

UNDERSTANDING CIRCLE THEOREMS-PART ONE.

Common terms:

- (a) **ARC**- Any portion of a circumference of a circle.
- (b) **CHORD**- A line that crosses a circle from one point to another. If this chord passes through the centre then it is referred to as a diameter
- (c) **A TANGENT**- A line that touches a circle at only one point.

Theorem 1.

The angle subtended at the centre of a circle is twice the angle subtended at the circumference by the same arc.

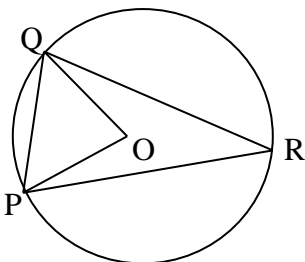
Theorem 2.

Angles subtended by an arc in the same segment of a circle are equal.

Example 1.

Given $\angle PQO = 65^\circ$

Find $\angle QRP$



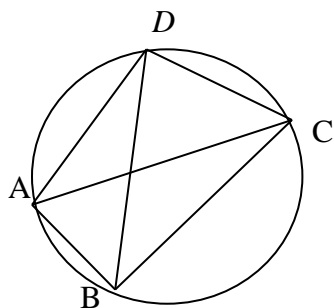
Triangle OQP is isosceles ($OP = OQ$, the radii)

$$\therefore \angle OPQ = 65^\circ \quad \therefore \angle QOP = 180^\circ - (65^\circ + 65^\circ) \text{ (angle sum of a triangle)}$$

$$= 50^\circ$$

$$\therefore \angle QRP = 25^\circ \text{ (half of angle at the centre).}$$

Example 2.



Given that $\angle BDC = 78^\circ$ and $\angle DCA = 56^\circ$.

Find angles $\angle BAC$ and $\angle DBA$.

Solution: $BAC = BDC = 78^\circ$. (both subtended by arc BC)

$DBA = DCA = 56^\circ$. (both subtended by arc AD)

Theorem3.

The opposite angles in a cyclic quadrilateral add up to 180° (the angles are supplementary).

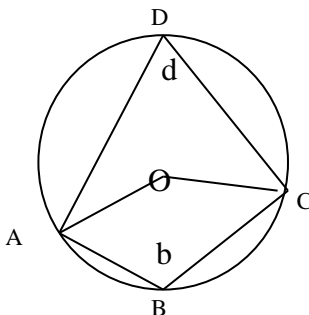
ABCD is a **cyclic quadrilateral** because all its vertices touch the circumference of the circle.(ABCO is not cyclic because O is not at the circumference).

Proof:

OA and OC are radii.

Let angle ADC = d

and angle ABC = b



AOC obtuse = $2d$ (angles at the centre)

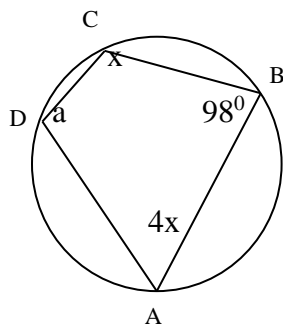
AOC reflex = $2b$ (angles at the centre)

$$\therefore 2d + 2b = 360^\circ \text{ (angles at a point)}$$

$$\therefore d + b = 180^\circ \text{ as required}$$

Example3.

Find a and x



$CDA = a$, $ABC = 98^\circ$, $DCB = x^\circ$, $DAB = 4x^\circ$

$$a = 180^\circ - 98^\circ \text{ (opposite angles of a cyclic quadrilateral)}$$

$$\therefore a = 82^\circ$$

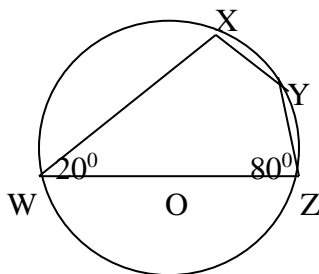
$$x + 4x = 180^\circ \text{ (opposite angles of a cyclic quadrilateral)}$$

$$5x = 180^\circ$$

$$x = 72^\circ$$

Exercise. 1.ABCD is a quadrilateral inscribed in circle, centre O, and AD is a diameter of the circle. If angle CDB = 46° and ADB = 31° . Calculate

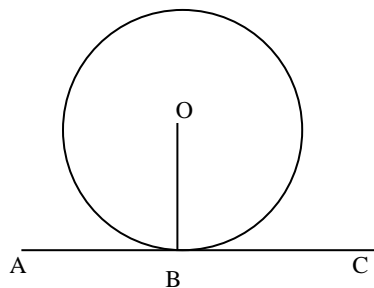
- (a) the angle ABC (b) the angle BCD (c) the angle BAD.
2. A circle has a radius of 155mm .AB is a chord of this circle which is 275mm long. What angle does AB subtend at the circumference of the circle.
3. Given angle XWZ = 20° , angle WZY = 80° and O is the centre of the circle
- (a) Find angle WXY
- (b) Show that WY bisects XWZ



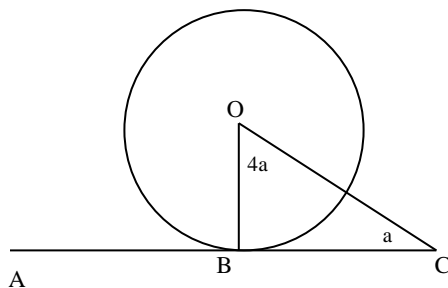
Theorem 4.

The angle between a tangent and the radius drawn to the point of contact is 90°

Line ABC is a tangent and angle ABO = 90°



Example 4. Find the angle BCO and angle BOC.



$$4a + a + 90^\circ = 180^\circ$$

$$5a = 90^\circ$$

$$a = 18^\circ.$$

Angle BCO = 18° and $\text{BOC} = 4 \times 18^{\circ} = 72^{\circ}$.

UNDERSTANDING CIRCLE THEOREMS –PART TWO.

Theorem 5.

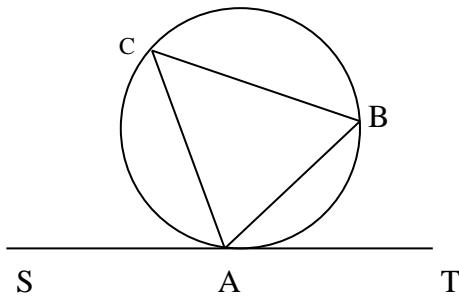
The tangents to a circle originating from a common point are equal in length.

Theorem 6.

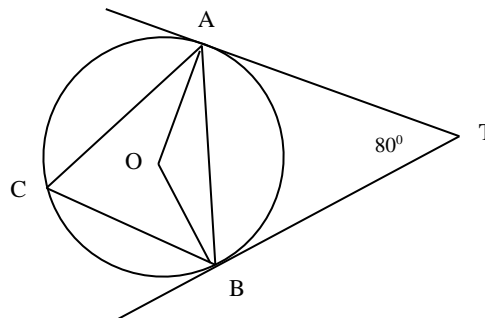
The Alternate segment theorem.

The angle between a tangent and chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

angle TAB = angle BCA and angle SAC = angle CBA



Example 1.



- a) $\triangle TBA$ is isosceles ($TA = TB$) ,angle TAB = angle TBA.

$$\begin{aligned} \therefore TBA &= \frac{1}{2} (180 - 80) \\ &= 50^{\circ} \end{aligned}$$

b) $OBT = 90^{\circ}$ (tangent and radius)

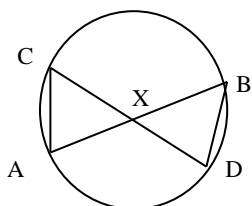
$$\begin{aligned} OBA &= 90^{\circ} - 50^{\circ} \\ &= 40^{\circ}. \end{aligned}$$

c) $ACB = ABT$ (alternate segment theorem)

$$ACB = 50^{\circ}$$

Theorem 7.

Intersecting chords theorem



$$\mathbf{AX \cdot BX = CX \cdot DX}$$

Proof:

In triangles AXC and BXD:

Angle ACX = angle DBX (same segment)

Angle CAX = angle BDX (same segment)

\therefore The triangles AXC and BXD are similar.

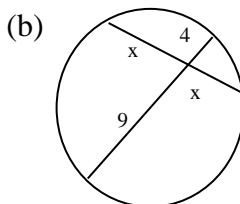
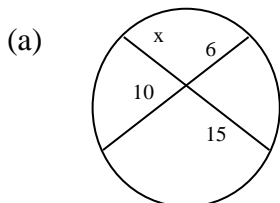
$$\frac{AX}{DX} = \frac{CX}{BX}$$

$$DX \cdot CX = AX \cdot BX$$

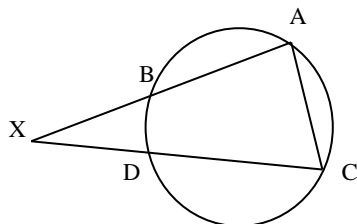
Thus $\mathbf{AX \cdot BX = CX \cdot DX}$

Exercise.

Find x



Theorem 8. **The intersecting secants theorem.**



Using triangles BXD and AXC;

Angle XAC = angle XDB, angle XCA = angle XBD. (Figure BACD is a cyclic quadrilateral). Thus triangles AXC and BXD have equal angles and are similar.

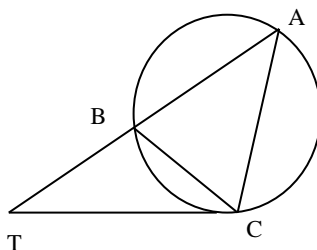
$$\frac{AX}{DX} = \frac{CX}{BX}$$

$$DX \cdot BX = AX \cdot CX$$

Thus $AX \cdot BX = CX \cdot DX$ (This is the intersecting secants theorem)

Theorem 9.

The secant/tangent theorem.



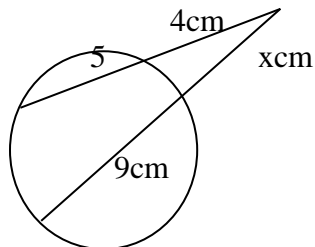
Angle BCT = angle BAC (alternate segment theorem). Triangles ATC and BTC share angle T and are similar triangles. (when triangles have two angles equal then they are similar).

In the triangles,

$$\frac{AT}{TC} = \frac{TC}{BT} \quad ; AT \cdot BT = TC^2 \text{ (This is the secant/tangent theorem).}$$

$$AT \cdot BT = TC^2$$

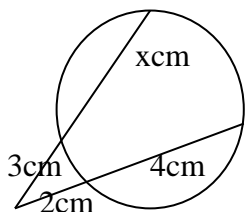
Example 2.



Solution: $4 \times 9 = x \cdot (9 + x)$
 $36 = 9x + x^2$
 $x^2 + 9x - 36 = 0$
 $x^2 + 12x - 3x - 36 = 0$
 $x(x + 12) - 3(x + 12) = 0$
 $(x - 3)(x + 12) = 0$; $x = 3$ or $x = -12$.

Since x cannot be negative then $x = 3$ cm.

Example 3.



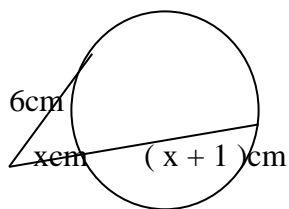
Solution: $3(3 + x) = 2 \times 6$
 $9 + 3x = 12$
 $3x = 3$
 $x = 1$ cm.

Exercise.

1. Two chords of a circle KL and MN intersect at X , and KL is produced to T . Given that $KX = 6$ cm, $XL = 4$ cm, $MX = 8$ cm and $LT = 8$ cm. calculate

- NX
- The length of the tangent from T to the circle
- The ratio of the areas of $\triangle KXM$ to $\triangle LXN$.

2.



Find x .

3. Chords AB and BC of a circle are produced to meet outside the circle at T . a tangent is drawn from T to touch the circle at E . Given $AB = 5$ cm, $BT = 4$ cm and $DC = 9$ cm,

Calculate

- CT
- TE
- the ratio of the areas of $\triangle ADT$ to $\triangle BCT$

d) the ratio of the areas of \triangle BET to \triangle AET