TOPIC: HEAT III

General Objective: The learner should able to determine the behaviour of gases using the gas law. SUB-TOPIC: Boyle's law

SPECIFIC OBJECTIVES: The learner should be able to;

- Carryout a simple experiment to demonstrate Boyle's law.
- Draw graphs of pressure, P, against volume, V and P against $\frac{1}{V}$ at constant temperature.

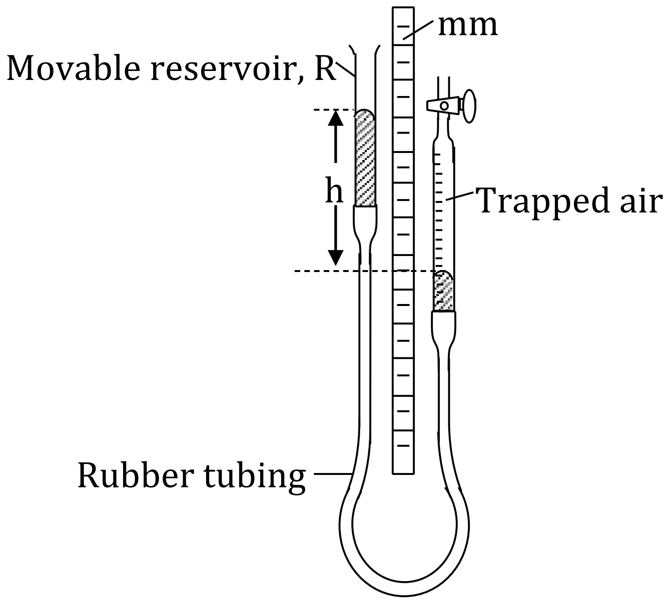
THE GAS LAWS

In this chapter we shall deal with the relationship between pressure, volume and temperature of a fixed mass of gas. The relationship is governed by certain laws known as the **gas laws.**

Boyle's Law

The volume of a fixed mass of gas at constant temperature is inversely proportional to the pressure.

Experiment: To Verify Boyle's Law



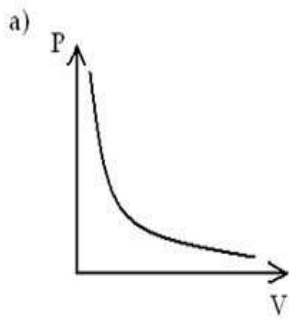
The apparatus consists of a graduated tube, T, connected by rubber tubing to a reservoir, R.

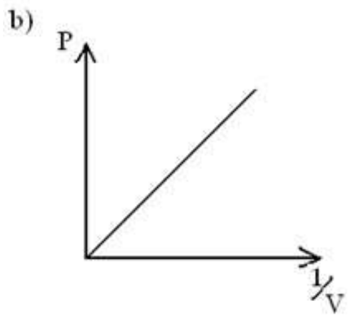
Dry air is trapped in the graduated tube over mercury as shown.

If H is the atmospheric pressure in mm of mercury, then the pressure acting on the air in T is P = H + h and p = (H – h) when the level of mercury in the reservoir is below the level of mercury in T.

The pressure is varied by altering the position of the reservoir, each time noting the height h and the volume V of the air in T.

A graph of P against V is plotted. A graph of P against $\frac{1}{V}$ is plotted.





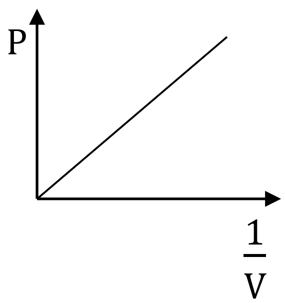
It is a straight line through the origin. Hence $P \propto \frac{1}{V}$.

Thus, the results may be summarised as
PV = C (a constant)

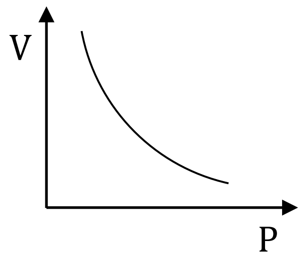
So, if a given mass of gas originally at a pressure P_1 and volume V_1, is compressed at constant temperature to a pressure P_2 and volume V_2, then

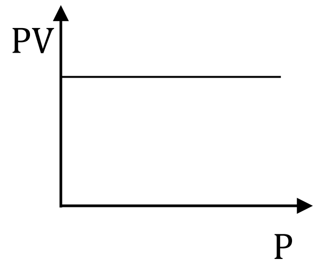
$$P_1V_1 = P_2V_2 = \text{constant}$$

NB: The air used must be dry because vapours generally do not obey the gas laws.



The following graphs also depict Boyle's law.





Examples

1. The volume of an air bubble at the bottom of a lake 80 m deep is 1.6 cm^3. What will be its volume just below the surface if the atmospheric pressure is equivalent to a height of 10 m of water? **Solution**

   $P_1 = 80 + 10 = 90$ m of water, $P_2 = 10$ m of water, $V_1 = 1.6\ cm^3$, $V_2 = ?$

   $$P_1V_1 = P_2V_2$$

   $$V_2 = \frac{1\ \ 1}{P_2} = \frac{x\ \ .}{10} = 14.4\ cm^3$$ P V 90 1 6

2. The volume of a fixed mass of gas at constant temperature is 20 cm^3 when the pressure is 75 cmHg. Find the new pressure when the volume increases to 50 cm^3.

   Solution:

   By Boyle's law PV = constant

   $\boldsymbol{P_1V_1 = P_2V_2}$

   $75 \times 20 = P_2 \times 50$

   P_2 = 30 cmHg

3. Air is trapped inside a glass tube by a thread of mercury 240mm long. When the tube is held horizontally, the length of the air column is 240mm.

   Diagram is on page 5 in longhorn bk4

   Assuming the atmospheric pressure is 750mmHg and the temperature is constant, calculate the length of the air column when the tube is held, (a) vertically with the open end up.

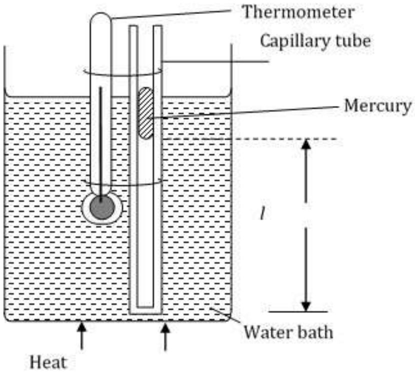
   (b) Vertically with the open end down.

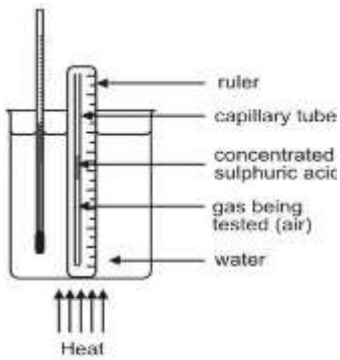
   (c) explain why the mercury does not fall out in (b)

Charles's Law

The volume of a fixed mass gas at constant pressure is directly proportional to its absolute temperature.

Experiment: To Verify Charles's Law





A capillary tube, closed at one end is warmed to drive away any moisture from it and then its open end is dipped into mercury so that, on cooling, a little mercury is sucked in. A thermometer is fastened on the capillary tube and the trapped air column is immersed in a water bath.

The temperature, θ, of the bath and the length *l* of the air column are measured and recorded.

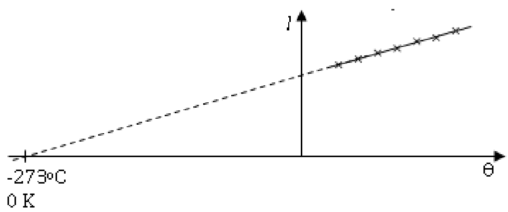
The temperature is varied by applying heat for an appropriate time.

The heating is stopped and then, after stirring, the new values of θ and *l* are noted.

The procedure is repeated for several different values of θ.

Now, the absolute temperature of the air in the capillary tube is $T = \theta + 273$ and, since the tube is of uniform cross-section, the length *l* is proportional to the volume of the trapped air.

A graph of *l* against θ is plotted. It is a straight line, which when extrapolated, it cuts the temperature axis at -273°C, i.e at T = 0 K (since $T = \theta + 273$)



Now, throughout the experiment the pressure on the trapped air was constant, being atmospheric plus that due to the weight of the mercury in the tube.

Therefore, the volume, V, is proportional to absolute temperature, T, at constant pressure.

Examples

1. The volume of a fixed mass of gas at constant pressure is 50 cm^3 at a temperature of 27 ^0C. What will the volume be at 42 ^0C?

Solution:

27 ^0C = 273 + 27 = 300 K

42 ^0C = 273 + 42 = 315 K

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = \frac{V_1 x T_2}{T_1}$$

$$V_2 = \frac{50 \times 315}{300}$$

V_2 = 52.5 cm^3

2. 800 cm^3 of gas at 20^oC is heated to 80^oC at constant pressure. W hat will the new volume be?

Solution

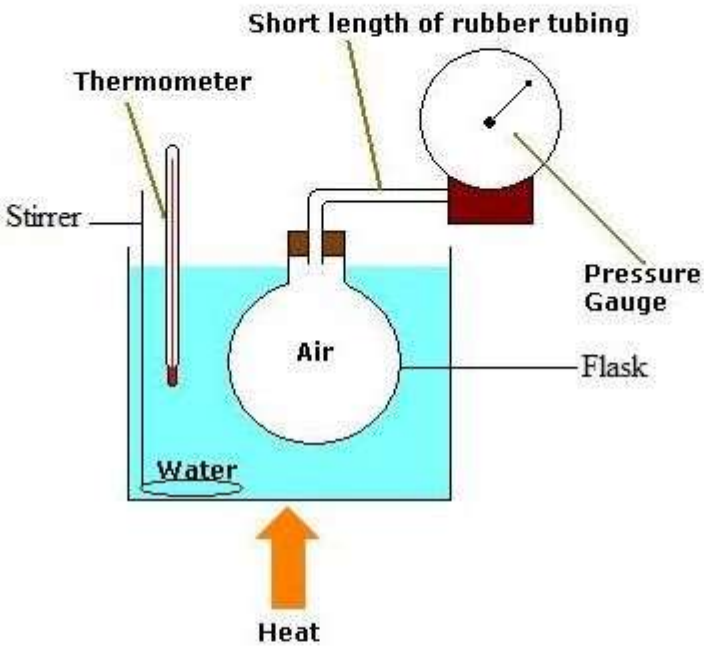
$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

$$\therefore V_2 = \frac{T_2}{T_1} V_1 = \frac{353 \text{ x } 800}{293} = 963.8\ cm^3$$

Pressure Law

The pressure of a fixed mass of gas at constant volume is directly proportional to its absolute temperature.

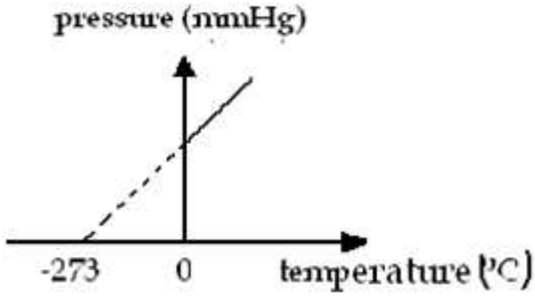
Experiment: To Verify the Pressure Law



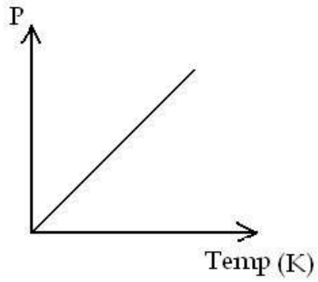
Dry air is trapped in a flask F and is connected to a pressure gauge.
F is fully immersed in a water bath.
The water is heated. The temperature of the air inside the flask and its pressure are recorded as shown in the table below.

Temperature (^0C)	Pressure (Pa)
-	-
-	-

The temperature of the gas in the flask is varied by heating the bath.
The water is stirred gently and sometime allowed before taking the liquid.
This ensures that the temperature of the surrounding is the same as the temperature of the air inside the flask.
The volume of the gas remains constant as it is not allowed to expand.
A graph of pressure against temperature is plotted.
A straight line graph is obtained with an intercept on the pressure axis.
When the line is extrapolated, it meets the temperature axis at approximately -273^0C.



When the graph above is re-drawn with temperature converted to absolute scale, the graph now passes through the origin.



Examples

1. The temperature of a gas with a pressure of 720mmHg was found to be 27^0C. What would be the final temperature of the gas if its pressure was raised to 760mmHg at constant volume?

2. Find the pressure of a gas whose temperature changes from 20^0C and pressure 680mmHg to 56^0C at constant volume.

Relation between Pressure, Volume and Absolute temperature (General gas law)

From Boyle's law PV = constant

From Charles's law $\frac{V}{T} = \text{constant}$

From pressure law $\frac{P}{T} = \text{constant}$

By combining the three equations we get

$$\frac{PV}{T} = \text{constant}$$

That is;

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

A gas that would obey the general gas law is called an **ideal gas.**

In practice however, gases only obey the general gas law approximately at low pressure and low temperatures.

At high temperature and high pressure all gases deviate appreciably from **the general gas law**. These are called real gases.

Standard temperature and pressure (s.t.p)

The temperature of 0^0C (273K) and 760mmHg have been chosen to be the standard states of the gas. They are called standard temperature and pressure (S.T.P) **Note:**

- ✓ All temperature must be in Kelvin
- ✓ In some calculations the volume of the gas has to be found at s.t.p (standard temperature and pressure). This is 0^0C and 760 mm Hg.
- ✓ Any units can be used for *P* and *V* so long as they are the same on both sides of the equation.

Examples

1. A balloon containing air at a pressure of 900 mmHg and temperature of 30^oC occupies a volume of 1500 cm^3. What will be its volume at s.t.p? **Solution**

P_1 = 900 mmHg, T_1 = 303 K, V_1 = 1500 cm^3

P_2 = 760 mmHg, T_2 = 273 K, V_2 = ?

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$\Box\ V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{900 \times 1500 \times 273}{303 \times 760} = 1600\ cm^3$$

2. Air in a 2.5 litres vessel at 127 ^{0}C exerts a pressure of 3 atm. Calculate the pressure that the same mass of air would exert if contained in a 4 liters vessel at – 43^0C.

Solution

$T_1 = 273 + 127 = 400K$, $T_2 = 273 - 43 = 230K$

$P_1 = 3\ atm$, $P_2 = ?\ atm$

$V_1 = 2.5\ L$ $V_2 = 4\ L$

$$\frac{P_1V_1}{T_1} \Box \frac{P_2V_2}{T_2}$$

$$P_2 \Box \frac{3 x 2.5 x 230}{400 x 4}$$

$$P_2 \Box \frac{P_1 x V_1 x T_2}{V_2 x T_1}$$

P2 = 1.078 atm

3. The volume of a gas is $90cm^3$ at 25 °C and 780mm Hg pressure. Calculate the volume of the gas at s.t.p.

Solution

T_2 = 0 °C = 273K

T_1 = 250°C = 25+273 = 298K

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2},$$

$$\frac{780x90}{298} = \frac{760xV_2}{273} \Rightarrow V_2 = 84.6cm^3$$

4. Calculate the volume at s.t.p of a gas whose volume is $100cm^3$ at -5°c and 746 mmHg

Solution

T_1 = -250 C = -25+273 = 248K

T_2 = 0°C = 273K V_1 = 100 cm^3 V_2 = ?

P_1= 746 mmHg ,

P_2 = 760mmHg

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2},$$

$$V_2 = \frac{P_1 x V_1 x T_2}{T_1 x P_2}$$

$$\frac{746 x 100 x 273}{248 x 760}$$

V_2 = $108cm^3$

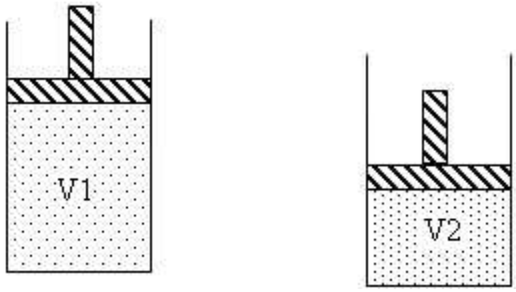
GASES AND THE KINETIC THEORY

The kinetic theory can explain the behavior of gases.

(a) **Cause of gas pressure:**

The pressure exerted by a gas is due to the molecules bombarding the wall of the container.

(b) **Boyle's Law**:



If the volume of a fixed mass of a gas is halved by halving the volume of the container, figure above, the number of molecules per cm^3 will be doubled. There will be twice as many collisions per second with the wall i.e. the pressure is doubled. This is Boyle's Law.

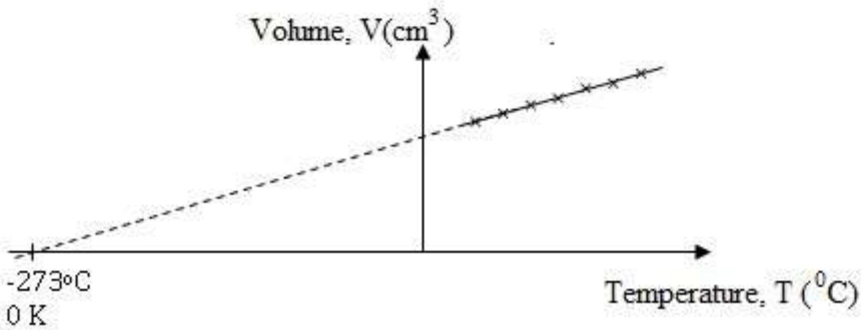
(c) **Temperature**:

When a gas is heated and its temperature rises, the average speed of its molecules increases. If the volume of the gas is to remain constant, its pressure increases due to more frequent and more violent collisions of the molecules with the walls. *(Pressure law)*

If the pressure of a gas is to remain constant, the volume must increase so that the number of collisions does not imcrease. *(Charles' law)*

(d) **Absolute Zero**:

This is the temperature at which the molecules have their lowest possible kinetic energy.



This temperature is called absolute zero temperature meaning that there is no chance of the gas attaining a temperature lower than this.

-273^0C is a theoretical temperature because by the time the volume of the gas falls to zero, the gas will have liquefied long before this temperature.

The absolute zero is called **zero kelvin (0K)** named after a scientist called Lord Kelvin.

REVISION EXERCISE

SECTION A

1. When air is pumped in a tube at constant temperature, the pressure increases because

A. the molecules are large

B. the molecules are moving faster

C. the molecules are closer together D. more molecules are hitting the tube.

2. The volume of a fixed mass of gas at 27.0^{0}c and a pressure of 750mm of mercury is 300cm^3. What is its volume when the pressure is raised to 900mm mercury and the temperature is 327^{0}c?

A. 125cm^3 B. 180cm^3 C. 500cm^3 D. 720cm^3

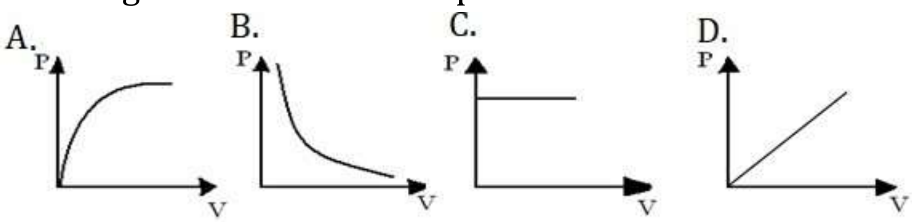
3. When the pressure of a mixed mass of a gas is reduced by half, its volume A. doubles at constant temperature.

B. is halved at constant temperature.

C. is halved if the temperature is also halved.

D. remains the same at constant temperature

4. Which of the following graphs shows the variation of the pressure of a gas as the volume changes at a constant temperature?



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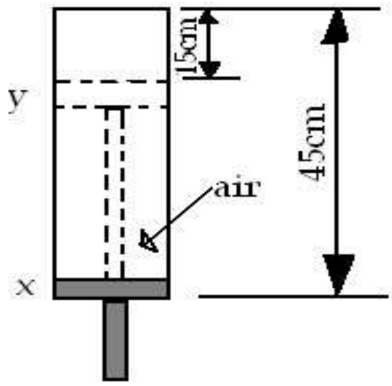


In figure above the piston is moved from X to Y at constant temperature. The air pressure is

A. trebled B. doubled C. reduced by a third D. unchanged

6. The pressure of a fixed mass of a gas at 17^0C is 10^5Pa. Find its pressure at 27^0C if the volume remains constant.

A. $\frac{27}{17} x 10^5 Pa$ B. $\frac{17}{27} x 10^5 Pa$ C. $\frac{300}{290} x 10^5 Pa$ D. $\frac{290}{300} x 10^5 Pa$

7. The pressure exerted by a gas decreases when its volume is increased at a constant temperature because the molecules

A. move faster B. move closer to one another.
C. hit the walls more often. D. hit the walls less frequently.

8. Which of the following statements is incorrect when a tin containing air tightly sealed is heated?

A. the average speed of molecules increases
B. the molecules of air hit the walls of the tin harder
C. The molecules of the air strike the walls of the tin less often.
D. The pressure inside the tin increases.

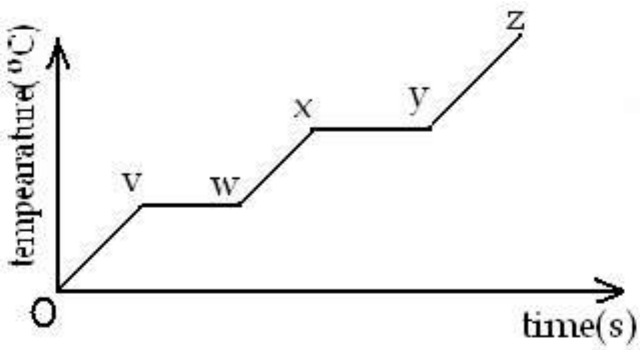
9. The particles in a solid at room temperature are A. close together and vibrating. B. close together and stationary.
C. far apart and moving at random.
D. close together and moving at random.

10. The temperature at which all the heat energy is removed from the substance is called

A. Kelvin temperature B. Celsius temperature
C. Freezing temperature D. Absolute zero temperature

11. Which one of the following statements is true?
A. The average kinetic energy of the molecules of a gas depends on temperature
B. Each molecule of a gas at a given temperature has a different speed
C. The pressure of a fixed mass of gas decreases as the temperature increases
D. The volume of a fixed mass of gas increases as the temperature decreases

12. The graph in the figure is that of the temperature against time for a substance, which is heated at a constant rate temperature



Which part of the graph corresponds to the situation when the molecules of the substance have the highest average kinetic energy?
A. OV B. VW C. VX D. YZ

13. At room temperature, air is less dense than water because air molecules
A. move faster
B. are smaller
C. have greater force of attraction
D. are more widely separated from each other.

14. When a liquid is heated
A. its density decreases
B. boiling occurs at all temperatures
C. its molecules move with the same speed
D. evaporation takes place throughout the liquid

15. It is more difficult to compress a liquid than a gas because
A. the speed of liquid molecules is lower than that of gas molecules B. liquid particles attract one another when compressed while gas particles repel each other
C. the distances between liquid particles are less than those between gas particles
D. liquid molecules repel one another when compressed while gas molecules repel one another

SECTION B

1. a) State Boyle`s Law and verify the law.
b) State Charles` Law and verify the law.
c) State Pressure Law and verify the law.

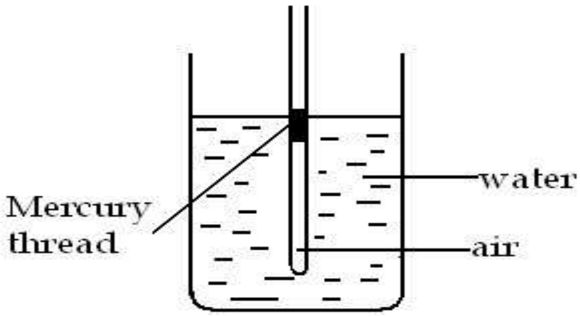
2. a) What is an equation of state of a gas?

b) (i) With the aid of a sketch graph, describe how absolute zero of temperature can be defined.
(ii) Use the kinetic theory of gases to explain the existence of absolute zero of temperature.

3. A gas of volume $1000cm^3$ at a pressure of 4.0×10^5 Pa and temperature 17 ^0C is heated to 89.5 ^0C at constant pressure. Find the new volume of the glass.
(Ans: 1250 cm^3)

4. A cylinder with a movable piston contains 0.35 m^3 of air at a temperature of 73 ^0C. Calculate the volume of the gas if it is cooled to 27 ^0C at constant pressure.
(Ans: 0.3 cm^3)

5.



A graduated glass-tube containing air trapped below a mercury thread is immersed in a beaker of water and the water is heated as shown in the figure above.

(i) Sketch a graph showing how the volume of the trapped air varies with absolute temperature.

(ii) When t he temperature of the water is 27 ^{0}C, the volume of the trapped air is $1.2x10^{-1}$ cm^3. Calculate the volume of air when its temperature is 77 ^{0}C. *(Ans: 0.14 cm^3)*

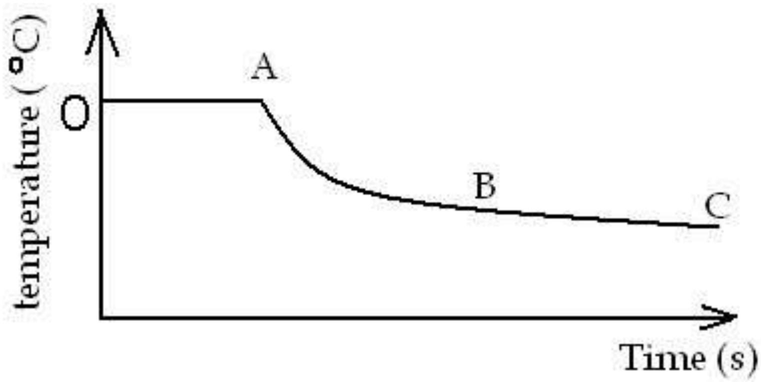
6. With aid of labeled diagram, describe an experiment to show how to volume of a gas varies with pressure at constant temperature.

7. With the aid of a labeled diagram, describe an experiment to show the relationship between the volume and temperature of a fixed mass of gas at atmospheric pressure.

8. A volume of a fixed mass of a gas increases from 300 cm^3 to 500 cm^3 at a constant temperature. Find the new pressure if the initial pressure is 70 cm Hg. *(Ans: 116.7 cmHg)*

9. A volume of 2500 cm^3 of hydrogen gas is collected at 77 ^{0}C at a pressure of 730 mmHg. Calculate the volume of the gas at s.t.p.
(Ans: 1873 cm^3)

10.



(a) The figure shows temperature versus time curve for a liquid. State what is happening along BC

(b) Use the kinetic theory of matter to explain what is happening along OA.

11. (a) Sketch the variation of volume with temperature in Kelvin, for a gas at constant pressure.

(b) State any two advantages of mercury as a thermometric substance.

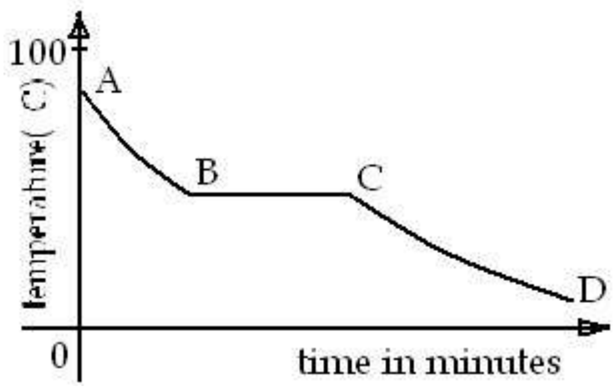
12. Explain the following observations;

(a) the increase in temperature of the gas

(b) the increases in the pressure of the gas

13. (a) State the kinetic theory of matter.

14.



The figure shows a cooling curve for a substance, which is in liquid form at 100^0C.

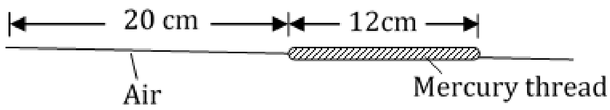
(i) In what stages is the substance over the regions AB, BC and CD of the curve?

(ii) Use the kinetic theory of matter to explain the difference between the stages of the substance over the regions AB and CD.

15. The volume of a gas is 100 cm^3 at a temperature of 27^oC and a pressure of 3.0×10^5 Pa. Find the volume of the gas at s.t.p.

16. A certain gas occupies a volume of 300 cm^3 at a temperature of 17^oC and a pressure of 1.0×10^5 Pa. What will be its volume at 57^oC if the pressure remains constant?

17. The diagram shows a horizontal tube of uniform cross-section closed at one end containing dry air which is trapped by a thread of mercury of length 12 cm.



When the tube is inverted, the length of the air column becomes 23.8 cm. Calculate the atmospheric pressure.

END.