have made a mistake somewhere. Correct your work.

4. Copy and complete the following table:

(a)

x	1	2	3	4	5
$3(x + 2)$					
$3x + 6$					
$3x + 2$					
$3(x - 2)$					
$3x - 6$					
$3x - 2$					

(b) If you do not get the same answers for the expressions $3(x + 2)$ and $3x + 6$, and for $3(x – 2)$ and $3x – 6$, you have made a mistake somewhere. Correct your work.

In algebra we normally write $3(x + 2)$ instead of $3 \times (x + 2)$. The expression $3 \times (x + 2)$ does not mean that you should first multiply by 3 when you evaluate the expression for a certain value of x. The brackets tell you that the first thing you should do is add the value(s) of x to 2 and then multiply the answer by 3.

However, instead of first adding the values within the brackets and then multiplying the answer by 3, we may just do the calculation $3 \times x + 3 \times 2 = 3x + 6$ as shown in the table.

(c) Which expressions amongst those given in the table on page 77 are equivalent? Explain.

(d) For what value(s) of x is $3(x + 2) = 3x + 2$?

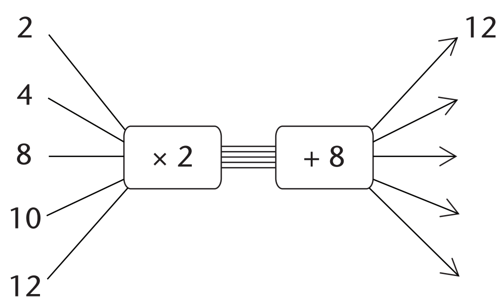
(e) Try to find a value of x such that $3(x + 2) \neq 3x + 6$.

If multiplication is the last step in evaluating an algebraic expression, then the expression is called a **product expression** or, briefly, a **product**. The way you evaluated the expression $3(x + 2)$ in the table is an example of a product expression.

5. (a) Determine the value of $5x + 15$ if $x = 6$.
   (b) Determine the value of $5(x + 3)$ if $x = 6$.
   (c) Can we use the expression $5x + 15$ to calculate the value of $5(x + 3)$ for any values of x? Explain.

6. Copy and complete the following flow diagrams:

(a)



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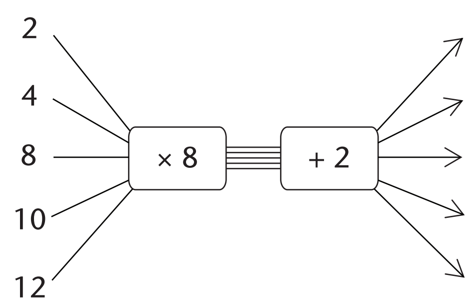

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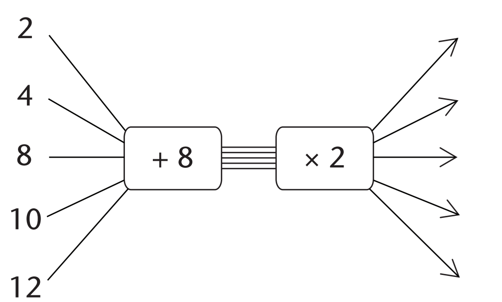

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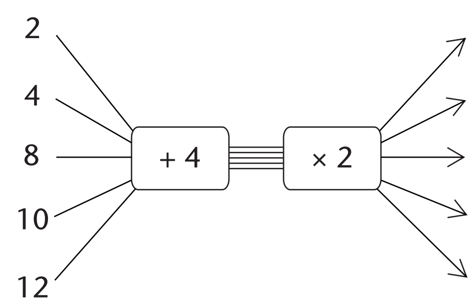

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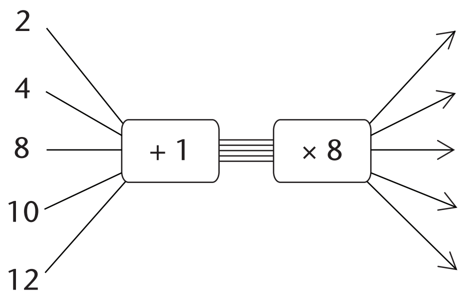

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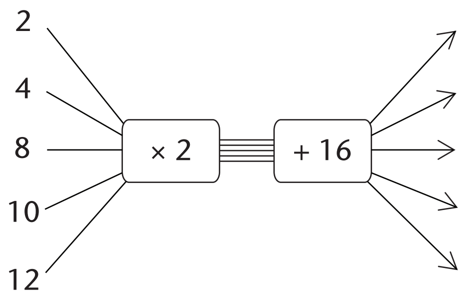

7. (a) Which of the flow diagrams in question 6(a) to (f) produce the same output numbers?
   (b) Write an algebraic expression for each of the flow diagrams in question 6.

PRODUCT EXPRESSIONS AND SUM EXPRESSIONS

1. Copy and complete the following:
   (a) $(3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)$
   $= \dots \times (\dots)$
   (b) $(3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)$
   $= (3 + 3 + \dots) + (\dots)$
   $= (\dots \times \dots) + (\dots \times \dots)$

2. Copy and complete the following:
   (a) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$
   $= \dots(\dots)$
   (b) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$
   $= (3x + 3x + \dots) + (\dots)$
   $= (\dots \times \dots) \dots (\dots \times \dots)$

3. In each case, write an expression without brackets that will give the same results as the given expression.
   (a) $3(x + 7)$
   (b) $10(2x + 1)$
   (c) $x(4x + 6)$
   (d) $3(2p + q)$
   (e) $t(t + 9)$
   (f) $x(y + z)$
   (g) $2b(b + a - 4)$
   (h) $k^2(k - m)$

The process of writing product expressions as sum expressions is called **expansion**. It is sometimes also referred to as **multiplication of algebraic expressions**.

4. (a) Copy and complete the following table for the given values of x, y and z.

	$3(x + 2y + 4z)$	$3x + 6y + 12z$	$3x + 2y + 4z$
$x = 1$ $y = 2$ $z = 3$			
$x = 10$ $y = 20$ $z = 30$			
$x = 23$ $y = 60$ $z = 100$			
$x = 14$ $y = 0$ $z = 1$			
$x = 5$ $y = 9$ $z = 32$			

(b) Which sum expression and product expression are equivalent?

5. For each expression, write an equivalent expression without brackets.

(a) $2(x^2 + x + 1)$
(b) $p(q + r + s)$
(c) $-3(x + 2y + 3z)$
(d) $x(2x^2 + x + 7)$
(e) $6x(8 - 2x)$
(f) $12x(4 - x)$
(g) $3x(8x - 5) - 4x(6x - 5)$
(h) $10x\mathbf{(}3x(8x - 5) - 4x(6x - 5)\mathbf{)}$

8.2 Simplifying algebraic expressions

EXPAND, REARRANGE AND THEN COMBINE LIKE TERMS

1. Write the shortest possible equivalent expression without brackets.

(a) $x + 2(x + 3)$
(b) $5(4x + 3) + 5x$
(c) $5(x + 5) + 3(2x + 1)$
(d) $(5 + x)^2$
(e) $-3(x^2 + 2x - 3) + 3(x^2 + 4x)$
(f) $x(x - 1) + x + 2$

When you are not sure if you have simplified an expression correctly, you should always check your work by evaluating the original expression and the simplified expression for some values of the variables.

2. (a) Evaluate $x(x + 2) + 5x^2 – 2x$ for $x = 10$.
   (b) Evaluate $6x^2$ for $x = 10$.
   (c) Can we use the expression $6x^2$ to calculate the values of the expression $x(x + 2) + 5x^2 – 2x$ for any given value of x? Explain.

This is how a sum expression for $x(x + 2) + 5x^2 – 2x$ can be made:

$x(x + 2) + 5x^2 – 2x = x \times x + x \times 2 + 5x^2 – 2x$
$= x^2 + 2x + 5x^2 – 2x$
$= x^2 + 5x^2 + 2x – 2x$ [Rearrange and combine like terms.]
$= 6x^2 + 0$
$= 6x^2$

3. Evaluate the following expressions for $x = –5$:
   (a) $x + 2(x + 3)$
   (b) $5(4x + 3) + 5x$
   (c) $5(x + 5) + 3(2x + 1)$
   (d) $(5 + x)^2$
   (e) $–3(x^2 + 2x – 3) + 3(x^2 + 4x)$
   (f) $x(x – 1) + x + 2$

4. Copy and complete the following table for the given values of x, y and z.

x	100	80	10	20	30
y	50	40	5	5	20
z	20	30	2	15	10
$x + (y – z)$					
$x – (y – z)$					
$x – y – z$					
$x – (y + z)$					
$x + y – z$					
$x – y + z$					

5. Say whether the following statements are true or false. Refer to the table in question 4. For any values of x, y and z:
   (a) $x + (y – z) = x + y – z$
   (b) $x – (y – z) = x – y – z$

6. Write the expressions without brackets. Do not simplify.
   (a) $3x – (2y + z)$
   (b) $–x + 3(y – 2z)$

We can simplify algebraic expressions by using properties of operations as shown:

$(5x + 3) – 2(x + 1)$
Hence $5x + 3 – 2x – 2$
Hence $5x – 2x + 3 – 2$
Hence $3x + 1$

$x – (y + z) = x – y – z$

Addition is both associative and commutative.

7. Write an equivalent expression without brackets for each of the following expressions and then simplify:

(a) $22x + (13x - 5)$ (b) $22x - (13x - 5)$

(c) $22x - (13x + 5)$ (d) $4x - (15 - 6x)$

8. Simplify:

(a) $2(x^2 + 1) - x - 2$ (b) $-3(x^2 + 2x - 3) + 3x^2$

Some of the techniques we have used so far to form equivalent expressions include:

- remove brackets
- rearrange terms
- combine like terms.

8.3 Simplifying quotient expressions

FROM QUOTIENT EXPRESSIONS TO SUM EXPRESSIONS

1. Copy and complete the table for the given values of x.

x	1	7	–3	–10
$7x^2 + 5x$				
$\frac{7x^2+5x}{x}$				
$7x + 5$				
$7x + 5x$				
$7x^2 + 5$				

2. (a) What is the value of $7x + 5$ for $x = 0$?

(b) What is the value of $\frac{7x^2+5x}{x}$ for $x = 0$?

(c) Which of the two expressions, $7x + 5$ or $\frac{7x^2 + 5x}{x}$, requires fewer calculations? Explain.

(d) Are the expressions $7x + 5$ and $\frac{7x^2 + 5x}{x}$ equivalent, $x = 0$ excluded? Explain.

(e) Are there any other expressions that are equivalent to $\frac{7x^2 + 5x}{x}$ from those given in the table? Explain.

If division is the last step in evaluating an algebraic expression, then the expression is called a **quotient expression** or an **algebraic fraction**.

The expression $\frac{7x^2 + 5x}{x}$ is an example of a quotient expression or algebraic fraction.

3. Copy and complete the following table for the given values of x:

x	5	10	–5	–10
$10x - 5x^2$	$50 - 125$ $= -75$			
$5x$	5×5 $= 25$			
$\frac{10x - 5x^2}{5x}$	$\frac{50 - 125}{25}$ $= \frac{-75}{25}$ $= -3$			
$2 - x$	$2 - 5$ $= -3$			

(a) What is the value of $2 - x$ for $x = 0$?

(b) What is the value of $\frac{10x - 5x^2}{5x}$ for $x = 0$?

(c) Are the expressions $2 - x$ and $\frac{10x - 5x^2}{5x}$ equivalent, $x = 0$ excluded? Explain.

(d) Which of the two expressions $2 - x$ or $\frac{10x - 5x^2}{5x}$ requires fewer calculations? Explain.

We have found that quotient expressions, such as $\frac{10x - 5x^2}{5x}$ can sometimes be manipulated to give equivalent expressions, such as $2 - x$.

The value of this is that these equivalent expressions require fewer calculations.

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The expressions $\frac{10x - 5x^2}{5x}$ and $2 - x$ are not quite equivalent because for $x = 0$, the value of $2 - x$ can be calculated, while the first expression has no value.

However, we can say that the two expressions are equivalent if they have the same values for all values of x admissible for both expressions.

How is it possible that $\frac{7x^2 + 5x}{x} = 7x + 5$ and $\frac{10x - 5x^2}{5x} = 2 - x$ for all admissible values of x? We say $x = 0$ is not an admissible value of x because division by 0 is not allowed.

One of the methods for finding equivalent expressions for algebraic fractions is by means of division:

$$\frac{7x^2 + 5x}{x} = \frac{1}{x}(7x^2 + 5x) \qquad \text{[just as } \frac{3}{5} = 3 \times \frac{1}{5}\text{]}$$

$$= (\frac{1}{x} \times 7x^2) + (\frac{1}{x} \times 5x) \qquad \text{[distributive property]}$$

$$= \frac{7x^2}{x} + \frac{5x}{x}$$

$$= 7x + 5 \qquad \text{[provided } x \neq 0\text{]}$$

4. Use the method shown on the previous page to simplify each fraction below:

(a) $\frac{8x + 10z + 6}{2}$

(b) $\frac{20x^2 + 16x}{4}$

(c) $\frac{9x^2y + xy}{xy}$

(d) $\frac{21ab - 14a^2}{7a}$

Simplifying a quotient expression can sometimes lead to a result which still contains quotients, as you can see in the following example:

$$\frac{5x^2 + 3x}{x^2}$$

$$= \frac{5x^2}{x^2} + \frac{3x}{x^2}$$

$$= 5 + \frac{3}{x}$$

5. (a) Evaluate $\frac{5x^2 + 3x}{x^2}$ for $x = -1$.

(b) For the expression $\frac{5x^2 + 3x}{x^2}$ to be equivalent to $5 + \frac{3}{x}$, which value of x must be excluded? Why?

6. Simplify the following expressions:
   (a) $\dfrac{8x^2 + 2x + 4}{2x}$ (b) $\dfrac{4n + 1}{n}$
7. Evaluate:
   (a) $\dfrac{8x^2 + 2x + 4}{2x}$ for $x = 2$ (b) $\dfrac{4n + 1}{n}$ for $n = 4$
8. Simplify:
   (a) $\dfrac{6x^4 - 12x^3 + 2}{2x}$ (b) $\dfrac{-6n^4 - 4n}{6n}$
9. When Natasha and Lebogang were asked to evaluate the expression $\dfrac{x^2 + 2x + 1}{x}$ for $x = 10$, they did it in different ways.

   Natasha's calculation:

   $10 + 2 + \dfrac{1}{10}$

   $= 12\dfrac{1}{10}$

   Lebogang's calculation:

   $\dfrac{100 + 20 + 1}{10}$

   $= \dfrac{121}{10}$

   $= 12\dfrac{1}{10}$

   Explain how each of them thought about evaluating the given expression.

8.4 Squares, cubes and roots of expressions

SIMPLIFYING SQUARES AND CUBES

Study the following example:

$(3x)^2 = 3x \times 3x$	Meaning of squaring
$= 3 \times x \times 3 \times x$	
$= 3 \times 3 \times x \times x$	Multiplication is commutative: $a \times b = b \times a$
$= 9x^2$	We say that $(3x)^2$ simplifies to $9x^2$

1. Simplify the following expressions:
   (a) $(2x)^2$ (b) $(2x^2)^2$ (c) $(-3y)^2$
2. Simplify the following expressions:
   (a) $25x - 16x$ (b) $4y + y + 3y$ (c) $a + 17a - 3a$
3. Simplify:
   (a) $(25x - 16x)^2$ (b) $(4y + y + 3y)^2$ (c) $(a + 17a - 3a)^2$

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Study the following example:

$(3x)^3 = 3x \times 3x \times 3x$	Meaning of cubing
$= 3 \times x \times 3 \times x \times 3 \times x$	
$= 3 \times 3 \times 3 \times x \times x \times x$	Multiplication is commutative: $a \times b = b \times a$
$= 27x^3$	We say that $(3x)^3$ simplifies to $27x^3$.

4. Simplify:
   (a) $(2x)^3$ (b) $(-x)^3$ (c) $(5a)^3$
   (d) $(7y^2)^3$ (e) $(-3m)^3$ (f) $(2x^3)^3$

5. Simplify:
   (a) $5a - 2a$ (b) $7x + 3x$ (c) $4b + b$

6. Simplify:
   (a) $(5a - 2a)^3$ (b) $(7x + 3x)^3$ (c) $(4b + b)^3$
   (d) $(13x - 6x)^3$ (e) $(17x + 3x)^3$ (f) $(20y - 14y)^3$

Always remember to test if the simplified expression is equivalent to the given expression for at least three different values of the given variable.

SQUARE AND CUBE ROOTS OF EXPRESSIONS

1. Thabang and his friend Vuyiswa were asked to simplify $\sqrt{2a^2 \times 2a^2}$.
   Thabang reasoned as follows:
   To find the square root of a number is the same as asking yourself the question: "Which number was multiplied by itself?" The number that is multiplied by itself is $2a^2$ and therefore $\sqrt{2a^2 \times 2a^2} = 2a^2$.
   Vuyiswa reasoned as follows:
   I should first simplify $2a^2 \times 2a^2$ to get $4a^4$ and then calculate $\sqrt{4a^4} = 2a^2$.
   Which of the two methods do you prefer? Explain why.

2. Say whether each of the following is true or false. Give a reason for your answer.
   (a) $\sqrt{6x \times 6x} = 6x$ (b) $\sqrt{5x^2 \times 5x^2} = 5x^2$

3. Simplify:
   (a) $y^6 \times y^6$ (b) $125x^2 + 44x^2$

4. Simplify:
   (a) $\sqrt{y^{12}}$ (b) $\sqrt{125x^2 + 44x^2}$
   (c) $\sqrt{25a^2 - 16a^2}$ (d) $\sqrt{121y^2}$

(e) $\sqrt{16a^2 + 9a^2}$ (f) $\sqrt{25a^2 - 9a^2}$

5. What does it mean to find the cube root of $8x^3$ written as $\sqrt[3]{8x^3}$?

6. Simplify:
   (a) $2a \times 2a \times 2a$ (b) $10b^3 \times 10b^3 \times 10b^3$
   (c) $3x^3 \times 3x^3 \times 3x^3$ (d) $-3x^3 \times -3x^3 \times -3x^3$

7. Determine the following:
   (a) $\sqrt[3]{1000b^9}$ (b) $\sqrt[3]{2a \times 2a \times 2a}$ (c) $\sqrt[3]{27x^3}$ (d) $\sqrt[3]{-27x^3}$

8. Simplify each of the following expressions:
   (a) $6x^3 + 2x^3$ (b) $-m^3 - 3m^3 - 4m^3$

9. Determine the following:
   (a) $\sqrt[3]{6x^3 + 2x^3}$ (b) $\sqrt[3]{-8m^3}$ (c) $\sqrt[3]{125y^3}$ (d) $\sqrt[3]{93a^3 + 123a^3}$

WORKSHEET

1. Simplify:
   (a) $2(3b + 1) + 4$ (b) $6 - (2 + 5e)$
   (c) $18mn + 22mn + 70mn$ (d) $4pqr + 3 + 9pqr$

2. Evaluate each of the following expressions for $m = 10$:
   (a) $3m^2 + m + 10$ (b) $5(m^2 - 5) + m^2 + 25$

3. (a) Simplify: $\dfrac{4b + 6}{2}$
   (b) Evaluate the expression $\dfrac{4b + 6}{2}$ for $b = 100$.

4. Simplify:
   (a) $(4g)^2$ (b) $(6y)^3$ (c) $(7s + 3s)^2$

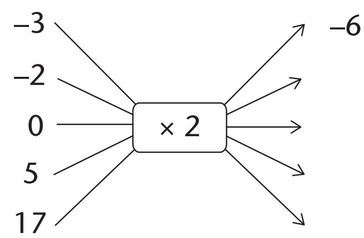
5. Determine the following:
   (a) $\sqrt{121b^2}$ (b) $\sqrt[3]{64y^3}$ (c) $\sqrt{63d^2 + 18d^2}$

Chapter 9
Algebraic equations 2

9.1 Thinking forwards and backwards

DOING AND UNDOING WHAT HAS BEEN DONE

1. Copy and complete the flow diagram on the right by finding the output values.



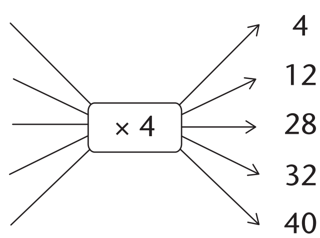
2. Copy and complete the following table:

x	–3	–2	0	5	17
$2x$					

3. Evaluate $4x$ if:

(a) $x = -7$ (b) $x = 10$ (c) $x = 0$

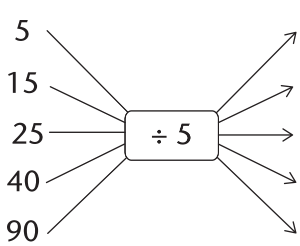
4. (a) Copy and complete the flow diagram on the right by finding the input values.
   (b) Puleng put another integer into the flow diagram and got –68 as an answer. Which integer did she put in? Show your calculation.
   (c) Explain how you worked to find the input numbers when you did question 4(a).

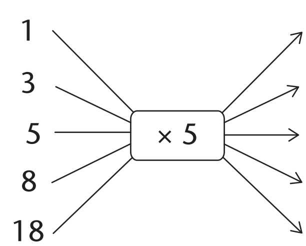


5. (a) Copy and complete the following table:

x					
$5x$	5	15	25	40	90

   (b) Copy and complete the following flow diagrams:





   (c) Explain how you completed the table.

One of the things we do in algebra is to **evaluate** expressions. When we evaluate expressions we replace a variable in the expression with an **input number** to obtain the value of the expression called the **output number**. We think of this process as a **doing process**.

However, in other cases we may need to undo what was done. When we know what output number was obtained but do not know what input number was used, we have to **undo** what was done in evaluating the expression. In such a case we say we are **solving an equation**.

6. Look again at questions 1 to 5. For each question, say whether the question required a doing or an undoing process. Give an explanation for your answer (for example: input to output).

7. (a) Copy and complete the following flow diagrams:

   1 → [+ 6] →   7 → [− 6] →

   (b) What do you observe?

8. (a) Copy and complete the following flow diagrams:

   2 → [× 5] →   10 → [÷ 5] →

   (b) What do you observe?

9. (a) Copy and complete the following flow diagrams:

   20 → [× 5] → [+ 5] →   105 → [− 5] → [÷ 5] →

   (b) What do you observe?

10. (a) Copy and complete the following flow diagram:

   64 → [÷ 8] → [+ 12] →

   (b) What calculations will you do to determine what the input number was when the output number is 20?

Solve the following problems by undoing what was done to get the answer:

11. When a certain number is multiplied by 10 the answer is 150. What is the number?

12. When a certain number is divided by 5 the answer is 1. What is the number?

13. When 23 is added to a certain number the answer is 107. What is the original number?

14. When a certain number is multiplied by 5 and 2 is subtracted from the answer, the final answer is 13. What is the original number?

Moving from the output value to the input value is called **solving the equation for the unknown**.

9.2 Solving equations using the additive and multiplicative inverses

FINDING THE UNKNOWN

Consider the equation $3x + 2 = 23$.

We can represent the equation $3x + 2 = 23$ in a flow diagram, where x represents an unknown number:

x → × 3 → + 2 → 23

When you reverse the process in the flow diagram, you start with the output number 23, then subtract 2 and then divide the answer by 3:

23 → – 2 → ÷ 3 →

We can write all of the above reverse process as follows:

Subtract 2 from both sides of the equation:

$$3x + 2 - 2 = 23 - 2$$
$$3x = 21$$

Divide both sides by 3:

$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

We say $x = 7$ is the solution of $3x + 2 = 23$, because $3 \times 7 + 2 = 23$. We say that $x = 7$ makes the equation $3x + 2 = 23$ true.

The numbers + 2 and – 2 are **additive inverses** of each other. When we add a number and its additive inverse we always get 0.

The numbers 3 and $\frac{1}{3}$ are **multiplicative inverses** of each other. When we multiply a number and its multiplicative inverse we always get 1, so $3 \times \frac{1}{3} = 1$.

The additive and multiplicative inverses help us to isolate the unknown value or the input value.

Solve the equations below by using the additive and multiplicative inverses. Check your answers.

1. $x + 10 = 0$
2. $49x + 2 = 100$
3. $2x = 1$
4. $20 = 11 - 9x$

Also remember:

- **The multiplicative property of 1:** the product of any number and 1 is that number.
- **The additive property of 0:** the sum of any number and 0 is that number.

In some cases you need to collect like terms before you can solve the equations using additive and multiplicative inverses, as in the example below:

Example: Solve for x: $7x + 3x = 10$

$$10x = 10$$
$$\frac{10x}{10} = \frac{10}{10}$$
$$x = 1$$

$7x$ and $3x$ are like terms and can be replaced with one equivalent expression: $(7 + 3)x = 10x$.

5. $4x + 6x = 20$
6. $5x = 40 + 3x$
7. $3x + 1 - x = 0$
8. $x + 20 + 4x = -55$

9.3 Solving equations involving powers

Solving an exponential equation is the same as asking the question: **To what exponent must the base be raised in order to make the equation true?**

1. Copy and complete the following table:

x	1	3	5	7
2^x				

2. Copy and complete the following table:

x		2		5
3^x	1		27	

Karina solved the equation $3^x = 27$ as follows:

$3^x = 27$

Hence $3^x = 3^3$

Hence $x = 3$

The number 27 can be expressed as 3^3 because $3^3 = 27$.

3. Now use Karina's method and solve for x in each of the following:
   (a) $2^x = 32$ (b) $4^x = 16$ (c) $6^x = 216$ (d) $5^{x+1} = 125$

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Chapter 10
Construction of geometric figures

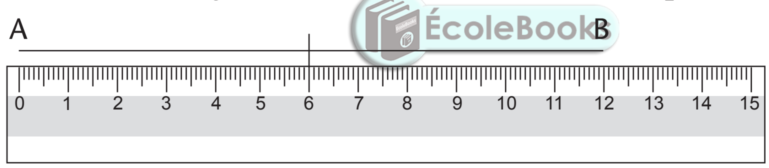
10.1 Bisecting lines

When we construct, or draw geometric figures, we often need to bisect lines or angles. To **bisect** means to cut something into two equal parts. There are different ways to bisect a line segment.

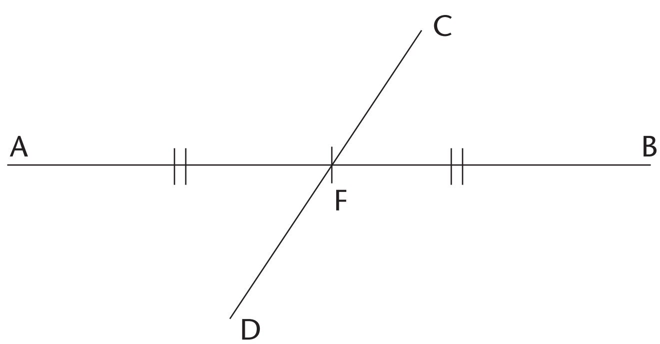
BISECTING A LINE SEGMENT WITH A RULER

1. Read through the following steps:

Step 1: Draw line segment AB and determine its midpoint.



Step 2: Draw any line segment through the midpoint.



The small marks on AF and FB show that AF and FB are equal.

CD is called a **bisector** because it bisects AB. AF = FB.

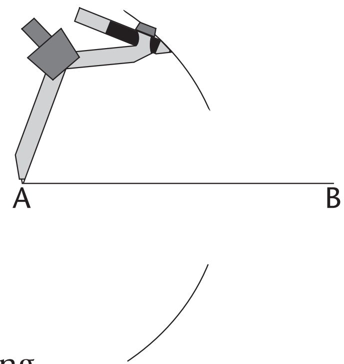
2. Use a ruler to draw and bisect the following line segments: AB = 6 cm and XY = 7 cm.

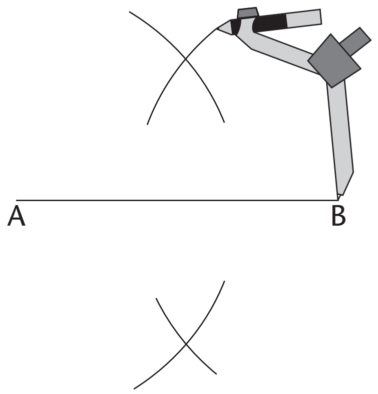
In Grade 6, you learnt how to use a compass to draw circles and parts of circles called arcs. We can use arcs to bisect a line segment.

BISECTING A LINE SEGMENT WITH A COMPASS AND RULER

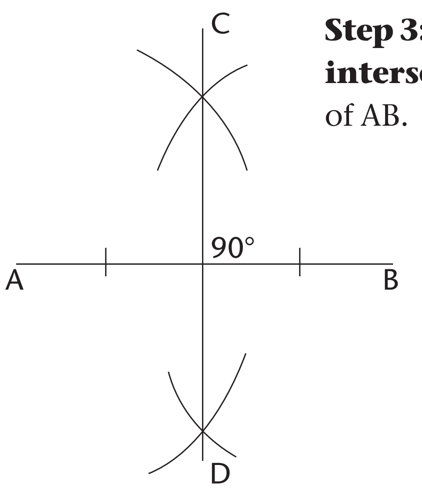
1. Read through the following steps:

Step 1: Place the compass on one endpoint of the line segment (point A). Draw an arc above and below the line. (Notice that all the points on the arc above and below the line are the same distance from point A.)





Step 2: Without changing the compass width, place the compass on point B. Draw an arc above and below the line so that the arcs cross the first two. (The two points where the arcs cross are the same distance away from point A and from point B.)



Step 3: Use a ruler to join the points where the arcs **intersect**. This line segment (CD) is the bisector of AB.

Intersect means to cross or meet.
A **perpendicular** is a line that meets another line at an angle of 90°.

Notice that CD is also **perpendicular** to AB. Therefore, CD is a **perpendicular bisector**.

2. Use a compass and a ruler to practise drawing perpendicular bisectors on line segments.

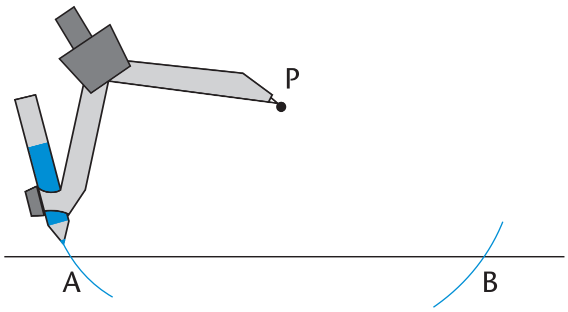
Try this!
Use only a protractor and ruler to draw a perpendicular bisector on a line segment. (Remember that we use a protractor to measure angles.)

10.2 Constructing perpendicular lines

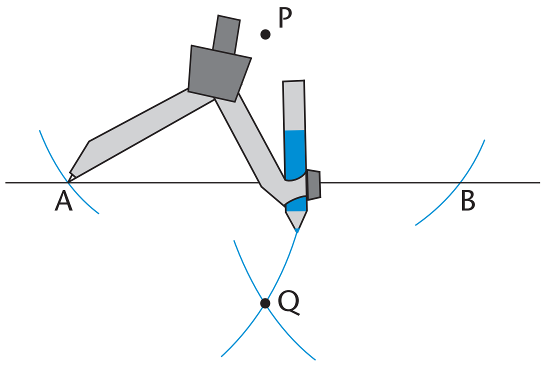
A PERPENDICULAR LINE FROM A GIVEN POINT

1. Read through the following steps:

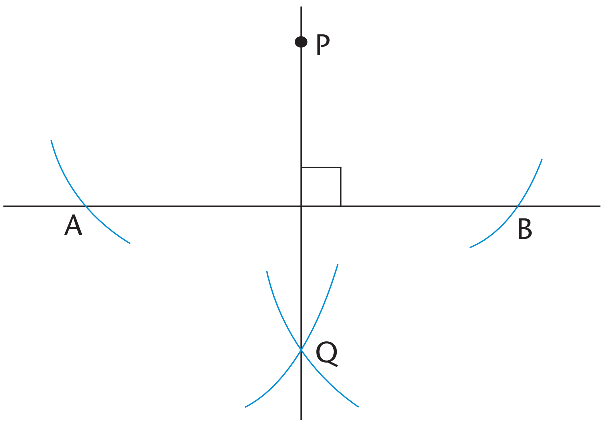
Step 1: Place your compass on the given point (point P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.



Step 2: From each arc on the line, draw another arc on the opposite side of the line from the given point (P). The two new arcs will intersect.

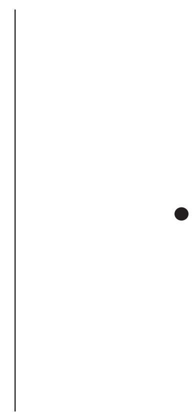


Step 3: Use your ruler to join the given point (P) to the point where the arcs intersect (Q).



PQ is perpendicular to AB.
We also write it like this: PQ ⊥ AB.

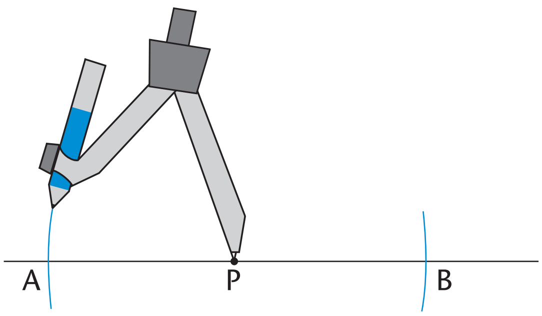
2. Use your compass and ruler to draw a perpendicular line from each given point to the line segment:



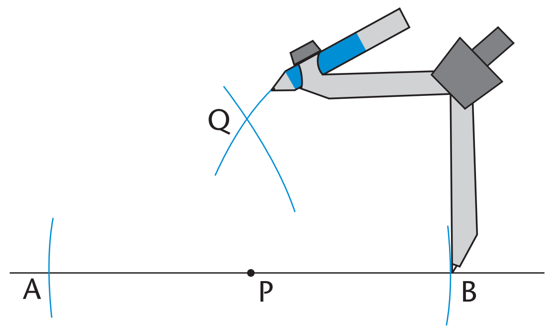
A PERPENDICULAR LINE AT A GIVEN POINT ON A LINE

1. Read through the following steps:

Step 1: Place your compass on the given point (P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.

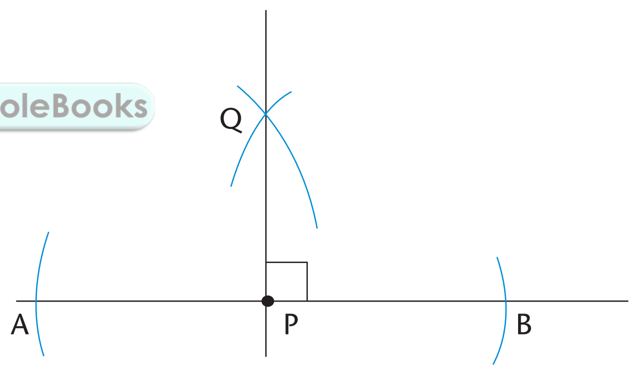


Step 2: Open your compass so that it is wider than the distance from one of the arcs to point P. Place the compass on each arc and draw an arc above or below the point P. The two new arcs will intersect.

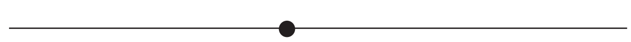


Step 3: Use your ruler to join the given point (P) and the point where the arcs intersect (Q).

PQ ⊥ AB



2. Copy the following diagrams. Use your compass and ruler to draw a perpendicular at the given point on each line:



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10.3 Bisecting angles

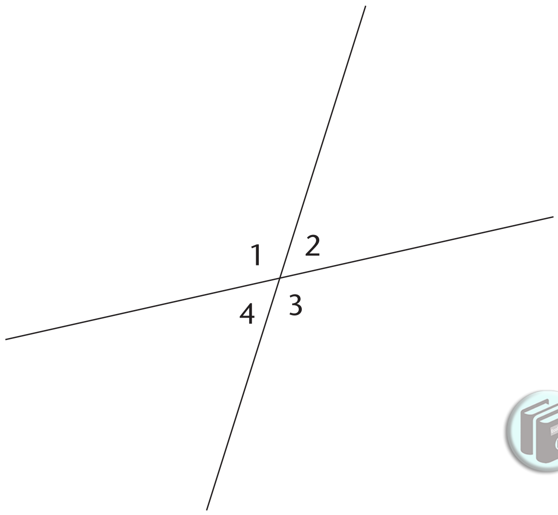
Angles are formed when any two lines meet. We use degrees (°) to measure angles.

MEASURING AND CLASSIFYING ANGLES

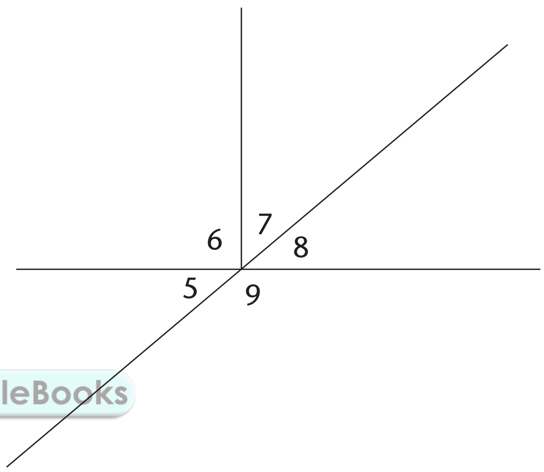
In the figures below, each angle has a number from 1 to 9.

1. Use a protractor to measure the sizes of all the angles in each figure. Write your answers for each figure.

(a)



(b)



2. Copy the statements below and use your answers to write the angle sizes:

$\hat{1}$ = ….. °

$\hat{1} + \hat{2}$ = ….. °

$\hat{1} + \hat{4}$ = ….. °

$\hat{2} + \hat{3}$ = ….. °

$\hat{3} + \hat{4}$ = ….. °

$\hat{1} + \hat{2} + \hat{4}$ = ….. °

$\hat{1} + \hat{2} + \hat{3} + \hat{4}$ = ….. °

$\hat{6}$ = ….. °

$\hat{7} + \hat{8}$ = ….. °

$\hat{6} + \hat{7} + \hat{8}$ = ….. °

$\hat{5} + \hat{6} + \hat{7}$ = ….. °

$\hat{6} + \hat{5}$ = ….. °

$\hat{5} + \hat{6} + \hat{7} + \hat{8}$ = ….. °

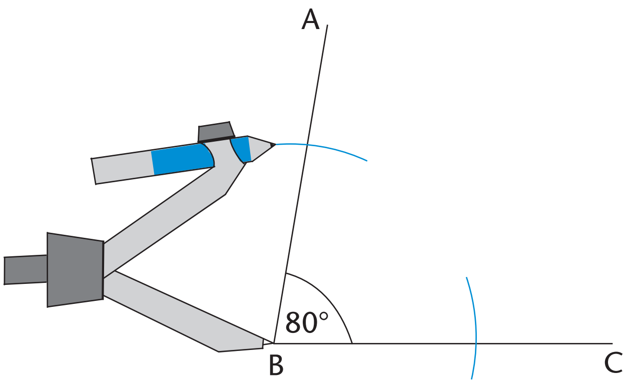
$\hat{5} + \hat{6} + \hat{7} + \hat{8} + \hat{9}$ = ….. °

3. Next to each of your answers above, write down what type of angle it is, namely acute, obtuse, right, straight, reflex or revolution.

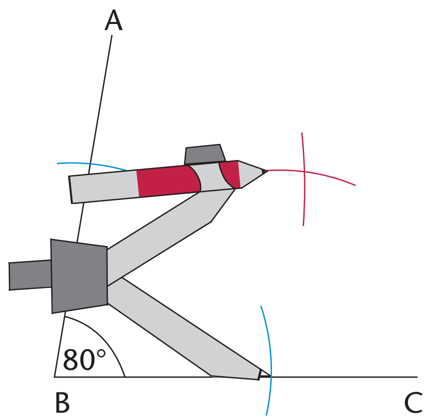
BISECTING ANGLES WITHOUT A PROTRACTOR

1. Read through the following steps:

Step 1: Place the compass on the vertex of the angle (point B). Draw an arc across each arm of the angle.

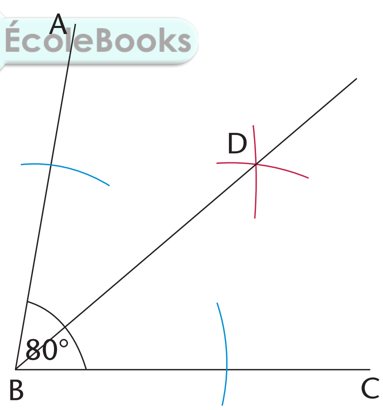


Step 2: Place the compass on the point where one arc crosses an arm and draw an arc inside the angle. Without changing the compass width, repeat for the other arm so that the two arcs cross.



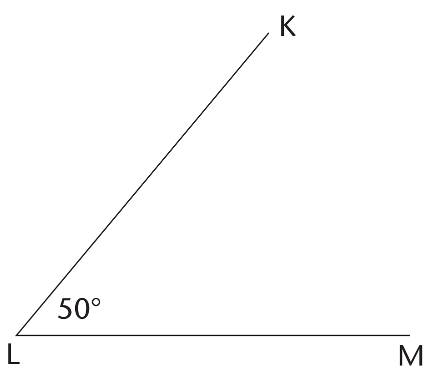
Step 3: Use a ruler to join the vertex to the point where the arcs intersect (D).

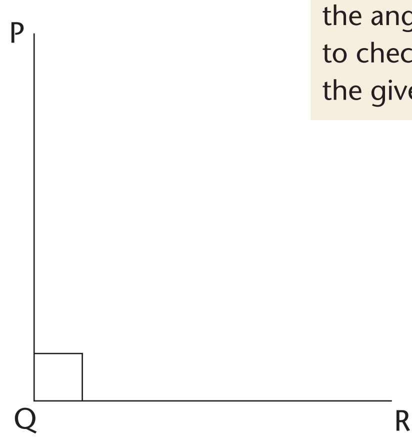
DB is the bisector of A$\hat{B}$C.



2. Copy the following angles and use your compass and ruler to bisect the angles.

You could measure each of the angles with a protractor to check if you have bisected the given angle correctly.



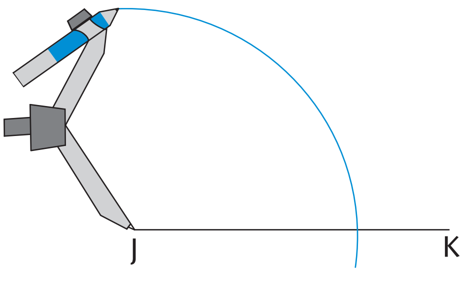


10.4 Constructing special angles without a protractor

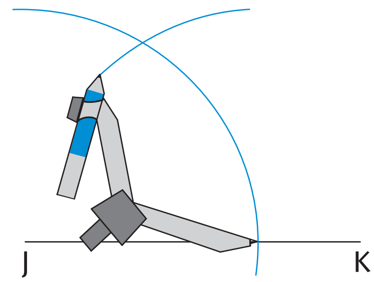
CONSTRUCTING ANGLES OF 60°, 30° AND 120°

1. Read through the following steps:

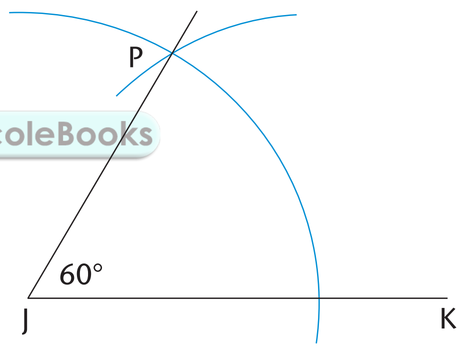
Step 1: Draw a line segment (JK). With the compass on point J, draw an arc across JK and up over above point J.



Step 2: Without changing the compass width, move the compass to the point where the arc crosses JK, and draw an arc that crosses the first one.



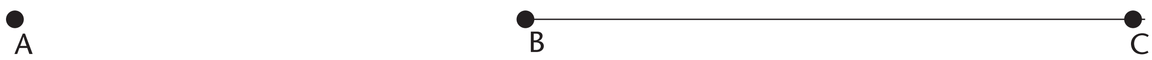
Step 3: Join point J to the point where the two arcs meet (point P).
$P\hat{J}K = 60°$



2. (a) Copy the drawing at the top of page 99. Construct an angle of 60° at point B.
   (b) Bisect the angle you constructed.
   (c) Do you notice that the bisected angle consists of two 30° angles?
   (d) Extend line segment BC to A. Then measure the angle adjacent to the 60° angle. What is its size?
   (e) What do the 60° angle and its adjacent angle add up to?

When you learn more about the properties of triangles later, you will understand why the method above creates a 60° angle. Or can you already work this out now? (Hint: What do you know about equilateral triangles?)

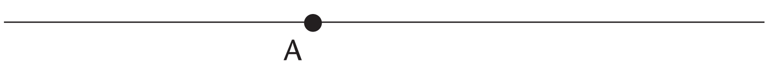
Adjacent means "next to".



CONSTRUCTING ANGLES OF 90° AND 45°

1. Construct an angle of 90° at point A. Go back to Section 10.2 if you need help.
2. Bisect the 90° angle to create an angle of 45°. Go back to Section 10.3 if you need help.

Challenge
Try to construct the following angles without using a protractor: 150°, 210° and 135°.



10.5 Constructing triangles

In this section, you will learn how to construct triangles. You will need a pencil, a protractor, a ruler and a compass.

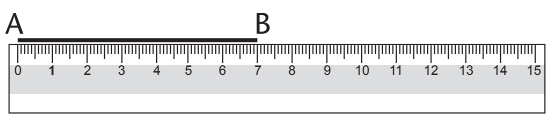
A triangle has three sides and three angles. We can construct a triangle when we know some of its measurements, that is, its sides, its angles, or some of its sides and angles.

CONSTRUCTING TRIANGLES

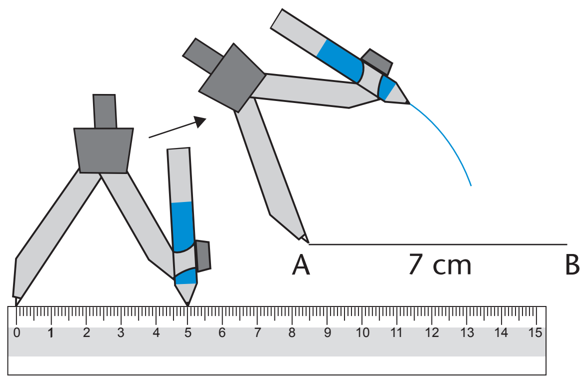
Constructing triangles when three sides are given

1. Read through the following steps. They describe how to construct ΔABC with side lengths of 3 cm, 5 cm and 7 cm.

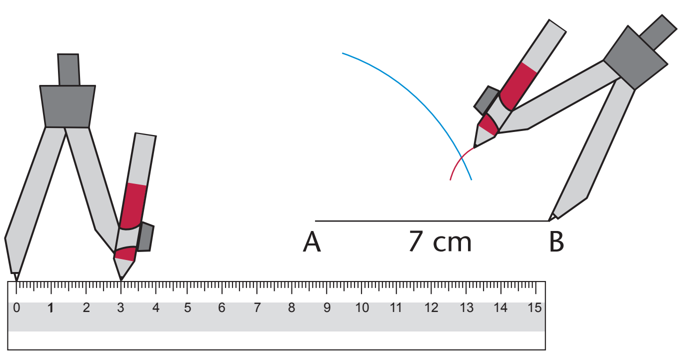
Step 1: Draw one side of the triangle using a ruler. It is often easier to start with the longest side.



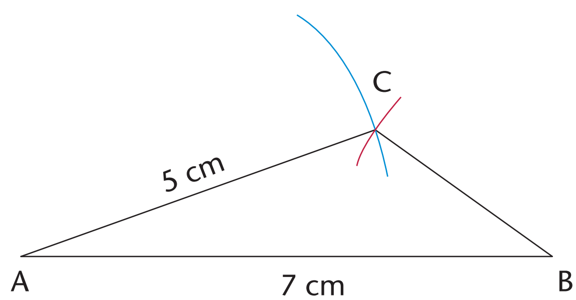
Step 2: Set the compass width to 5 cm. Draw an arc 5 cm away from point A. The third vertex of the triangle will be somewhere along this arc.



Step 3: Set the compass width to 3 cm. Draw an arc from point B. Note where this arc crosses the first arc. This will be the third vertex of the triangle.



Step 4: Use your ruler to join points A and B to the point where the arcs intersect (C).

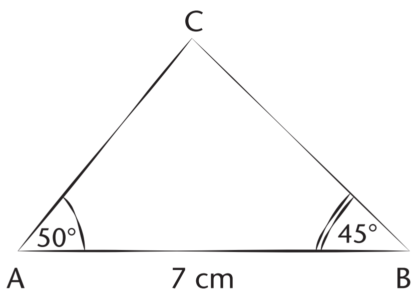


2. Follow the steps above to construct the following triangles:
   (a) ΔABC with sides 6 cm, 7 cm and 4 cm
   (b) ΔKLM with sides 10 cm, 5 cm and 8 cm
   (c) ΔPQR with sides 5 cm, 9 cm and 11 cm

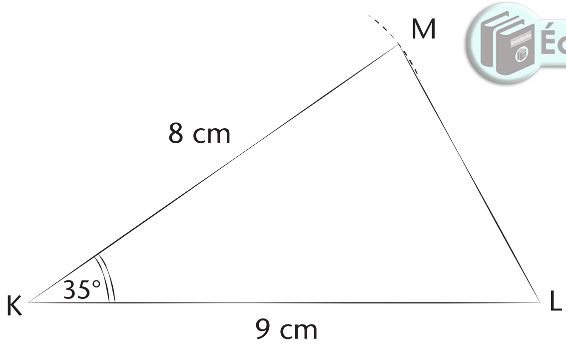
Constructing triangles when certain angles and sides are given

3. Use the rough sketches in (a) to (c) below to construct accurate triangles, using a ruler, compass and protractor. Make sure that:
   - the dotted lines show where you have to use a compass to measure the length of a side
   - you use a protractor to measure the size of the given angles.

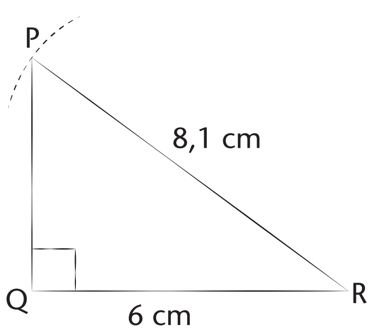
   (a) Construct ΔABC, with **two angles and one side given**.



   (b) Construct a ΔKLM, with **two sides and an angle given**.



   (c) Construct a right-angled ΔPQR, with the **hypotenuse and one other side given**.



4. Measure the missing angles and sides of each triangle in 3(a) to (c) on the previous page. Write the measurements next to your completed constructions.
5. Compare each of your constructed triangles in question 3(a) to (c) with a classmate's triangles. Are the triangles exactly the same?

If triangles are exactly the same, we say they are **congruent**.

Challenge

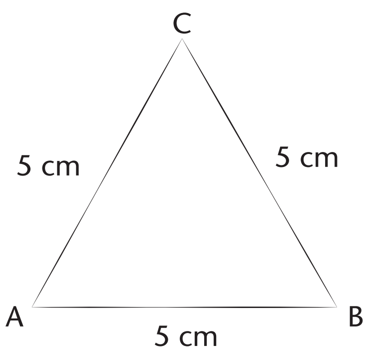
1. Construct these triangles:
   (a) ΔSTU, with three angles given: $\hat{S} = 45°$, $\hat{T} = 70°$ and $\hat{U} = 65°$.
   (b) ΔXYZ, with two sides and the angle opposite one of the sides given: $\hat{X} = 50°$, XY = 8 cm and XZ = 7 cm.
2. Can you find more than one solution for each triangle above? Explain your findings to a classmate.

10.6 Properties of triangles

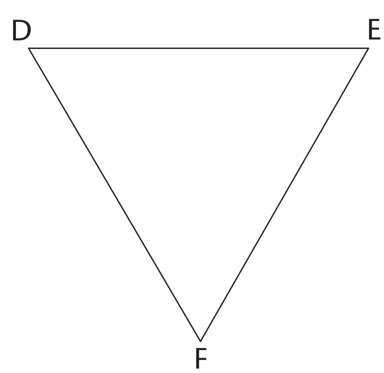
The angles of a triangle can be the same size or different sizes. The sides of a triangle can be the same length or different lengths.

PROPERTIES OF EQUILATERAL TRIANGLES

1. (a) Construct ΔABC.
   (b) Measure and label the sizes of all its sides and angles.



2. Measure and write down the sizes of the sides and angles of ΔDEF on the right.
3. Both triangles in questions 1 and 2 are called **equilateral triangles**. Discuss with a classmate if the following is true for an equilateral triangle:
   - All the sides are equal.
   - All the angles are equal to 60°.



PROPERTIES OF ISOSCELES TRIANGLES

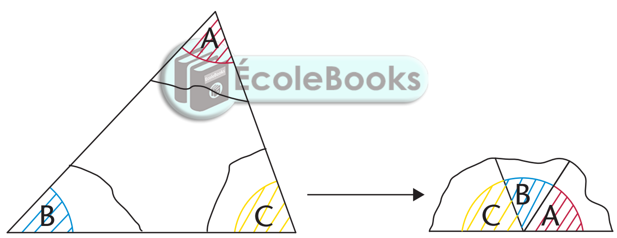
1. (a) Construct ΔDEF with EF = 7 cm, $\hat{E} = 50°$ and $\hat{F} = 50°$.
   Also construct ΔJKL with JK = 6 cm, KL = 6 cm and $\hat{J} = 70°$.
   (b) Measure and label all the sides and angles of each triangle.

2. Both triangles above are called **isosceles triangles**. Discuss with a classmate whether the following is true for an isosceles triangle:
   - Only two sides are equal.
   - Only two angles are equal.
   - The two equal angles are opposite the two equal sides.

THE SUM OF THE ANGLES IN A TRIANGLE

1. Look at your constructed triangles ΔABC, ΔDEF and ΔJKL above and on the previous page. What is the sum of the three angles each time?

2. Did you find that the sum of the interior angles of each triangle is 180°? Do the following to check if this is true for other triangles.



   (a) On a clean sheet of paper, construct any triangle. Label the angles A, B and C and cut out the triangle.
   (b) Neatly tear the angles off the triangle and fit them next to one another.
   (c) Notice that $\hat{A}$, $\hat{B}$ and $\hat{C}$ form a straight angle. Complete: $\hat{A} + \hat{B} + \hat{C} =$ ..... °

We can conclude that the interior angles of a triangle always add up to 180°.

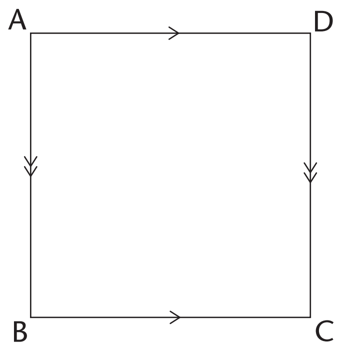
10.7 Properties of quadrilaterals

A quadrilateral is any closed shape with four straight sides. We classify quadrilaterals according to their sides and angles. We note which sides are parallel, perpendicular or equal. We also note which angles are equal.

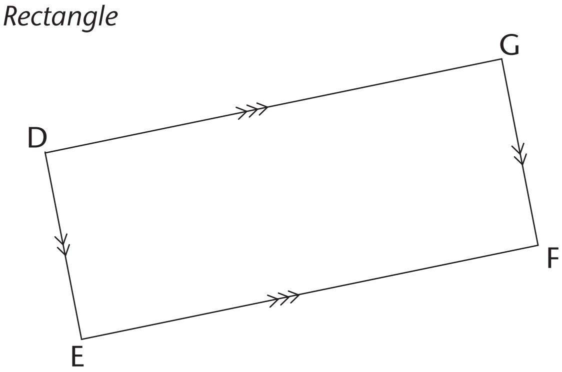
PROPERTIES OF QUADRILATERALS

1. Measure and write down the sizes of all the angles and the lengths of all the sides of each of the following quadrilaterals:

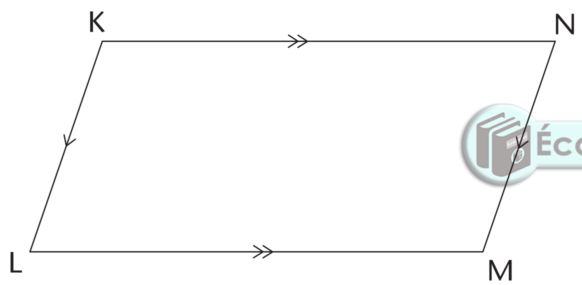
Square



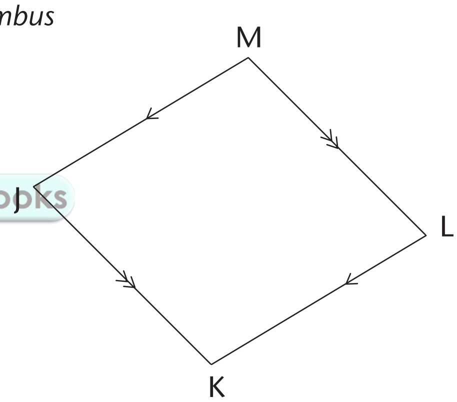
Rectangle



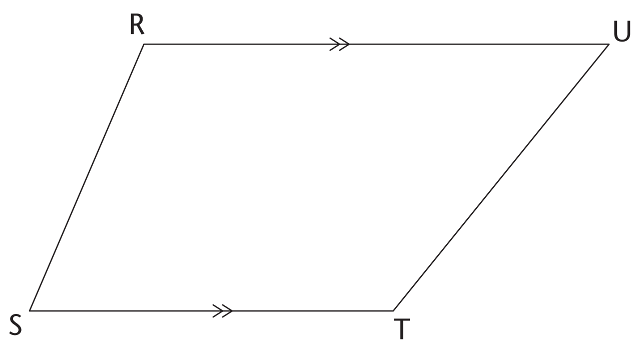
Parallelogram



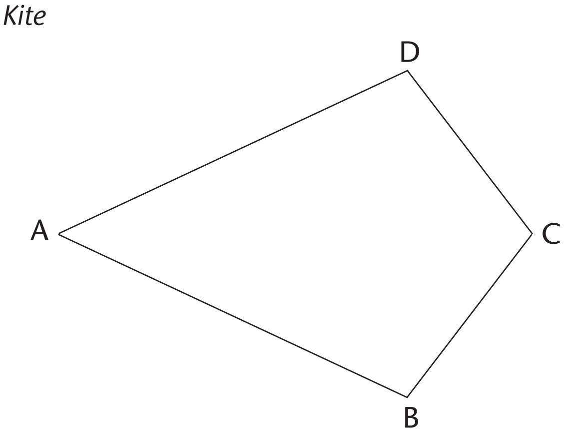
Rhombus



Trapezium



Kite



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2. Use your answers in question 1. Copy the following table and place a ✓ in the correct box to show which property is correct for each shape.

Properties	**Parallelogram**	**Rectangle**	**Rhombus**	**Square**	**Kite**	**Trapezium**
Only one pair of sides are parallel						
Opposite sides are parallel						
Opposite sides are equal						
All sides are equal						
Two pairs of adjacent sides are equal						
Opposite angles are equal						
All angles are equal						

SUM OF THE ANGLES IN A QUADRILATERAL

1. Add up the four angles of each quadrilateral on the previous page. What do you notice about the sum of the angles of each quadrilateral?
2. Did you find that the sum of the interior angles of each quadrilateral equals 360°? Do the following to check if this is true for other quadrilaterals:
   (a) On a clean sheet of paper, use a ruler to construct any quadrilateral.
   (b) Label the angles A, B, C and D. Cut out the quadrilateral.
   (c) Neatly tear the angles off the quadrilateral and fit them next to one another.
   (d) What do you notice?

We can conclude that the interior angles of a quadrilateral always add up to 360°.

10.8 Constructing quadrilaterals

You learnt how to construct perpendicular lines in Section 10.2. If you know how to construct parallel lines, you should be able to construct any quadrilateral accurately.

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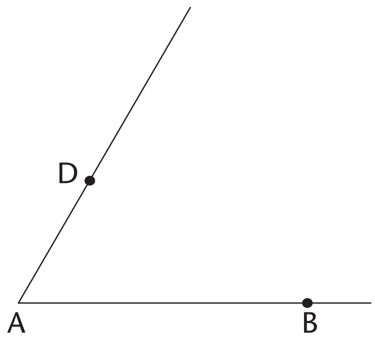
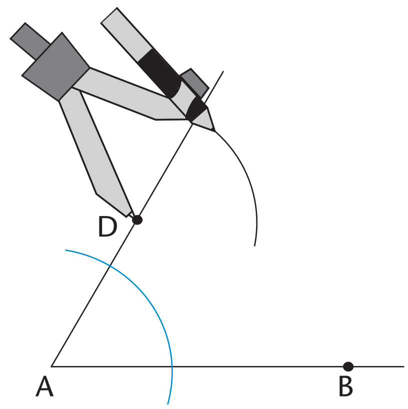
CONSTRUCTING PARALLEL LINES TO DRAW QUADRILATERALS

1. Read through the following steps:

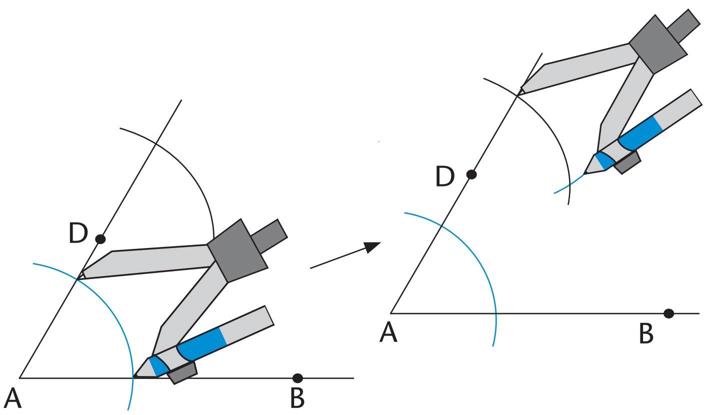
Step 1: From line segment AB, mark a point D. This point D will be on the line that will be parallel to AB. Draw a line from A through D.



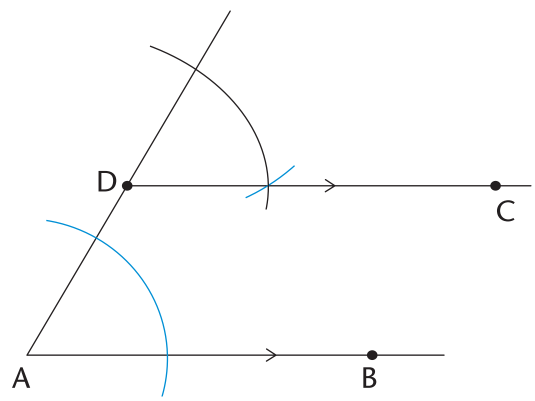
Step 2: Draw an arc from A that crosses AD and AB. Keep the same compass width and draw an arc from point D as shown.



Step 3: Set the compass width to the distance between the two points where the first arc crosses AD and AB. From the point where the second arc crosses AD, draw a third arc to cross the second arc.



Step 4: Draw a line from D through the point where the two arcs meet. DC is parallel to AB.



2. Practise drawing a parallelogram, square and rhombus.
3. Use a protractor to draw quadrilaterals with at least one set of parallel lines.

WORKSHEET

1. Do the following construction:

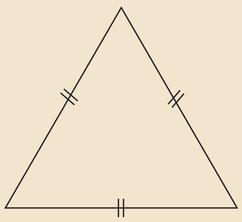
   (a) Use a compass and ruler to construct equilateral $\triangle ABC$ with sides of 9 cm.

   (b) Without using a protractor, bisect $\hat{B}$. Let the bisector intersect AC at point D.

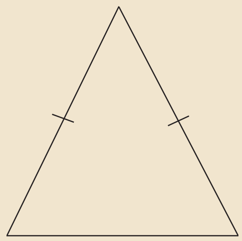
   (c) Use a protractor to measure $A\hat{D}B$. Write the measurement on the drawing.

2. Name the following types of triangles and quadrilaterals:

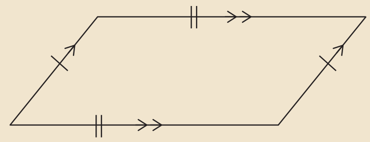
A



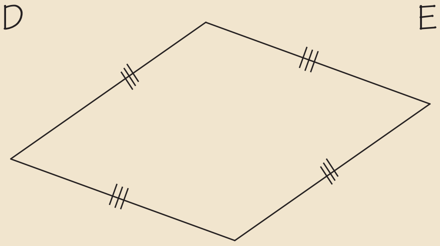
B



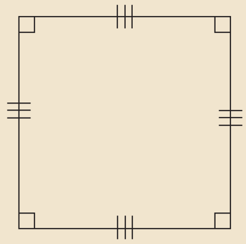
C



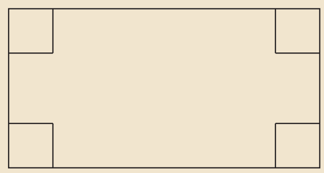
D



E



F



3. Which of the following quadrilaterals matches each description below? (There may be more than one answer for each.)

parallelogram; rectangle; rhombus; square; kite; trapezium

   (a) All sides are equal and all angles are equal.

   (b) Two pairs of adjacent sides are equal.

   (c) One pair of sides is parallel.

   (d) Opposite sides are parallel.

   (e) Opposite sides are parallel and all angles are equal.

   (f) All sides are equal.

Chapter 11
Geometry of 2D shapes

11.1 Types of triangles

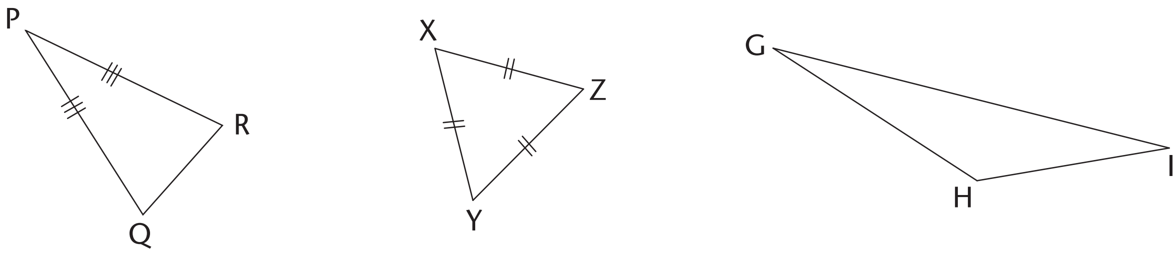
By now, you know that a triangle is a closed 2D shape with three straight sides. We can classify or name different types of triangles according to the lengths of their sides and according to the sizes of their angles.

NAMING TRIANGLES ACCORDING TO THEIR SIDES

1. Copy the table and match the name of each type of triangle with its correct description:

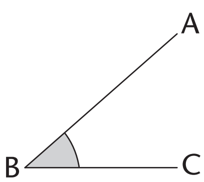
Name of triangle	Description of triangle
Isosceles triangle	All the sides of a triangle are equal.
Scalene triangle	None of the sides of a triangle are equal.
Equilateral triangle	Two sides of a triangle are equal.

2. Name each type of triangle by looking at its sides:

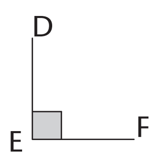


NAMING TRIANGLES ACCORDING TO THEIR ANGLES

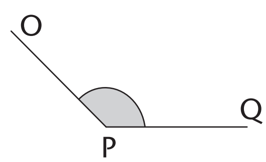
Remember the following types of angles:



Acute angle
(< 90°)

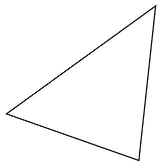


Right angle
(= 90°)

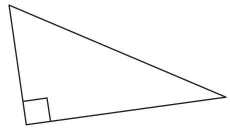


Obtuse angle
(between 90° and 180°)

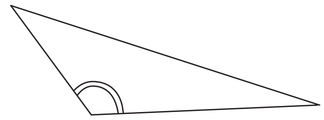
Study the following triangles and then answer the questions:



Acute triangle



Right-angled triangle



Obtuse triangle

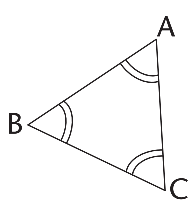
1. Are all the angles of a triangle always equal?
2. When a triangle has an obtuse angle, what kind of triangle is it called?
3. When a triangle has only acute angles, what kind of triangle is it called?
4. What size must one of the angles of a triangle be for it to be called a right-angled triangle?

INVESTIGATING THE ANGLES AND SIDES OF TRIANGLES

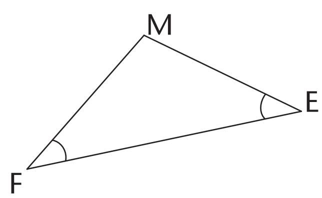
1. (a) What is the sum of the interior angles of a triangle?
   (b) Can a triangle have two right angles? Explain your answer.
   (c) Can a triangle have more than one obtuse angle? Explain your answer.

If you cannot work out the answers in question 1(b) and (c), try to construct the triangles to find the answers.

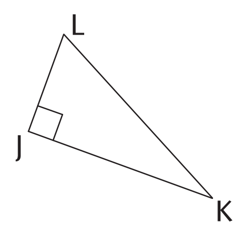
2. Look at the triangles below. The arcs show which angles are equal.



Equilateral triangle



Isosceles triangle



Right-angled triangle

   (a) ΔABC is an equilateral triangle. What do you notice about its angles?
   (b) ΔFEM is an isosceles triangle. What do you notice about its angles?
   (c) ΔJKL is a right-angled triangle. Is its longest side opposite the 90° angle?
   (d) Construct any three right-angled triangles on a sheet of paper. Is the longest side always opposite the 90° angle?

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Properties of triangles:

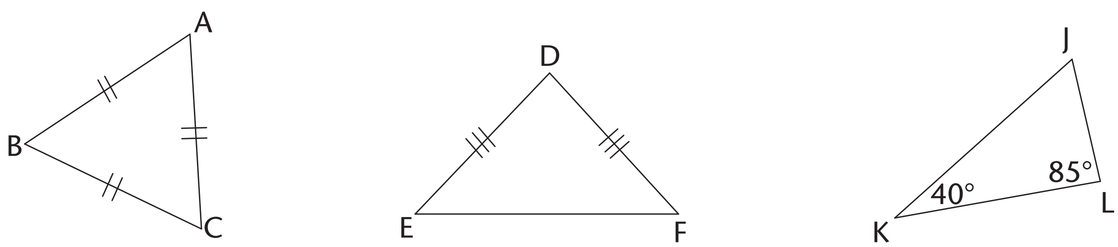
- The **sum of the interior angles** of a triangle is 180°.
- An **equilateral triangle** has all sides equal and each interior angle is equal to 60°.
- An **isosceles triangle** has two equal sides and the angles opposite the equal sides are equal.
- A **scalene triangle** has no sides equal.
- A **right-angled triangle** has a right angle (90°).
- An **obtuse triangle** has one obtuse angle (between 90° and 180°).
- An **acute triangle** has three acute angles (< 90°).

Interior angles are the angles inside a closed shape; not the angles outside of it.

11.2 Unknown angles and sides of triangles

You can use what you know about triangles to obtain other information. When you work out new information, you must always give reasons for the statements you make.

Look at the following examples of working out unknown angles and sides when certain information is given. The reason for each statement is written in square brackets.



$\hat{A} = \hat{B} = \hat{C} = 60°$ [Angles in an equilateral Δ = 60°]
DE = DF [Given]
$\hat{E} = \hat{F}$ [Angles opposite the equal sides of an isosceles Δ are equal]
$\hat{J} = 55°$ [The sum of the interior angles of a Δ = 180°; so $\hat{J} = 180° - 40° - 85°$]

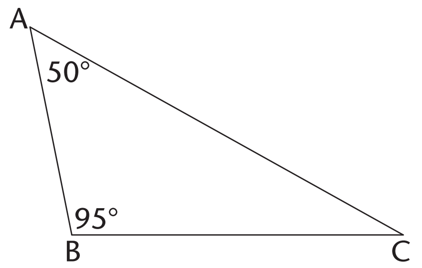
You can shorten the following reasons in the ways shown:

- Sum of interior angles (∠s) of a triangle (Δ) = 180°: **Interior ∠s of Δ**
- Isosceles triangle has two sides and two angles equal: **Isosceles Δ**
- Equilateral triangle has three sides and three angles equal: **Equilateral Δ**
- Angles forming a straight line = 180°: **Straight line**

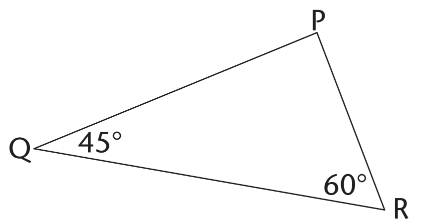
WORKING OUT UNKNOWN ANGLES AND SIDES

Find the sizes of unknown angles and sides in the following triangles. Always give reasons for every statement.

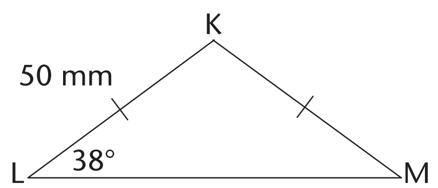
1. Find $\hat{C}$.



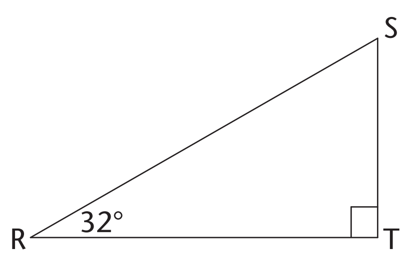
2. Find $\hat{P}$.



3. (a) Find KM.
   (b) Find $\hat{K}$.



4. What is the size of $\hat{S}$?



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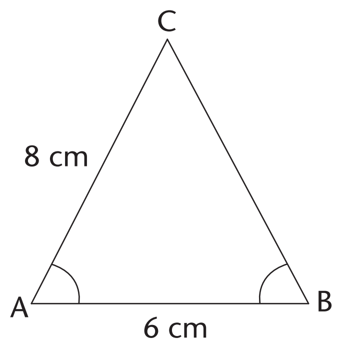
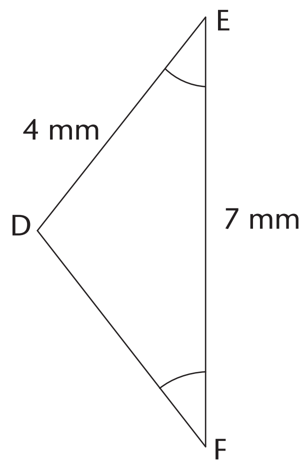
5. (a) Find CB.
   (b) Find $\hat{C}$ if $\hat{A} = 50°$.

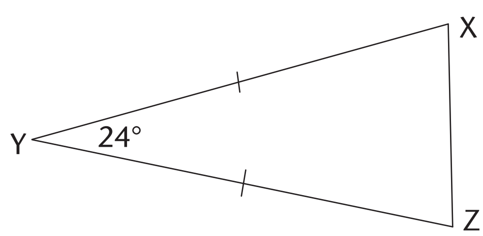


6. (a) Find DF.
   (b) Find $\hat{E}$ if $\hat{D} = 100°$.

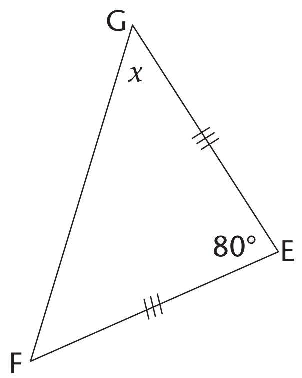


WORKING OUT MORE UNKNOWN ANGLES AND SIDES

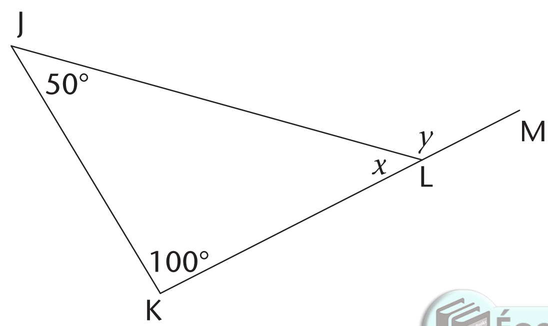
1. Calculate the size of $\hat{X}$ and $\hat{Z}$.



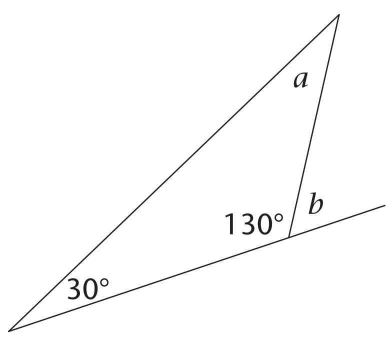
2. Calculate the size of x.



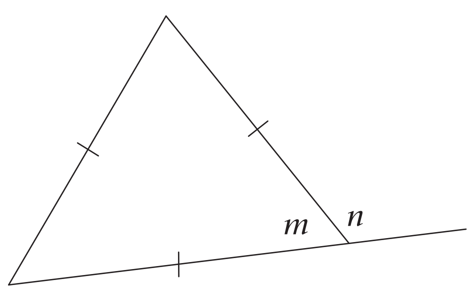
3. KLM is a straight line. Calculate the size of x and y.



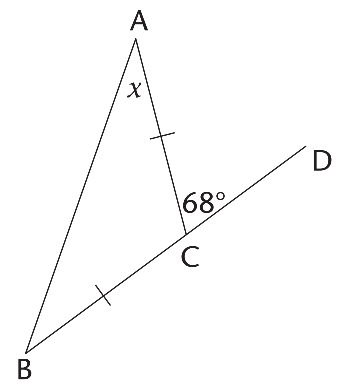
4. Angle b and an angle with a size of 130° form a straight angle. Calculate the size of a and b.



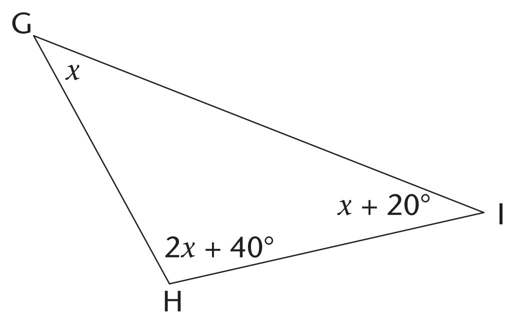
5. Angle m and n form a straight angle. Calculate the size of m and n.



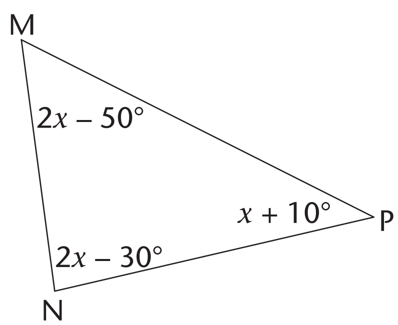
6. BCD is a straight line segment. Calculate the size of x.



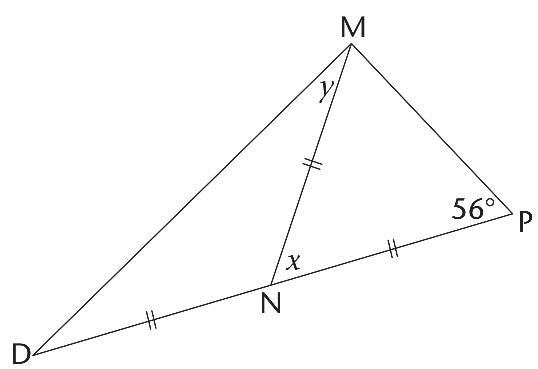
7. Calculate the size of x and then the size of $\hat{H}$.



8. Calculate the size of $\hat{N}$.



9. DNP is a straight line. Calculate the size of x and y.



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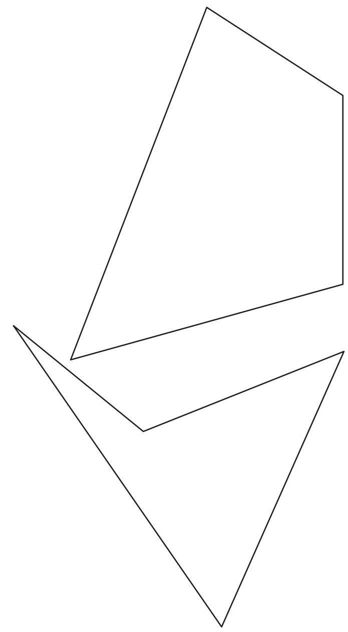
11.3 Types of quadrilaterals and their properties



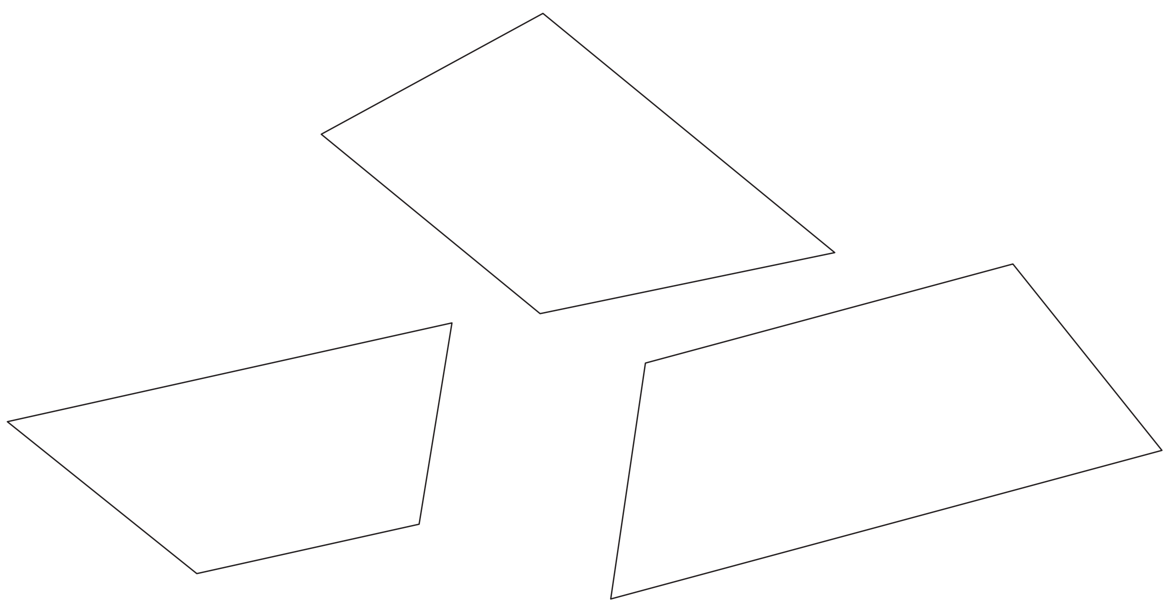
A quadrilateral is a figure with four straight sides which meet at four vertices. In many quadrilaterals all the sides are of different lengths and all the angles are of different sizes.

> You have previously worked with these types of quadrilaterals, in which some sides have the same lengths, and some angles may be of the same size:
>
> parallelograms
> rectangles
> kites
> rhombuses
> squares
> trapeziums

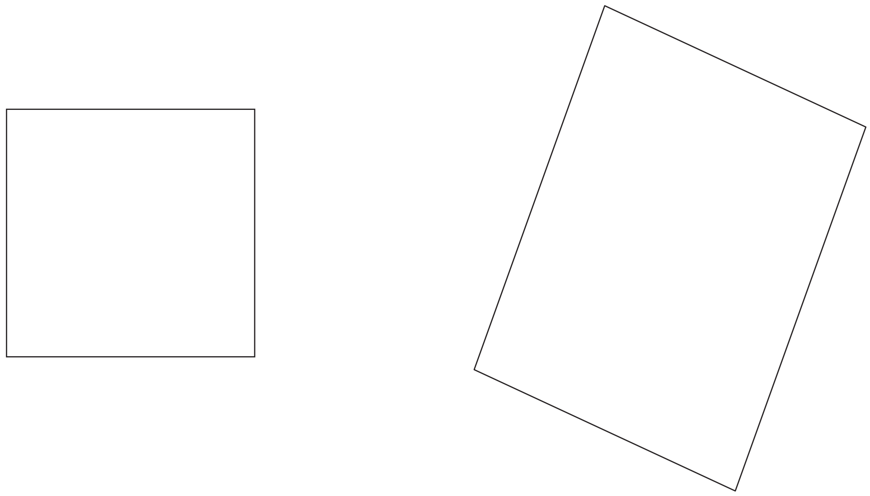
THE PROPERTIES OF DIFFERENT TYPES OF QUADRILATERALS

1. In each of the following questions, different examples of a certain type of quadrilateral are given. In each case identify which kind of quadrilateral it is. Describe the properties of each type by making statements about the lengths and directions of the sides and the sizes of the angles of each type. You may have to take some measurements to be able to do this.

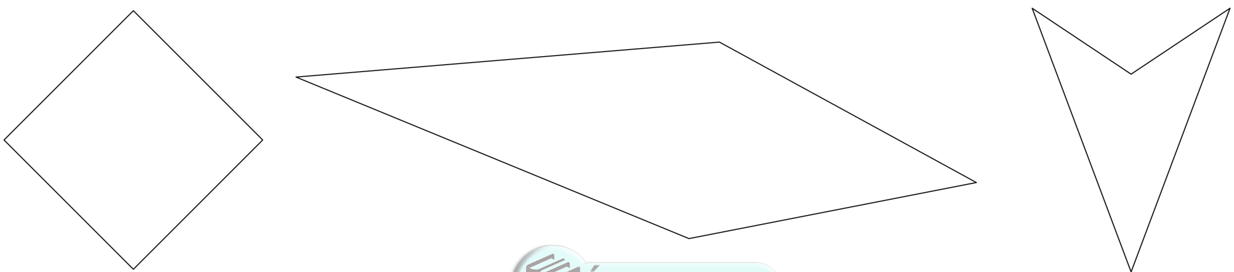
(a)



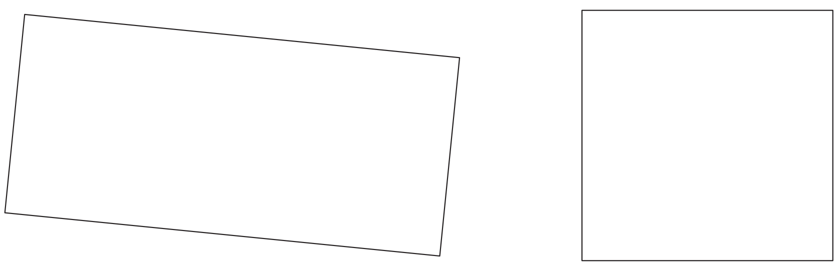
(b)



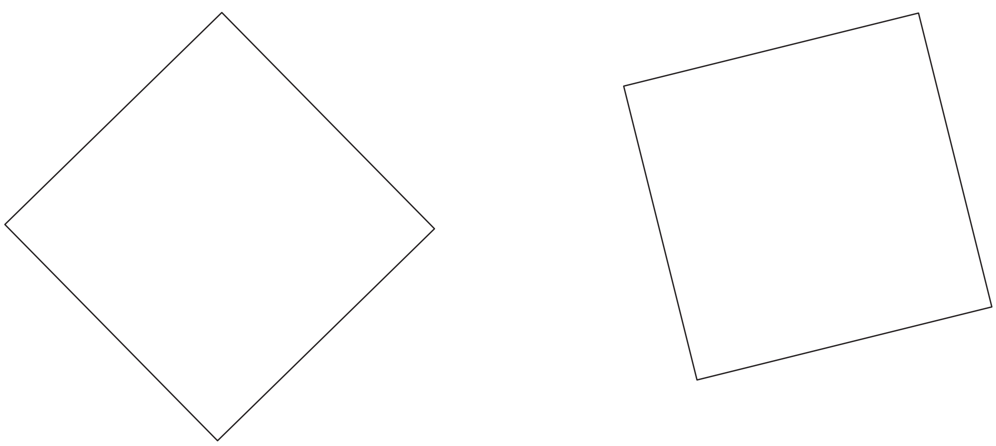
(c)



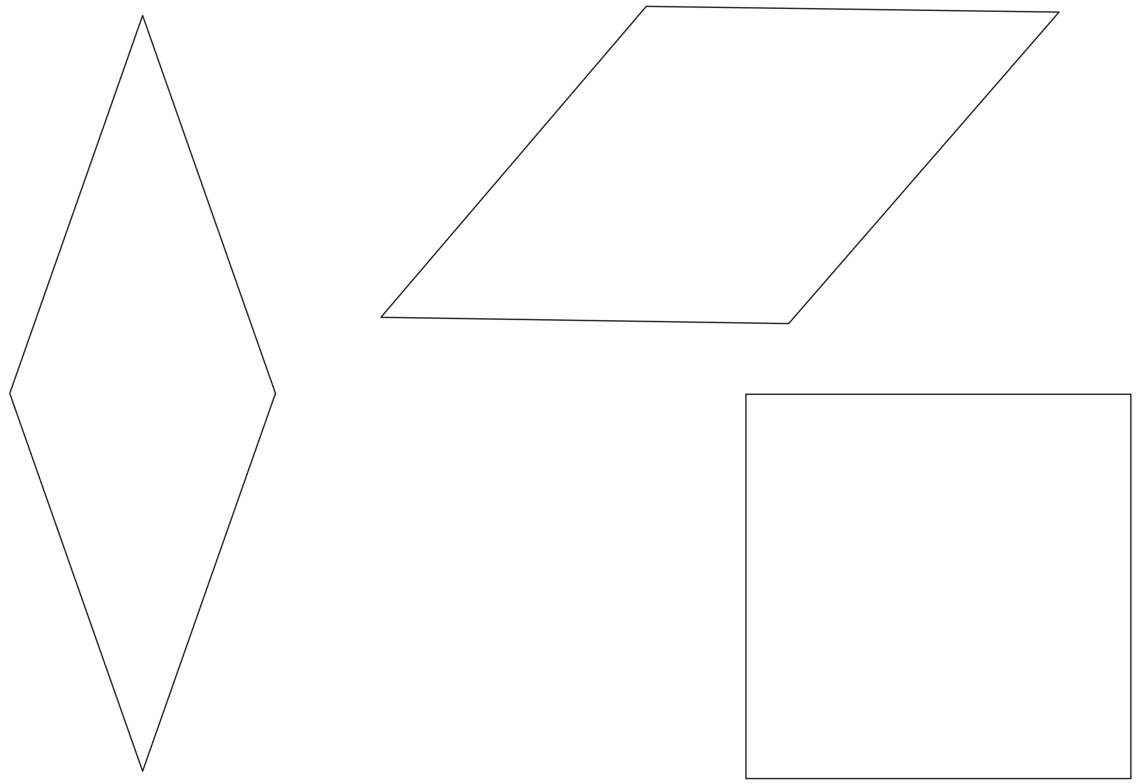
(d)



(e)



(f)



2. Use your completed lists and the drawings in question 1 to determine if the following statements are true or false:

   (a) A rectangle is a parallelogram.
   (b) A square is a parallelogram.
   (c) A rhombus is a parallelogram.
   (d) A kite is a parallelogram.
   (e) A trapezium is a parallelogram.
   (f) A square is a rhombus.
   (g) A square is a rectangle.
   (h) A square is a kite.
   (i) A rhombus is a kite.
   (j) A rectangle is a rhombus.
   (k) A rectangle is a square.

If a quadrilateral has *all* the properties of another quadrilateral, you can define it in terms of the other quadrilateral, as you have found above.

A **convention** is something (such as a definition or method) that most people agree on, accept and follow.

3. Here are some conventional definitions of quadrilaterals:
   - A **parallelogram** is a quadrilateral with two opposite sides parallel.
   - A **rectangle** is a parallelogram that has all four angles equal to 90°.
   - A **rhombus** is a parallelogram with all four sides equal.
   - A **square** is a rectangle with all four sides equal.
   - A **trapezium** is a quadrilateral with one pair of opposite sides parallel.
   - A **kite** is a quadrilateral with two pairs of adjacent sides equal.

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Write down other definitions that work for the following quadrilaterals:

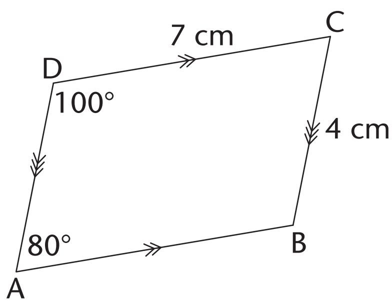
(a) Rectangle (b) Square (c) Rhombus
(d) Kite (e) Trapezium

11.4 Unknown angles and sides of quadrilaterals

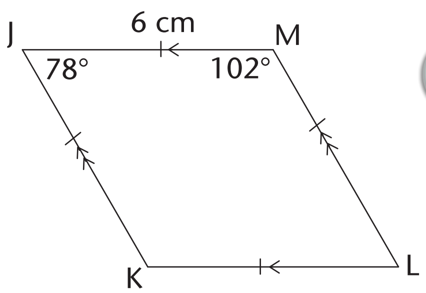
FINDING UNKNOWN ANGLES AND SIDES

Find the length of all the **unknown sides** and **angles** in the following quadrilaterals. Give reasons to justify your statements. (Also recall that the sum of the angles of a quadrilateral is 360°.)

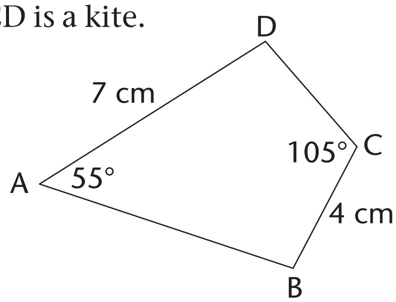
1.



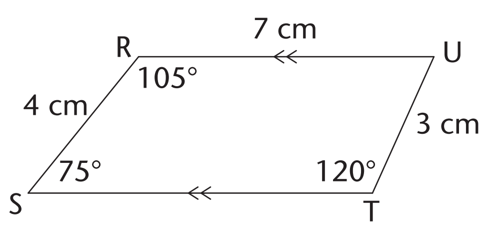
2.



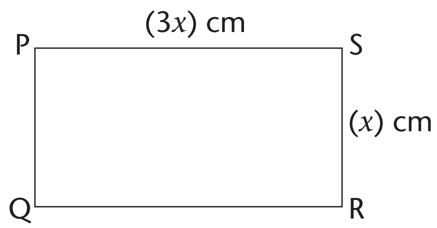
3. ABCD is a kite.



4. The perimeter of RSTU is 23 cm.

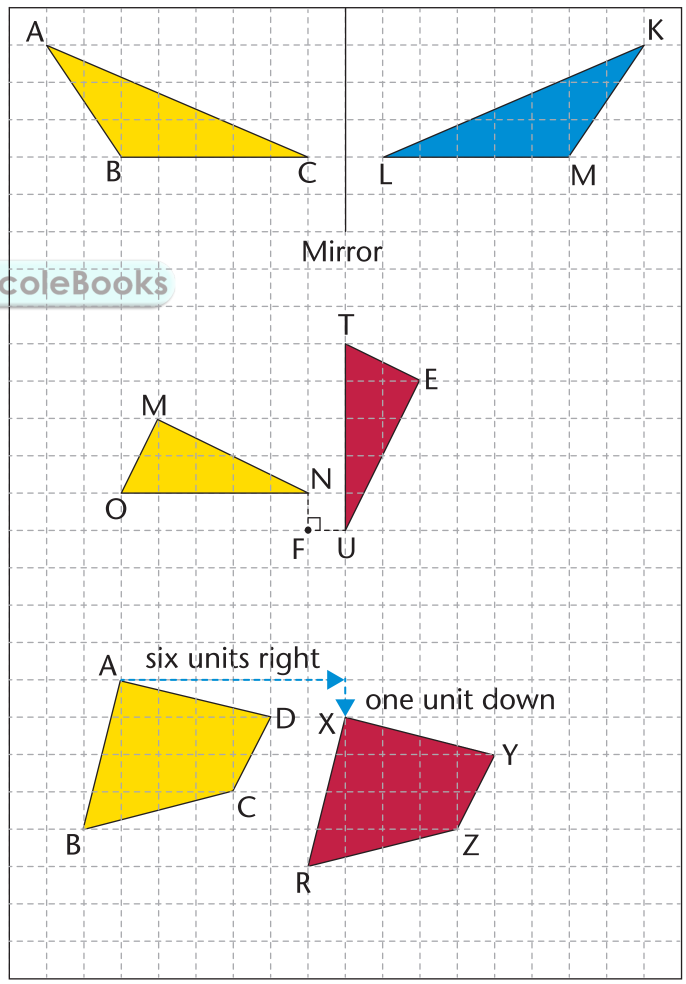


5. PQRS is a rectangle and has a perimeter of 40 cm.



11.5 Congruency

WHAT IS CONGRUENCY?



1. ΔABC is reflected in the vertical line (mirror) to give ΔKLM.
   Are the sizes and shapes of the two triangles exactly the same?

2. ΔMON is rotated 90° around point F to give you ΔTUE.
   Are the sizes and shapes of ΔMON and ΔTUE exactly the same?

3. Quadrilateral ABCD is translated six units to the right and one unit down to give quadrilateral XRZY.
   Are ABCD and XRZY exactly the same?

In the previous activity, each of the figures was transformed (reflected, rotated or translated) to produce a second figure. The second figure in each pair has **the same angles**, **side lengths**, **size and area** as the first figure. The second figure is therefore an **accurate copy** of the first figure.

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When one figure is an image of another figure, we say that the two figures are **congruent**.
The symbol for congruency is: ≡

The word **congruent** comes from the Latin word *congruere*, which means "to agree". Figures are congruent if they match up perfectly when laid on top of each other.

Notation of congruent figures

When we name shapes that are congruent, we name them so that the matching, or corresponding, angles are in the same order. For example, in **ΔABC** and **ΔKLM** on the previous page:

$\hat{A}$ is congruent to (matches and is equal to) $\hat{K}$.
$\hat{B}$ is congruent to $\hat{M}$.
$\hat{C}$ is congruent to $\hat{L}$.

We therefore use this notation: **ΔABC ≡ ΔKML**.

We cannot assume that, when the angles of polygons are equal, the polygons are congruent. You will learn about the conditions of congruence in Grade 9.

Similarly, for the other pairs of figures on the previous page:
ΔMON ≡ **ΔTUE** and **ABCD** ≡ **XRZY**.

The notation of congruent figures also shows which sides of the two figures correspond and are equal. For example, ΔABC ≡ ΔKML shows that:

AB = KM, BC = ML and AC = KL.

The incorrect notation ΔABC ≡ ΔKLM would show the following incorrect information:
$\hat{B} = \hat{L}$, $\hat{C} = \hat{M}$, AB = KL and AC = KM.

IDENTIFYING CONGRUENT ANGLES AND SIDES

Copy the following table and write down which angles and sides are equal between each pair of congruent figures:

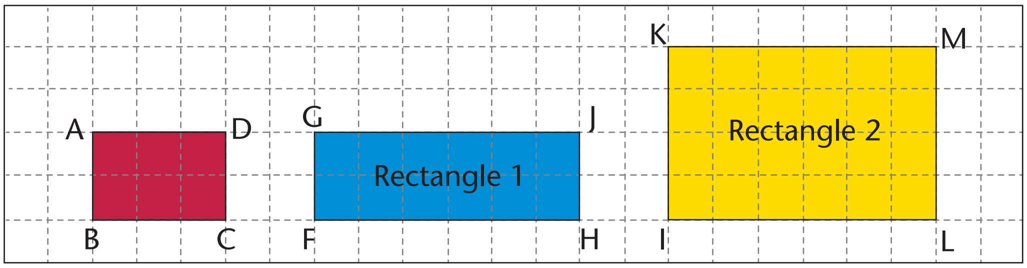
1. ΔPQR ≡ ΔUCT	2. ΔKLM ≡ ΔUWC
3. ΔGHI ≡ ΔQRT	4. ΔKJL ≡ ΔPOQ

11.6 Similarity

In Grade 7, you learnt that two figures are **similar** when they have the **same shape** (their angles are equal) but they may be **different sizes**. The sides of one figure are proportionally longer or shorter than the sides of the other figure; that is, the length of each side is multiplied or divided by the same number. We say that one figure is an enlargement or a reduction of the other figure.

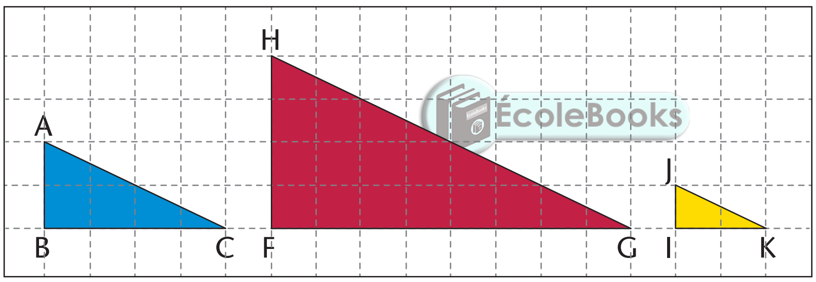
CHECKING FOR SIMILARITY

1. Look at the rectangles below and answer the questions that follow:



(a) Look at rectangle 1 and ABCD. How many times is FH longer than BC? How many times is GF longer than AB?
(b) Look at rectangle 2 and ABCD. How many times is IL longer than BC? How many times is LM longer than CD?
(c) Is rectangle 1 or rectangle 2 an enlargement of rectangle ABCD? Explain your answer.

2. Look at the triangles below and answer the questions that follow:



(a) How many times is:
- FG longer than BC?
- HF longer than AB?
- HG longer than AC?
- IK shorter than BC?
- JI shorter than AB?
- JK shorter than AC?

(b) Is ΔHFG an enlargement of ΔABC? Explain your answer.
(c) Is ΔJIK a reduction of ΔABC? Explain your answer.

In the previous activity, rectangle KILM is an enlargement of rectangle ABCD. Therefore, ABCD is similar to KILM. The symbol for “is similar to” is: |||. So we write: ABCD ||| KILM.

The triangles on the previous page are also similar. ΔHFG is an enlargement of ΔABC, and ΔJIK is a reduction of ΔABC.

In ΔABC and ΔHFG, $\hat{A} = \hat{H}$, $\hat{B} = \hat{F}$ and $\hat{C} = \hat{G}$. We therefore write it like this: ΔABC ||| ΔHFG.

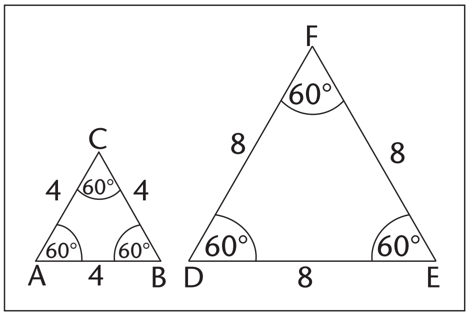
In the same way, ΔABC ||| ΔJIK.

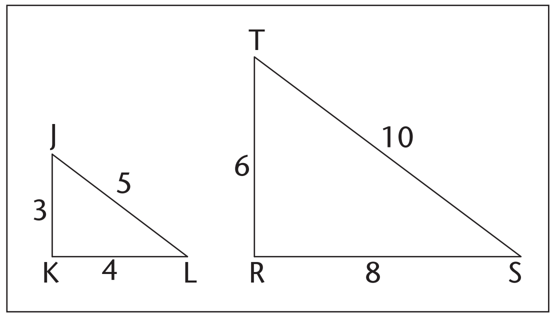
When you enlarge or reduce a polygon, you need to enlarge or reduce all its sides proportionally, or by the same ratio. This means that you multiply or divide each length by the same number.

> **Similar figures** are figures that have the same angles (same shape) but are not necessarily the same size.

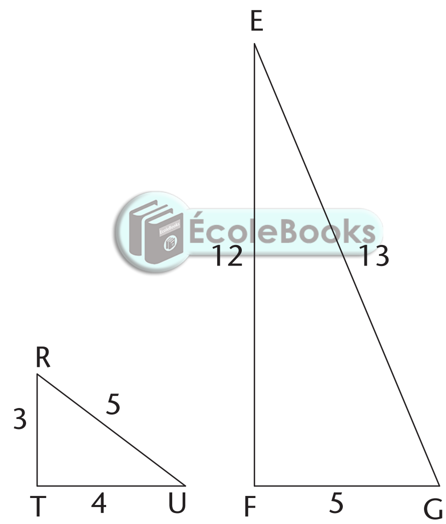
USING PROPERTIES OF SIMILAR AND CONGRUENT FIGURES

1. Are the triangles in each pair similar? Give a reason for each answer.

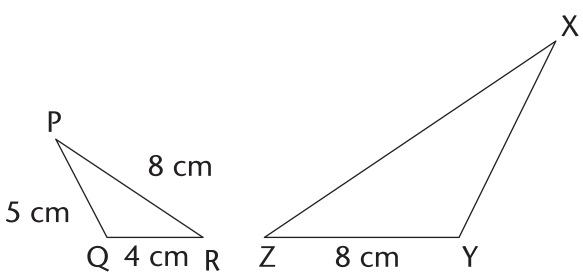




2. Is ΔRTU ||| ΔEFG? Give a reason for your answer.



3. ΔPQR ||| ΔXYZ. Determine the length of XZ and XY.



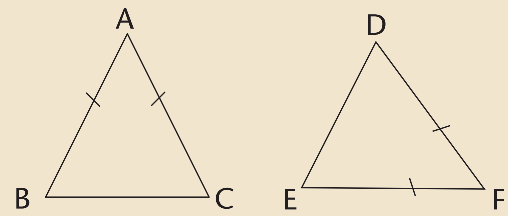
4. Are the following statements true or false? Explain your answers.
   (a) Figures that are congruent are similar.
   (b) Figures that are similar are congruent.
   (c) All rectangles are similar.
   (d) All squares are similar.

WORKSHEET

1. Study the triangles below and answer the following questions:

   (a) Choose the correct answer and write it down.
   ΔABC is:

   - ☐ acute and equilateral
   - ☐ obtuse and scalene
   - ☐ acute and isosceles
   - ☐ right-angled and isosceles



   (b) If AB = 40 mm, what is the length of AC?

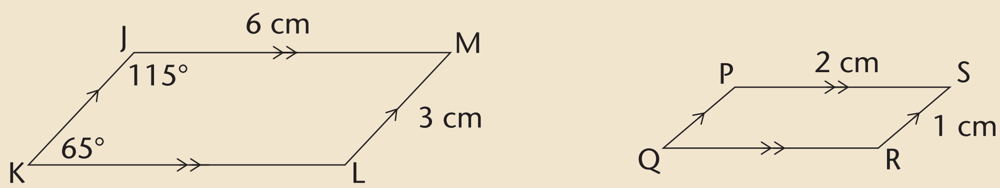
   (c) If $\hat{B} = 80°$, what is the size of $\hat{C}$ and of $\hat{A}$?

   (d) $\Delta ABC \equiv \Delta FDE$. Name all the sides in the two triangles that are equal to AB.

   (e) Name the side that is equal to DE.

   (f) If $\hat{F}$ is 40°, what is the size of $\hat{B}$?

2. Look at figures JKLM and PQRS. (Give reasons for your answers below.)



   (a) What type of quadrilateral is JKLM?

   (b) Is JKLM ||| PQRS?

   (c) What is the size of $\hat{L}$?

   (d) What is the size of $\hat{S}$?

   (e) What is the length of KL?

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Chapter 12
Geometry of straight lines

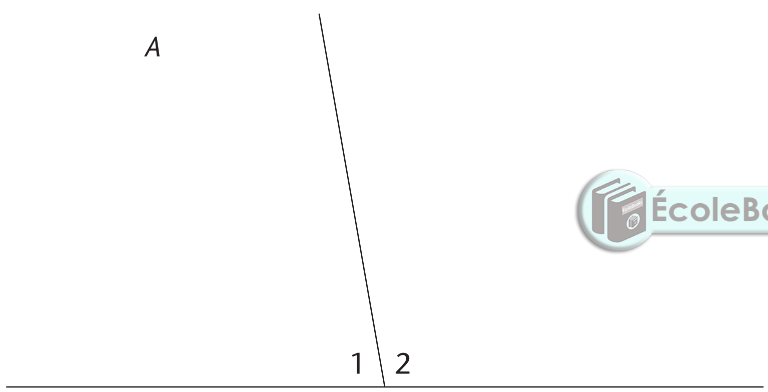
12.1 Angles on a straight line

SUM OF ANGLES ON A STRAIGHT LINE

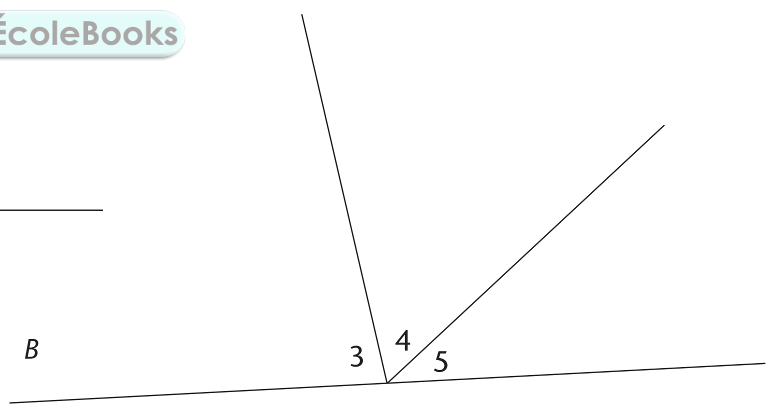
In the figures below, each angle is given a label from 1 to 5.

1. Use a protractor to measure the sizes of all the angles in each figure. Write down your answers.

A



B



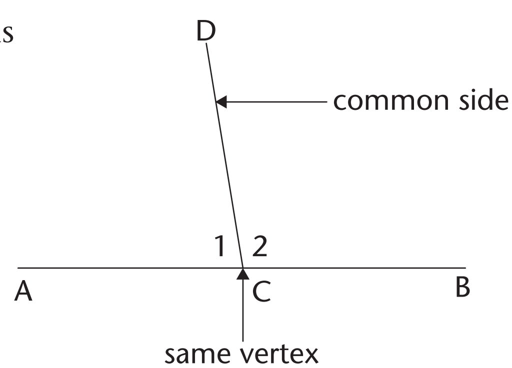
2. Use your answers to determine the following sums:

   (a) $\hat{1} + \hat{2}$ (b) $\hat{3} + \hat{4} + \hat{5}$

The sum of angles that are formed on a straight line is equal to 180°. (We can shorten this property as: ∠s on a straight line.)

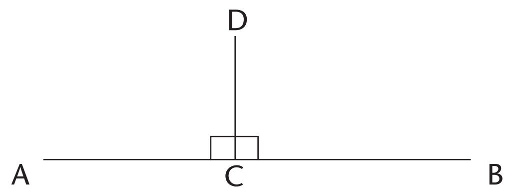
Two angles whose sizes add up to 180° are also called **supplementary** angles, for example $\hat{1} + \hat{2}$.

Angles that share a vertex and a common side are said to be **adjacent**. So $\hat{1} + \hat{2}$ are therefore also called **supplementary adjacent angles**.



When two lines are perpendicular, their adjacent supplementary angles are each equal to 90°.

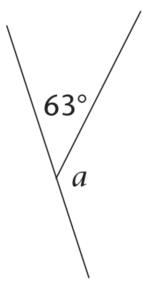
In the drawing alongside, $D\hat{C}A$ and $D\hat{C}B$ are adjacent supplementary angles because they are next to each other (adjacent) and they add up to 180° (supplementary).



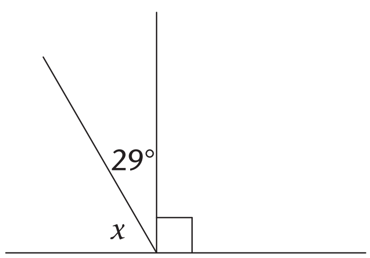
FINDING UNKNOWN ANGLES ON STRAIGHT LINES

Work out the sizes of the unknown angles below. Build an equation each time as you solve these geometric problems. Always give a reason for every statement you make.

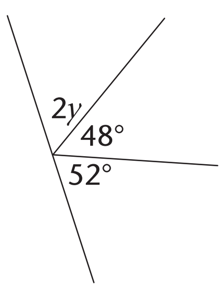
1. Calculate the size of *a*.



2. Calculate the size of *x*.

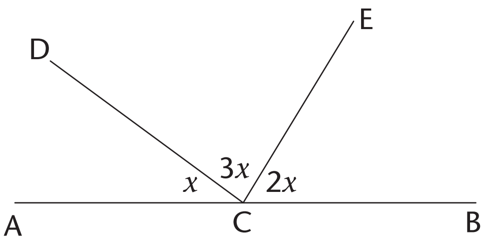


3. Calculate the size of *y*.

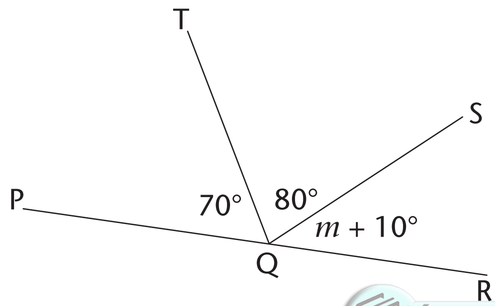


FINDING MORE UNKNOWN ANGLES ON STRAIGHT LINES

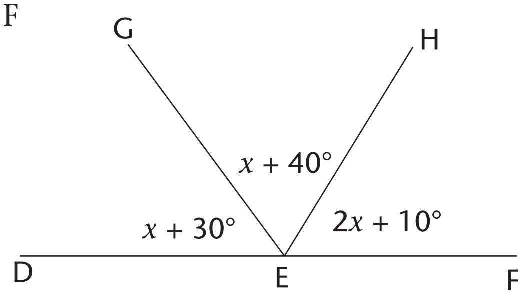
1. Calculate the size of:
   (a) x
   (b) $E\hat{C}B$



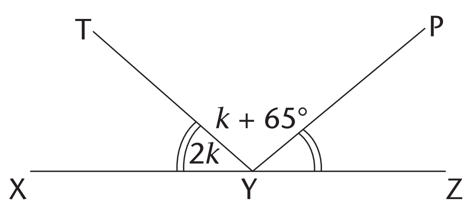
2. Calculate the size of:
   (a) m
   (b) $S\hat{Q}R$



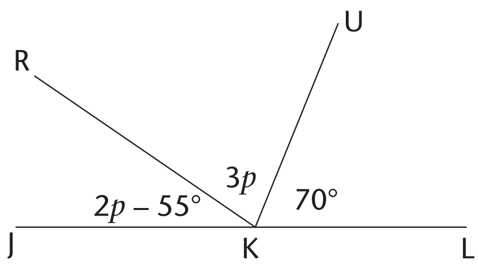
3. Calculate the size of:
   (a) x
   (b) $H\hat{E}F$



4. Calculate the size of:
   (a) k
   (b) $T\hat{Y}P$



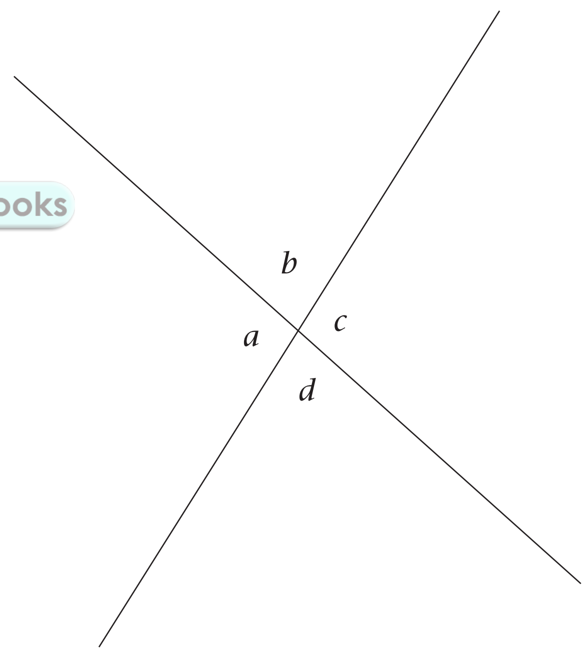
5. Calculate the size of:
   (a) p
   (b) $J\hat{K}R$



12.2 Vertically opposite angles

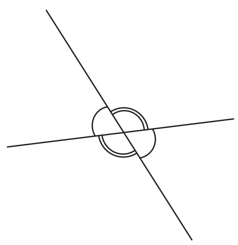
WHAT ARE VERTICALLY OPPOSITE ANGLES?

1. Use a protractor to measure the sizes of all the angles in the figure. Write down your answers.



2. Notice which angles are equal and how these equal angles are formed.

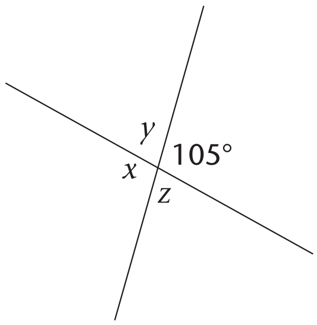
Vertically opposite angles (**vert. opp. ∠s**) are the angles opposite each other when two lines intersect.
Vertically opposite angles are **always equal**.



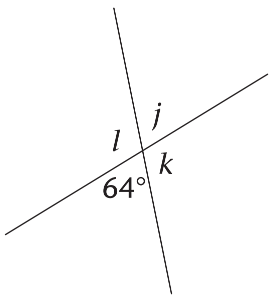
FINDING UNKNOWN ANGLES

Calculate the sizes of the unknown angles in the following figures. Always give a reason for every statement you make.

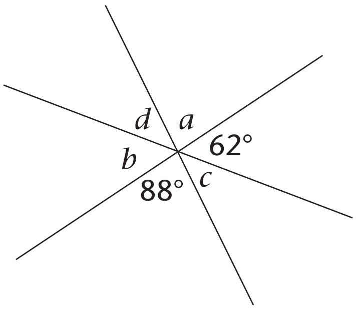
1. Calculate x, y and z.



2. Calculate j, k and l.



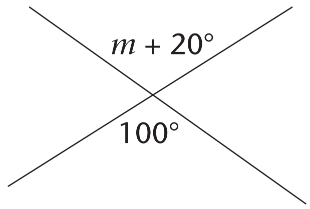
3. Calculate a, b, c and d.



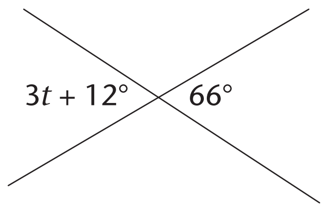
EQUATIONS USING VERTICALLY OPPOSITE ANGLES

Vertically opposite angles are always equal. We can use this property to build an equation. Then we solve the equation to find the value of the unknown variable.

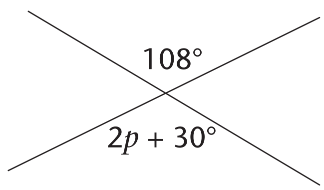
1. Calculate the value of m.



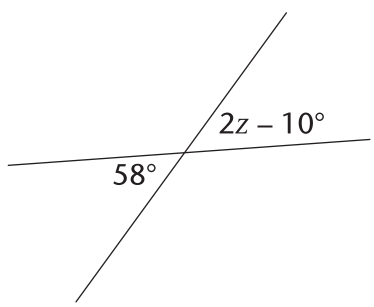
2. Calculate the value of t.



3. Calculate the value of p.



4. Calculate the value of z.



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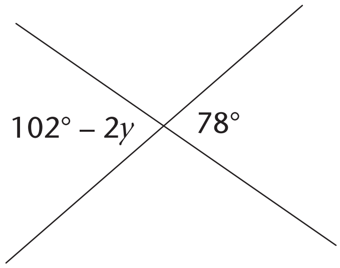
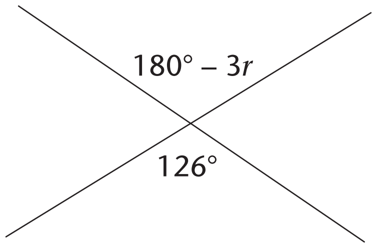
5. Calculate the value of y.



6. Calculate the value of r.

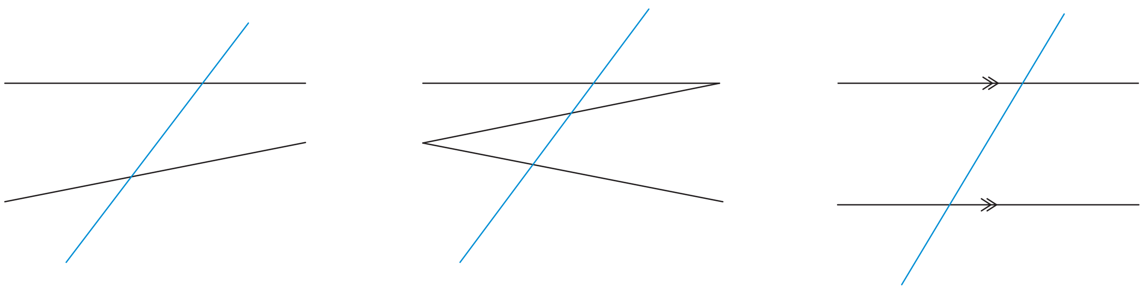


12.3 Lines intersected by a transversal

PAIRS OF ANGLES FORMED BY A TRANSVERSAL

A **transversal** is a line that crosses at least two other lines.

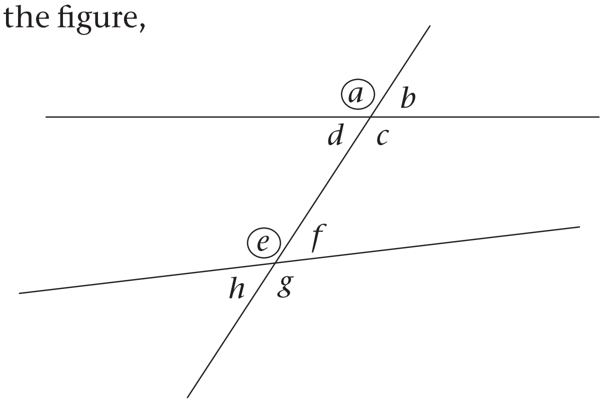
The blue line is the transversal.



When a transversal intersects two lines, we can compare the sets of angles on the two lines by looking at their positions.

The angles that lie on the same side of the transversal and are in matching positions are called **corresponding angles** (**corr. ∠s**). In the figure, these are corresponding angles:

- *a* and *e*
- *b* and *f*
- *d* and *h*
- *c* and *g*.



1. In the figure, *a* and *e* are both left of the transversal and above a line.

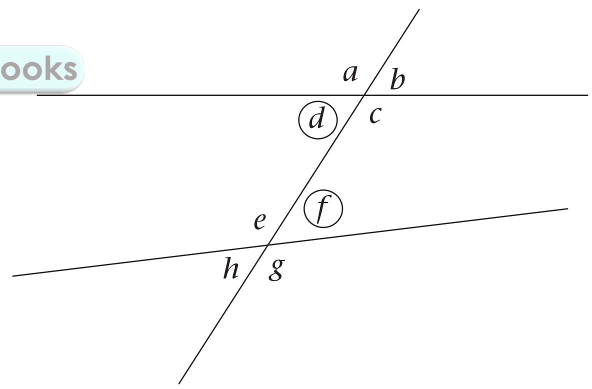
   Write down the location of the following corresponding angles. The first one has been done for you.

   b and *f*: Right of the transversal and above the lines.

   d and *h*   *c* and *g*

Alternate angles (**alt. ∠s**) lie on opposite sides of the transversal, but are not adjacent or vertically opposite. When the alternate angles lie between the two lines, they are called **alternate interior angles**. In the figure, these are alternate interior angles:

- *d* and *f*
- *c* and *e*.



When the alternate angles lie outside of the two lines, they are called **alternate exterior angles**. In the figure, these are alternate exterior angles:

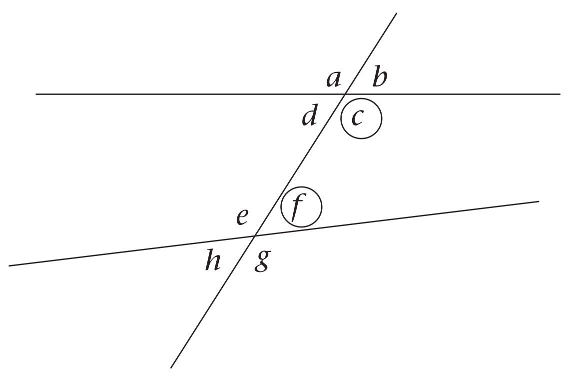
- *a* and *g*
- *b* and *h*.

2. Write down the location of the following alternate angles:

   d and *f*   *c* and *e*   *a* and *g*   *b* and *h*

Co-interior angles (**co-int. ∠s**) lie on the same side of the transversal and between the two lines. In the figure, these are co-interior angles:

- *c* and *f*
- *d* and *e*.

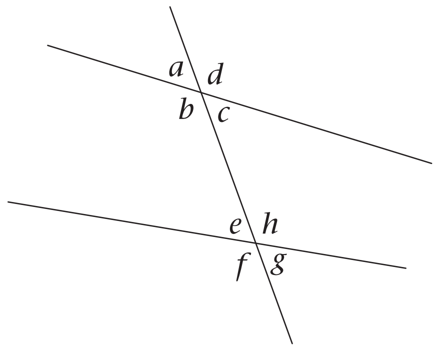


3. Write down the location of the following co-interior angles:

   d and *e*   *c* and *f*

IDENTIFYING TYPES OF ANGLES

Two lines are intersected by a transversal, as shown below.



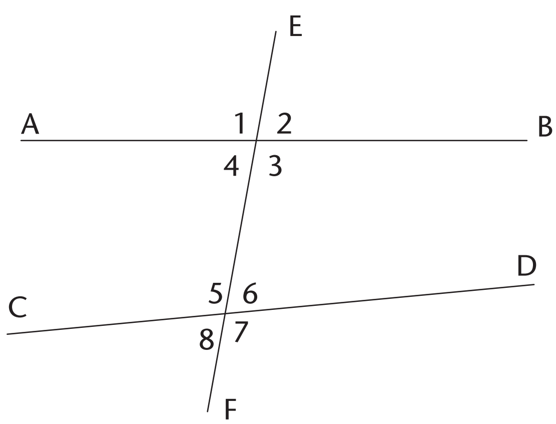
Write down the following pairs of angles:

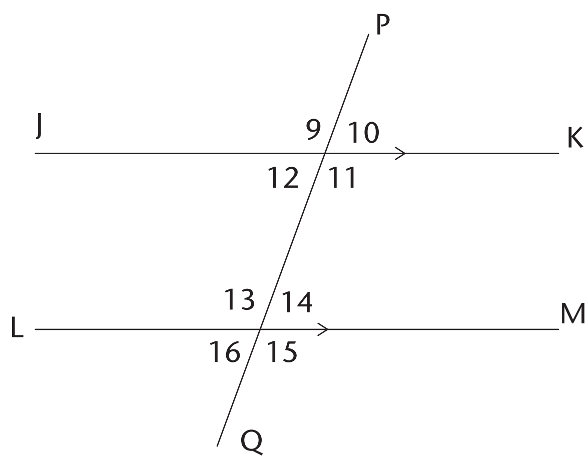
1. two pairs of corresponding angles
2. two pairs of alternate interior angles
3. two pairs of alternate exterior angles
4. two pairs of co-interior angles
5. two pairs of vertically opposite angles

12.4 Parallel lines intersected by a transversal

INVESTIGATING ANGLE SIZES

In the figure below on the left-hand side, EF is a transversal to AB and CD. In the figure below on the right-hand side, PQ is a transversal to parallel lines JK and LM.





1. Use a protractor to measure the sizes of all the angles in each figure. Write down the measurements of each angle.

2. Copy the table and use your measurements to complete it.

Angles	**When two lines are not parallel**	**When two lines are parallel**
Corr. ∠s	$\hat{1}$ = ...; $\hat{5}$ = ... $\hat{4}$ = ...; $\hat{8}$ = ... $\hat{2}$ = ...; $\hat{6}$ = ... $\hat{3}$ = ...; $\hat{7}$ = ...	$\hat{9}$ = ...; $\widehat{13}$ = ... $\widehat{12}$ = ...; $\widehat{16}$ = ... $\widehat{10}$ = ...; $\widehat{14}$ = ... $\widehat{11}$ = ...; $\widehat{15}$ = ...
Alt. int. ∠s	$\hat{4}$ = ...; $\hat{6}$ = ... $\hat{3}$ = ...; $\hat{5}$ = ...	$\widehat{12}$ = ...; $\widehat{14}$ = ... $\widehat{11}$ = ...; $\widehat{13}$ = ...
Alt. ext. ∠s	$\hat{1}$ = ...; $\hat{7}$ = ... $\hat{2}$ = ...; $\hat{8}$ = ...	$\hat{9}$ = ...; $\widehat{15}$ = ... $\widehat{10}$ = ...; $\widehat{16}$ = ...
Co-int. ∠s	$\hat{4} + \hat{5}$ = ... $\hat{3} + \hat{6}$ = ...	$\widehat{12} + \widehat{13}$ = ... $\widehat{11} + \widehat{14}$ = ...

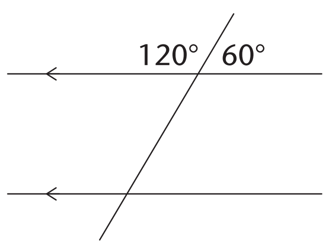
3. Look at your completed table in question 2. What do you notice about the angles formed when a transversal intersects parallel lines?

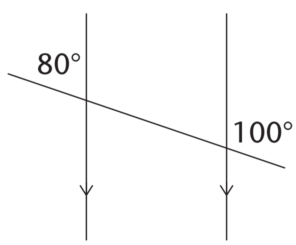
When lines are parallel:

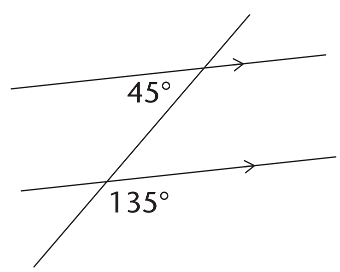
- corresponding angles are equal
- alternate interior angles are equal
- alternate exterior angles are equal
- co-interior angles add up to 180°.

IDENTIFYING ANGLES ON PARALLEL LINES

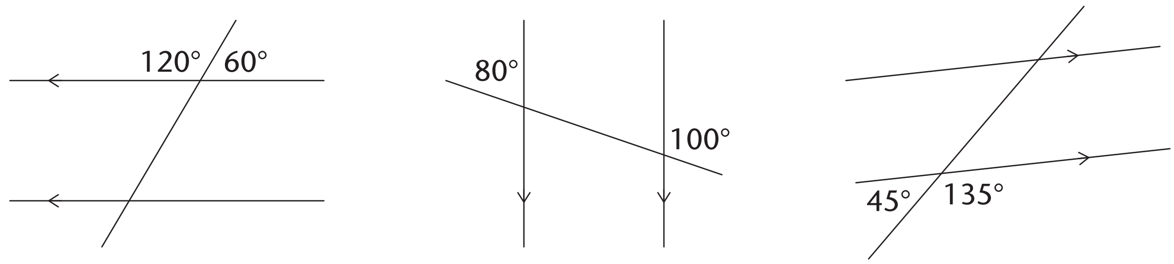
1. Copy these drawings and fill in the corresponding angles to those given:



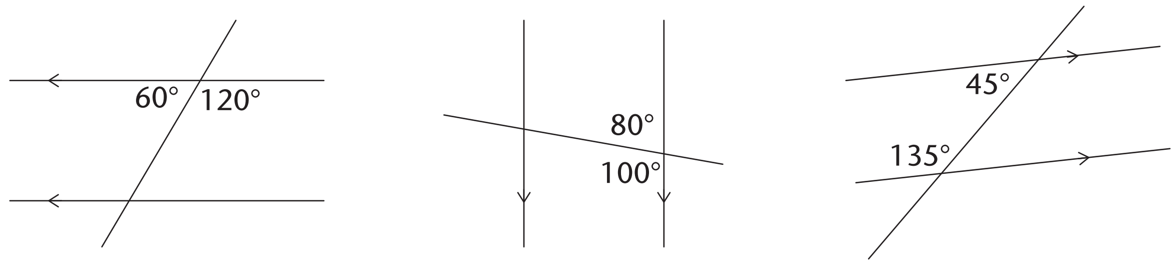




2. Copy the following drawings and fill in the alternate exterior angles:

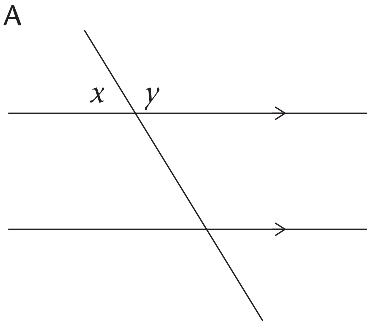


3. (a) Copy the drawings and fill in the alternate interior angles.
   (b) Circle the two pairs of co-interior angles in each figure.

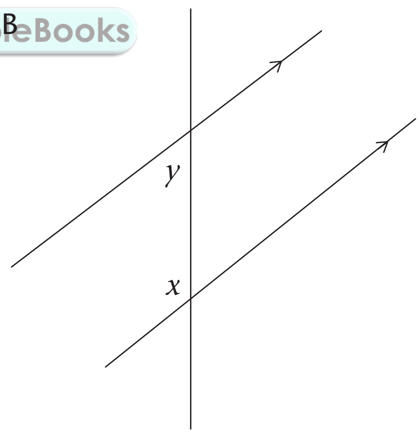


4. (a) Copy the drawings below. Without measuring, fill in all the angles in the following figures that are equal to x and y.
   (b) Explain your reasons for each x and y that you filled in to your partner.

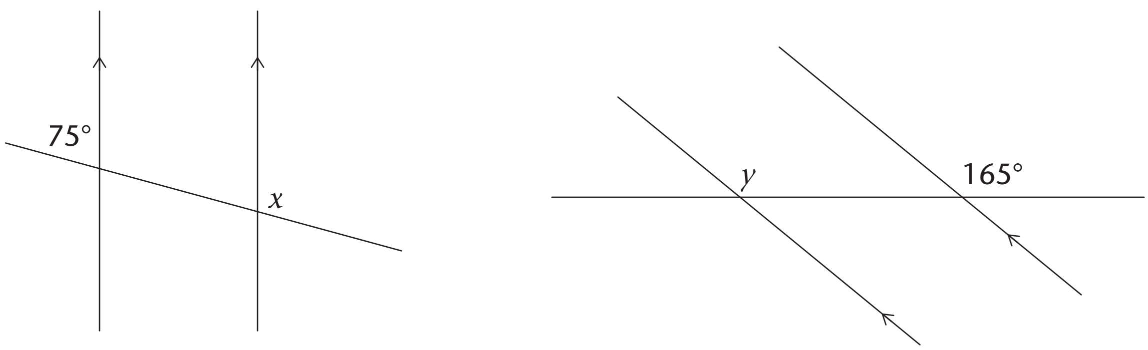
A



B



5. Give the value of x and y below:

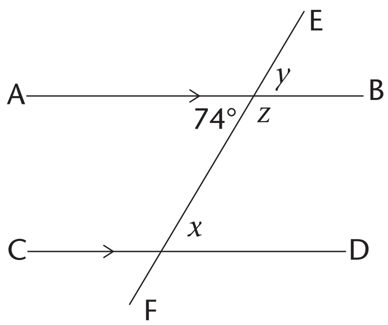


12.5 Finding unknown angles on parallel lines

WORKING OUT UNKNOWN ANGLES

Work out the sizes of the unknown angles. Give reasons for your answers. (The first one has been done as an example.)

1. Find the sizes of x, y and z.

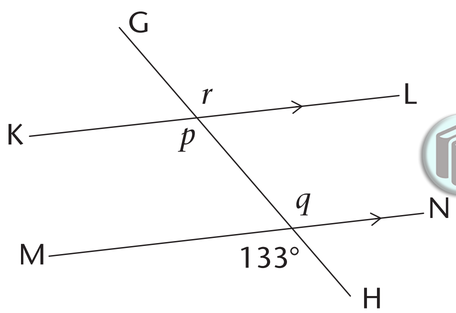


$x = 74°$ [alt. ∠ with given 74°; AB || CD]

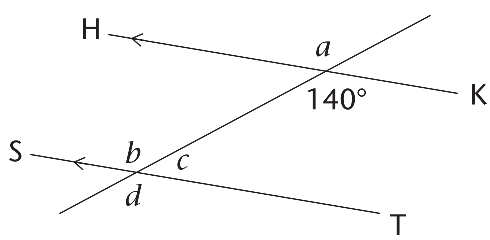
$y = 74°$ [corr. ∠ with x; AB || CD]
or $y = 74°$ [vert. opp. ∠ with given 74°]

$z = 106°$ [co-int. ∠ with x; AB || CD]
or $z = 106°$ [∠s on a straight line]

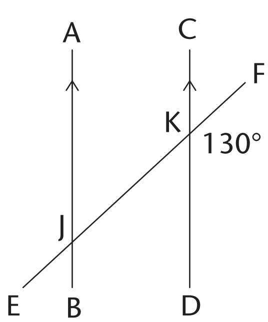
2. Work out the sizes of p, q and r.



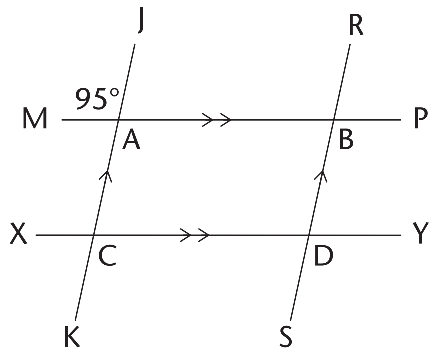
3. Find the sizes of a, b, c and d.



4. Find the sizes of all the angles in this figure.

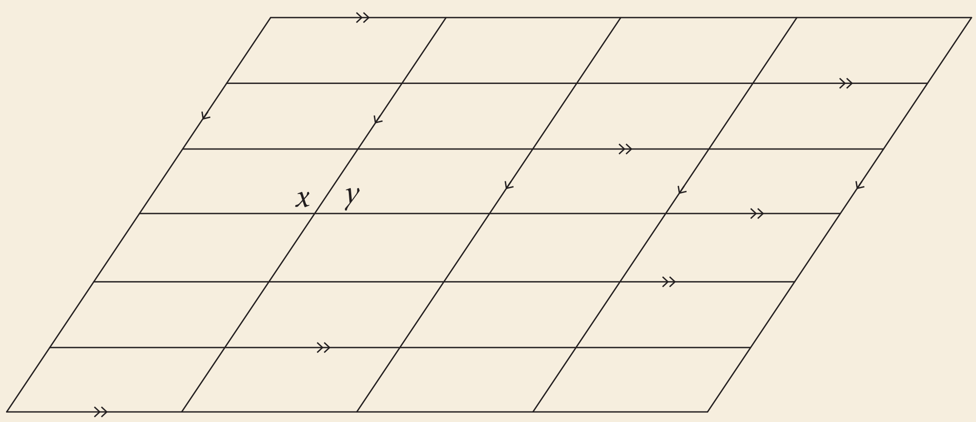


5. Find the sizes of all the angles. (Can you see two transversals and two sets of parallel lines?)



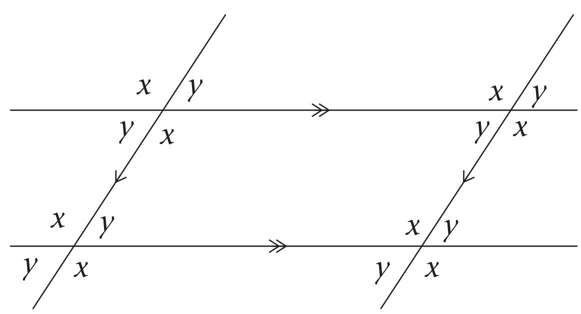
EXTENSION

Two angles in the following diagram are given as x and y. Copy the diagram and fill in all the angles that are equal to x and y.



SUM OF THE ANGLES IN A QUADRILATERAL

The diagram on the right is a section of the previous diagram.



1. What kind of quadrilateral is in the diagram? Give a reason for your answer.
2. Look at the top left intersection. Complete the following equation:

   Angles around a point = 360°

   $\therefore \quad x + y + \ldots\ldots + \ldots\ldots = 360°$

3. Look at the interior angles of the quadrilateral on page 136. Copy and complete the following equations:

Sum of angles in the quadrilateral = $x + y + \ldots\ldots + \ldots\ldots$

From question 2: $x + y + \ldots\ldots + \ldots\ldots = 360°$

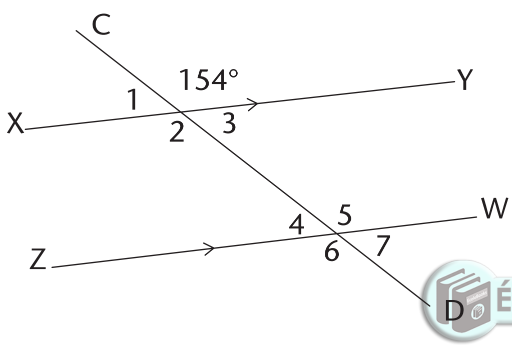
$\therefore$ Sum of angles in a quadrilateral = $\ldots\ldots°$

Can you think of another way to use the diagram on page 136 to work out the sum of the angles in a quadrilateral?

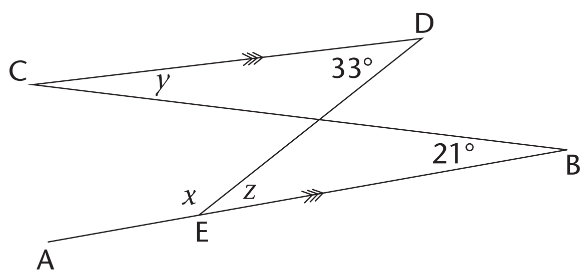
12.6 Solving more geometric problems

ANGLE RELATIONSHIPS ON PARALLEL LINES

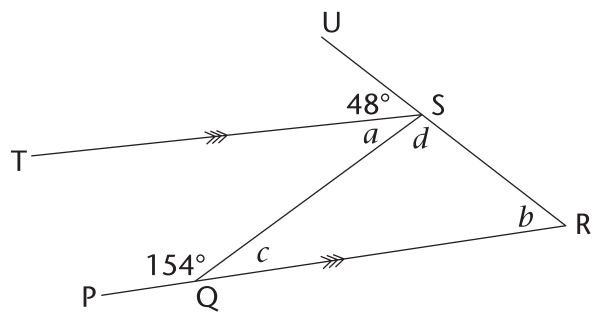
1. Calculate the sizes of $\hat{1}$ to $\hat{7}$.



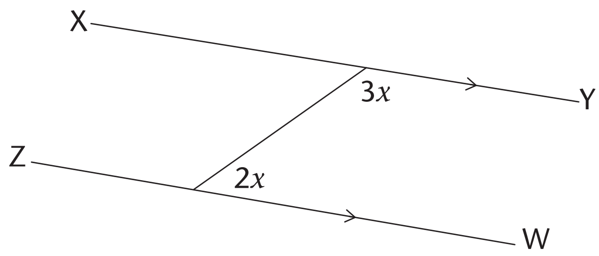
2. Calculate the sizes of x, y and z.



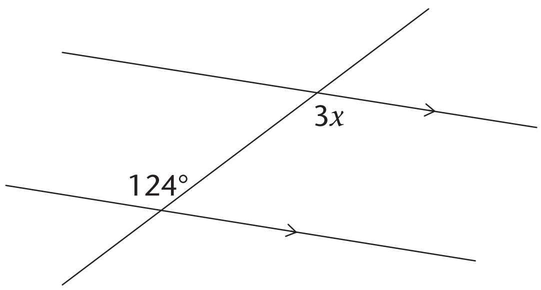
3. Calculate the sizes of a, b, c and d.



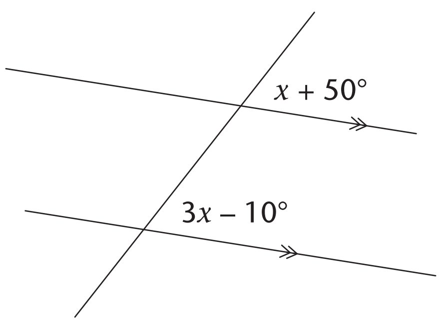
4. Calculate the size of x.



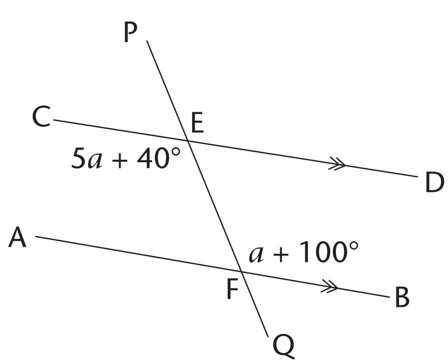
5. Calculate the size of x.



6. Calculate the size of x.

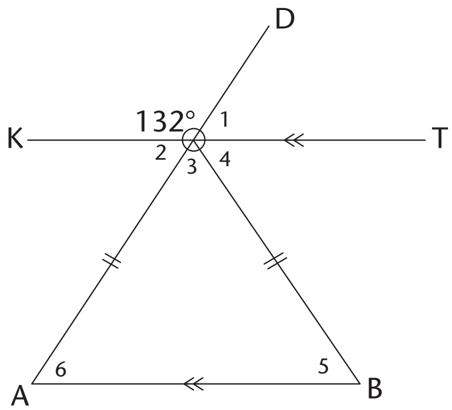


7. Calculate the sizes of a and $C\hat{E}P$.

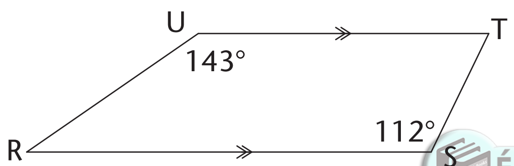


INCLUDING PROPERTIES OF TRIANGLES AND QUADRILATERALS

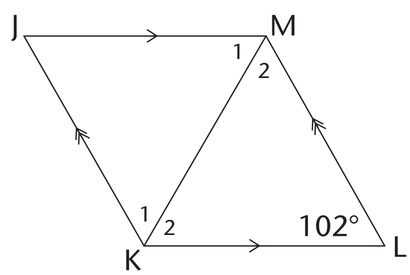
1. Calculate the sizes of $\hat{1}$ to $\hat{6}$.



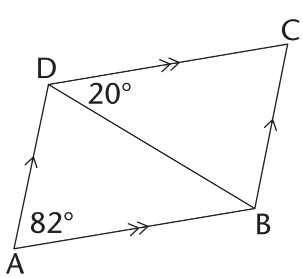
2. RSTU is a trapezium. Calculate the sizes of $\hat{T}$ and $\hat{R}$.



3. JKLM is a rhombus. Calculate the sizes of $J\widehat{M}L$, $\widehat{M}_2$ and $\hat{K}_1$.

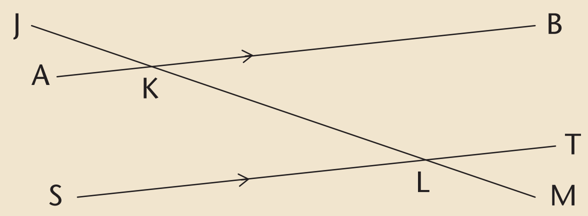


4. ABCD is a parallelogram. Calculate the sizes of $A\hat{D}B$, $A\hat{B}D$, $\hat{C}$ and $D\hat{B}C$.



WORKSHEET

1. Look at the drawing below. Name the items listed alongside it.



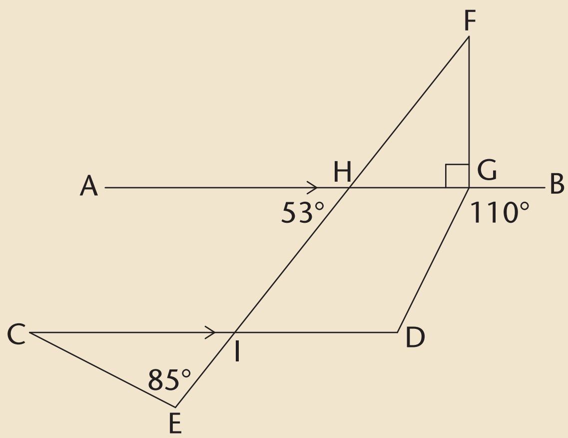
(a) a pair of vertically opposite angles

(b) a pair of corresponding angles

(c) a pair of alternate interior angles

(d) a pair of co-interior angles

2. In the diagram, AB || CD. Calculate the sizes of $F\hat{H}G$, $\hat{F}$, $\hat{C}$ and $\hat{D}$. Give reasons for your answers.



3. In the diagram, OK = ON, KN || LM, KL || MN and $L\hat{K}O = 160°$.

Calculate the value of x. Give reasons for your answers.