MATHEMATICS

Grade 9 - Term 1

CAPS

Learner Book

Revised edition

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CHAPTER 1
Whole numbers

1.1 Properties of numbers

DIFFERENT TYPES OF NUMBERS

The natural numbers

The numbers that we use to count are called **natural numbers**:

1 2 3 4 5 6 7 8 9 10 11 12 13 14

Natural numbers have the following properties:
When you add two or more natural numbers, you get a natural number again.
When you multiply two or more natural numbers, you get a natural number again.

Mathematicians describe this by saying: The system of natural numbers is **closed under addition and multiplication**.

However, when a natural number is *subtracted* from another natural number, the answer is not always a natural number again. For example, there is no natural number that provides the answer to 5 – 20.

Similarly, when a natural number is *divided* by another natural number, the answer is not always a natural number again. For example, there is no natural number that provides the answer to 10 ÷ 3.

The system of natural numbers is **not closed under subtraction or division**.

When subtraction or division is done with natural numbers, the answers are not always natural numbers.

1. (a) Is there a smallest natural number, in other words, a natural number that is smaller than all other natural numbers? If so, what is it?
   (b) Is there a largest natural number, in other words, a natural number that is larger than all other natural numbers? If so, what is it?

2. In each of the following cases, say whether the answer is a natural number or not:
   (a) 100 + 400 (b) 100 – 400
   (c) 100 × 400 (d) 100 ÷ 400

The whole numbers

Although we do not use 0 for counting, we need it to write numbers. Without 0, we would need a special symbol for 10, all multiples of 10 and some other numbers. For example, all the numbers that belong in the yellow cells below would need a special symbol.

	41	42	43	44	45	46	47	48	49
	51	52	53	54	55	56	57	58	59
	61	62	63	64	65	66	67	68	69
	71	72	73	74	75	76	77	78	79
	81	82	83	84	85	86	87	88	89
	91	92	93	94	95	96	97	98	99
	111	112	113	114	115	116	117	118	119

The natural numbers combined with 0 is called the system of **whole numbers**.

If you are working with natural numbers and you add two numbers, the answer will always be different from any of the two numbers added. For example: 21 + 25 = 46 and 24 + 1 = 25. If you are working with whole numbers, in other words including 0, this is not the case. When 0 is added to a number the answer is just the number you start with: 24 + 0 = 24.

For this reason, 0 is called the **identity element** for addition. In the set of natural numbers there is no identity element for addition.

3. Is there an identity element for multiplication in the whole numbers? Explain your answer.
4. (a) What is the smallest natural number?
   (b) What is the smallest whole number?

The integers

In the set of whole numbers, no answer is available when you subtract a number from a number smaller than itself. For example, there is no whole number that is the answer for 5 – 8. But there is an answer to this subtraction in the system of integers.

For example: 5 – 8 = –3. The number –3 is read as "negative 3" or "minus 3".

Whole numbers start with 0 and extend in one direction:

0 1 2 3 4 5 6 → → → ......

Integers extend in both directions:

...... ← ← ← –5 –4 –3 –2 –1 0 1 2 3 4 5 6 → → → ......

All whole numbers are also **integers**. The set of whole numbers forms part of the set of integers. For each whole number, there is a negative number that corresponds with it. The negative number –5 corresponds to the whole number 5 and the negative number –120 corresponds to the whole number 120.

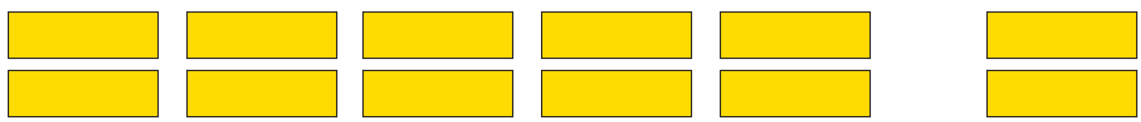
Within the set of integers, the sum of two numbers can be 0.

For example 20 + (–20) = 0 and 135 + (–135) = 0.

20 and –20 are called **additive inverses** of each other.

5. Calculate the following without using a calculator:
   (a) 100 – 165 (b) 300 – 700
6. You may use a calculator to calculate the following:
   (a) 123 – 765 (b) 385 – 723

The rational numbers



7. Five people share 12 slabs of chocolate equally among them.
   (a) Will each person get more or less than two full slabs of chocolate?
   (b) Can each person get another half of a slab?
   (c) How much more than two full slabs can each person get, if the two remaining slabs are divided as shown here?

   (d) Will each person get 2,4 or $2\frac{2}{5}$ slab?

The system of integers does not provide an answer for all possible division questions. For example, as we see above, the answer for 12 ÷ 5 is not an integer.

To have answers for all possible division questions, we have to extend the number system to include fractions and negative fractions, in other words, numbers of the form $\frac{\text{integer}}{\text{integer}}$. This system of numbers is called **rational numbers**. We can represent rational numbers as common fractions or as decimal numbers.

8. Express the answers for each of the following division problems in two ways. Firstly, using the common fraction notation and secondly, using the decimal notation for fractions.
   (a) 23 ÷ 10 (b) 23 ÷ 5
   (c) 230 ÷ 100 (d) 8 ÷ 10

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9. Copy the table and answer the statement by writing "yes" or "no" in the appropriate cell.

Statement	Natural numbers	Whole numbers	Integers	Rational numbers
The sum of two numbers is a number of the same kind (closed under addition).				
The sum of two numbers is always bigger than either of the two numbers.				
When one number is subtracted from another, the answer is a number of the same kind (closed under subtraction).				
When one number is subtracted from another, the answer is always smaller than the first number.				
The product of two numbers is a number of the same kind (closed under addition).				
The product of two numbers is always bigger than either of the two numbers.				
The quotient of two numbers is a number of the same kind (closed under division).				
The quotient of two numbers is always smaller than the first of the two numbers.				

Irrational numbers

Rational numbers do not provide for all situations that may occur in Mathematics. For example, there is no rational number which will produce the answer 2 when it is multiplied by itself.

(number) × (same number) = 2

$2 \times 2 = 4$ and $1 \times 1 = 1$, so clearly, this number must be between 1 and 2.

But there is no number which can be expressed as a fraction, in either the common fraction or the decimal notation, which will solve this problem. Numbers like these are called **irrational numbers**.

Here are some more examples of irrational numbers:

$\sqrt{5}$ $\sqrt{10}$ $\sqrt{3}$ $\sqrt{7}$ π

Rational and irrational numbers together, are called **real numbers**.

1.2 Calculations with whole numbers

Do **not** use a calculator in Section 1.2, unless told to do so.

ESTIMATING, ROUNDING OFF AND COMPENSATING

1. A shop owner wants to buy chickens from a farmer. The farmer wants R38 for each chicken. Answer the following questions without doing written calculations:
   (a) If the shop owner has R10 000 to buy chickens, do you think he can buy more than 500 chickens?
   (b) Do you think he can buy more than 200 chickens?
   (c) Do you think he can buy more than 250 chickens?

What you were trying to do in question 1 is called **estimation**. To estimate, when working with numbers, means to try to get close to an answer without actually doing the calculations. However, you can do other, simpler calculations to estimate.

When the goal is not to get an accurate answer, numbers may be rounded off. For example, the cost of 51 chickens at R38 each may be **approximated** by calculating 50 × 40. This is clearly much easier than calculating 51 × R38.

To approximate something means to try find out more or less how much it is, without measuring or calculating it precisely.

2. (a) How much is 5 × 4?
   (b) How much is 5 × 40?
   (c) How much is 50 × 40?

The cost of 51 chickens at R38 each is therefore, approximately R2 000.

This approximation was obtained by rounding both 51 and 38 off to the nearest multiple of 10, and then calculating with the multiples of 10.

3. In each case, estimate the cost by rounding off to calculate the approximate cost, without using a calculator. In each case, make two estimates. First make a rough estimate by rounding the numbers off to the nearest 100 before calculating. Then make a better estimate by rounding the numbers off to the nearest 10 before calculating.
   (a) 83 goats are sold for R243 each
   (b) 121 chairs are sold for R258 each
   (c) R5 673 is added to R3 277
   (d) R874 is subtracted from R1 234

Suppose you have to calculate R823 – R273.
An estimate can be made by rounding the numbers off to the nearest 100:
R800 – R300 = R500.

4. (a) By working with R800 instead of R823, an error was introduced into your answer. How can this error be corrected: by adding R23 to the R500, or by subtracting it from R500?

(b) Correct the error to get a better estimate.
(c) Now also correct the error that was made by subtracting R300 instead of R273.

What you did in question 4 is called **compensating for errors**.

5. Estimate each of the following by rounding off the numbers to the nearest 100:

(a) 812 – 342	(b) 2 342 – 1 876
(c) 812 + 342	(d) 2 342 + 1 876
(e) 9 + 278	(f) 3 231 – 1 769
(g) 8 234 – 2 776	(h) 5 213 – 3 768

6. Find the exact answer for each of the calculations in question 5, by working out the errors caused by rounding, and compensating for them.

ADDING IN COLUMNS

1. (a) Write 8 000 + 1 100 + 130 + 14 as a single number.
   (b) Write 3 000 + 700 + 50 + 8 as a single number.
   (c) Write 5 486 in expanded notation, as shown in question 1(b).

You can calculate 3 758 + 5 486 as shown on the left below.

	3 758
	5 486
Step 1	8 000
Step 2	1 100
Step 3	130
Step 4	14
	9 244

You can do this in short, as shown on the right. This is a bit harder on the brain, but it saves paper!

3 758
5 486
9 244

2. Explain how the numbers in each of Steps 1 to 4 are obtained.

It is only possible to use the shorter method if you add the units first, then add the tens, then the hundreds and finally, the thousands. You can then do what you did in question 1(a), without writing the separate terms of the expanded form.

3. Calculate each of the following:

(a) 3 878 + 3 784	(b) 298 + 8 594
(c) 10 921 + 2 472	(d) 1 298 + 18 782

4. A farmer buys a truck for R645 840, a tractor for R783 356, a plough for R83 999 and a bakkie for R435 690.

(a) Estimate to the nearest R100 000 how much these items will cost altogether.
(b) Use a calculator to calculate the total cost.

5. An investor makes R543 682 in one day on the stock market and then loses R264 359 on the same day.
   (a) Estimate to the nearest R100 000 how much money she has made in total on that day.
   (b) Use a calculator to determine how much money she has made.

MULTIPLYING IN COLUMNS

1. (a) Write 3 489 in expanded notation.
   (b) Write an expression without brackets that is equivalent to
   $7 \times (3\,000 + 400 + 80 + 9)$.

$7 \times 3\,489$ may be calculated as shown on the left below.

	3 489
	× 7
Step 1	63
Step 2	560
Step 3	2 800
Step 4	21 000
	24 423

A shorter method is shown on the right.

3 489
× 7
24 423

2. Explain how the numbers in each of Steps 1 to 4 on the above left are obtained.

$47 \times 3\,489$ may be calculated as shown on the left below.

	3 489
	× 47
Step 1	63
Step 2	560
Step 3	2 800
Step 4	21 000
Step 5	360
Step 6	3 200
Step 7	16 000
Step 8	120 000
	163 983

A shorter method is shown on the right.

3 489
× 47
24 423
139 560
163 983

3. Explain how the numbers in each of Steps 5 to 8 on the left on page 7 are obtained.
4. Explain how the number 139 560 that appears in the shorter form on the right on page 7 is obtained.

SUBTRACTING IN COLUMNS

1. Write each of the following as a single number:
   (a) 8 000 + 400 + 30 + 2
   (b) 7 000 + 1 300 + 120 + 12
   (c) 3 000 + 900 + 50 + 7

2. If you worked correctly you should have obtained the same answers for questions 1(a) and 1(b). If this was not the case, redo your work.

The expression 7 000 + 1 300 + 120 + 12 was formed from 8 000 + 400 + 30 + 2 by:

- taking 1 000 away from 8 000 and adding it to the hundreds term to get 1 400
- taking 100 away from 1 400 and adding it to the tens term to get 130
- taking 10 away from 130 and adding it to the units term to get 12.

3. Form an expression like the expression in question 1(b) for each of the following:
   (a) 8 000 + 200 + 100 + 4 (b) 3 000 + 400 + 30 + 1

4. Write expressions like in question 1(b) for the following numbers:
   (a) 7 214 (b) 8 103

8 432 – 3 957 can be calculated as shown below:

	8 432
	– 3 957
Step 1	5
Step 2	70
Step 3	400
Step 4	4 000
Step 5	4 475

To do the subtraction in each column, you need to think of 8 432 as 8 000 + 400 + 30 + 2; in fact, you have to think of it as 7 000 + 1 300 + 120 + 12.

In Step 1, the 7 of 3 957 is subtracted from 12.

5. (a) How is the 70 in Step 2 obtained?
   (b) How is the 400 in Step 3 obtained?
   (c) How is the 4 000 in Step 4 obtained?
   (d) How is the 4 475 in Step 5 obtained?

Because of the zeros obtained in Steps 2, 3 and 4, the answers need not be written separately as shown above. The work can actually be shown in the short way on the right.

8 432
– 3 957
4 475

6. Calculate each of the following:
   (a) 9 123 – 3 784 (b) 8 284 – 3 547

7. Use a calculator **only** to check your answers. If your answers are wrong, try again.

8. Calculate each of the following:
   (a) 7 243 – 3 182 (b) 6 221 – 1 888

You may use a calculator to do the questions below.

9. Bettina has R87 456 in her savings account. She withdraws R44 800 to buy a car. How much money is left in her savings account?

10. Liesbet starts a savings account by making a deposit of R40 000. Over a period of time she does the following transactions on the savings account:
    - a withdrawal of R4 000
    - a withdrawal of R2 780
    - a deposit of R1 200
    - a deposit of R7 550
    - a withdrawal of R5 230
    - a deposit of R8 990
    - a deposit of R1 234

    How much money does she have in her savings account now?

11. (a) R34 537 – R13 267 (b) R135 349 – R78 239

LONG DIVISION

Study this method for calculating 13 254 ÷ 56:

	13 254	
200 × 56 = 11 200	11 200	(200 is a rough estimate of the answer for 13 254 ÷ 56)
	2 054	(2 054 remains after 11 200 is taken from 13 254)
30 × 56 = 1 680	1 680	(30 is a rough estimate of the answer for 2 054 ÷ 56)
	374	(374 remains after 1 680 is taken from 2 054)
6 × 56 = 336	336	(6 is an estimate of the answer for 374 ÷ 56)
236 × 56 = 13 216	38	(38 remains)

So, 13 254 ÷ 56 = 236 remainder 38, or $13\ 254 \div 56 = 236\frac{38}{56} = 236\frac{19}{28}$, which can also be written as 236,68 (correct to two decimal figures).

The work can also be set out as follows:

```
          6
         30
        200
56 | 13 254        or more briefly as        236
     11 200                           56 | 13 254
      2 054                                11 200
      1 680                                 2 054
        374                                 1 680
        336                                   374
         38                                   336
                                               38
```

1. (a) Mlungisi's work to do a certain calculation is shown on the right. What is the question that Mlungisi tries to answer?
   (b) Where does the number 31 200 in Step 1 come from? How did Mlungisi obtain it, and for what purpose did he calculate it?
   (c) Explain Step 2 in the same way as you explained Step 1.
   (d) Explain Step 3.

```
                 463
          78 | 36 177
Step 1         31 200
Step 2          4 977
Step 3          4 680
Step 4            297
Step 5            234
                   63
```


2. Calculate each of the following without using a calculator:
   (a) 33 030 ÷ 63 (b) 18 450 ÷ 27

3. Use a calculator to check your answers to question 2. If your answers are wrong, try again. It is important that you learn to do long division correctly.

4. Calculate each of the following:
   (a) 76 287 ÷ 287 (b) 65 309 ÷ 44

Use your calculator to do questions 5 and 6 below.

5. A municipality has budgeted R85 000 for putting up new street name boards. The street name boards cost R72 each. How many new street name boards can be put up, and how much money will be left in the budget?

6. A furniture dealer quoted R840 000 for supplying 3 450 school desks. A school supply company quoted R760 000 for supplying 2 250 of the same desks. Which provider is cheapest, and what do the two providers actually charge for one school desk?

1.3 Multiples and factors

LOWEST COMMON MULTIPLES AND PRIME FACTORISATION

1. Consecutive multiples of 6, starting at 6 itself, are shown in the following table:

6	12	18	24	30	36	42	48	54	60
66	72	78	84	90	96	102	108	114	120
126	132	138	144	150	156	162	168	174	180
186	192	198	204	210	216	222	228	234	240

(a) The following table also shows multiples of a number. What is the number?

15	30	45	60	75	90	105	120	135	150
165	180	195	210	225	240	255	270	285	300
315	330	345	360	375	390	405	420	435	450
465	480	495	510	525	540	555	570	585	600

(b) Copy both tables. Draw rough circles around all the numbers that occur in both tables.

(c) What is the smallest number that occurs in both tables?

90 is a multiple of 6; it is also a multiple of 15.

90 is called a **common multiple** of 6 and 15; it is a multiple of both.

The smallest number that is a multiple of both 6 and 15 is the number 30.

30 is called the **lowest common multiple** or **LCM** of 6 and 15.

2. Calculate, without using a calculator:

(a) 2 × 3 × 5 × 7 × 11
(b) 2 × 2 × 5 × 7 × 13
(c) 2 × 3 × 3 × 3 × 5 × 13
(d) 3 × 5 × 5 ×17

Check your answers by using a calculator or by comparing with some classmates.

The number 2 is a factor of each of the numbers 2 310, 1 820 and 3 510.

Another way of saying this is: 2 is a **common factor** of 2 310, 1 820 and 3 510.

3. (a) Is 2 × 3, in other words, 6, a common factor of 2 310 and 3 510?
(b) Is 2 × 3 × 5, in other words, 30, a common factor of 2 310 and 3 510?
(c) Is there any bigger number than 30 that is a common factor of 2 310 and 3 510?

30 is called the **highest common factor** or **HCF** of 2 310 and 3 510.

In question 2 you can see the list of **prime factors** of the numbers 2 310, 1 820, 3 510 and 1 275.

> The LCM of two numbers can be found by multiplying all the prime factors of both numbers, without repeating (except where a number is repeated as a factor in one of the numbers).
>
> The HCF of two numbers can be found by multiplying the factors that are common to the two numbers, i.e. in the list of prime factors of both numbers.

4. In each case, find the HCF and LCM of the numbers:

   (a) 1 820 and 3 510
   (b) 2 310 and 1 275
   (c) 1 820 and 3 510 and 1 275
   (d) 2 310 and 1 275 and 1 820
   (e) 780 and 7 700
   (f) 360 and 1 360

1.4 Solving problems about ratio, rate and proportion

RATIO AND RATE PROBLEMS

You **may** use a calculator in this section.

1. Moeneba collects apples in the orchard. She picks about five apples each minute. Approximately how many apples will Moeneba pick in each of the following periods of time?

   (a) eight minutes
   (b) 11 minutes
   (c) 15 minutes
   (d) 20 minutes

In the situation described in question 1, Moeneba picks apples **at a rate of** about five apples **per minute**.

2. Garth and Kate also collect apples in the orchard, but they both work faster than Moeneba. Garth collects at a rate of about 12 apples per minute, and Kate collects at a rate of about 15 apples per minute. Copy and complete the following table to show approximately how many apples they will each collect in different periods of time:

Period of time in min	1	2	3	8	10	20
Moeneba	5			40		
Garth	12					
Kate	15					
The three together	32					

In this situation, the number of apples picked is **directly proportional** to the time taken.

If you filled the table in correctly, you will notice that during any period of time, Kate collected three times as many apples as Moeneba. We can say that during any time interval, the **ratio** between the numbers of apples collected by Moeneba and Kate is **3 to 1**, which can be written as **3 : 1**. For any period of time, the ratio between the numbers of apples collected by Garth and Moeneba is 12 : 5.

3. (a) What is the ratio between the numbers of apples collected by Kate and Garth during a period of time?
   (b) Would it be correct to also say that the ratio between the numbers of apples collected by Kate and Garth is 5 : 4? Explain your answer.

4. To make biscuits of a certain kind, five parts of flour are to be mixed with two parts of oatmeal, and one part of cocoa powder. How much oatmeal and how much cocoa powder must be used if 500 g of flour is used?

5. A motorist covers a distance of 360 km in exactly four hours.
   (a) Approximately how far did the motorist drive in one hour?
   (b) Do you think the motorist covered exactly 90 km in each of the four hours? Explain your answer briefly.
   (c) Approximately how far will the motorist drive in seven hours?
   (d) Approximately how long will the motorist need to travel 900 km?

Some people use these formulae to do calculations like those in question 5:

$\textbf{average speed} = \dfrac{\textbf{distance}}{\textbf{time}}$, which means distance ÷ time

$\textbf{distance} = \textbf{average speed} \times \textbf{time}$

$\textbf{time} = \dfrac{\textbf{distance}}{\textbf{average speed}}$, which means distance ÷ average speed

6. For each of questions 5(c) and 5(d), state which formula will produce the correct answer.

7. A motorist completes a journey in three sections, making two long stops to eat and relax between sections. During section A he covers 440 km in four hours. During section B he covers 540 km in six hours. During section C he covers 280 km in four hours.
   (a) Calculate his average speed over each of the three sections.
   (b) Calculate his average speed for the journey as a whole.
   (c) On the next day, the motorist has to travel 874 km. How much time (stops excluded) will he need to do this? Justify your answer with calculations.

8. Different vehicles travel at different average speeds. A large transport truck with a heavy load travels much slower than a passenger car. A small bakkie is also slower than a passenger car. In the table on the following page, some average speeds and the

times needed are given for different vehicles that all have to be driven for the same distance of 720 km. Copy and complete the table:

Time in hours	12	9	8	6	5
Average speed in km/h	60				

9. Look at the table you have just completed.
   (a) What happens to the time needed if the average speed increases?
   (b) What happens to the average speed if the time is reduced?
   (c) What can you say about the product average speed × time, for the numbers in the table?

In the situation above, the average speed is said to be **indirectly proportional** to the time needed for the journey.

1.5 Solving problems in financial contexts

You **may** use a calculator in this section.

DISCOUNT, PROFIT AND LOSS

1. (a) R12 800 is divided equally between 100 people.
   How much money does each person get?
   (b) How much money do eight of the people together get?

Another word for hundredths is **per cent**.

Instead of $\frac{5}{100}$ we can write 5%. The symbol % means exactly the same as $\frac{\ }{100}$.
In question 1(a) you calculated $\frac{1}{100}$ or 1% of R12 800, and in question 1(b) you calculated $\frac{8}{100}$ or 8% of R12 800.

> The amount that a dealer pays for an article is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after the discount is the **selling price**.

2. The marked prices of some articles are given below. A discount of 15% is offered to customers who pay cash. In each case, calculate how much a customer who pays cash will actually pay:
   (a) R850
   (b) R140
   (c) R32 600
   (d) R138

Lina bought a couch at a sale. It was marked R3 500 but she paid only R2 800.

She was given a discount of R700.

What percentage discount was given to Lina?

This question means:

How many hundredths of the marked price were taken off?

To answer the question we need to know how much $\frac{1}{100}$ (one hundredth) of the marked price is.

3. (a) How much is $\frac{1}{100}$ of R3 500?
   (b) How many hundredths of R3 500 is the same as R700?
   (c) What percentage discount was given to Lisa: 10% or 20%?

4. The cost price, marked price and selling price of some articles are listed below:
   Article A: Cost price = R240; marked price = R360; selling price = R324.
   Article B: Cost price = R540; marked price = R700; selling price = R560.
   Article C: Cost price = R1 200; marked price = R2 000; selling price = R1 700.
   The profit is the difference between the cost price and the selling price.
   For each of the above articles, calculate the percentage discount and profit.

5. Remey decided to work from home and bought herself a sewing machine for R750. She planned to make 40 covers for scatter cushions. The fabric and other items she needed cost her R3 600. Remey planned to sell the covers at R150 each.
   (a) How much profit could Remey make if she sold all 40 covers at this price?
   (b) Remey managed to sell only 25 of the covers and decided to sell the rest at R100 each. Calculate her percentage profit.

6. Zadie bakes and sells pies to earn some extra income. The cost of the ingredients for one chicken pie comes to about R68. She sold the pies for R60 each. Did she make a profit or a loss? Calculate the percentage loss or profit.

HIRE PURCHASE

Sometimes you need an item but do not have enough money to pay the full amount immediately. One option is to buy the item on **hire purchase** (**HP**). You will have to pay a deposit and sign an agreement in which you undertake to pay monthly instalments until you have paid the full amount. Therefore:

HP price = deposit + total of instalments

The difference between the HP price and the cash price is the interest that the dealer charges you for allowing you to pay off the item over a period of time.

1. Sara buys a flat screen television on HP. The cash price is R4 199. She has to pay a deposit of R950 and 12 monthly instalments of R360.

(a) Calculate the total HP price.
(b) How much interest does she pay?

2. Susie buys a car on HP. The car costs R130 000. She pays a 10% deposit on the cash price and will have to pay monthly instalments of R4 600 for a period of three years. David buys the same car, but chooses another option where he has to pay a 35% deposit on the cash price and monthly instalments of R3 950 for two years.
   (a) Calculate the HP price for both options.
   (b) Calculate the difference between the total price paid by Susie and by David.
   (c) Calculate the interest that Susie and David have to pay as a percentage of the cash price.

SIMPLE INTEREST

When interest is calculated for a number of years on an amount (i.e. a fixed deposit), without the interest being added to the amount each year for the purpose of later interest calculations, it is referred to as **simple interest**. If the amount is invested for part of a year, the time must be written as a fraction of a year.

Example:

R2 000 invested at 10% per annum simple interest over 2 years:

End of first year: Amount = R2 000 + R200 interest of original amount = R2 200

End of second year: Amount = R2 200 + R200 interest of original amount = R2 400

1. Interest rates are normally expressed as percentages. This makes it easier to compare rates. Express each of the following as a percentage:
   (a) A rate of R5 for every R100
   (b) A rate of R7,50 for every R50
   (c) A rate of R20 for every R200
   (d) A rate of x rands for every a rands

Per annum means "per year".

2. Annie deposits R8 345 into a savings account at Bonus Bank. The interest rate is 9% per annum.
   (a) How much interest will she have earned at the end of the first year?
   (b) Annie decides to leave the deposit of R8 345 with the bank for an indefinite period, and to withdraw only the interest at the end of every year. How much interest does she receive over a period of five years?

3. Maxi invested R3 500 at an interest rate of 5% per annum. Her total interest was R875. For what period did she invest the amount?

4. Money is invested for one year at an interest rate of 8% per annum. Copy and complete the table of equivalent rates:

Sum invested (R)	1 000	2 500	8 000	20 000	90 000	x
Interest earned (R)						

5. Interest on overdue accounts is charged at a rate of 20% per annum. Calculate the interest due on an account that is ten days overdue if the amount owing is R260. (Give your answer to the nearest cent.)
6. A sum of money invested in the bank at 5% per annum, i.e. simple interest, amounted to R6 250 after five years. This final amount includes the interest. Thuli figured out that the final amount is (1 + 0,05 × 5) × amount invested.
   (a) Explain Thuli's thinking.
   (b) Calculate the amount that was invested.

COMPOUND INTEREST

When the interest earned each year is added to the original amount, and the interest for the following year is calculated on this new amount, the result is known as **compound interest**.

Example:

R2 000 is invested at 10% per annum compound interest:

End of first year:	Amount = R2 000 + R200 interest = R2 200
End of second year:	Amount = R2 200 + R220 interest = R2 420
End of third year:	Amount = R2 420 + R242 interest = R2 662

1. An amount of R20 000 is invested at 5% per annum compound interest.
   (a) What is the total value of the investment after one year?
   (b) What is the total value of the investment after two years?
   (c) What is the total value of the investment after three years?
2. Bonus Bank is offering an investment scheme over two years at a compound interest rate of 15% per annum. Mr Pillay wishes to invest R800 in this way.
   (a) How much money will be due to him at the end of the two-year period?
   (b) How much interest will have been earned during the two years?
3. Andrew and Zinzi are arguing about interest on money that they have been given for Christmas. They each received R750. Andrew wants to invest his money in ABC Building Society for two years at a compound interest rate of 14% per annum, while Zinzi claims that she will do better at Bonus Bank, earning 15% simple interest per annum over two years. Who is correct?

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4. Mr Martin invests an amount (P) of R12 750 at 5,3% (r) compound interest over a period (n) of four years. Use the formula: $A = P(1 + \frac{r}{100})^n$ and calculate the final amount (A) that his investment will be worth after four years.
   (a) How many conversion periods will his investment have altogether?
   (b) How much is his investment worth after four years?
   (c) Calculate the total interest that he earns on his initial investment.
5. Calculate the interest generated by an investment (P) of R5 000 at 10% (r) compound interest over a period (n) of three years. A is the final amount. Use the formula: $A = P(1 + \frac{r}{100})^n$ to calculate the interest.

EXCHANGE RATE AND COMMISSION

1. (a) Tim bought £650 at the foreign exchange desk at Gatwick Airport in the UK at a rate of R15,66 per £1. The desk also charged 2,5% commission on the transaction. How much did Tim spend to buy the pounds?
   (b) What was the value of R1 in British pounds on that day?
2. Mandy wants to order a book from the internet. The price of the book is $25,86. What is the price of the book in rands? Say, for example, that the exchange rate is R9,95 for $1.
3. Bongani is a car salesperson. He earns a commission of 3% on the sale of a car with the value of R220 000. Calculate how much commission he earned.

CHAPTER 2
Integers

2.1 Which numbers are smaller than 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

Numbers such as –7 and –500, the additive inverses of whole numbers, are included with all the whole numbers and are called **integers**.
Fractions can be negative too, for example: $-\frac{3}{4}$ and –3,46.

Natural numbers are used for counting and fractions (rational numbers) are used for measuring. Why do we also have negative numbers?

When a larger number is subtracted from a smaller number, the answer may be a negative number: 5 – 12 = –7. This number is called **negative 7**.

One of the most important reasons for inventing negative numbers was to provide solutions for equations like the following:

Equation	Solution	Required property of negative numbers
$17 + x = 10$	$x = -7$ because $17 + (-7) = 17 - 7$ $= 10$	1. Adding a negative number is just like subtracting the corresponding positive number
$5 - x = 9$	$x = -4$ because $5 - (-4) = 5 + 4 = 9$	2. Subtracting a negative number is just like adding the corresponding positive number
$20 + 3x = 5$	$x = -5$ because $3 \times (-5) = -15$	3. The product of a positive number and a negative number is a negative number

PROPERTIES OF INTEGERS

1. In each case, state what number will make the equation true. Also state which of the properties of integers in the table above, is demonstrated by the equation:
   (a) $20 - x = 50$
   (b) $50 + x = 20$
   (c) $20 - 3x = 50$
   (d) $50 + 3x = 20$

2.2 Adding and subtracting with integers

Addition and subtraction of negative numbers

Examples: (–5) + (–3) and (–20) – (–7)

This is done in the same way as the addition and subtraction of positive numbers.

(–5) + (–3) can also be written as –5 + (–3) or as –5 + –3

(–5) + (–3) = –8 and –20 – (–7) = –13

This is just like 5 + 3 = 8 and 20 – 7 =13, or R5 + R3 = R8, and R20 – R7 = R13.

Subtraction of a larger number from a smaller number

Examples: 5 – 9 and 29 – 51

Let us first consider the following:

5 + (–5) = 0   10 + (–10) = 0   and   20 + (–20) = 0

If we subtract 5 from 5, we get 0, but then we still have to subtract 4:

$$\begin{aligned} 5 - 9 &= \underbrace{5 - 5} - 4 \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

We know that –9 = (–4) + (–5)

Suppose the numbers are larger, for example 29 – 51:

29 – 51 = 29 – 29 – 22

–51 = (–29) + (–22)

How much will be left of the 51, after you have subtracted 29 from 29 to get 0?

How can we find out? Is it 51 – 29?

Addition of a positive and a negative number

Examples: 7 + (–5); 37 + (–45) and (–13) + 45

The following statement is true if the unknown number is 5:

20 – (a certain number) = 15

We also need numbers that will make sentences like the following true:

20 + (a certain number) = 15

But to go from 20 to 15 you have to subtract 5.

The number we need to make the sentence *20 + (a certain number) = 15* true, must have the following strange property:

If you **add** this number, it should have the **same effect** as **subtracting 5**.

So, mathematicians agreed that the number called negative 5 will have the property that if you add it to another number, the effect will be the same as subtracting the natural number 5.

This means that mathematicians agreed that 20 + (–5) is equal to 20 – 5.

In other words, instead of adding *negative 5* to a number, you may subtract 5.

> Adding a negative number has the same effect as subtracting a corresponding natural number.
> For example: 20 + (–15) = 20 – 15 = 5.

Subtraction of a negative number

We have dealt with cases like –20 – (–7) on the previous page.

The following statement is true if the number is 5:

25 + (a certain number) = 30

We also need a number to make this statement true:

25 – (a certain number) = 30

If you subtract this number, it should have the same effect as adding 5.

It was agreed that: *25 – (–5) is equal to 25 + 5*.

Instead of subtracting the negative number, you add the corresponding positive number (the additive inverse):

$$8 - (-3) = 8 + 3$$
$$= 11$$
$$-5 - (-12) = -5 + 12$$
$$= 7$$

> We may say that for each "positive" number there is a **corresponding** or **opposite** negative number.
> Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**.

Subtraction of a positive number from a negative number

For example: –7 – 4 actually means (–7) – 4.

Instead of subtracting a positive number, you add the corresponding negative number.

For example: –7 – 4 can be seen as (–7) + (–4) = –11.

CALCULATIONS WITH INTEGERS

Calculate each of the following:

1. –7 + 18
2. 24 – 30 – 7
3. –15 + (–14) – 9
4. 35 – (–20)
5. 30 – 47
6. (–12) – (–17)

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate each of the following:
   (a) –7 + –7 + –7 + –7 + –7 + –7 + –7 + –7 + –7 + –7
   (b) –10 + –10 + –10 + –10 + –10 + –10 + –10
   (c) 10 × (–7)
   (d) 7 × (–10)

2. Say whether you agree (✓) or (✗) disagree with each statement:
   (a) 10 × (–7) = 70
   (b) 9 × (–5) = (–9) × 5
   (c) (–7) × 10 = 7 × (–10)
   (d) 9 × (–5) = –45
   (e) (–7) × 10 = 10 × (–7)
   (f) 5 × (–9) = 45

Multiplication of integers is commutative:
(–20) × 5 = 5 × (–20)

THE DISTRIBUTIVE PROPERTY

1. Calculate each of the following. Note that brackets are used for two purposes in these expressions, i.e. to indicate that certain operations are to be done first, and to show the integers.
   (a) 20 + (–5)
   (b) 4 × **(**20 + (–5)**)**
   (c) 4 × 20 + 4 × (–5)
   (d) (–5) + (–20)
   (e) 4 × **(**(–5) + (–20)**)**
   (f) 4 × (–5) + 4 × (–20)

2. If you worked correctly, your answers for question 1 should be 15; 60; 60; –25; –100 and –100. If your answers are different, check to see where you went wrong and correct your work.

3. Calculate each of the following where you can:
   (a) 20 + (–15)
   (b) 4 × **(**20 + (–15)**)**
   (c) 4 × 20 + 4 × (–15)
   (d) (–15) + (–20)
   (e) 4 × **(**(–15) + (–20)**)**
   (f) 4 × (–15) + 4 × (–20)
   (g) 10 + (–5)
   (h) (–4) × **(**10 + (–5)**)**
   (i) (–4) × 10 + **(**(–4) × (–5)**)**

4. What property of integers is demonstrated in your answers for questions 3(a) and (g)? Explain your answer.

In question 3(i) you had to multiply two negative numbers. What was your guess? We can calculate (–4) × **(**10 + (–5)**)** as in (h). It is (–4) × 5 = –20.

If we want the distributive property to be true for integers, then (–4) × 10 + (–4) × (–5) must be equal to –20.

(–4) × 10 + (–4) × (–5) = –40 + (–4) × (–5)

Then (–4) × (–5) must be equal to 20.

5. Calculate each of the following:
   (a) 10 × 50 + 10 × (–30)
   (b) 50 + (–30)
   (c) 10 × **(**50 + (–30)**)**
   (d) (–50) + (–30)
   (e) 10 × (–50) + 10 × (–30)
   (f) 10 × **(**(–50) + (–30)**)**

- The product of two positive numbers is a positive number, for example: 5 × 6 = 30.
- The product of a positive number and a negative number is a negative number, for example: 5 × (–6) = –30.
- The product of a negative number and a positive number is a negative number, for example: (–5) × 6 = –30.

6. (a) Write out only the numerical expressions below which you would expect to have the same answers. Do not do the calculations.

   16 × (53 + 68)   53 × (16 + 68)   16 × 53 + 16 × 68   16 × 53 + 68

   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?

7. Consider your answers for question 5.
   (a) Does multiplication distribute over addition in the case of integers?
   (b) Illustrate your answer with two examples.

8. Write out only the numerical expression below which you would expect to have the same answers. Do not do the calculations now.

   10 × **(**(–50) – (–30)**)**   10 × (–50) – (–30)   10 × (–50) – 10 × (–30)

9. Do the three sets of calculations given in question 8.

10. Calculate (–10) × **(**5 + (–3)**)**.

11. Now consider the question of whether or not multiplication by a negative number distributes over addition and subtraction of integers. For example, would (–10) × 5 + (–10) × (–3) also have the answer of –20, like (–10) × **(**5 + (–3)**)**?

To make sure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that:

(a negative number) × (a negative number) is a positive number.

For example: (–10) × (–3) = 30.

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12. Calculate each of the following:

(a) $(-20) \times (-6)$
(b) $(-20) \times 7$
(c) $(-30) \times (-10) + (-30) \times (-8)$
(d) $(-30) \times ((-10) + (-8))$
(e) $(-30) \times (-10) - (-30) \times (-8)$
(f) $(-30) \times ((-10) - (-8))$

Here is a summary of the properties of integers that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0. For example, $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by calculating $3 + 10$ is 13.
- The product of a positive and a negative integer is negative. For example, $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive. For example, $(-15) \times (-6) = 90$.

DIVISION WITH INTEGERS

1. Calculate each of the following:

(a) $5 \times (-7)$
(b) $(-3) \times 20$
(c) $(-5) \times (-10)$
(d) $(-3) \times (-20)$

2. Use your answers in question 1 to determine the following:

(a) $(-35) \div 5$
(b) $(-35) \div (-7)$
(c) $(-60) \div 20$
(d) $(-60) \div (-3)$
(e) $50 \div (-5)$
(f) $50 \div (-10)$
(g) $60 \div (-20)$
(h) $60 \div (-3)$

- The quotient of a positive number and a negative number is a negative number.
- The quotient of two negative numbers is a positive number.

MIXED CALCULATIONS WITH INTEGERS

1. Calculate each of the following:

(a) $20(-50 + 7)$
(b) $20 \times (-50) + 20 \times 7$
(c) $20(-50 + -7)$
(d) $20 \times (-50) + 20 \times -7$
(e) $-20(-50 + -7)$
(f) $-20 \times -50 + -20 \times -7$

2. Calculate each of the following:
   (a) $40 \times (-12 + 8) - 10 \times (2 + -8) - 3 \times (-3 - 8)$
   (b) $(9 + 10 - 9) \times 40 + (25 - 30 - 5) \times 7$
   (c) $-50(40 - 25 + 20) + 30(-10 + 7 + 13) - 40(-16 + 15 - 2)$
   (d) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$
   (e) $-3 \times (-14 - 6 + 5) \times (-13 - 7 + 10) \times (20 - 10 - 15)$

2.4 Powers, roots and word problems

Answer all questions in this section **without** using a calculator.

1. Copy and complete the following tables:

(a)

x	1	2	3	4	5	6	7	8	9	10	11	12
x^2												
x^3												

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
x^2												
x^3												

3^2 is 9 and $(-3)^2$ is also 9.
3^3 is 27 and $(-5)^3$ is -125.
Both (–3) and 3 are **square roots** of 9.
3 may be called the **positive square root** of 9, and (–3) may be called the **negative square root** of 9.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.
10^2 is 100 and $(-10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.

The symbol $\sqrt{\ }$ means that you must take the **positive square root** of the number.

2. Calculate each of the following:
   (a) $\sqrt{4} - \sqrt{9}$
   (b) $\sqrt[3]{27} + (-\sqrt[3]{64})$
   (c) $-(3^2)$
   (d) $(-3)^2$
   (e) $4^2 - 6^2 + 1^2$
   (f) $3^3 - 4^3 - 2^3 - 1^3$
   (g) $\sqrt{81} - \sqrt{4} \times \sqrt[3]{125}$
   (h) $-(4^2)(-1)^2$
   (i) $\dfrac{(-5)^2}{\sqrt{37-12}}$
   (j) $\dfrac{-\sqrt{36}}{-1^3 - 2^3}$

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3. Determine the answer to each of the following:
   (a) The overnight temperature in Polokwane drops from 11 °C to –2 °C. By how many degrees has the temperature dropped?
   (b) The temperature in Escourt drops from 2 °C to –1 °C in one hour, and then another two degrees in the next hour. How many degrees in total did the temperature drop over the two hours?
   (c) A submarine is 75 m below the surface of the sea. It then rises by 21 m. How far below the surface is it now?
   (d) A submarine is 37 m below the surface of the sea. It then sinks a further 15 m. How far below the surface is it now?

CHAPTER 3
Fractions

3.1 Equivalent fractions

THE SAME NUMBER IN DIFFERENT FORMS

1. How much money is each of the following amounts?
   (a) $\frac{1}{5}$ of R200 (b) $\frac{2}{10}$ of R200 (c) $\frac{4}{20}$ of R200

Did you notice that all the answers are the same? That is because $\frac{1}{5}$, $\frac{2}{10}$ and $\frac{4}{20}$ are **equivalent fractions**. They are different ways of writing the same number.

Consider this bar. It is divided into five equal parts.

Each piece is **one fifth** of the whole bar.

2. Now copy the bar and draw lines on the bar so that it is approximately divided into ten equal parts.

   (a) What part of the whole bar is each of your ten parts?
   (b) How many tenths is the same as one fifth?
   (c) How many tenths is the same as two fifths?
   (d) How many fifths is the same as eight tenths?

3. Copy the bar below and draw lines on the bar below so that it is approximately divided into 25 equal parts.

   (a) How many twenty-fifths is the same as two fifths?
   (b) How many fifths is the same as 20 twenty-fifths?

In question 3(b) you found that $\frac{4}{5}$ is equivalent to $\frac{20}{25}$: these are just two different ways to describe the same part of the bar.

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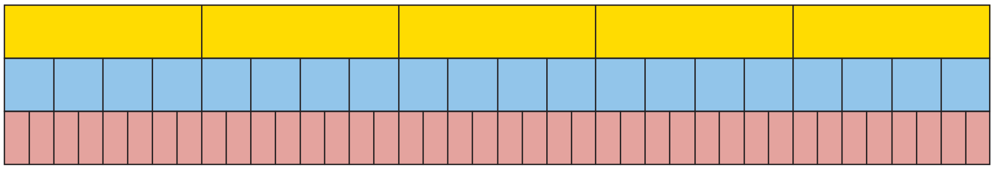
This can be expressed by writing $\frac{4}{5} = \frac{20}{25}$ which means that $\frac{4}{5}$ and $\frac{20}{25}$ are equivalent to each other.

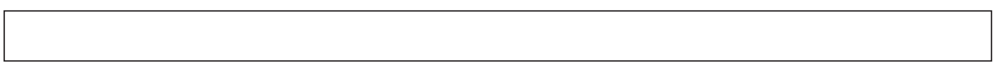
4. Write down all the other pairs of equivalent fractions which you found while doing questions 2 and 3.

The yellow bar is divided into fifths.



5. (a) Into what kind of fraction parts is the blue bar divided?
   (b) Into what kind of fraction parts is the red bar divided?
   (c) If you want to mark the yellow bar in twentieths (like the blue bar), into how many parts do you have to divide each of the fifths?
   (d) If you want to mark the yellow bar in fortieths (like the red bar), into how many parts do you have to divide each of the fifths?
   (e) If you want to mark the yellow bar in eightieths, into how many parts do you have to divide each of the fifths?
   (f) If you want to mark the blue bar in eightieths, into how many parts do you have to divide each of the twentieths?

6. Suppose this bar is divided into four equal parts, in other words, quarters.



   (a) If the bar is also divided into 20 equal parts, how many of these smaller parts will there be in each quarter?
   (b) If each quarter is divided into six equal parts, what part of the whole bar will each small part be?

7. Copy and complete this table of equivalent fractions, as far as you can using whole numbers. All the fractions in each column must be equivalent.

sixteenths	8	4	2	10	14	12
eighths						
quarters						
twelfths						
twentieths						

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Equivalent fractions can be formed by multiplying the numerator and denominator by the same number. For example: $\frac{1}{5} = \frac{4 \times 1}{4 \times 5} = \frac{4}{20}$

8. Write down five different fractions that are equivalent to $\frac{3}{4}$.
9. Express each of the following numbers as twelfths:

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

You may divide the numerator and denominator by the same number, instead of multiplying the numerator and denominator by the same number. This gives you a simpler fraction.

The **simplest form** of a fraction has no common factors. For example, you find the simplest form of the fraction $\frac{4}{12}$ is $\frac{1}{3}$ by dividing both the numerator and denominator by the common factor of 4.

10. Convert each of the following fractions to their simplest form:

(a) $\frac{40}{100}$ (b) $\frac{4}{16}$

(c) $\frac{5}{25}$ (d) $\frac{6}{30}$

(e) $\frac{6}{24}$ (f) $\frac{8}{88}$

CONVERTING BETWEEN MIXED NUMBERS AND FRACTIONS

Numbers that have both whole number and fraction parts are called **mixed numbers**.

Examples of mixed numbers: $3\frac{4}{5}$, $2\frac{7}{8}$ and $8\frac{3}{10}$

Mixed numbers can be written in expanded notation, for example:

$3\frac{4}{5}$ means $3 + \frac{4}{5}$ $2\frac{7}{8}$ means $2 + \frac{7}{8}$ $8\frac{3}{10}$ means $8 + \frac{3}{10}$.

To add and subtract mixed numbers, you can work with the whole number parts and the fraction parts separately, for example:

$3\frac{4}{5} + 13\frac{3}{5}$
$= 16\frac{7}{5}$
$= 17\frac{2}{5}$

$13\frac{3}{5} - 3\frac{4}{5}$ (we need to "borrow" a unit from 13, because we cannot subtract $\frac{4}{5}$ from $\frac{3}{5}$)
$= 12\frac{8}{5} - 3\frac{4}{5}$
$= 9\frac{4}{5}$

However, this method can be difficult to do with some examples, and it does not work with multiplication and division.

An alternative and preferred method is to convert the mixed number to an **improper fraction**, as shown in the example below:

$3\frac{4}{5}$

$= 3 + \frac{4}{5}$

$= \frac{15}{5} + \frac{4}{5}$

$= \frac{19}{5}$

NOTE
You can obtain the numerator of 19 in one step by multiplying the denominator (5) by the whole number (3), and then adding the numerator (4).

So, you can calculate $3\frac{4}{5} + 13\frac{3}{5}$ using this method:

$3\frac{4}{5} + 13\frac{3}{5}$

$= \frac{19}{5} + \frac{68}{5}$

$= \frac{87}{5}$

The answer must be converted to a mixed number again: $\frac{87}{5} = 17\frac{2}{5}$

1. Convert each of the following mixed numbers to improper fractions:
   (a) $5\frac{3}{5}$ (b) $2\frac{3}{8}$ (c) $3\frac{4}{7}$ (d) $4\frac{5}{12}$

2. Convert each of the following improper fractions to mixed numbers:
   (a) $\frac{32}{5}$ (b) $\frac{25}{8}$ (c) $\frac{24}{9}$ (d) $\frac{37}{20}$

3.2 Adding and subtracting fractions

To add or subtract two fractions, they have to be expressed with the *same* denominators first. To achieve that, one or more of the given fractions may have to be replaced with equivalent fractions.

$\frac{3}{20} + \frac{2}{5}$

$= \frac{3}{20} + \frac{2 \times 4}{5 \times 4}$

$= \frac{3}{20} + \frac{8}{20}$

$= \frac{11}{20}$

We will refer to this as the LCM method.

$\frac{5}{12} + \frac{7}{20}$

$= \frac{5 \times 20}{12 \times 20} + \frac{7 \times 12}{20 \times 12}$

$= \frac{100}{240} + \frac{84}{240}$

$= \frac{184}{240}$

$= \frac{23}{30}$

We will later refer to this method of adding or subtracting fractions as Method A.

In the case of $\frac{5}{12} + \frac{7}{20}$, multiplying by 20 and by 12 was a sure way of making equivalent fractions of the same kind, in this case two hundred-and-fortieths. However, the numbers became quite big. Just imagine how big the numbers will become if you use the same method to calculate $\frac{17}{75} + \frac{13}{85}$!

Fortunately, there is a method of keeping the numbers smaller (in many cases) when making equivalent fractions, so that fractions can be added or subtracted. In this method you first calculate the **lowest common multiple** or LCM of the denominators. In the case of $\frac{5}{12} + \frac{7}{20}$, the smaller multiples of the denominators are:

12:	12	24	36	48	60	72	84
20:	20	40	60	80	100	120	140

The smallest number that is a multiple of both 12 and 20 is 60.

Both $\frac{5}{12}$ and $\frac{7}{20}$ can be expressed in terms of sixtieths:

$\frac{5}{12} = \frac{5 \times 5}{12 \times 5} = \frac{25}{60}$ because to make twelfths into sixtieths you have to divide each twelfth into five equal parts, to get $12 \times 5 = 60$ equal parts, i.e. sixtieths.

Similarly, $\frac{7}{20} = \frac{7 \times 3}{20 \times 3} = \frac{21}{60}$.

Hence $\frac{5}{12} + \frac{7}{20} = \frac{25}{60} + \frac{21}{60} = \frac{46}{60} = \frac{23}{30}$

We may call this method the LCM method of adding or subtracting fractions.

ADDING AND SUBTRACTING FRACTIONS

1. Which method of adding and subtracting fractions do you think will be the easiest and quickest for you, Method A or the LCM method? Explain.

2. Calculate each of the following:

   (a) $\frac{3}{8} + \frac{2}{5}$ (b) $\frac{3}{10} + \frac{7}{8}$

   (c) $3\frac{2}{5} + 2\frac{3}{10}$ (d) $7\frac{3}{8} + 3\frac{11}{12}$

3. Calculate each of the following:

   (a) $\frac{13}{20} - \frac{2}{5}$ (b) $\frac{7}{12} - \frac{1}{4}$

   (c) $5\frac{1}{2} - 3\frac{3}{8}$ (d) $4\frac{1}{9} - 5\frac{2}{3}$

4. Paulo and Sergio buy a pizza. Paulo eats $\frac{1}{3}$ of the pizza and Sergio eats two fifths. How much of the pizza is left over?

5. Calculate each of the following. State whether you use Method A or the LCM method.

(a) $\frac{7}{15} + \frac{11}{24}$

(b) $\frac{73}{100} - \frac{7}{75}$

(c) $\frac{3}{25} + \frac{13}{40}$

(d) $\frac{9}{16} - \frac{3}{10}$

(e) $\frac{1}{18} + \frac{7}{20}$

(f) $\frac{11}{35} - \frac{3}{14}$

(g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

3.3 Multiplying and dividing fractions

THINK ABOUT MULTIPLICATION AND DIVISION WITH FRACTIONS

1. Read the questions below, but do not answer them now. Just describe in each case what calculations you think must be done to find the answer to the question. You can think later about how the calculations may be done.

   (a) Ten people come to a party and each of them must get $\frac{5}{8}$ of a pizza. How many pizzas must be bought to provide for all of them?

   (b) $\frac{5}{8}$ of the cost of a new clinic must be carried by the ten doctors who will work there. What part of the cost of the clinic must be carried by each of the doctors, if they have agreed to share the cost equally?

   (c) If a whole pizza costs R10, how much does $\frac{5}{8}$ of a pizza cost?

   (d) The owner of a spaza shop has ten whole pizzas. How many portions of $\frac{5}{8}$ of a pizza each can he make up from the ten pizzas?

2. Look at the different sets of calculations shown below.

   (a) Which set of calculations is a correct way to find the answer for question 1(a)?

   (b) Which set of calculations is a correct way to find the answer for question 1(b)?

   (c) Which set of calculations is a correct way to find the answer for question 1(c)?

   (d) Which set of calculations is a correct way to find the answer for question 1(d)?

Set A: $\frac{10}{10} \times \frac{5}{8} = \frac{50}{80}$

Set B: $\frac{5}{8} = \frac{50}{80}$. 50 eightieths ÷ 10 = $\frac{5}{80}$

Set C: How many eighths in ten wholes? 80 eighths. How many five-eighths in 80? 80 ÷ 5 = 16

Set D: $\frac{5}{8}$ is five eighths. 10 × five eighths = $\frac{50}{8}$

Set E: $\frac{5}{8} \div 10 = \frac{5}{8} \times \frac{10}{1} = \frac{50}{8}$

Multiply a fraction by a whole number

Example:

$8 \times \frac{3}{5} = 8 \times 3 \text{ fifths} = 24 \text{ fifths} = \frac{24}{5} = 4\frac{4}{5}$

Divide a fraction by a whole number

You can divide a fraction by converting it to an equivalent fraction with a numerator that is a multiple of the divisor.

Example:

$\frac{2}{3} \div 5 = \frac{10}{15} \div 5 = 10 \text{ fifteenths} \div 5 = 2 \text{ fifteenths} = \frac{2}{15}$

A fraction of a whole number, and a fraction of a fraction

Examples:

A $\frac{7}{12}$ of R36.

$\frac{1}{12}$ of R36 is the same as R36 ÷ 12 = R3, so $\frac{7}{12}$ of R36 is 7 × R3 = R21.

B $\frac{7}{12}$ of 36 fiftieths.

$\frac{1}{12}$ of 36 fiftieths is the same as 36 fiftieths ÷ 12 = 3 fiftieths,

so $\frac{7}{12}$ of 36 fiftieths is 7 × 3 fiftieths = 21 fiftieths.

$\frac{7}{12} \times \frac{36}{50}$ means $\frac{7}{12}$ of $\frac{36}{50}$, it is the same.

$\frac{1}{12}$ of $\frac{36}{50}$ is the same as $\frac{36}{50} \div 12 = \frac{3}{50}$, so $\frac{7}{12}$ of $\frac{36}{50}$ is $7 \times \frac{3}{50} = \frac{21}{50}$.

3. (a) You calculated $\frac{7}{12} \times \frac{36}{50}$ in the example above. What was the answer?

   (b) Calculate $\frac{7 \times 36}{12 \times 50}$, and simplify your answer.

Example:

$\frac{2}{3} \times \frac{5}{8} = \frac{2}{3} \text{ of } \frac{15}{24} = \frac{1}{3} \text{ of } \frac{30}{24} = \frac{10}{24} = \frac{5}{12}$

The same answer is obtained by calculating $\frac{2 \times 5}{3 \times 8}$.

To multiply two fractions, you may simply multiply the numerators and the denominators.

$\frac{2}{3} \times \frac{9}{20} = \frac{2 \times 9}{3 \times 20} = \frac{18}{60} = \frac{3}{10}$

Division by a fraction

When we divide by a fraction, we have a very different situation. Think about this:

If you have 40 pizzas, how many learners can have $\frac{3}{5}$ a pizza each?

To find the number of fifths in 40 pizzas: 40 × 5 = 200 fifths of a pizza.

To find the number of three fifths: 200 ÷ 3 = 66 portions of $\frac{3}{5}$ pizza and two fifths of a pizza is left over.

Since the portion for each learner is three fifths, the two fifths of a pizza that remains is two thirds of a portion.

So, to calculate $40 \div \mathbf{\frac{3}{5}}$, we multiplied by **5** and divided by **3**, and that gave us 66 and two thirds of a portion.

In fact, we calculated $40 \times \frac{5}{3}$.

> Division is the inverse of multiplication.
> So, to divide by a fraction, you multiply by its inverse.

Example:

$\frac{18}{60} \div \frac{2}{3} = \frac{18}{60} \times \frac{3}{2} = \frac{54}{120} = \frac{9}{20}$

MULTIPLYING AND DIVIDING FRACTIONS

1. Calculate each of the following:

   (a) $\frac{3}{4}$ of $\frac{12}{25}$

   (b) $\frac{3}{4} \times \frac{12}{100}$

   (c) $\frac{3}{4}$ of $\frac{13}{25}$

   (d) $\frac{3}{4} \times 1\frac{1}{2}$

   (e) $\frac{3}{20} \times \frac{5}{6}$

   (f) $\frac{3}{20}$ of $\frac{3}{20}$

2. A small factory manufactures copper pans for cooking. Exactly $\frac{3}{50}$ kg of copper is needed to make one pan.

   (a) How many pans can they make if $\frac{18}{50}$ kg of copper is available?

   (b) How many pans can they make if $\frac{20}{50}$ kg of copper is available?

   (c) How many pans can they make if $\frac{2}{5}$ kg of copper is available?

   (d) How many pans can they make if $\frac{3}{4}$ kg of copper is available?

(e) How many pans can be made if $\frac{144}{50}$ kg of copper is available?

(f) How many pans can be made if 5 kg of copper is available?

3. Calculate each of the following:

(a) $\frac{18}{50} \div \frac{3}{50}$

(b) $\frac{9}{25} \div \frac{3}{50}$

(c) $\frac{144}{50} \div \frac{3}{50}$

(d) $2\frac{44}{50} \div \frac{3}{50}$

(e) $2\frac{22}{25} \div \frac{3}{50}$

(f) $\frac{5}{8} \div \frac{3}{50}$

(g) $20 \div \frac{3}{50}$

(h) $2 \div \frac{3}{50}$

(i) $1 \div \frac{3}{50}$

(j) $\frac{1}{2} \div \frac{3}{50}$

4. A rectangle is $3\frac{5}{8}$ cm long and $2\frac{3}{5}$ cm wide.

(a) What is the area of this rectangle?

(b) What is the perimeter of this rectangle?

5. A rectangle is $5\frac{5}{6}$ cm long and its area is $8\frac{1}{6}$ cm^2.

How wide is this rectangle?

6. Calculate each of the following:

(a) $2\frac{3}{8}$ of $5\frac{4}{5}$

(b) $3\frac{2}{7} \times 2\frac{7}{12}$

(c) $8\frac{2}{5} \div 3\frac{3}{10}$

(d) $3\frac{3}{10} \times 3\frac{3}{10}$

(e) $2\frac{5}{8} \div 5\frac{7}{10}$

(f) $\frac{3}{5} \times 1\frac{2}{3} \times 1\frac{3}{4}$

7. Calculate each of the following:

(a) $\frac{2}{3}(\frac{3}{4} + \frac{7}{10})$

(b) $\frac{2}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{7}{10}$

(c) $\frac{5}{8}(\frac{4}{5} - \frac{1}{3})$

(d) $\frac{5}{8} \times \frac{4}{5} - \frac{5}{8} \times \frac{1}{3}$

8. A piece of land with an area of 40 ha is divided into 30 equal plots. The total price of the land is R45 000. Remember that "ha" is the abbreviation for hectares.

(a) Jim buys $\frac{2}{5}$ of the land.

(i) How many plots is this and how much should he pay?

(ii) What is the area of the land that Jim buys?

(b) Charlene buys $\frac{1}{3}$ of the land. How many plots is this and how much should she pay?

(c) Bongani buys the rest of the land. Determine the fraction of the land that he buys.

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

1. Calculate each of the following:

(a) $\frac{3}{4} \times \frac{3}{4}$

(b) $\frac{7}{10} \times \frac{7}{10}$

(c) $2\frac{5}{8} \times 2\frac{5}{8}$

(d) $1\frac{5}{12} \times 1\frac{5}{12}$

(e) $3\frac{5}{7} \times 3\frac{5}{7}$

(f) $10\frac{3}{4} \times 10\frac{3}{4}$

$\frac{9}{16}$ is the square of $\frac{3}{4}$, because $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. $\frac{3}{4}$ is the square root of $\frac{9}{16}$.

2. Find the square root of each of the following numbers:

(a) $\sqrt{\frac{25}{49}}$

(b) $\sqrt{\frac{36}{121}}$

(c) $\sqrt{\frac{64}{25}}$

(d) $\sqrt{2\frac{46}{49}}$

3. Calculate each of the following:

(a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

(b) $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

(c) $\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

(d) $\frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$

4. Find the cube root of each of the following numbers:

(a) $\sqrt[3]{\frac{27}{1\,000}}$

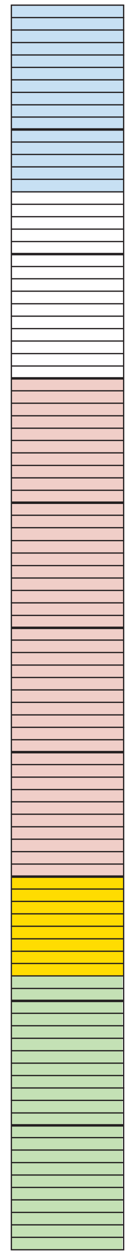
(b) $\sqrt[3]{\frac{125}{216}}$

(c) $\sqrt[3]{\frac{1\,000}{216}}$

(d) $\sqrt[3]{15\frac{5}{8}}$

3.4 Equivalent forms

FRACTIONS, DECIMALS AND PERCENTAGE FORMS



1. The rectangle on the right is divided into small parts.
   - (a) How many of these small parts are there in the rectangle?
   - (b) How many of these small parts are there in one tenth of the rectangle?
   - (c) What fraction of the rectangle is blue?
   - (d) What fraction of the rectangle is pink?

Instead of "six hundredths" we may say "6 per cent" or, in short, "6%". It means the same thing.

15 per cent of the rectangle on the right is blue.

2. (a) What percentage of the rectangle is green?
   (b) What percentage of the rectangle is pink?

0,37 and 37% and $\frac{37}{100}$ are different ways of writing the same value (**37 hundredths**).

3. Express each of the following in three ways, namely as a decimal, a percentage and a fraction (in simplest form):

(a) three tenths	(b) seven hundredths
(c) 37 hundredths	(d) seven tenths
(e) two fifths	(f) seven twentieths

4. Copy the table and fill in the missing values.

Decimal	Percentage	Common fraction (simplest form)
0,2		
	40%	
		$\frac{3}{8}$
0,05		

5. (a) Jannie eats a quarter of a watermelon. What percentage of the watermelon is this?
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk in the bottle has he drunk?
   (c) Jem used 0,18 of the paint in a tin. If he uses half of the remaining amount the next time he paints, what fraction (in simplest form) is left over?

Chapter 4
The decimal notation for fractions

4.1 Equivalent forms

Decimal fractions and common fractions are simply different ways of expressing the same number. They are different **notations** showing the same value.

To write a decimal fraction as a common fraction: Write the decimal with a denominator that is a power of ten (10, 100, 1 000, etc.) and then simplify it if possible.

For example: $0{,}35 = \frac{35}{100} = \frac{7}{20} \times \frac{5}{5} = \frac{7}{20}$

To write a common fraction as a decimal fraction: Change the common fraction to an equivalent fraction with a power of ten as a denominator.

For example: $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0{,}75$

If you are permitted to use your calculator, simply type in 3 ÷ 4 and the answer will be given as 0,75. On some calculators you will need to press an additional button to convert the exact fraction to a decimal.

Notation means a set of symbols that are used to show a special thing.

COMMON FRACTIONS, DECIMAL FRACTIONS AND PERCENTAGES

Do *not* use a calculator in this exercise.

1. Write the following decimal fractions as common fractions in their simplest form:

(a) 0,56
(b) 3,87
(c) 1,9
(d) 5,205

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2. Write the following common fractions as decimal fractions:
   (a) $\frac{9}{20}$
   (b) $\frac{7}{5}$
   (c) $\frac{24}{25}$
   (d) $2\frac{3}{8}$

3. Write the following percentages as common fractions in their simplest form:
   (a) 70%
   (b) 5%
   (c) 12,5%

4. Write the following decimal fractions as percentages:
   (a) 0,6
   (b) 0,43
   (c) 0,08
   (d) 0,265
   (e) 0,005

5. Write the following common fractions as percentages:
   (a) $\frac{7}{10}$
   (b) $\frac{3}{4}$
   (c) $\frac{33}{50}$
   (d) $\frac{60}{60}$
   (e) $\frac{2}{25}$
   (f) $\frac{29}{50}$

6. Jane and Devi are in different schools. At Jane's school the year mark for Mathematics was out of 80, and Jane got 60 out of 80. At Devi's school the year mark was out of 50 and Devi got 40 out of 50.
   (a) What fraction of the total marks, in its simplest form, did Devi obtain at her school?
   (b) What percentage did Devi and Jane get for Mathematics?
   (c) Who performed better, Jane or Devi?

7. During a basketball game, Lebo tried to score 12 times. Only four of her attempts were successful.
   (a) What fraction of her attempts was successful?
   (b) What percentage of her attempts was not successful?

4.2 Calculations with decimals

When you **add** and **subtract** decimal fractions:

- Add tenths to tenths.
- Subtract tenths from tenths.
- Add hundredths to hundredths.
- Subtract hundreds from hundredths.

And so on!

When you **multiply** decimal fractions, you change the decimals to whole numbers, do the calculation and lastly, change them back to decimal fractions.

Example: To calculate 13,1 × 1,01, you first calculate 131 × 101 (which equals 13 231). Then, since you have multiplied the 13,1 by 10, and the 1,01 by 100 in order to turn them into whole numbers, you need to divide this answer by 10 × 100 (i.e. 1 000). Therefore, the final answer is 13,231.

When you **divide** decimal fractions, you can use equivalent fractions to help you.

Example: $21{,}7 \div 0{,}7 = \frac{21{,}7}{0{,}7} = \frac{21{,}7}{0{,}7} \times \frac{10}{10} = \frac{217}{7} = 31$

Notice how you multiply both the numerator and denominator of the fraction by the same number (in this case, 10). Always multiply by the *smallest* power of ten that will convert both values to whole numbers.

CALCULATIONS WITH DECIMALS

Do *not* use a calculator in this exercise. Ensure that you show all steps of your working.

1. Calculate the value of each of the following:
   (a) 3,3 + 4,83
   (b) 0,6 + 18,3 + 4,4
   (c) 9,3 + 7,6 – 1,23
   (d) (16,0 – 7,6) – 0,6
   (e) 9,43 – (3,61 + 1,14)
   (f) 1,21 + 2,5 – (2,3 – 0,23)
2. Calculate the value of each of the following:
   (a) 4 × 0,5
   (b) 15 × 0,02
   (c) 0,8 × 0,04
   (d) 0,02 × 0,15
   (e) 1,07 × 0,2
   (f) 0,016 × 0,02
3. Calculate the value of each of the following:
   (a) 7,2 ÷ 3
   (b) 12 ÷ 0,3
   (c) 0,15 ÷ 0,5
   (d) 10 ÷ 0,002
   (e) 0,3 ÷ 0,006
   (f) 0,024 ÷ 0,08
4. Write down the value that is equal to or closest to the answer to each calculation:
   (a) 3 × 0,5
   A: 6
   B: 1,5
   C: 0,15
   (b) 4,4 ÷ 0,2
   A: 8,8
   B: 2,2
   C: 22

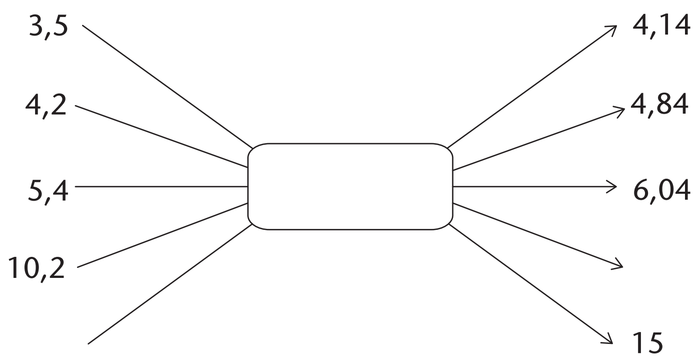
(c) 56 × 1,675

A: more than 56

B: more than 84

C: more than 112

5. Copy the diagram. Determine the operator and the unknown numbers and fill them in:



6. Calculate each of the following:

(a) $(0,1)^2$ (b) $(0,03)^2$ (c) $(2,5)^2$

(d) $\sqrt{0,04}$ (e) $\sqrt{0,16}$ (f) $\sqrt{0,49}$

(g) $(0,2)^3$ (h) $(0,4)^3$ (i) $(0,03)^3$

(j) $\sqrt[3]{0,064}$ (k) $\sqrt[3]{0,125}$ (l) $\sqrt[3]{0,216}$

7. Calculate each of the following:

(a) 2,5 × 2 ÷ 10 (b) 4,2 – 5 × 1,2

(c) $\frac{5,4 + 7,35}{0,05}$ (d) 4,2 ÷ 0,21 + 0,45 × 0,3

4.3 Solving problems

ALL KINDS OF PROBLEMS

Do *not* use a calculator in this exercise. Ensure that you show all steps of working.

1. Is 6,54 × 0,81 = 0,654 × 8,1? Explain your answer.

2. You are given that 45 × 24 = 1 080. Use this result to determine:

(a) 4,5 × 2,4 (b) 4,5 × 24 (c) 4,5 × 0,24

(d) 0,045 × 24 (e) 0,045 × 0,024 (f) 0,045 × 24

3. Without actually dividing, choose which answer in brackets is the correct answer, or the closest to the correct answer.
   (a) 14 ÷ 0,5 (7; 28; 70)
   (b) 0,58 ÷ 0,7 (8; 80; 0,8)
   (c) 2,1 ÷ 0,023 (10; 100; 5)

4. (a) John is asked to calculate 6,5 ÷ 0,02. He does the following:
   Step 1: 6,5 ÷ 2= 3,25
   Step 2: 3,25 × 100 = 325
   Is he correct? Why?
   (b) Use John's method in part (a) to calculate:
   (i) 4,8 ÷ 0,3
   (ii) 21 ÷ 0,003

5. Given: 0,174 ÷ 0,3 = 0,58. Using this fact, write down the answers for the following without doing any further calculations:
   (a) 0,3 × 0,58
   (b) 1,74 ÷ 3
   (c) 17,4 ÷ 30
   (d) 174 ÷ 300
   (e) 0,0174 ÷ 0,03
   (f) 0,3 × 5,8

4.4 More problems

MORE PROBLEMS AND CALCULATIONS

You *may* use a calculator for this exercise.

1. Calculate the following, rounding off all answers correct to two decimal places:
   (a) 8,567 + 3,0456
   (b) 2,781 – 6,0049
   (c) 1,234 × 4,056
   (d) $\frac{5,678 + 3,245}{1,294 - 0,994}$

2. What is the difference between 0,890 and 0,581?

3. If a rectangle is 12,34 cm wide and 31,67 cm long:
   (a) What is the perimeter of the rectangle?
   (b) What is the area of the rectangle? Round off your answer to two decimal places.

4. Alison buys a cooldrink for R5,95, a chocolate for R3,25 and a packet of chips for R4,60. She pays with a R20 note.
   (a) How much did she spend?
   (b) How much change did she get?

5. A tractor uses 11,25 ℓ of fuel in 0,75 hours. How many litres does it use in one hour?

6. Mrs Ruka received her municipal bill.
   (a) Her water consumption charge for one month is R32,65. The first 5,326 kℓ are free, then she pays R5,83 per kilolitre for every kilolitre thereafter.
   How much water did the Ruka household use?
   (b) The electricity charge for Mrs Ruka for the same month was R417,59. The first 10 kWh are free. For the next 100 kWh the charge is R1,13 per kWh, and thereafter for each kWh the charge is R1,42.
   How much electricity did the Ruka household use?
7. A roll of material is 25 m long. To make one dress, you need 1,35 m of material. How many dresses can be made out of a roll of material and how much material is left over?
8. If one litre of petrol weighs 0,679 kg, what will 28,6 ℓ of petrol weigh?
9. The reading on a water meter at the beginning of the month is 321,573 kℓ. At the end of the month the reading is 332,523 kℓ. How much water (in ℓ) was used during this month?

4.5 Decimals in algebraic expressions and equations

DECIMALS IN ALGEBRA

1. Simplify the following:
   (a) $\sqrt{0,09x^{36}}$
   (b) $7,2x^3 - 10,4x^3$
   (c) $(2,4x^2y^3)(10y^3x)$
   (d) $11,75x^2 - 1,2x \times 5x$
   (e) $\dfrac{3,4x - 1,2x}{1,1x \times 4}$
   (f) $\sqrt[3]{0,008x^{12}} + \sqrt{0,16x^8}$
   (g) $3x^2 + 0,1x^2 - 45,6 + 3,9$
   (h) $\dfrac{0,4y + 1,2y}{0,6x - 3x}$
2. Simplify the following:
   (a) $\dfrac{0,5x^9}{0,02x^3}$
   (b) $\dfrac{0,325}{x^2} - \dfrac{1,675}{x^2}$
   (c) $\dfrac{3,6x}{1,5y^3} \times \dfrac{5y}{0,6x}$
   (d) $\dfrac{9,5x^2}{1,2y^2} \div \dfrac{0,05x}{0,04y^8}$
3. Solve the following equations:
   (a) $0,24 + x = 0,31$
   (b) $x + 5,61 = 7,23$
   (c) $x - 3,14 = 9,87$
   (d) $4,21 - x = 2,74$
   (e) $0,96x = 0,48$
   (f) $x \div 0,03 = 1,5$

WORKSHEET

You are *not* permitted to use a calculator in this exercise, *except* for question 5. Ensure that you show all steps of working, where relevant.

1. Copy and complete the following table:

Percentage	Common fraction	Decimal fraction
2,5%		
	$\frac{15}{250}$	
		0,009

2. Calculate each of the following:

   (a) $6,78 - 4,92$ (b) $1,7 \times 0,05$ (c) $7,2 \div 0,36$

   (d) $4,2 - 0,4 \times 1,2 + 7,37$ (e) $(0,12)^2$ (f) $\frac{3\sqrt{0,04}}{\sqrt[3]{0,027}}$

3. $36 \times 19 = 684$. Use this result to determine:

   (a) $3,6 \times 1,9$ (b) $0,036 \times 0,19$ (c) $68,4 \div 0,19$

4. Simplify:

   (a) $(4,95x - 1,2) - (3,65x + 3,1)$ (b) $\frac{2,75x^{50}}{0,005x^{25}}$

5. Mulalo went to the shop and purchased two tubes of toothpaste for R6,98 each; three cans of cooldrink for R6,48 each, and five tins of baked beans for R7,95 each. If he pays with a R100 note, how much change should he get?

Chapter 5
Exponents

5.1 Revision

Remember that exponents are a shorthand way of writing repeated multiplication of the same number by itself. For example: $5 \times 5 \times 5 = 5^3$. The **exponent**, which is 3 in this example, stands for however many times the value is being multiplied. The number that is being multiplied, which is 5 in this example, is called the **base**.

If there are mixed operations, then the powers should be calculated before multiplication and division. For example: $5^2 \times 3^2 = 25 \times 9$.

You learnt these laws for working with exponents in previous grades:

Law	Example
$a^m \times a^n = a^{m+n}$	$3^2 \times 3^3 = 3^{2+3} = 3^5$
$a^m \div a^n = a^{m-n}$	$5^4 \div 5^2 = 5^{4-2} = 5^2$
$(a^m)^n = a^{m \times n}$	$(2^3)^2 = 2^{2 \times 3} = 2^6$
$(a \times t)^n = a^n \times t^n$	$(3 \times 4)^2 = 3^2 \times 4^2$
$a^0 = 1$	$32^0 = 1$

THE EXPONENTIAL FORM OF A NUMBER

1. Write the following in exponential notation:

(a) $2 \times 2 \times 2 \times 2 \times 2$ (b) $s \times s \times s \times s$ (c) $(-6) \times (-6) \times (-6)$
(d) $2 \times 2 \times 2 \times 2 \times s \times s \times s \times s$ (e) $3 \times 3 \times 3 \times 7 \times 7$ (f) $500 \times (1{,}02) \times (1{,}02)$

2. Write each of the numbers in exponential notation in some different ways, if possible:

(a) 81 (b) 125 (c) 1 000
(d) 64 (e) 216 (f) 1 024

ORDER OF OPERATIONS

1. Calculate the value of $7^2 - 4$.
Bathabile did the calculation like this: $7^2 - 4 = 14 - 4 = 10$
Nathaniel did the calculation differently: $7^2 - 4 = 49 - 4 = 45$
Which learner did the calculation correctly? Give reasons for your answer.

2. Calculate: $5 + 3 \times 2^2 - 10$, with explanations.

3. Explain how to calculate $2^6 - 6^2$.

4. Explain how to calculate $(4 + 1)^2 + 8 \times \sqrt[3]{64}$.

LAWS OF EXPONENTS

1. Use the laws of exponents to simplify the following (leave answer in exponential form):
   (a) $2^2 \times 2^4$ (b) $3^4 \div 3^2$ (c) $3^0 + 3^4$
   (d) $(2^3)^2$ (e) $(2 \times 5)^2$ (f) $(2^2 \times 7)^3$

2. Copy and complete the table. Substitute the given number for y. The first column has been done as an example.

	y	2	3	4	5
(a)	$y \times y^4$	2×2^4 $= 2^{1+4}$ $= 2^5$ $= 32$			
(b)	$y^2 \times y^3$	$2^2 \times 2^3$ $= 2^{2+3}$ $= 4 \times 8$ $= 32$			
(c)	y^5	$2^5 = 32$			

3. Are the expressions $y \times y^4$, $y^2 \times y^3$ and y^5 equivalent? Explain.

4. Copy and complete the table. Substitute the given number for y.

	y	2	3	4	5
(a)	$y^4 \div y^2$	$2^4 \div 2^2$ $= 16 \div 4$ $= 4$			
(b)	$y^3 \div y^1$	$2^3 \div 2^1$ $= 8 \div 2$ $= 4$			
(c)	y^2	$2^2 = 4$			

5. (a) From the table, is $y^4 \div y^2 = y^3 \div y^1 = y^2$? Explain.
   (b) Evaluate $y^3 \div y^1$ for $y = 15$.

6. Copy and complete the following table:

	x	2	3	4	5
(a)	2×5^x	2×5^2 $= 2 \times 25$ $= 50$			
(b)	$(2 \times 5)^x$	$(2 \times 5)^2$ $= 10^2$ $= 100$			
(c)	$2^x \times 5^x$	$2^2 \times 5^2$ $= 4 \times 25$ $= 100$			

7. (a) From the table above, is $2 \times 5^x = (2 \times 5)^x$? Explain.
   (b) Which expressions in question 6 are equivalent? Explain.

8. Below is a calculation that Wilson did as homework. Mark each problem correct or incorrect and explain the mistakes.
   (a) $b^3 \times b^8 = b^{24}$
   (b) $(5x)^2 = 5x^2$
   (c) $(-6a) \times (-6a) \times (-6a) = (-6a)^3$

5.2 Integer exponents

5^4 means $5 \times 5 \times 5 \times 5$. The exponent 4 indicates the number of appearances of the repeated factor.

What may a negative exponent mean, however? For example, what may 5^{-4} mean?

Mathematicians have decided to use negative exponents to indicate repetition of the multiplicative inverse of the base, for example 5^{-4} is used to indicate $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ or $(\frac{1}{5})^4$, and x^{-3} is used to indicate $(\frac{1}{x})^3$, which is $\frac{1}{x} \times \frac{1}{x} \times \frac{1}{x}$.

This decision was not taken blindly – mathematicians were well aware that it makes good sense to use negative exponents in this meaning. One major advantage is that the negative exponents, when used in this meaning, have the same properties as positive exponents, for example:

$2^{-3} \times 2^{-4} = 2^{(-3)+(-4)} = 2^{-7}$ because $2^{-3} \times 2^{-4}$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ which is $(\frac{1}{2})^7$.

$2^{-3} \times 2^4 = 2^{(-3)+4} = 2^1$ because $2^{-3} \times 2^4$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (2 \times 2 \times 2 \times 2)$ which is 2.

NEGATIVE EXPONENTS

1. Express each of the following in the exponential notation in two ways: with positive exponents and with negative exponents:
   (a) $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$
   (b) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$
2. In each case, check whether the statement is true or false. If it is false, write a correct statement. If it is true, give reasons why you say so.
   (a) $10^{-3} = 0{,}001$
   (b) $3^{-5} \times 9^2 = 3$
   (c) $25^2 \times 10^{-6} \times 2^6 = 5$
   (d) $(\frac{1}{5})^{-4} = 5^4$
3. Calculate each of the following, without using a calculator:
   (a) $10^{-3} \times 20^4$
   (b) $(\frac{1}{5})^{-4}$
4. (a) Use a scientific calculator to determine the decimal values of the given powers.

   Example: To find 3^{-1} on your calculator, use the key sequence: 3 y^x 1 ± =

Power	2^{-1}	5^{-1}	$(-2)^{-1}$	$(0{,}3)^{-1}$	0^{-1}	10^{-1}	10^{-2}
Decimal value							

   (b) Explain the meaning of 10^{-3}.

5. Determine the value of each of the following in two ways:
   A. By using the definition of powers (for example, $5^2 \times 5^0 = 25 \times 1 = 25$).
   B. By using the properties of exponents (for example, $5^2 \times 5^0 = 5^{2+0} = 5^2 = 25$).

   (a) $(3^3)^{-2}$
   (b) $4^2 \times 4^{-2}$
   (c) $5^{-2} \times 5^{-1}$
6. Calculate the value of each of the following. Express your answers as common fractions.
   (a) 2^{-3}
   (b) $3^2 \times 3^{-2}$
   (c) $(2 + 3)^{-2}$
   (d) $3^{-2} \times 2^{-3}$
   (e) $2^{-3} + 3^{-3}$
   (f) 10^{-3}
   (g) $2^3 + 2^{-3}$
   (h) $(3^{-1})^{-1}$
   (i) $(2^{-3})^2$
7. Which of the following are true? Correct any false statement.
   (a) $6^{-1} = -6$
   (b) $3x^{-2} = \frac{1}{3x^2}$
   (c) $3^{-1}x^{-2} = \frac{1}{3x^2}$
   (d) $(ab)^{-2} = \frac{1}{a^2b^2}$
   (e) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$
   (f) $\left(\frac{1}{3}\right)^{-1} = 3$

5.3 Solving simple exponential equations

An exponential equation is an equation in which the variable is in the exponent. So, when you solve exponential equations, you are solving questions of the form; *"To what power must the base be raised for the statement to be true?"*

To solve this kind of equation, remember that:

If $a^m = a^n$, then $m = n$.

In other words, if the base is the same on either side of the equation, then the exponents are the same.

Example:

$3^x = 243$

$3^x = 3^5$ (rewrite using the same base)

$x = 5$ (since the bases are the same, we equate the exponents)

Some exponential equations are slightly more complex:

Examples:

$3^{x+3} = 243$

$3^{x+3} = 3^5$ (rewrite using the same base)

$x + 3 = 5$ (equate the exponents)

$x = 2$

Check: LHS $3^{2+3} = 3^5 = 243$

$3^{x+3} = 1$ (remember $1 = 3^0$)

$3^{x+3} = 3^0$ (rewrite using the same base)

$x + 3 = 0$ (equate the exponents)

$x = -3$

Remember that the exponent can also be negative. However, you follow the same method to solve these kinds of equations.

Example: $2^x = \frac{1}{32}$

$2^x = 2^{-5}$ (rewrite using the same base)

$x = -5$ (equate the exponents)

SOLVING EXPONENTIAL EQUATIONS

1. Use the following table to answer questions that follow:

x	2	3	4	5
2^x	4	8	16	32
3^x	9	27	81	243
5^x	25	125	625	3 125

Find the value of x:

(a) $2^x = 32$ (b) $3^x = 81$ (c) $5^x = 3\,125$

(d) $2^x = 8$ (e) $5^x = 625$ (f) $3^x = 9$

(g) $5^{x+1} = 25$ (h) $3^{x+2} = 27$ (i) $2^{x-1} = 8$

2. Solve these exponential equations. You may use your calculator if necessary.

(a) $4^x = \frac{1}{64}$ (b) $6^{2x} = 1\,296$ (c) $2^{x-1} = \frac{1}{8}$

(d) $3^{x+2} = \frac{1}{729}$ (e) $5^{x+1} = 15\,625$ (f) $2^{x+3} = \frac{1}{4}$

(g) $4^{x+3} = \frac{1}{256}$ (h) $3^{2-x} = 81$ (i) $5^{3x} = \frac{1}{125}$

5.4 Scientific notation

Scientific notation is a way of writing numbers that are too big or too small to be written clearly in decimal form. The diameter of a hydrogen atom, for example, is a very small number. It is 0,000000053 mm. The distance from the sun to the earth is, on average, 150 000 000 km.

In scientific notation, the diameter of the hydrogen molecule is written as $5{,}3 \times 10^{-8}$ and the distance from the sun to the earth is written as $1{,}5 \times 10^{8}$. It is easier to compare and to calculate numbers like these, as it is very cumbersome to count the zeros when you work with these numbers.

Look at more examples below:

Decimal notation	Scientific notation
6 130 000	$6{,}13 \times 10^{6}$
0,00001234	$1{,}234 \times 10^{-5}$

A number written in scientific notation is written as the product of two numbers, in the form $\pm a \times 10^{n}$. Here, a is a decimal number between 1 and 10, and n is an integer.

Any number can be written in scientific notation, for example:

$40 = 4{,}0 \times 10$

$2 = 2 \times 10^{0}$

The decimal number 324 000 000 is written as $3{,}24 \times 10^{8}$ in scientific notation, because the decimal comma is moved eight places to the left to form 3,24.

The decimal number 0,00000065 written in scientific notation is $6{,}5 \times 10^{-7}$, because the decimal point is moved seven places to the right to form the number 6,5.

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WRITING VERY SMALL AND VERY LARGE NUMBERS

1. Express the following numbers in scientific notation:
   (a) 134,56
   (b) 0,0000005678
   (c) 876 500 000
   (d) 0,0000000000321
   (e) 0,006789
   (f) 89 100 000 000 000
   (g) 0,001
   (h) 100
2. Express the following numbers in ordinary decimal notation:
   (a) $1{,}234 \times 10^{6}$
   (b) 5×10^{-1}
   (c) $4{,}5 \times 10^{5}$
   (d) $6{,}543 \times 10^{-11}$
3. Why do we say that 34×10^{3} is not written in scientific notation? Rewrite it in scientific notation.
4. Is each of these numbers written in scientific notation? If not, rewrite it so that it is in scientific notation.
   (a) $90{,}3 \times 10^{-5}$
   (b) 100×10^{2}
   (c) $1{,}36 \times 10^{5}$
   (d) $2{,}01 \times 10^{-2}$
   (e) $0{,}01 \times 10^{3}$
   (f) $0{,}6 \times 10^{8}$

CALCULATIONS USING SCIENTIFIC NOTATION

Example: $123\,000 \times 4\,560\,000$

$= 1{,}23 \times 10^{5} \times 4{,}56 \times 10^{6}$ (write in scientific notation)

$= 1{,}23 \times 4{,}56 \times 10^{5} \times 10^{6}$ (multiplication is commutative)

$= 5{,}6088 \times 10^{11}$ (Use a calculator to multiply the decimals, but add the powers mentally.)

1. Use scientific notation to calculate each of the following. Give the answer in scientific notation.
   (a) 135 000 × 246 000 000
   (b) 987 654 × 123 456
   (c) 0,000065 × 0,000216
   (d) 0,000000639 × 0,0000587

Example: $5 \times 10^{3} + 4 \times 10^{4}$

$= 0{,}5 \times 10^{4} + 4 \times 10^{4}$ (Form like terms)

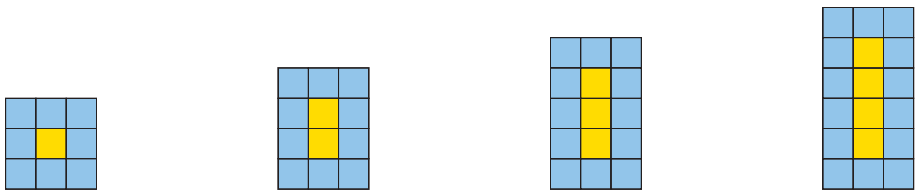
$= 4{,}5 \times 10^{4}$ (Combine like terms)

2. Calculate the following. Leave the answer in scientific notation.
   (a) $7{,}16 \times 10^{5} + 2{,}3 \times 10^{3}$
   (b) $2{,}3 \times 10^{-4} + 6{,}5 \times 10^{-3}$
   (c) $4{,}31 \times 10^{7} + 1{,}57 \times 10^{6}$
   (d) $6{,}13 \times 10^{-10} + 3{,}89 \times 10^{-8}$

CHAPTER 6
Patterns

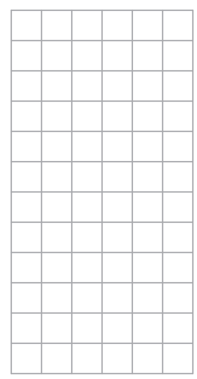
6.1 Geometric patterns

INVESTIGATING AND EXTENDING



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

1. Blue and yellow square tiles are combined to form the above arrangements.
   (a) How many yellow tiles are there in each arrangement?
   (b) How many blue tiles are there in each arrangement?
   (c) If more arrangements are made in the same way, how many blue tiles and how many yellow tiles will there be in arrangement 5? Check your answer by drawing the arrangement onto grid paper.

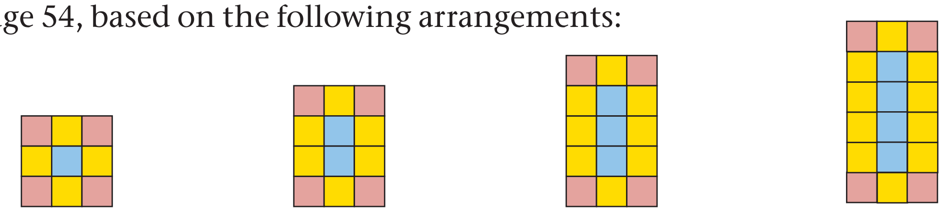


   (d) Copy and complete the following table:

Number of yellow tiles	1	2	3	4	5	8
Number of blue tiles						

   (e) How many blue tiles will there be in a similar arrangement with 26 yellow tiles?
   (f) How many blue tiles will there be in a similar arrangement with 100 yellow tiles?
   (g) Describe how you thought to produce your answer for (f)?

2. (a) In these arrangements there are red tiles too. Copy and complete the table on page 54, based on the following arrangements:



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

Number of blue tiles	1	2	3	4	5	6	7
Number of yellow tiles							
Number of red tiles							

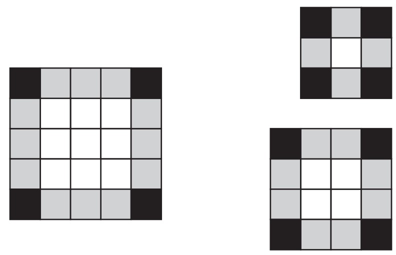
(b) How many red tiles are there in each arrangement?
(c) How many yellow tiles are there in each arrangement?

The number of red tiles in arrangements like those in question 2, is **constant.** It is always four, no matter how many blue and yellow tiles there are.

The number of blue tiles is different for different arrangements. We can say the number of blue tiles **varies**. We can also say the number of blue tiles is a **variable**.

3. Is the number of yellow tiles in the above arrangements a constant or is it a variable?

4. Look at the arrangements on the right. They consist of black squares, grey squares and white squares.



   (a) Draw another arrangement of the same kind, but with a different length, on grid paper.
   (b) Describe what is constant in these arrangements.
   (c) What are the variables in these arrangements?

The smallest arrangement above may be called arrangement 1, the next bigger one may be called arrangement 2, and so on.

5. (a) Copy and complete the table for arrangements like those in question 4.

Arrangement number	1	2	3	4	5	6	7	10	20
Number of black squares									
Number of grey squares									
Number of white squares									

   (b) How many grey squares do you think there will be in arrangement 15? Explain your answer.
   (c) How many black squares do you think there will be in arrangement 15? Explain your answer.

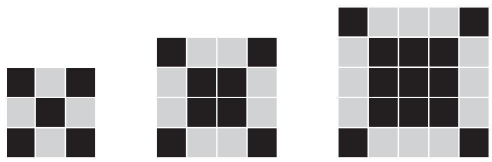
(d) How many white squares do you think there will be in arrangement 15? Explain your answer.

The numbers of grey squares in the different arrangements in question 4 form a pattern: 4; 8; 12; 16; 20; 24; . . . , and so on.

The numbers of white squares in the different arrangements also form a pattern: 1; 4; 9; 16; 25; 36; 49; . . . , and so on.

6. What are the next five numbers in each of the above patterns?

7. (a) On grid paper, draw the next arrangement that follows the same pattern:



(b) How many black tiles are there in the arrangement you have drawn?

(c) How many black tiles will there be in each of the next four arrangements?

DO SOMETHING MORE

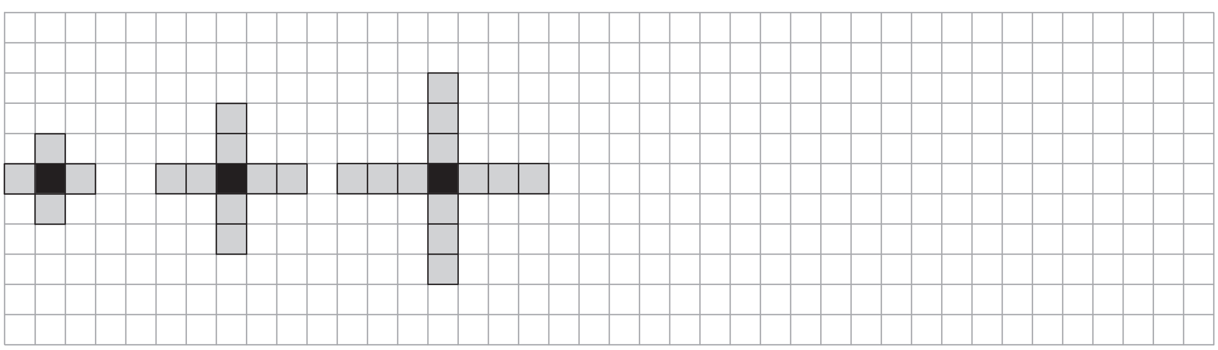
Consider the arrangements in question 4 on page 54 again. If there are 20 grey tiles in such an arrangement, how many white tiles are there? Copy and complete the table, entering your answer in the table.

Number of grey squares	20	36	52			
Number of white squares				256	225	625

6.2 More patterns

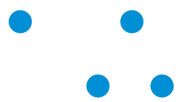
DRAWING AND INVESTIGATING

1. (a) On grid paper, make two more arrangements of black and grey squares so that a pattern is formed.



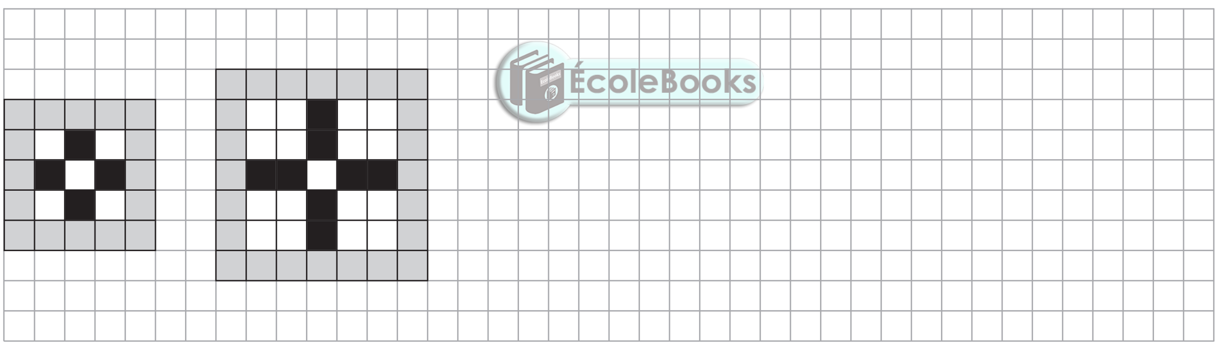
(b) Is there a constant in your pattern? If yes, what is its value?
(c) Is there a variable in your pattern? If yes, give the values of the variable.

2. (a) Make three more arrangements with dots to form the sequence 1; 3; 6; 10; 15 . . .



(b) How many dots will there be in the sixth and seventh arrangements? Explain how you got your answer.
(c) How many dots are there in arrangements 1 and 2 together?
(d) How many dots are there in arrangements 2 and 3 together?
(e) How many dots are there in arrangements 3 and 4 together?
(f) How many dots are there in arrangements 4 and 5 together?
(g) Describe the pattern in your answers for (c), (d), (e) and (f).

3. (a) On grid paper, draw two more arrangements to make a pattern.



(b) What are the variables in your pattern?
(c) The number of black squares is a variable in these arrangements. The value of this variable is four in the first arrangement and eight in the second arrangement. What is the value of this variable in the third arrangement?
(d) What are the values of each of the variables in the fifth arrangement in your pattern? Explain your answers.

4. (a) Now, on grid paper, make a pattern of your own.

(b) Copy this table and use it to describe the variables in your pattern, and their values:

Arrangement number	1	2	3	4	5	6

6.3 Different kinds of patterns in sequences

DO THE SAME THING REPEATEDLY

1. (a) Write the next three numbers in each of the sequences below.

Sequence A: 5 9 13 17 21 ....

Sequence B: 5 10 20 40 80 ....

Sequence C: 5 10 17 26 37 ....

(b) Describe the differences in the ways in which the three sequences are formed.

2. You will now make a sequence with the first term 5. Write 5 on the left on the line below. Then add 8 to the first term (5) to form the second term of your sequence. Write the second term next to the first term (5) in the line below. Now add 8 to the second term to form the third term. Continue like this to form ten more terms.

The numbers in a sequence are also called the **terms** of the sequence.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between consecutive terms in a sequence is **constant**.

A sequence can be formed by repeatedly multiplying or dividing. In this case the **ratio** between consecutive terms is **constant**.

A sequence can also be formed in such a way that neither the difference nor the ratio between consecutive terms is constant.

To write more terms of sequence A in question 1(a), you **added** 4 **repeatedly**.

To write more terms of sequence B in question 1(a), you **multiplied** by 2 **repeatedly**.

To write more terms of sequence C in question 1(a) you did not add the same number each time, nor did you multiply by the same number.

3. Write the next three terms of each sequence. In each case also describe what the pattern is, for example "there is a constant difference of –5 between consecutive terms".

(a) 16; 8; 0; –8; ...
(b) 1; 4; 9; 16; ...
(c) 2; 8; 18; 32; ...
(d) 3; 6; 11; 18; ...
(e) 640; 320; 160; ...
(f) 1; 2; 4; 7; 11; ...

4. In each case, follow the instruction to make a sequence with eight terms.
   (a) Start with 1 and multiply by 2 repeatedly.
   (b) Start with 256 and subtract 32 repeatedly.
   (c) Start with 256 and divide by 2 repeatedly.

The sequence that you made in question 2 can be represented with a table like the one shown below:

Term number	1	2	3	4	5	6	7	8	9	10
Term value	5	13	21	29	37	45	53	61	69	77

5. In each case make a sequence by following the instructions. Copy the tables and write the term numbers and the term values in the tables.
   (a) Term 1 = 10. Add 15 repeatedly.

Term number								
Term value								

   (b) Term 1 = 10. Term value = 15 × term number – 5.

Term number								
Term value								

   (c) Term 1 = 10. Multiply by 2 repeatedly.

Term number						
Term value						

   (d) Term 1 = 20. Term value = $10 \times 2^{\text{term number}}$

Term number						
Term value						

(e) Term 1 = 10. Term value = $10 \times 2^{\text{term number} - 1}$

Term number						
Term value						

(f) Term 4 = 30. Add 5 repeatedly.

Term number								
Term value								

6. Instructions for forming a sequence are given in two different ways in question 5. How would you describe the two different ways for giving instructions to form a sequence?

6.4 Formulae for sequences

The formula for a number sequence can be written in two different ways:

- A description of the **relationship between consecutive terms**: In other words, the calculations that you do to a term to produce the next term, as in questions 5(a), (c) and (f) on the previous page. The first (or another) term must be given. This kind of formula has two parts: the first term and the relationship between terms.
- A description of the **relationship between the value of the term and its position in the sequence**: This relationship describes the calculations that can be done **on the term number** to produce the **term value**, as in question 5(b), (d) and (e) on the previous page.

MAKE TWO FORMULAE FOR THE SAME SEQUENCE

1. Choose any whole number smaller than 10 as the first term of a sequence.
   (a) Copy the table. Use your chosen first term to form a sequence by adding 5 repeatedly.
   (b) Multiply each term number below by 5 to form a sequence:

Term number	1	2	3	4	5	6	7	8
Term value								

   (c) What is similar about the two sequences you have formed?

(d) Now fill in your own sequence in the same table:

Term number	1	2	3	4	5	6	7	8
Term value in (b)								
Term value of your own sequence in (a)								

(e) What must you add to or subtract from each term value in (b) to get the same sequence as the one you made in (a)?

(f) Copy and fill in the following to write a formula for each sequence:
For the sequence in (b): Term value = ... (term number)
For the sequence in (a): Term value = ... (term number)

2. Now you are going to repeat what you did in question 1, with a different set of sequences. In this sequence, the term number is multiplied by 3 to get the term value.

Term number	1	2	3	4	5	6	7	8
Term value	3	6	9	12	15	18	21	24

Now make a formula describing the relationship of the **term value** to the **term number** for each of these sequences:

(a) The sequence that starts with 8 and is formed by adding 3 repeatedly.
(b) The sequence that starts with 12 and is formed by adding 3 repeatedly.
(c) The sequence that starts with 2 and is formed by adding 3 repeatedly.

3. Copy the tables. Write the first eight terms of each of the following sequences and in each case, describe how each term can be calculated from the previous term.

(a) Term value = 10 × term number + 5

Term number	1	2	3	4	5	6	7	8
Term value								

(b) Term value = 5 × term number – 3

Term number	1	2	3	4	5	6	7	8
Term value								

4. For each sequence, write a formula to obtain each term from the previous term. Try to write a formula which relates each term to its position in the sequence. Check both your formulae by applying them, and write the results in a table like the one below.
   (a) 7 11 15 19 23 27 31 35 39 43
      A. Relationship between consecutive terms

      B. Relationship between term value and its position in sequence

Term number	1	2	3	4	5
Term value using A					
Term value using B					

   (b) 60 57 54 51 48 45 42 39 36
      A. Relationship between consecutive terms

      B. Relationship between term value and its position in sequence

Term number	1	2	3	4	5
Term value using A					
Term value using B					

   (c) 1 2 4 8 16 32 64 128
      A. Relationship between consecutive terms

      B. Relationship between term value and its position in sequence

Term number	1	2	3	4	5
Term value using A					
Term value using B					

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Chapter 7
Functions and relationships

7.1 Find output numbers for given input numbers

TWO DIFFERENT SETS OF INPUT NUMBERS

In this activity you will do some calculations with:

- Set A: the natural numbers smaller than 10, i.e. 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- Set B: multiples of 10 that are bigger than 10 but smaller than 100, i.e. the numbers 20, 30, 40, 50, 60, 70, 80 and 90.

1. You are going to choose a number, multiply it by 5, and subtract the answer from 50.
   (a) Choose any number from set A and do the above calculations.
   (b) Choose any number from set B and do the above calculations.
   (c) If you choose any other number from set B, do you think the answer will also be a negative number?

Output numbers are numbers that you obtain when you apply the rule to the input numbers.

2. (a) Write down all the different output numbers that will be obtained when the calculations $50 - 5x$ are performed on the different numbers in set A.
   (b) Write down the output numbers that will be obtained when the formula $50 - 5x$ is applied to set B.

3. (a) Copy and complete the following table for set A:

Input numbers	1	2	3	4	5	6	7	8	9
Values of $50 - 5x$									

   (b) Copy and complete the following table for set B:

Input numbers	20	30	40	50	60	70	80	90
Values of $50 - 5x$								

4. In this question your set of input numbers will be the even numbers: 2; 4; 6; 8; 10; ...
   (a) What will all the output numbers be if the rule $2n + 1$ is applied to the set of even numbers? Write a list.

(b) What will the output numbers be if the rule $2n - 1$ is applied?
(c) What will the output numbers be if the rule $2n + 5$ is applied?
(d) What will the output numbers be if the rule $3n + 1$ is applied?

5. (a) What kind of output numbers will be obtained by applying the rule $x - 1\,000$ to natural numbers smaller than 1 000?
(b) What kind of output numbers will be obtained by applying the rule $\frac{x}{10} + 10$ to natural numbers smaller than 10?
(c) If you use the rule $30x + 2$, and use input numbers that are positive fractions with denominators 2, 3 and 5, what kind of output numbers will you obtain?

7.2 Different ways to represent the same relationship

Consider the work that you did in Section 6.4 of Chapter 6. In each question, there were two variable quantities.

A quantity that changes is called a **variable quantity** or just a **variable**.

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. You can sometimes work out which number is linked to a specific value of the other variable.

An algebraic expression, such as $10x + 5$, describes what calculations must be done to find the output number that corresponds to a given input number.

The output number can also be called the output value, or the value of the expression, which is $10x + 5$ in this case.

For any input number, application of the rule $10x + 5$ produces only one output number, and it is very clear what that number is. For instance, if the formula is applied to $x = 3$, the output number is 35.

A relationship between two variables in which there is only one output number for each input number, is called a **function**.

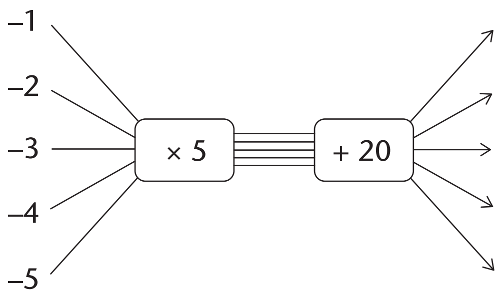
Functions can be represented in:

- a table that shows some values of the two variables as it clearly shows which value of the output variable corresponds to each particular value of the input variable
- a flow diagram, which shows what calculations are to be done to calculate the output number that corresponds to a given input variable

- a formula, which also describes what calculations are to be done to calculate the output number that corresponds to a given input variable
- a graph.

Examples of these four ways of describing a function are given on the next two pages.

1. Copy and complete the following flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you completed shows:

- that each input number is multiplied by 5, then 20 is added, to produce the output numbers
- which output numbers correspond to which input numbers.

The calculations that need to be done can also be described with an expression. The expression $5x + 20$ describes the calculations that you did in question 1. You can also write this as a formula:

- A **verbal formula**:
  output number = 5 × input number + 20
- An **algebraic formula**:
  output number = $5x + 20$

The output numbers of a function are also called **function values**. Hence the formula can also be written as *function value* = $5x + 20$.

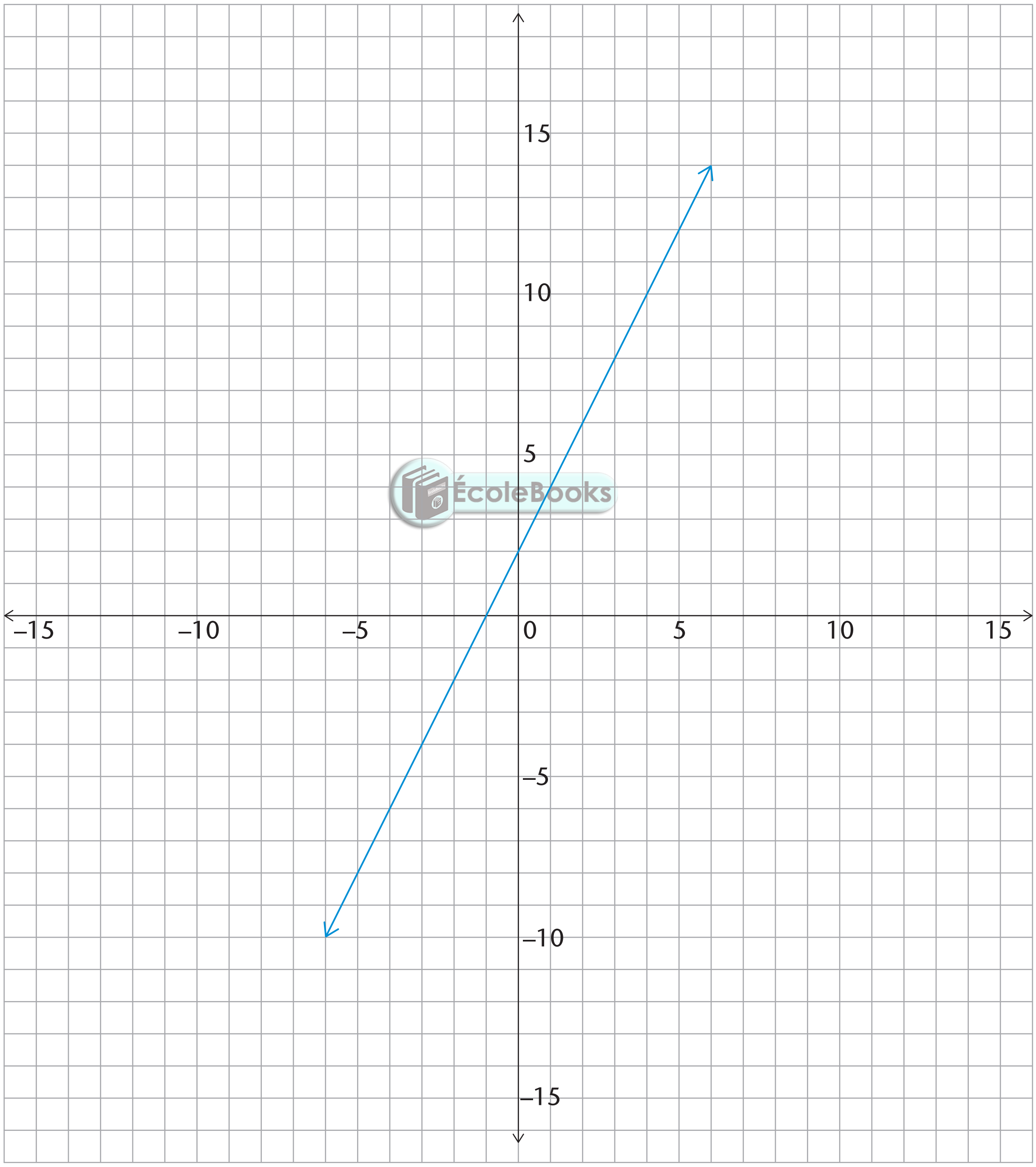
2. Copy and complete this table for the function described by $5x + 20$:

Input numbers	–1	–2	–3	–4	–5
Function values					

3. Draw a graph of this function discussed in question 1 and 2 on graph paper.

4. A graph of a certain function is given below. Copy and complete the table for this function:

Input numbers					
Function values					



7.3 Different representations of the same relationship

On separate pages, represent each of the following functions with the following:

(a) a flow diagram
(b) a table of values for the set of integers from –5 to 5
(c) a graph

1. The relationship described by the expression $3x + 4$.
2. The relationship described by the expression $2x - 5$.
3. The relationship described by the expression $\frac{1}{2}x + 2$.
4. The relationship described by the expression $-3x + 4$.
5. The relationship described by the expression $2{,}5x + 1{,}5$.
6. The relationship described by the expression $0{,}2x + 1{,}4$.
7. The relationship described by the expression $-2x - 4$.

Chapter 8
Algebraic expressions

8.1 Algebraic language

WORDS, DIAGRAMS AND EXPRESSIONS

1. Copy and complete the following table:

	Words	Flow diagram	Expression
	Multiply a number by 5 and then subtract 3 from the answer.	→ [× 5] → [− 3] →	$5x - 3$
(a)	Add 5 to a number and then multiply the answer by 3.	→ [ ] → [ ] →	
(b)		→ [− 3] → [× 5] →	
(c)			$3(2x + 3)$

An **algebraic expression** indicates **a sequence of operations** that can also be described in words. In some cases they can be described with flow diagrams.

Expressions in brackets should always be calculated first. If there are no brackets in an algebraic expression, it means that multiplication and division must be done first, and addition and subtraction afterwards.

For example, if $x = 5$ the expression $12 + 3x$ means "multiply 5 by 3, then add 12". It does **not** mean "add 12 and 3, then multiply by 5".

If you wish to say "add 12 and 3, then multiply by 5", the numerical expression should be $5 \times (12 + 3)$ or $(12 + 3) \times 5$.

2. Describe each of these sequences of calculations with an algebraic expression:
   (a) Multiply a number by 10, subtract 5 from the answer, and multiply the answer by 3.

(b) Subtract 5 from a number, multiply the answer by 10, and multiply this answer by 3.

3. Evaluate each of these expressions for $x = 10$:

(a) $200 - 5x$
(b) $(200 - 5)x$
(c) $5x + 40$
(d) $5(x + 40)$
(e) $40 + 5x$
(f) $5x + 5 \times 40$

SOME WORDS WE USE IN ALGEBRA

- An expression with one term only, like $3x^2$, is a **monomial**.
- An expression which is a sum of two terms, like $5x + 4$, is called a **binomial**.
- An expression which is a sum of three terms, like $3x^3 + 2x + 9$, is called a **trinomial**.

The symbol x is often used to represent the **variable** in an algebraic expression, but other letter symbols may also be used.

In the monomial $3x^2$, the 3 is called the **coefficient** of x^2.

In the binomial $5x + 4$, and the trinomial $3x^2 + 2x + 9$, the numbers 4 and 9 are called **constants**.

1. Copy and complete the table, using the completed first row as an example:

Expression	Type of expression	Symbol used to represent the variable	Constant	Coefficient of ...
$x^2 + 6x + 10$	Trinomial	x	10	the second term is: 6
$6s^3 + s^2 + 5$				s^2 is:
$\frac{k}{3} + 12$				the first term is:
$4p^{10}$				p^{10} is:

2. Consider the polynomial pattern starting with $7x^5 + 5x^4 + 3x^3 + x^2 + \ldots$

(a) What is the coefficient of the fourth term?
(b) What is the exponent value of the fifth term?
(c) Do you think the sixth term will be a constant? Why?

EQUIVALENT ALGEBRAIC EXPRESSIONS

1. Copy and complete the table on page 69 by doing the necessary calculations. Calculate the numerical value of the expressions for the various values of x.

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	x	–2	–1	0	1	2
(a)	$3x + 2$					
(b)	$2x – 3$					
(c)	$3x + 2 + 2x – 3$					
(d)	$2x – 3 + 3x + 2$					
(e)	$5x – 1$					
(f)	$(3x + 2)(2x – 3)$					
(g)	$3x(2x – 3) + 2(2x – 3)$					
(h)	$6x^2 – 5x – 6$					
(i)	$\frac{(3x+2)(2x-3)}{3x+2}$					
(j)	$\frac{6x^2-5x-6}{3x+2}$					

2. Although they may look different, make a list of all the algebraic expressions above which have the same numerical value for the same value of x.

Equivalent expressions are algebraic expressions that have different sequences of operations, but have the same numerical value for any given value of x.

It is often convenient not to work with a given expression, but to **replace** it with an equivalent expression.

3. Copy and complete the following table:

x	2	3	5	10	–5	–10
$12x – 7 + 3x + 10 – 5x$						

4. Copy and complete the following table:

x	2	3	5	10	–5	–10
$10x + 3$						

5. (a) Is $10x + 3$ equivalent to $12x – 7 + 3x + 10 – 5x$? Explain your answer.

(b) Suppose you need to know how much $12x - 7 + 3x + 10 - 5x$ is for $x = 37$ and $x = -43$. What do you think is the easiest way to find out?

CONVENTIONS FOR WRITING ALGEBRAIC EXPRESSIONS

Here are some things that mathematicians have agreed upon, and it makes mathematical work much easier if all people stick to these agreements.

A **convention** is something that people have agreed to do in the same way.

The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$, and $4(x - 5)$ instead of $4 \times (x - 5)$.

It is a convention to write a known number first in a product, i.e. we write $3 \times x$ rather than $x \times 3$, and we write $3x$ but **not** $x3$.

1. Rewrite each of the following in the normal way of writing algebraic expressions:
   (a) $x \times 4 + x \times y - y \times 3$
   (b) $7 \times (10 - x) + (5 \times x + 3)10$

People all over the world have agreed that, in expressions that do not contain brackets, addition and subtraction should be performed as they appear from left to right.

According to this convention, $x - y + z$ means that you first have to subtract y from x, then add z. For example if $x = 10$, $y = 5$ and $z = 3$, $x - y + z$ is $10 - 5 + 3$ and it means $10 - 5 = 5$, then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10 - 8 = 2$.

2. Calculate $50 - 20 + 30$ and $50 + 30 - 20$ and $50 - 30 + 20$.

3. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$:
   (a) $x + y - z$
   (b) $x - z + y$
   (c) $10y - 3x + 5z - 4y$
   (d) $10y - 3x - 5z + 4y + 3x$

People have also agreed that, in expressions that do not contain brackets, we should do multiplication (and division) **before** addition and subtraction.

Hence, $5 + 3 \times 4$ should be understood as "multiply 4 by 3, then add the answer to 5"; not as "add 5 and 3 then multiply the answer by 4".

Also, $3 \times 4 + 5$ should be understood as "multiply 4 by 3, then add 5 to the answer"; not as "add 4 and 5 then multiply the answer by 3".

4. Do each of the following calculations:
   (a) Multiply 4 by 3, then add 5 to the answer.
   (b) Add 4 and 5, then multiply the answer by 3.
   (c) Multiply 4 by 3, then add the answer to 5.
   (d) Add 5 and 3, then multiply the answer by 4.

5. Rewrite the instructions in 4(a) and 4(c) without using words.

6. Calculate each of the following:
   (a) $10 \times 5 + 30$ (b) $30 + 10 \times 5$
   (c) $10 \times 5 - 30$ (d) $30 - 10 \times 5$

7. (a) Add 4 and 5, then subtract the answer from 20.
   (b) Subtract 4 from 20 and then add 5.
   (c) Add 4 and 5, then multiply the answer by 3.
   (d) Multiply 3 by 5 and then add the answer to 4.

If we want to specify the calculations in 7(a) and 7(c) without using words, we will face challenges.

We cannot write $20 - 4 + 5$ for "*add 4 and 5 then subtract the answer from 20*", because that would mean "*subtract 4 from 20, then add 5*". We need a way to indicate, without using words, that we want the addition to be performed before the subtraction in this case.

Similarly, we cannot write $4 + 5 \times 3$ for "*add 4 and 5 then multiply the answer by 3*", because that would mean "*multiply 3 by 5 and then add the answer to 4*". We need a way to indicate, without using words, that we want the addition to be performed *before* the multiplication in this case.

Mathematicians have agreed to use brackets to address the above challenges. The following convention is used all over the world:

> Whenever there are brackets in an expression, the calculations within the brackets should be performed first.

Hence, $20 - (4 + 5)$ means *"add 4 and 5 then subtract the answer from 20"*, but $20 - 4 + 5$ means "*subtract 4 from 20, then add 5*".

$(4 + 5) \times 3$ or $3 \times (4 + 5)$ means "*add 4 and 5 then multiply the answer by 3*", but $4 + 5 \times 3$ means "*multiply 3 by 5, then add the answer to 4*".

$10 + 2(5 + 9)$ means "*add 5 and 9, multiply the answer by 2, then add this answer to 10*":

$5 + 9 = 14$ $14 \times 2 = 28$ $28 + 10 = 38$

8. Calculate each of the following:

(a) $100 + 50 - 30$
(b) $100 + (50 - 30)$
(c) $100 - 50 + 30$
(d) $100 - (50 + 30)$
(e) $3(10 - 4) + 2$
(f) $10(5 + 7) + 3(18 - 8)$
(g) $250 - 10 \times (18 + 2) + 35$
(h) $(20 + 20) \times (20 - 10)$
(i) $(250 - 10) \times (18 + 2) + 35$
(j) $20 + 20 \times (20 - 10)$
(k) $200 + \mathbf{(}100 \times 2(15 + 5)\mathbf{)}$
(l) $(200 + 100) \times 2 \times 15 + 5$

In algebra, we normally write $3(x + 2y)$ instead of $(x + 2y) \times 3$, and we write $3(x - 2y)$ instead of $(x - 2y) \times 3$. Do not let this conventional way of writing in algebra confuse you. The expression $3(x + 2y)$ does not mean that multiplication by 3 is the first thing you should do when you evaluate the expression for certain values of x and y. The first thing you should do is to add the values of x and y. That is what the brackets tell you!

However, performing the instructions $3(x + 2y)$ is not the only way in which you can find out how much $3(x + 2y)$ is for any given values of x and y. Instead of working out $3(x + 2y)$, you may work out $3x + 6y$. In this case you will multiply each term before you add them together.

9. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$:

(a) $xy + z$
(b) $x(y + z)$
(c) $x + yz$
(d) $xy + xz$
(e) $xy - z$
(f) $x(y - z)$
(g) $x - yz$
(h) $xy - yz$
(i) $x + (y - z)$
(j) $x - (y - z)$
(k) $x - (y + z)$
(l) $x - y - z$
(m) $x + y - z$
(n) $x - y + z$

8.2 Properties of operations

1. Calculate each of the following:

(a) $5(3 + 4)$
(b) $5 \times 3 + 5 \times 4$
(c) $6 \times 3 + (4 + 6)$
(d) $(6 + 4) + 3 \times 6$
(e) $3 \times (4 \times 5)$
(f) $(3 \times 4) \times 5$

You should have noticed that for each row the results are the same. This is because operations with numbers have certain properties, namely the **distributive**, **commutative** and **associative** properties.

The **distributive** property is used each time you multiply a number in parts. For example:

The number thirty-four is actually 30 + 4. You may calculate 5 × 34 by calculating 5 × 30 and 5 × 4, and then adding the two answers:

$$5 \times 34 = 5 \times 30 + 5 \times 4$$

The word "distribute" means to spread out. The distributive property may be described as follows:

$a(b + c) = ab + ac$

where a, b and c can be any numbers.

We may say: "multiplication distributes over addition".

2. Calculate each of the following:
   (a) $5(x - y)$ for $x = 10$ and $y = 8$
   (b) $5x - 5y$ for $x = 10$ and $y = 8$
   (c) $5(x - y)$ for $x = 100$ and $y = 30$
   (d) $5x - 5y$ for $x = 100$ and $y = 30$
   (e) $5(x - y + z)$ for $x = 10$, $y = 3$ and $z = 2$
   (f) $5x - 5y + 5z$ for $x = 10$, $y = 3$ and $z = 2$

3. We say "multiplication distributes over addition". Does multiplication also distribute over subtraction? Give examples to support your answer.

For any values of x and y:

- $x + y$ and $y + x$ give the same answers, and
- xy and yx give the same answers.

This is called the **commutative property** of addition and multiplication.

4. We say "addition is commutative" and "multiplication is commutative".
   Is subtraction also commutative? Demonstrate your answer with an example.

The **associative property** allows you to arrange three or more numbers in any sequence when adding or multiplying. For any values of x, y and z, the following expressions all have the same answer:

$$x + y + z \qquad y + x + z \qquad z + y + x$$

5. Calculate 16 + 33 + 14 + 17 in the easiest possible way.

The associative property of multiplication allows you to simplify something like the following:

$$abc + bca + cba$$

Because the order of multiplication does not change the result we can rewrite this expression as: $abc + abc + abc$.

This then can be simplified by adding like terms to be $3abc$. You will be able to use these properties throughout this chapter and when you do algebraic manipulations.

When you form an expression that is equivalent to a given expression, you say that you *manipulate* the expression.

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6. Replace each of the following expressions with a simpler expression that will give the same answer. **Do not do any calculations now**. In each case, state why your replacement will be easier to do.
   (a) $17 \times 43 + 17 \times 57$
   (b) $7 \times 5 \times 8 \times 4 + 12 \times 8 \times 4 \times 7 - 9 \times 4 \times 5 \times 8$
   (c) $43 \times 17 + 57 \times 17$
   (d) $43x + 57x$ (for $x = 213$ or any other value)

7. Which properties of operations did you use in each part of question 6?

8.3 Combining like terms in algebraic expressions

REARRANGE TERMS, THEN COMBINE LIKE TERMS

To check whether two expressions are possibly equivalent, you can evaluate both expressions for several different values of the variable.

1. In each case below, copy the tables, then predict whether the two expressions are equivalent. Check by evaluating both for $x = 1$, $x = 10$, $x = 2$ and $x = -2$ in the tables.
   (a) $x(x + 3)$ and $x^2 + 3$

   (b) $x(x + 3)$ and $x^2 + 3x$

Some expressions can be simplified by rearranging the terms and combining "like terms".

In the expression $5x^2 + 13x + 7 + 2x^2 - 8x - 12$, the terms $5x^2$ and $2x^2$ are like terms.

> Two or more like terms can be combined to form a single term.

$5x^2 + 2x^2$ can be replaced by $7x^2$ because for any value of x, for example $x = 2$ or $x = 10$, calculating $5x^2 + 2x^2$ and $7x^2$ will produce the same output value. Try it!

2. Copy and complete the following table:

x	10	2	5	1
$5x^2 + 2x^2$				
$7x^2$				
$13x - 8x$				
$5x$				

It is difficult to see the like terms in a long expression like $3x^2 + 13x + 7 + 2x^2 - 8x - 12$. Fortunately, you can rearrange the terms in an expression so that the like terms are next to each other.

3. (a) Copy the table and complete the second and third rows of the table. You will complete the next two rows when you do question 3(g).

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$				
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$				

(b) What do you observe?
(c) How does the one expression in the above table differ from the other one?
(d) Combine like terms in $3x^2 + 2x^2 + 13x - 8x + 7 - 12$ to make a shorter equivalent expression.
(e) Evaluate your shorter expression for $x = 10$, $x = 2$ and $x = 5$.
(f) Is your shorter expression equivalent to $3x^2 + 13x + 7 + 2x^2 - 8x - 12$? Explain how you know whether it is or is not.
(g) Evaluate $5x^2 + 5x - 5$ and $5(x^2 + x - 1)$ for $x = 10$, $x = 2$, $x = 5$ and $x = 1$, and write your answers in the last two rows of the table.

4. Simplify each expression:
(a) $(3x^2 + 5x + 8) + (5x^2 + x + 4)$
(b) $(7x^2 + 3x + 5) + (2x^2 - x - 2)$
(c) $(6x^2 - 7x - 4) + (4x^2 + 5x + 5)$
(d) $(2x^2 - 5x - 9) - (5x^2 - 2x - 1)$
(e) $(-2x^2 + 5x - 3) + (-3x^2 - 9x + 5)$
(f) $(y^2 + y + 1) + (y^2 - y - 1)$

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5. Copy and complete the table. (Hint: Save yourself some work by simplifying first!)

x	2,5	3,7	6,4	12,9	35	–4,7	–0,04
$(3x + 6,5) + (7x + 3,5)$							
$(13x - 6) + (26 - 12x)$							

6. Simplify:
   (a) $(2r^2 + 3r - 5) + (7r^2 - 8r - 12)$
   (b) $(2r^2 + 3r - 5) - (7r^2 - 8r - 12)$
   (c) $(2x + 5xy + 3y) - (12x - 2xy - 5y)$
   (d) $(2x + 5xy + 3y) + (12x - 2xy - 5y)$
7. Evaluate the following expressions for $x = 3$, $x = -2$, $x = 5$ and $x = -3$:
   (a) $2x(x^2 - x - 1) + 5x(2x^2 + 3x - 5) - 3x(x^2 + 2x + 1)$
   (b) $(3x^2 - 5x + 7) - (7x^2 + 3x - 5) + (5x^2 - 2x + 8)$
8. Write equivalent expressions without brackets:
   (a) $3x^4 - (x^2 + 2x)$
   (b) $3x^4 - (x^2 - 2x)$
   (c) $3x^4 + (x^2 - 2x)$
   (d) $x - (y + z - t)$
9. Write equivalent expressions without brackets, rearrange so that like terms are grouped together, and then combine the like terms:
   (a) $2y^2 + (y^2 - 3y)$
   (b) $3x^2 + (5x + x^2)$
   (c) $6x^2 - (x^4 + 3x^2)$
   (d) $2t^2 - (3t^2 - 5t^3)$
   (e) $6x^2 + 3x - (4x^2 + 5x)$
   (f) $2r^2 - 5r + 7 + (3r^2 - 7r - 8)$
   (g) $5(x^2 + x) + 2(x^2 + 3x)$
   (h) $2x(x - 3) + 5x(x + 2)$
10. Write equivalent expressions without brackets and simplify these expressions as far as possible.

    Example: $5r^2 - 2r(r + 5) = 5r^2 - 2r^2 - 10r$
    $= 3r^2 - 10r$

    (a) $3x^2 + x(x + 3)$
    (b) $5x + x(7 - 2x)$
    (c) $6r^2 - 2r(r - 5)$
    (d) $2a(a + 3) + 5a(a - 2)$
    (e) $6y(y + 1) - 3y(y + 2)$
    (f) $4x(2x - 3) - 3x(x + 2)$
    (g) $2x^2(x - 5) - x(3x^2 - 2)$
    (h) $x(x - 1) + x(2x + 3) - 2x(3x + 1)$

8.4 Multiplication of algebraic expressions

MULTIPLY POLYNOMIALS BY MONOMIALS

1. (a) Calculate 3 × 38 and 3 × 62, and add the two answers.
   (b) Add 38 and 62, then multiply the answer by 3.

(c) If you do not get the same answer for (a) and (b), you have made a mistake. Rework until you get it right.

The fact that if you work correctly, you get the same answer in questions 1(a) and (b), is a demonstration of the **distributive property**.

The distributive property may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

What you saw in question 1 was that:
$3 \times 100 = 3 \times 38 + 3 \times 62$.

This can also be expressed by writing $3(38 + 62) = 3 \times 38 + 3 \times 62$.

2. (a) Calculate 10×56.
   (b) Calculate $10 \times 16 + 10 \times 40$.
3. (a) Write down any two numbers smaller than 100. Let us call them x and y. Add your two numbers and multiply the answer by 3.
   (b) Calculate $3 \times x$ and $3 \times y$, and add the two answers.
   (c) If you do not get the same answers for (a) and (b), you have made a mistake somewhere. Correct your work.
4. Copy and complete the following table:

x	12	50	5
y	4	30	10
$5x - 5y$			
$5(x - y)$			
$5x + 5y$			
$5(x + y)$			

Performing the instructions $5(x + y)$ is not the only way in which you can find out how much $5(x + y)$ is for any given values of x and y. Instead of doing $5(x + y)$, you may do $5x + 5y$. In this case you will multiply first, and again, before you add.

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5. (a) For $x = 10$ and $y = 20$, evaluate $8(x + y)$ by first adding 10 and 20, and then multiplying by 8.
   (b) Now evaluate $8(x + y)$ by doing $8x + 8y$; in other words, first calculate 8×10 and 8×20.

6. In question 5 you evaluated $8(x + y)$ in two different ways for the given values of x and y. Now also evaluate $20(x - y)$ in two different ways, for $x = 5$ and $y = 3$.

7. Use the distributive property in each of the following cases to make a different expression that is equivalent to the given expression:
   (a) $a(b + c)$
   (b) $a(b + c + d)$
   (c) $x(x + 1)$
   (d) $x(x^2 + x + 1)$
   (e) $x(x^3 + x^2 + x + 1)$
   (f) $x^2(x^2 - x + 3)$
   (g) $2x^2(3x^2 + 2)$
   (h) $3x^3(2x^2 + 4x - 5)$
   (i) $-2x^4(x^3 - 2x^2 - 4x + 5)$
   (j) $a^2b(a^3 - a^2 + a + 1)$
   (k) $x^2y^3(3x^2y + xy^2 - y)$
   (l) $-2x(x^3 - y^3)$
   (m) $2a^2b(3a^2 + 2a^2b^2 + 4b^2)$
   (n) $2ab^2(3a^3 - 1)$

What you do in this question is sometimes called "multiplication of a polynomial by a monomial".
One may also say that in each case you **expand** the expression, or you write an equivalent expression in **expanded form**.

8. Expand the parts of each expression and simplify. Then evaluate the expression for $x = 5$.
   (a) $5(x - 2) + 3(x + 4)$
   (b) $x(x + 4) - 4(x + 4)$
   (c) $x(x - 4) + 4(x - 4)$
   (d) $x(x^2 + 3x + 9) - 3(x^2 + 3x + 9)$
   (e) $x(x^2 - 3x + 9) + 3(x^2 - 3x + 9)$
   (f) $x^2(x^2 - 3x + 4) - x(x^3 + 4x^2 + 2x + 3)$

9. Write in expanded form:
   (a) $x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2)$
   (b) $x^2y(x^2 - 2xy + y^2) - xy^2(2x^2 - 3xy - y^2)$
   (c) $ab^2c(b^2c^2 - ac) + b^2c^4(a^2 + abc^2)$
   (d) $p^2q(pq^2 + p + q) + pq(p - q^2)$

SQUARES AND CUBES AND ROOTS OF MONOMIALS

1. Evaluate each of the following expressions for $x = 2$, $x = 5$ and $x = 10$:
   (a) $(3x)^2$
   (b) $9x^2$
   (c) $(2x)^2$
   (d) $4x^2$
   (e) $(2x)^3$
   (f) $8x^3$
   (g) $(2x + 3x)^2$
   (h) $(10x - 7x)^2$

2. In each case, write an equivalent monomial without brackets:
   (a) $(5x)^2$
   (b) $(5x)^3$
   (c) $(20x)^2$
   (d) $(10x)^3$
   (e) $(2x + 7x)^2$
   (f) $(20x - 13x)^3$

The square root of $16x^2$ is $4x$, because $(4x)^2 = 16x^2$.

3. Write down the square root of each of the following expressions:
   (a) $\sqrt{(7x)^2}$
   (b) $\sqrt{9x^2}$
   (c) $\sqrt{(20x)^2}$
   (d) $\sqrt{100x^2}$
   (e) $\sqrt{(20x - 15x)^2}$
   (f) $\sqrt{16x^2 + 9x^2}$
   (g) $\sqrt{(21x - 16x)^2}$
   (h) $\sqrt{(5x)^2}$

The cube root of $64x^3$ is $4x$, because $(4x)^3 = 64x^3$.

4. Write down the cube root of each of the following expressions:
   (a) $\sqrt[3]{(7x)^3}$
   (b) $\sqrt[3]{27x^3}$
   (c) $\sqrt[3]{(20x)^3}$
   (d) $\sqrt[3]{1\,000x^3}$
   (e) $\sqrt[3]{(20x - 15x)^3}$
   (f) $\sqrt[3]{125x^3}$

8.5 Dividing polynomials by integers and monomials

1. Copy and complete the following table:

x	20	10	5	–5	–10	–20
$(100x - 5x^2) \div 5x$						
$20 - x$						

   Can you explain your observations?

2. (a) R240 prize money must be shared equally between 20 netball players. How much should each one get?
   (b) Mpho decided to do the calculations below. Do not do Mpho's calculations, but think about this: Will Mpho get the same answer that you got for question (a)?
   $(140 \div 20) + (100 \div 20)$
   (c) Gert decided to do the calculations below. Without doing the calculations, say whether or not Gert will get the same answer that you got for question (a).
   $(240 \div 12) + (240 \div 8)$

3. Do the necessary calculations to find out whether the following statements are true or false:
   (a) $(140 + 100) \div 20 = (140 \div 20) + (100 \div 20)$
   (b) $240 \div (12 + 8) = (240 \div 12) + (240 \div 8)$
   (c) $(300 - 60) \div 20 = (300 \div 20) - (60 \div 20)$

Division is **right-distributive** over addition and subtraction, for example:
$(2 + 3) \div 5 = (2 \div 5) + (3 \div 5)$.

The division symbol is to the right of the brackets; it is not left-distributive, for example:
$10 \div (2 + 4) \neq (10 \div 2) + (10 \div 4)$.

For example: $(200 + 40) \div 20 = (200 \div 20) + (40 \div 20) = 10 + 2 = 12$, and
$(500 + 200 - 300) \div 50 = (500 \div 50) + (200 \div 50) - (300 \div 50)$

4. Evaluate each expression for $x = 2$ and $x = 10$:
   (a) $(10x^2 + 5x) \div 5$
   (b) $(10x^2 \div 5) + (5x \div 5)$
   (c) $2x^2 + x$
   (d) $(10x^2 + 5x) \div 5x$
   (e) $(10x^2 \div 5x) + (5x \div 5x)$
   (f) $2x + 1$

The distributive property of division can be expressed in the following way:
$(x + y) \div z = (x \div z) + (y \div z)$
$(x - y) \div z = (x \div z) - (y \div z)$

5. (a) Do not do any calculations. Which of the following expressions do you *think* will have the same value as $(10x^2 + 20x - 15) \div 5$, for $x = 10$ as well as $x = 2$?

   $2x^2 + 20x - 15$ $\quad$ $10x^2 + 20x - 3$ $\quad$ $2x^2 + 4x - 3$

   (b) Do the necessary calculations to check your answer.

6. Simplify:
   (a) $(2x + 2y) \div 2$
   (b) $(4x + 8y) \div 4$
   (c) $(20xy + 16x) \div 4x$
   (d) $(42x - 6) \div 6$
   (e) $(28x^4 - 7x^3 + x^2) \div x^2$
   (f) $(24x^2 + 16x) \div 8x$
   (g) $(30x^2 - 24x) \div 3x$

7. Simplify:
   (a) $(9x^2 + xy) \div xy$
   (b) $(48a - 30ab + 16ab^2) \div 2a$
   (c) $(3a^3 + a^2) \div a^2$
   (d) $(13a - 17ab) \div a$
   (e) $(3a^2 + 5a^3) \div a$
   (f) $(39a^2b + 13ab + ab^2) \div ab$

The instruction $72 \div 6$ may also be written as $\frac{72}{6}$.

This notation, which looks just like the common fraction notation, is often used to indicate division.

Hence, instead of $(10x^2 + 20x - 15) \div 5$, we may write $\frac{10x^2 + 20x - 15}{5}$.

Since $(10x^2 + 20x - 15) \div 5$ is equivalent to $(10x^2 \div 5) + (20x \div 5) - (15 \div 5)$,
$\frac{10x^2 + 20x - 15}{5}$ is equivalent to $\frac{10x^2}{5} + \frac{20x}{5} - \frac{15}{5}$.

8. Find a simpler equivalent expression for each of the following expressions (clearly, these expressions do not make sense if $x = 0$):

(a) $\dfrac{16x^2 - 12x}{4x}$ (b) $\dfrac{16x^3 - 12x}{4x}$

(c) $\dfrac{16x^3 - 12x^2}{4x}$ (d) $\dfrac{16x^3 - 12x^2}{4x^2}$

(e) $\dfrac{16x^3 - 12x^2}{2x}$ (f) $\dfrac{16x^3 - 12x}{8x}$

9. In each case check if the statement is true for $x = 10$, $x = 100$, $x = 5$, $x = 1$ and $x = -2$.

(a) $\dfrac{x^2}{x} = x$ (b) $\dfrac{x^3}{x} = x^2$ (c) $\dfrac{x^3}{x^2} = x$

(d) $\dfrac{5x^3}{x} = 5x^2$ (e) $\dfrac{5x^3}{x} = 5^3$ (f) $\dfrac{5x}{x^2} = \dfrac{5}{x}$

10. Explain why the equations below are true:

(a) $\dfrac{100x - 5x^2}{5x} = 20 - x$ for all values of x, except $x = 0$.

(b) $\dfrac{15x^2 - 10x}{5x}$ is equivalent to $3x - 2$, excluding $x = 0$.

11. Copy and complete the following table:

x	1,5	2,8	−3,1	0,72
$\dfrac{3x+12}{3}$				
$\dfrac{18x^2+6}{6}$				
$\dfrac{5x^2+7x}{x}$				

(Hint: Simplify the expressions first to save yourself some work!)

12. Simplify each expression to the equivalent form requiring the fewest operations:

(a) $\dfrac{3a + a^2}{a}$ (b) $\dfrac{x^3 + 2x^2 - x}{x}$ (c) $\dfrac{2a + 12ab}{2a}$

(d) $\dfrac{12x^2 + 10x}{2x}$ (e) $\dfrac{21ab - 14a^2}{7a}$ (f) $\dfrac{15a^2b + 30ab^2}{5ab}$

(g) $\dfrac{7x^3 + 21x^2}{7x^2}$ (h) $\dfrac{3x^2 + 9x}{3x}$

13. Solve the equations:

(a) $\frac{3x^2 + 15x}{3x} = 20$ (b) $\frac{30x - 18x^2}{6x} = 2$

14. Copy and complete the following table:

	x	1,1	1,2	1,3	1,4	1,5
(a)	$\frac{x^3 + 2x^2 - x}{x}$					
(b)	$\frac{7x^3 + 21x^2}{7x^2}$					
(c)	$\frac{50x^2 + 5x}{5x}$					

15. Simplify the following expressions:

(a) $\frac{3x(5x + 4) + 6x(5x + 3)}{5x}$ (b) $\frac{14x^2 - 28x}{7x} + \frac{24x - 18x^2}{3x}$

8.6 Products and squares of binomials

How can we obtain the expanded form of $(x + 2)(x + 3)$?

In order to expand $(x + 2)(x + 3)$, you can first keep $(x + 2)$ as it is, and apply the distributive property:

$(\boldsymbol{x} + \mathbf{2})(x + 3)$
$= (\boldsymbol{x} + \mathbf{2})x + (\boldsymbol{x} + \mathbf{2})3$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

1. Describe how can you check if $(x + 2)(x + 3)$ is actually equivalent to $x^2 + 5x + 6$.

To expand $(x - y)(x + 3y)$ it can be written as $(x - y)x + (x - y)3y$, and the two parts can then be expanded.

$(x - y)(x + 3y)$
$= (x - y)x + (x - y)3y$
$= x^2 - xy + 3xy - 3y^2$
$= x^2 + 2xy - 3y^2$

2. Do some calculations to check whether $(x - y)(x + 3y)$ and $x^2 + 2xy - 3y^2$ are equivalent. Write the results of your calculations in a table like the one on page 83.

x					
y					

3. Expand each of these expressions:
   (a) $(x + 3)(x + 4)$
   (b) $(x + 3)(4 - x)$
   (c) $(x + 3)(x - 5)$
   (d) $(2x^2 + 1)(3x - 4)$
   (e) $(x + y)(x + 2y)$
   (f) $(a - b)(2a + 3b)$
   (g) $(k^2 + m)(k^2 + 2m)$
   (h) $(2x + 3)(2x - 3)$
   (i) $(5x + 2)(5x - 2)$
   (j) $(ax - by)(ax + by)$

4. Expand each of these expressions:
   (a) $(a + b)(a + b)$
   (b) $(a - b)(a - b)$
   (c) $(x + y)(x + y)$
   (d) $(x - y)(x - y)$
   (e) $(2a + 3b)(2a + 3b)$
   (f) $(2a - 3b)(2a - 3b)$
   (g) $(5x + 2y)(5x + 2y)$
   (h) $(5x - 2y)(5x - 2y)$
   (i) $(ax + b)(ax + b)$
   (j) $(ax - b)(ax - b)$

5. Can you guess the answer to each of the following questions without working it out as you did in question 3? Try them out and then check your answers.
   Expand the following expressions:
   (a) $(m + n)(m + n)$
   (b) $(m - n)(m - n)$
   (c) $(3x + 2y)(3x + 2y)$
   (d) $(3x - 2y)(3x - 2y)$

All the expressions in questions 4 and 5 are **squares of binomials**, for example $(ax + b)^2$ and $(ax - b)^2$.

6. Expand:
   (a) $(ax + b)^2$
   (b) $(ax - b)^2$
   (c) $(2s + 5)^2$
   (d) $(2s - 5)^2$
   (e) $(ax + by)^2$
   (f) $(ax - by)^2$
   (g) $(2s + 5r)^2$
   (h) $(2s - 5r)^2$

7. Expand and simplify:
   (a) $(4x + 3)(6x + 4) + (3x + 2)(8x + 5)$
   (b) $(4x + 3)(6x + 4) - (3x + 2)(8x + 5)$

8.7 Substitution into algebraic expressions

1. In question 2 you have to find the values of different expressions, for some given values of x. Look carefully at the different expressions in the table. Do you think some of them may be equivalent?

   Simplify the longer expression to check whether you end up with the shorter expression.

2. Copy and complete the following table:

	x	13	–13	2,5	10
(a)	$(2x + 3)(3x - 5)$				
(b)	$10x^2 + 5x - 7 + 3x^2 - 4x - 3$				
(c)	$3(10x^2 - 5x + 2) - 5x(6x - 4)$				
(d)	$13x^2 + x - 10$				
(e)	$6x^2 - x - 15$				
(f)	$5x + 6$				

3. Copy and complete the following table:

	x	1	2	3	4
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

4. Describe any patterns that you observe in your answers for question 3.

5. Copy and complete the following table:

	x	1,5	2,5	3,5	4,5
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

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Chapter 9
Equations

9.1 Solving equations by inspection

1. Six equations are listed in the table below. Use the table to find out for which of the given values of x will be true that the left-hand side of the equation is equal to the right-hand side.

"Searching" for the solution of an equation by using tables is called **solution by inspection**.

x	–3	–2	–1	0	1	2	3	4
$2x + 3$	–3	–1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	–11	–8	–5	–2	1	4	7	10
$10x - 7$	–37	–27	–17	–7	3	13	23	33
$5x + 3$	–12	–7	–2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	–2

(a) $2x + 3 = 5x + 3$
(b) $5x + 3 = 9 - x$
(c) $2x + 3 = x + 4$
(d) $10x - 7 = 5x + 3$
(e) $3x - 2 = x + 4$
(f) $9 - x = 2x + 3$

Two equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

9.2 Solving equations using additive and multiplicative inverses

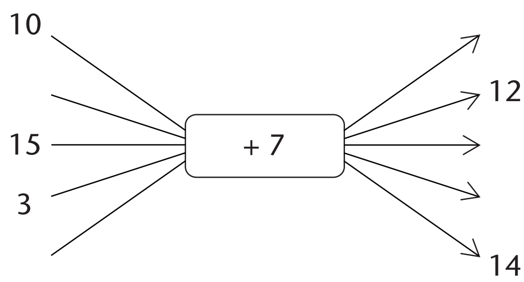
1. In each case find the value of x:

(a) x → [+ 7] → 10

(b) x → [× 2] → [+ 3] → 13

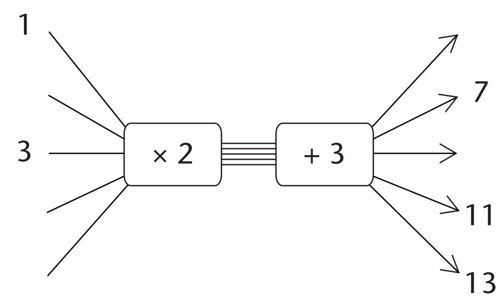
2. Copy and complete the flow diagrams. Fill in all the missing numbers.

(a)



To find the second input number you may say to yourself, *"After I added 7, I had 12. What did I have before I added 7?"*

(b)

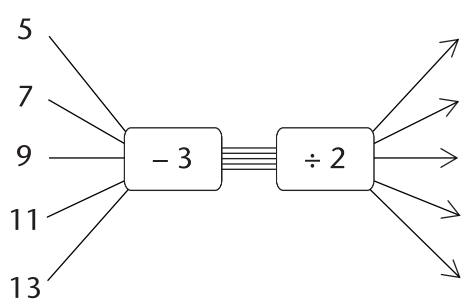


To find the input number that corresponds to 13, you may ask yourself, "*What did I have before I added 3?*" and then, "*What did I have before I multiplied by 2?*"

3. Use your answers for question 2 to check your answers for question 1.

4. Describe the instructions in flow diagram 2(b) in words, and also with a symbolic expression.

5. Copy and complete the following flow diagram:



This flow diagram is called the **inverse** of the flow diagram in question 2(b).

6. Compare the input numbers and the output numbers of the flow diagrams in question 2(b) and question 5. What do you notice?

7. (a) Add 5 to any number and then subtract 5 from your answer. What do you get?
   (b) Multiply any number by 10 and then divide the answer by 10. What do you get?

If you add a number and then subtract the same number, you are back where you started. This is why addition and subtraction are called **inverse operations**.

If you multiply by a number and then divide by the same number, you are back where you started. This is why multiplication and division are called **inverse operations**.

The expression $5x - 3$ says *"multiply by 5 then subtract 3"*. This instruction can also be given with a flow diagram: $\longrightarrow \boxed{\times 5} \longrightarrow \boxed{-3} \longrightarrow$

The equation $5x - 3 = 47$ can also be written as a flow diagram:

$\longrightarrow \boxed{\times 5} \longrightarrow \boxed{-3} \longrightarrow 47$

8. Solve the equations below. You may do this by using the inverse operations. You may write a flow diagram to help you to see the operations.
   (a) $2x + 5 = 23$
   (b) $3x - 5 = 16$
   (c) $5x - 60 = -5$
   (d) $\frac{1}{3}x + 11 = 19$
   (e) $10(x + 3) = 88$
   (f) $2(x - 13) = 14$

9.3 Setting up equations

CONSTRUCTING EQUATIONS

You can easily make an equation that has 5 as the solution. Here is an example:

Start by writing the solution	$x = 5$
Add 3 to both sides	$x + 3 = 8$
Multiply both sides by 5	$5x + 15 = 40$

1. What is the solution of the equation $5x + 15 = 40$?

2. Make your own equation with the solution $x = 3$.

3. Bongile worked like this to make the equation $2(x + 8) = 30$, but he rubbed out part of his work:

Start by writing the solution	$x =$
Add 8 to both sides	$= 15$
Multiply both sides by 2	$2(x + 8) = 30$

   Copy and complete Bongile's writing to solve the equation $2(x + 8) = 30$.

4. This is how Bongile made a more difficult equation:

Start by writing the solution	$x =$
Multiply by 3 on both sides	$3x =$
Subtract 9 from both sides	$3x - 9 = 6$
Add $2x$ to both sides	$5x - 9 = 2x + 6$

   (a) What was on the right-hand side before Bongile subtracted 9?
   (b) What is the solution of $5x - 9 = 2x + 6$?

5. Bongile started with a solution and he ended up with an equation. Write down the steps that Bongile took to make the equation, and solve the equation:

$$x =$$
$$8x =$$
$$8x + 3 =$$
$$3x + 3 = 35 - 5x$$

SOLVING EQUATIONS

To make an equation, you can apply the same operation on both sides.

To solve an equation, you can apply the inverse operation on both sides.

Make (↓)		Solve (↑)
	$x = 4$	
Multiply by 8	$8x = 32$	Divide by 8
Add 3	$8x + 3 = 35$	Subtract 3
Subtract $5x$	$3x + 3 = 35 - 5x$	Add $5x$

Use any appropriate method to solve the following equations:

1. (a) $5x + 3 = 24 - 2x$ (b) $2x + 4 = -9$
   (c) $3 - x = x - 3$ (d) $6(2x + 1) = 0$

2. (a) $4(1 - 2x) = 12 - 7x$ (b) $8(1 - 3x) = 5(4x + 6)$
   (c) $7x - 10 = 3x + 7$ (d) $1{,}6x + 7 = 3{,}5x + 3{,}2$

NUMBER PATTERNS AND EQUATIONS

1. (a) Which of the following rules will produce the number pattern given in the second row of the table below?
   A. Term value = $8n$ where n is the term number
   B. Term value = $6n - 1$ where n is the term number
   C. Term value = $6n + 2$ where n is the term number
   D. Term value = $10n - 2$ where n is the term number
   E. Term value = $5n + 3$ where n is the term number

Term number	1	2	3	4	5	6	7	8	9
Term value	8	13	18	23	28	33	38	43	48

   (b) The sixth term of the sequence has the value 33. Which term will have the value 143? You may set up and solve an equation to find out.
   (c) Apply rule E to your answer, to check if your answer is correct.

2. (a) Write the rule that will produce the number pattern in the second row of this table. You may have to experiment to find out what the rule is.

Term number	1	2	3	4	5	6	7	8	9
Term value	5	8	11	14	17	20	23	26	29

(b) Which term will have the value 221?

3. The rule for number pattern A is $4n + 11$, and the rule for pattern B is $7n - 34$.
(a) Copy and complete the following table for the two patterns:

Term number	1	2	3	4	5	6	7	8	9
Pattern A									
Pattern B									

(b) For which value of n are the terms of the two patterns equal?

9.4 Equation and situations

1. Consider this situation:
To rent a room in a certain building, you have to pay a deposit of R400 and then R80 per day.
(a) How much money do you need to rent the room for ten days?
(b) How much money do you need to rent the room for 15 days?

2. Which of the following best describes the method that you used to do question 1(a) and (b)?
A. Total cost = R400 + R80
B. Total cost = 400(number of days + 80)
C. Total cost = 80 × number of days + 400
D. Total cost = (80 + 400) × number of days

3. For how many days can you rent the room described in question 1, if you have R2 800 to pay?

If you want to know for how many days you can rent the room if you have R720, you can set up an equation and solve it.

Example: You know the total cost is R720 and you know that you can work out the total cost like this:

Total cost = $80x + 400$, where x is the number of days.
So, $80x + 400 = 720$ and x = four days.

In each of the cases on page 90 (given in questions 4 to 7), find the unknown number by setting up an equation and solving it.

4. To rent a certain room, you have to pay a deposit of R300 and then R120 per day.
   (a) For how many days can you rent the room if you can pay a total of R1 740? (If you experience trouble in setting up the equation, it may help you to decide first how you will work out what it will cost to rent the room for six days.)
   (b) What will it cost to rent the room for ten days, 11 days and 12 days?
   (c) For how many days can you rent the room if you have R3 300 available?
   (d) For how many days can you rent the room if you have R3 000 available?

5. Ben and Thabo decide to do some calculations with a certain number. Ben multiplies the number by 5 and adds 12. Thabo gets the same answer as Ben when he multiplies the number by 9 and subtracts 16. What is the number they worked with?

6. The cost of renting a certain car for a period of x days can be calculated with the following formula:

   Rental cost in rands = $260x + 310$

   What information about renting this car will you get, if you solve the equation $260x + 310 = 2\,910$?

7. Sarah paid a deposit of R320 for a stall at a market, and she also pays R70 per day rental for the stall. She sells fruit and vegetables at the stall, and finds that she makes about R150 profit each day. After how many days will she have earned as much as she has paid for the stall, in total?

9.5 Solving equations by using the laws of exponents

You may need to look back at Chapter 5 to remember the laws of exponents.

One kind of exponential equation that you deal with in Grade 9 has one or more terms with a base that is raised to a power containing a variable.

Example: $2^x = 16$

When we need to find the unknown value, we are asking the question: *"To what power must the base be raised for the statement to be true?"*

Example: $2^x = 16$ Make sure that the terms with x are on their own on one side.

$2^x = 2^4$ Write the known term in the same base as the term with the exponent.

$x = 4$ Equate the exponents.

In the example above, we can equate the exponents because the two numbers are equal only when they are raised to the same power.

1. Solve for x:

(a) $5^{x-1} = 125$
(b) $2^{x+3} = 8$
(c) $10^x = 10\,000$
(d) $4^{x+2} = 64$
(e) $7^{x+1} = 1$
(f) $x^0 = 1$

Example: Solve for x: $3^x = \frac{1}{27}$

$3^x = 3^{-3}$ (Rewrite $\frac{1}{27}$ as a number to base 3.)

$x = -3$ (Equate the exponents.)

2. Solve for x:

(a) $7^x = \frac{1}{49}$
(b) $10^x = 0{,}001$
(c) $6^x = \frac{1}{216}$
(d) $10^{x-1} = 0{,}001$
(e) $4^{-x} = \frac{1}{16}$
(f) $7^x = 7^{-3}$

In another kind of equation involving exponents, the variable is in the base.

> When we need to find the unknown value, we ask the question: *"Which number must be raised to the given power for the statement to be true?"*

For these equations, you should remember what you know about the powers of numbers such as 2, 3, 4, 5 and 10.

SOLVING EQUATIONS WITH A VARIABLE IN THE BASE

1. Copy and complete the table below and answer the questions that follow:

	x	2	3	4	5
(a)	x^3	$2^3 = 8$			
(b)	x^5	$2^5 = 32$			
(c)	x^4	$2^4 = 16$			

For what value of x is:

(a) $x^3 = 64$
(b) $x^5 = 32$
(c) $x^4 = 256$
(d) $x^3 = 8$
(e) $x^4 = 16$
(f) $x^5 = 3\,125$

2. Solve for x and give a reason:

(a) $x^3 = 216$
(b) $x^2 = 324$
(c) $x^4 = 10\,000$
(d) $8^x = 512$
(e) $18^x = 324$
(f) $6^x = 216$

WORKSHEET

1. Ahmed multiplied a number by 5, added 3 to the answer, and then subtracted the number he started with. The answer was 11. What number did he start with?

2. Use any appropriate method to solve the equations:

   (a) $3(x - 2) = 4(x + 1)$

   (b) $5(x + 2) = -3(2 - x)$

   (c) $1{,}5x = 0{,}7x - 24$

   (d) $5(x + 3) = 5x + 12$

   (e) $2{,}5x = 0{,}5(x + 10)$

   (f) $7(x - 2) = 7(2 - x)$

   (g) $\frac{1}{2}(2x - 3) = 5$

   (h) $2x - 3(3 + x) = 5x + 9$

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