VECTORS Summary:

1.Avectorhasbothmagnitudeanddirection.

2. $OP = \begin{pmatrix} x \\ y \end{pmatrix}$ isthepositionvectorofpoint $P(x, y)$.

3.ThemagnitudeorlengthormodulusofvectorOPisdenotedby

$|OP| = \sqrt{x^2 + y^2}$.

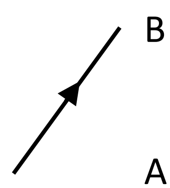
4.Toaddtwovectorsweaddthecorrespondingnumbers

5.Tosubtracttwovectorswesubtractthecorrespondingnumbers

6.Ascalarkmultipliedbyvector $OP = \begin{pmatrix} x \\ y \end{pmatrix}$ istreatedasfollows:

$$kOP = k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

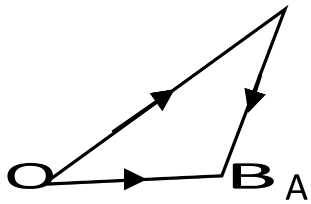
7.AdisplacementvectorABisrepresentedbyadirectedlinesegmentAB as shown:



ThevectorsABandBAareequalinlengthbutoppositeindirection

□BA=□AB

8.InthetriangleOAB,thedisplacementOAfollowedbyABisequaltoa single displacementOB.



OB=OA+AB

□AB=OB□OA"Thevectortriangleequation"

9.IfvectorABisparalleltoCD,then $AB = kCD$

10.IfABCDisaparallelogram,thenthetwooppositesidesareparalleland

alsoequalinlength(AB=DCandAD=BC).

11.IfABisparalleltoBCwithacommonpointB,thenthepointsA,BandC arecollinear()

$$AB = kBC$$

EXAMPLES:

1.GiventhepointsA(4,1)andB(12,16),findthe:

(i)columnvectorAB

(ii)lengthofAB

2.ThepositionvectorsofPandQare $\begin{pmatrix}-2\\13\end{pmatrix}$ and $\begin{pmatrix}4\\5\end{pmatrix}$ respectively,findthe magnitudeofPQ

3.FindthedistancebetweenthepointsP(□8,2)andQ(4,7)

4.Giventhat $OA = \begin{pmatrix}3\\7\end{pmatrix}$ and $AB = \begin{pmatrix}5\\-1\end{pmatrix}$, findthe:

(i)positionvectorofB

(ii) $|OB|$

5.Giventhat $OB = \begin{pmatrix}1\\9\end{pmatrix}$ and $AB = \begin{pmatrix}-2\\5\end{pmatrix}$, findthe:

(i)coordinatesofA

(ii)modulusofOA

6.GiventhepointsP(□2,3)andQ(3,6),findthecoordinatesofR,if

$$OR = 3OP + \frac{1}{3}OQ.$$

7.GiventhepointsA(3,4)andB(9,2),findthecoordinatesofT,if

$$OT = OA + \frac{1}{2}AB.$$

8.Giventhat $a = \begin{pmatrix}-2\\-9\end{pmatrix}$, $b = \begin{pmatrix}7\\2\end{pmatrix}$ and $m = a + 2b$, findthemagnitudeof m.

9.Giventhevectors $a = \begin{pmatrix}-2\\7\end{pmatrix}$, $b = \begin{pmatrix}8\\11\end{pmatrix}$ and $c = \begin{pmatrix}4\\-5\end{pmatrix}$, findthelengthof $a + 2b + c.$

10.Giventhevectors $AB = \begin{pmatrix}9\\-7\end{pmatrix}$ and $BC = \begin{pmatrix}-6\\3\end{pmatrix}$, findthe:

(i)columnvectorAC

(ii)modulusofAC

11.Giventhevectors $PQ = \begin{pmatrix} 13 \\ 4 \end{pmatrix}$ and $RQ = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$, find:

(i)vectorPR

(ii)thelengthofPR

12.Giventhevectors $AB = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$ $AC = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, and findthemagnitudeofBC.

13.If $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $r = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$, and findthevaluesofaandbsuchthat

$ap + bq = r.$

14.If $a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $c = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$, findthevaluesofxandysuch that

$xa + yb = c.$

15.If $u = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $w = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$, findthevaluesofxandysuchthat

$xu + yv = w.$

15.ABCDisaparallelogramwithA(□2,□2),B(6,□2)andC(2,1).Findthe coordinatesofD.

16.ABCDisaparallelogramwithA(2,1),B(3,4)andC(□1,2).Findthe coordinatesofD

17.PQRSisaparallelogramwithP(1,1),Q(5,3)andR(7,7).Findthe:
(i)columnvectorPS (ii)coordinatesofS.

18.ABCDisaquadrilateralwithA(4,1),B(2,□2,C(□2,0)andD(0,3).Show thatABCDisaparallelogram

19.Thevectors $a = \begin{pmatrix} 2 \\ \lambda \end{pmatrix}$ and $b = \begin{pmatrix} 8 \\ -12 \end{pmatrix}$ areparalleltoeachother.Find the valueof λ

20.Thevectors $p = \begin{pmatrix} \lambda \\ 2 \end{pmatrix}$ and $q = \begin{pmatrix} 15 \\ 6 \end{pmatrix}$ areparalleltoeachother.Findthe valueof λ

21.ShowthatthepointsA(□1,3),B(2,1)andC(8,□3)arecollinear.

22.ShowthatthepointsP(□1,□5),Q(0,□2)andR(2,4)arecollinear.

23.GiventhepointsA(□2,□1),B(1,5)andC(2,7),findthevalueofksuch that

$AB = kAC,$ hencestatetheratio $AB : AC.$

24.InthevectortriangleOAB,MisapointonABsuchthatAM:AB=2:5.

Express:

(i)AMintermsofAB (ii)MBintermsofAB

(iii)ABintermsofAM

(iv)ABintermsofMB

(v)OMintermsofOAandAB

(vi)OMintermsofOBandAB

25.InthevectortriangleOAB,KisapointonABsuchthat3AK=2KB.

Express:

(i)AKintermsofAB (ii)KBintermsofAB

(iii)AKintermsofKB (iv)KBintermsofAK

(v)OKintermsofOAandAB

(vi)OKintermsofOBandAB

26.InthevectortriangleOAB,NisthemidpointofAB.Express:

(i)ONintermsofOAandAB

(ii)ONintermsofOBandAB

27.ThepositionvectorsofthepointsAandBare and $\begin{pmatrix}12\\-11\end{pmatrix}$ $\begin{pmatrix}6\\4\end{pmatrix}$

respectively.PointMisonABsuchthatAM:AB=2:3,findthe:

(i)columnvectorAB

(ii)columnvectorAM

(iii)positionvectorofM.

28.Giventhat $OA = \begin{pmatrix}2\\7\end{pmatrix}, \quad OB = \begin{pmatrix}7\\-3\end{pmatrix}$ andMisapointonABsuchthat

3AM=2MB,findthe:

(i)coordinatesofM

(ii)magnitudeofOM

29.Giventhat $OA = \begin{pmatrix}-2\\2\end{pmatrix}, \quad OB = \begin{pmatrix}8\\4\end{pmatrix}$ andMisthemidpointofAB,findthe:

(i)columnvectorAB

(ii)positionvectorofM

30.Giventhat $OA = \begin{pmatrix} -1 \\ -15 \end{pmatrix}, \; OB = \begin{pmatrix} 7 \\ -11 \end{pmatrix}$ andpointEdividesABintheratio

1:3,findthepositionvectorofE.

31.Giventhat $OA = a, \; OB = b$ andMisthemidpointofAB,

(a)DrawavectordiagramshowingvectorAB

(b)Expressthefollowingvectorsintermsofaandb:

(i)AB

(ii)AM

(iii)OM

32.InatriangleOAB, $OA = a, \; OB = b$ andpointKdividesABintheratio
1:2,

Expressthefollowingvectorsintermsofaandb:

(i)AB

(ii)AK

(iii)OK

33.InatriangleOAB, $OA = a, \; OB = b$ andNisapointonABsuchthat

2AB=3NB.ExpressvectorONintermsofaandb.

34.InatriangleOAB,pointCdividesABintheratio2:3 and $OA = a$, $OB = b$,
DisthemidpointofOC.

(a)Expressthefollowingvectorsintermsofaandb:

(i)AB

(ii)OC

(iii)BD

(b)TakingOastheorigin,pointA(□15,20)andB(10,0),findthe:

(i)positionvectorofCin(a)(i)above.

(ii)coordinatesofC. (iii)lengthofOC.

35.InatriangleOAB,MandNaremidpointsofOAandOBrespectively.

$OB = b, \quad ON = b$ andPisapointonABsuchthat4AP=3AB.

(a)Expressthefollowingvectorsintermsofaandb:

(i)AB (ii)OP

(iii)MB

(iv)NP

(b)ShowthatABisparalleltoMN.

36.InatriangleOAB,MandNaremidpointsofABandOBrespectively.

$OB = b, \quad ON = b$ andPisapointonOMsuchthat3OP=2OM.

(a)Expressthefollowingvectorsintermsofaandb:

(i)AB

(ii)OM

(iii)PB

(iv)AP

(b)(i)ShowthatthepointsA,PandNarecollinear.

(ii)FindtheratioinwhichPdividesAN.

37.InatriangleOAB,PandQarepointsonOAandAB ($OA = a$, $OB = b$)
respectivelysuchthat3OP=PA,AQ=2QBandNisthemidpointofOQ.
ANMisastraightlinewhichissuchthatAN=5NM.Givenalsothat $OM = h\,OB$,
wherehisascalar.

(a)Expressthefollowingvectorsintermsofaandb:

(i)OQ

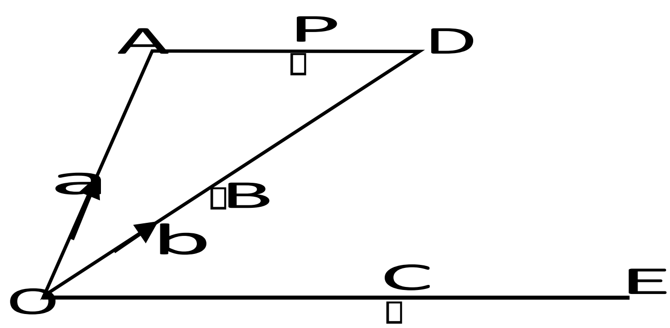
(ii)AN

(iii)PN

(iv)NB

(b)ShowthatthepointsP,NandBarecollinear (c)Findthevalueofh.

38.Inthefigurebelow,PisapointonADsuchthatPD:AP=1:2, $OB = b$, $OB = b$,
3OB=2BDandOC=3CE=3AP.



(a)Expressthefollowingvectorsintermsofaandb:

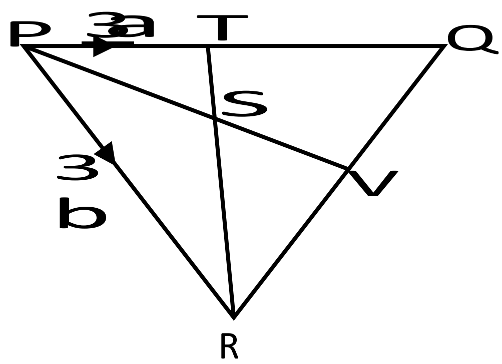
(i)AD

(ii)BP

(iii)DC

(b)ShowthatAD:OE=3:8

39.Inthefigurebelow, $PT = 3a$, $PR = 3b$, $PQ = 4PT$, $2PS = PV$ and $3RS = 2RT$.



(a)Expressthefollowingvectorsintermsofaandb:

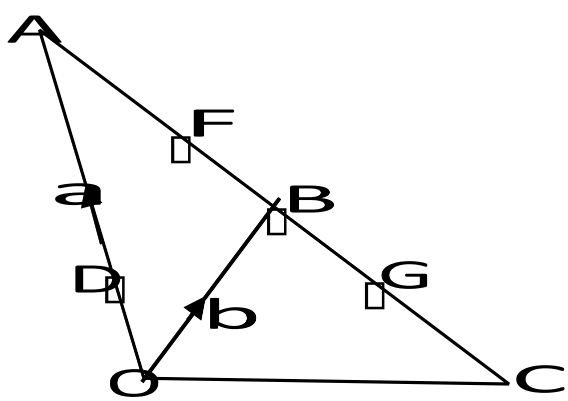
(i)RS

(ii)PV

(iii)RQ

(b)FindtheratioofRVtoRQ.

37.Inthefigurebelow, $OA = a$, $OB = b$, FandGarepointsonACsuchthat AF:AB=3:4andAG:AC=2:3.PointDisonOAsuchthat OD:DA=FB:BG=1:2.



(a)ExpressAGandACintermsofAB.Hencefindthefollowingvectors in termsofaandb: (i)AB

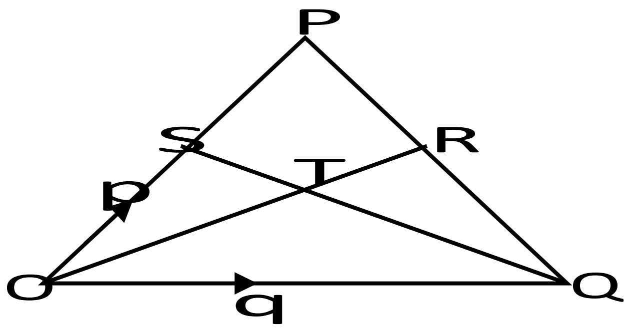
(ii)AC

(iii)DG

(iv)OF

(b)FindtheratioDG:OC

39.Inthefigurebelow, $OP = p,\ OQ = q,\ OS = \frac{3}{4}OP$ andPR:RQ=2:1



(a)Expressthefollowingvectorsintermsofpandq:

(i)PQ

(ii)OR

(iii)SQ

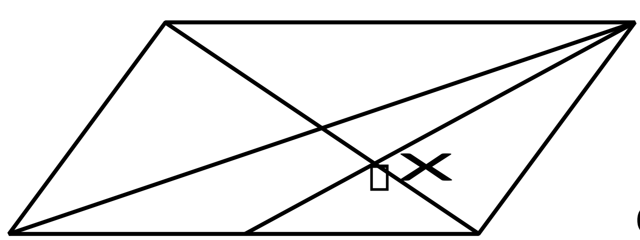
(b)LineORandSQmeetatpointTsuchthatOT=hORandST=kSQ.

(i)ByexpressingOTintwodifferentways,findthevaluesofhandk

(ii)DeterminetheratioinwhichTdividesSQ

40.Inthefigurebelow,OABCisaparallelogramwhereOA=aand

AB=b.PointNisonOAsuchthatON:NA=1:2.



C B

O N A

(a)Expressthefollowingvectorsintermsofaandb:

(i)AC

(ii)BN

(b)LineACandBNmeetatpointXsuchthatAX=hACandBX=kBN

(i)ByexpressingOXintwodifferentways,findthevaluesofhandk

(ii)DeterminetheratioinwhichXdividesAC

35.InatriangleOAB,NandMarepointsofABandOB $OA = a$, $OB = b$,

respectively.LineONandAMmeetatpointTsuchthatAT=TMand $OT = \frac{3}{4} ON.$

GiventhatOM=xOBandAN=yAB,Expressthevectors:

(i)AMandOTintermsofa,bandx.

(ii)ONandOTintermsofa,bandy,hencefindthevaluesofxandy.

EER:

1.GiventhepointsA(3,4)andB(9,1),findthecoordinatesofP,if

$$OP = OA + \frac{1}{3} AB.$$

2.Giventhevectors $a = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ m \end{pmatrix}$ and $c = \begin{pmatrix} n \\ -2 \end{pmatrix}$, findthevaluesof mandnforwhich $4a + 2b = 3c.$

$p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$ap + bq = r.$

3.Giventhevectors and $r = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, findthevaluesof aandbforwhich

4.Giventhatvector $OA = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $OB = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$, findthemagnitudeof vector $P = OA + \frac{2}{3} AB.$

5.Giventhatvector $p = \begin{pmatrix} x \\ x+2 \end{pmatrix}$, findthepossiblevaluesofxforwhich $|p| = 10.$

6.ThepositionvectorsofthepointsAandBare $\begin{pmatrix} 7 \\ -11 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 15 \end{pmatrix}$ respectively.IfpointEdividesABintheratio1:3,findtheposition vectorofE.

7.FindthedistancebetweenthepointsP(□8,2)andQ(4,7)

8.GiventhepointsA(□1,2),B(2,8),C(□2,□5)andD(4,y),findthevalueof yforwhichABisparalleltoCD.

9.Giventhevectors $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $r = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, and findthevaluesof $ap + bq = r.$

$AB = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ $CB = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$

aandbforwhich

1.Giventhevectors and findthe:

(i)vectorAC

(ii)magnitudeofAC

13.Thevectors $OP = \begin{pmatrix} -1 \\ -15 \end{pmatrix}$ $OQ = \begin{pmatrix} 7 \\ -11 \end{pmatrix}$, and $PN = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

(a)Findthe:

(i)positionvectorofN (02marks)

(ii)lengthofON (02marks)

(iii)coordinatesofpointE,whereEdividesPQintheratio1:3.(03marks)

(b)UsethevectormethodtoshowthatNliesonPQ.Hencestatethe

ratioPN:PQ. (05marks)

5.GiventhatOA=a,OB=bandCisthemidpointofAB,

(a)DrawavectordiagramshowingvectorAB. (01mark)

(b)Expressintermsofaandbthevectors:

(i)AB (01mark)

(ii)OC (02marks)

37.InthetriangleOAB,Cisapointon ABsuchthat [OA = a, OB = b,]

AC:AB=1:3andDisthemidpointofOB.

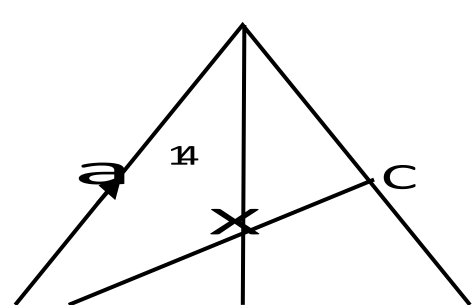
A

(a)Expressthefollowingvectorsintermsofaandb:

(i)AB

(ii)OC

(iii)AD



(b)XisapointonADsuchthatAX:AD=4:5.Findintermsofaandbthe vectors:

(i)AX

(ii)OX

(c)FindinsimplestformtheratioOX:OC.

TRANSLATION Summary:

1.Translationdealswithmovementofanobjecttoanewposition

2.Atranslation $T = \begin{pmatrix} a \\ b \end{pmatrix}$, meansthatanobjectismovedadistanceainthe

x□directionandadistancebinthey□direction

3.Atranslation $T = \begin{pmatrix} a \\ b \end{pmatrix}$, movespointP(x,y)toanewposition $P^{I}(x + a,\ y + b)$.
Thus,Translation+object=image

EXAMPLES:

1.Atranslation $T = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ mapsthepointsP(3,7)andQ(6,1)ontothe pointsand P^{I} Q^{I}

respectively.Findthecoordinatesofand P^{I} Q^{I}

2.AtrianglewithverticesA(2,1)B(2,3)andC(4,1)ismappedontoits imagebyatranslation $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Findthecoordinatesoftheimageofthe triangleABC.

3.Atranslation $T = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ mapspointPonto $P'(-1, 4)$. Findthe coordinatesofpointP

4.FindthetranslationthatmapspointA(2,6)onto $A'(3, 8)$.

5.AtranslationTmapspointP(2,5)onto $P'(3, 2)$. FindtheimageofQ(5, 7)undertranslationT

6.AtrianglewithverticesA(1,2)B(3,4)andC(5,2)ismappedontoits imagebyatranslation $T_1 = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ followedbyatranslation $T_2 = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.
Find: (i)asingletranslationrepresentingthetwosuccessive translations

(ii)thecoordinatesoftheimageofthetriangleABC.

7.AtrianglewithverticesA(2,0)B(1,□3)andC(□2,1)undergoesa translation $T_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ togivetriangle $A'B'C'$. Triangle isthen $A'B'C'$ mappedontotriangle $A''B''C''$ byatranslation $T_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

(a)Findthecoordinatesofthe verticesof:

(i)triangle $A'B'C'$

(ii)triangle $A''B''C''$

(b)PlottriangleABCanditsimagesonthesameaxes.

8.Atranslation $T = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, mapstheliney=2x+1ontoitsimage.Findthe equationoftheimageline