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MATHEMATICS
PAPER I
JULY 2017
2 ½ HOURS

ST. JOSEPH OF NAZARETH HIGH SCHOOL
UGANDA CERTIFICATE OF EDUCATION
INTERNAL MOCKS EXAMINATION 2017
MATHEMATICS
PAPER I
2 ½ HOURS

INSTRUCTIONS TO CANDIDATES:

- Answer all questions in Section A and any five questions from Section B.
- Any additional question(s) answered will not be marked.
- All necessary calculations must be done in the answer booklet provided. Therefore, no paper should be given for rough work.
- Graph paper is provided.
- Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

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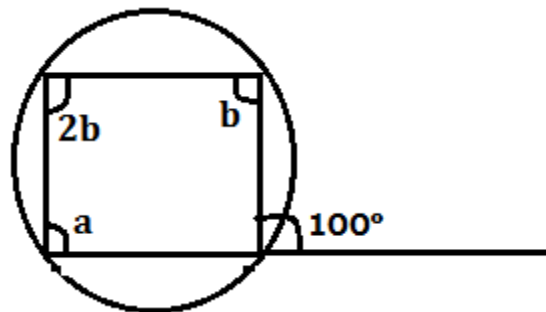
SECTION A (40 MARKS)

Attempt all questions in this.

1. Make x the subject of the formula $A = (px^2 - q)^{\frac{1}{3}}$. (4 marks)

2. Find the integral values of x which satisfies the inequalities :
 $2x - 1 < 7 + x \leq 3x + 1$ (4 marks)

3. Find the value of a and b in



(4 marks)

4. A point $P(-3,2)$ is reflected through the mirror line $y - x = 0$. Find the coordinates of the point image P^1 . (4 marks)

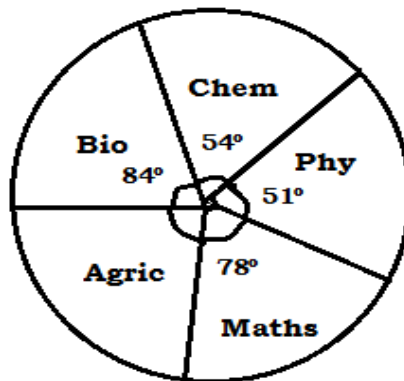
5. Two doctors are chosen randomly one at a time without replacement from a medical staff consisting of 2 women and 3 men to attend an HIV/AIDS workshop. Calculate the probability that the two doctors chosen are:
 (i) of the same sex (2 marks)
 (ii) of opposite sex (2 marks)

6. Solve for x in $(x + 3)(x - 2) = 14$ (4 marks)

7. Determine the value of y for which the matrix below has no inverse.
 $\begin{pmatrix} y & 2 \\ y^2 & 1 \end{pmatrix}$ (4 marks)

8. Given that $m * n = m^2 - 2n$
 Find (i) $3 * 2$ (2 marks)
 (ii) $(3 * 2) * 5$ (2 marks)
9. If $\tan \theta = \frac{3}{4}$ for $0^\circ \leq \theta \leq 90^\circ$, find the value of; $\sin \theta + \cos \theta$ (4 marks)

10. The total marks scored by a student in five subjects was 240.
 The pie - chart below represents the marks scored in each subjects.



What was the score in Agriculture? (4 marks)

SECTION B (60 MARKS)
Attempt any five questions from this section.

11. The table below shows the masses to the nearest kg of 80 animals on a certain farm.

Mass (Kg)	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
No. of animals	5	13	32	27	3

- (a) State the modal class
 (b) Calculate the mean mass of the animals.
 (c) Draw a histogram to represent the above data and use it to find the modal mass (12 marks)

12. (a) Given that $A = \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$ and $C = 2AB - A^2$.
Determine matrix C . (4 marks)
- (b) Find the inverse of $\begin{pmatrix} 2 & 5 \\ 3 & -2 \end{pmatrix}$. Hence or otherwise.
Solve the equations; $2x + 5y = 11$ and $3x - 2y = 7$. (4 marks)
- (c) Opio bought 5 Biology books and 6 Chemistry books for a total of shs. 2440. Magambo bought 7 Biology books and 9 Chemistry books for a total of shs. 3560.
- (i) Form a matrix equation to represent the above information.
(ii) Use matrix method to find the price of one Biology book. (4 marks)
13. Four towns P, Q, R and S are such that Q is 84km directly to the north of P and R is on a bearing of 295° from P at a distance of 60km . S is on a bearing of 340° from R and at a distance of 30km .
- (a) Using a scale of 1cm to represent 10km make an accurate scale drawing to show the relative position of the towns. (7 marks)
- (b) Find;
- (i) The distance and bearing of Q from R
(ii) The distance and bearing of S from Q .
(iii) The bearing of P from S (5 marks)
14. (a) Use the graphical method to solve the simultaneous equations,
 $y = 8 - 2x - x^2$ and $y = 4 - 2x$ for $-5 \leq x \leq 3$. (12 marks)
- (b) Find also the roots of the equation $8 - 2x - x^2 = 0$ from the graph
15. (a) Factorise: $p^2 - q^2$. Hence, find the exact value of $2557^2 - 2547^2$. (4 marks)
- (b) If $x = 8.52 \times 10^{-1}$, $y = 2.4 \times 10^{-2}$ and $z = 4.0 \times 10^{-3}$, without using tables or calculator evaluate $\frac{x-y}{z}$, give your answer in standard form. (4 marks)
- (c) Kamoga spent $\frac{1}{3}$ of his net salary on school fees. He also spent $\frac{1}{4}$ of the remainder on rent. He then spent $\frac{1}{8}$ of what was left on transport. If finally he had shs. 4200. What was his net salary? (4 marks)

16. The coordinates of the vertices of rectangle $ABCD$ are $A(1,1)$, $B(6,1)$, $C(6,4)$ and $D(1,4)$.

(a) (i) Find the coordinates of the vertices of its image, $A^1B^1C^1D^1$ under the transformation defined by the matrix $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

(ii) Draw the object and its image on the graph paper provided.

(iii) On the same graph, draw the image, $A^{11}B^{11}C^{11}D^{11}$ of $A^1B^1C^1D^1$ under the transformation given by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b) Find a single matrix which would map $A^{11}B^{11}C^{11}D^{11}$ onto $ABCD$.

(12 marks)

17. Nyakana makes two types of shoes A and B . He takes 3 hours to make one shoe of type A and 4 hours to make one shoe of type B . He works for a maximum of 120 hours to make x pairs of type A and y pairs of type B .

It costs him sh. 400 to make a pair of type A and sh. 150 to make a pair of type B .

His total cost does not exceed shs. 9000. He must make at least 8 pairs of type A , and more than 12 pairs of type B .

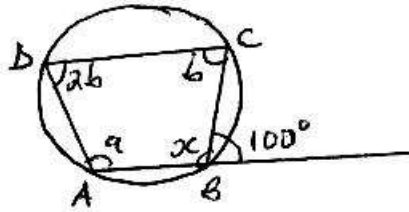
(a) Write down four inequalities representing the given information.

(b) On a graph paper, draw the inequalities and shade the unwanted regions.

(c) Nyakana makes a profit of shs. 40 on each pair of type A and shs. 70 on each pair of type B shoes.

Use your graph to determine the maximum possible profit he makes. (12 marks)

~END~

	ST. JOSEPH OF NAZARETH HIGH SCHOOL INTERNAL MOCK EXAMS 2017 S.H MATHS P.1 MARKING GUIDE	①
No.	SOLUTION	MKS COMMENT
1.	$A^3 = [(px^2 - q)^{\frac{1}{3}}]^3$ $A^3 = px^2 - q$ $px^2 = A^3 + q$ $x = \left(\frac{A^3 + q}{p}\right)^{\frac{1}{2}} \text{ or } x = \sqrt{\frac{A^3 + q}{p}}$	M ₁ B ₁ B ₁ A ₁ 04
2.	$2x - 1 < 7 + x \leq 3x + 1$ $2x - 1 < 7 + x \text{ and } 7 + x \leq 3x + 1$ $x < 8$ $x < 8 \text{ B}_1$ $6 \leq 2x$ $3 \leq x \text{ B}_1$ $\therefore 3 \leq x < 8$ $x = \{3, 4, 5, 6, 7\} \text{ A}_1$	M ₁ B ₁ B ₁ A ₁ 04
3.	 <p>Let $\angle ABC = x$ $x + 100^\circ = 180^\circ$ (\angles on straight line) $x = 180^\circ - 100^\circ = 80^\circ \text{ B}_1$ $2b + 80^\circ = 180^\circ$ (Opp. \angles of cyclic quad.) M₁ $\frac{2b}{2} = \frac{100^\circ}{2}$ $\therefore b = 50^\circ \text{ A}_1$ $a + b = 180^\circ$ (Opp. \angles of cyclic quad.) $a = 180^\circ - 50^\circ$ $a = 130^\circ \text{ A}_1$</p>	04

2

4. Matrix for reflection, $R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 in $y = x$

Transformation \times Obj = Image

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0+2 \\ -3+0 \end{pmatrix}$$

$$OP' = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

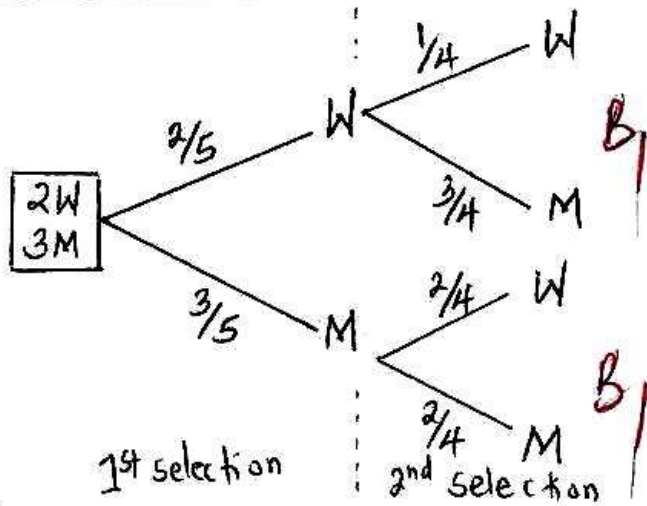
\therefore Coordinates of P' (2, -3)

B₁

M/M

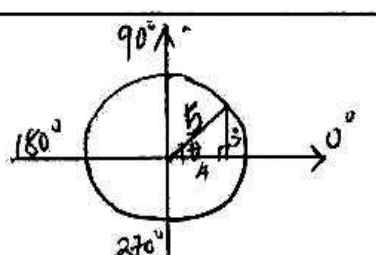
A₁
0/1

5.



(i) $P(\text{Both of same sex}) = P(W_1 \cap W_2) + P(M_1 \cap M_2)$
 $= \left(\frac{2}{5} \times \frac{1}{4}\right) + \left(\frac{3}{5} \times \frac{2}{4}\right)$
 $= \frac{1}{10} + \frac{3}{10}$
 $= \frac{2}{5}$ A₁

(ii) $P(\text{of opposite sex}) = P(W_1 \cap M_2) + P(M_1 \cap W_2)$
 $= \left(\frac{2}{5} \times \frac{3}{4}\right) + \left(\frac{3}{5} \times \frac{2}{4}\right)$
 $= \frac{3}{10} + \frac{3}{10}$
 $= \frac{3}{5}$ A₁

<p>6.</p>	$x^2 - 2x + 3x - 6 = 14$ $x^2 + x - 20 = 0$ $(-4, 5) \quad M_1$ $x^2 - 4x + 5x - 20 = 0$ $x(x-4) + 5(x-4) = 0 \quad M_1$ $(x+5)(x-4) = 0$ <p>Either $x = -5$ or $x = 4$ A_1</p>	<p style="text-align: right;">(3)</p>
<p>7.</p>	<p>for singular matrix; $\det = 0$</p> <p>Determinant $= (yx) - (2y^2) \quad M_1$</p> $0 = y - 2y^2$ $0 = y(1 - 2y) \quad M_1$ <p>Either $y = 0$ or $y = \frac{1}{2}$ A_1</p>	<p style="text-align: right;">04</p>
<p>8.</p>	$m * n = m^2 - 2n$ <p>(i) $3 * 2 = 3^2 - 2(2) \quad M_1$</p> $= 9 - 4$ $= 5 \quad A_1$ <p>(ii) $(3 * 2) * 5 = 5 * 5$</p> $= 5^2 - 2(5) \quad M_1$ $= 25 - 10$ <p>$\therefore (3 * 2) * 5 = 15 \quad A_1$</p>	<p style="text-align: right;">04</p>
<p>9.</p>	 $h = \sqrt{4^2 + 3^2}$ $= \sqrt{16 + 9}$ $= \sqrt{25}$ $h = 5 \quad B_1$ <p>$\sin \theta = \frac{3}{5} \quad B_1; \quad \cos \theta = \frac{4}{5} \quad B_1$</p> <p>$\sin \theta + \cos \theta = \frac{3}{5} + \frac{4}{5} = \frac{7}{5} \quad A_1$</p>	<p>SOH CAH TOA</p>

			(4)
10.	$\begin{aligned} \text{Agriculture Sector} &= 360^\circ - [84^\circ + 54^\circ + 51^\circ + 78^\circ] \\ &= 360^\circ - 267^\circ \\ &= 93^\circ \end{aligned}$ $\begin{aligned} \text{Score in Agric} &= \frac{93^\circ}{360^\circ} \times 240 \\ &= 62 \text{ marks} \end{aligned}$	M1 B1 M1 A1	
		04	

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NO.	SOLUTION	MKS	COMMENT
12.	<p>(a) $A = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$</p> <p>$C = 2 \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix}^2$</p> <p>$C = 2 \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \left[\begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 3 \end{pmatrix} \right]$</p> <p>$C = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -8 & 9 \end{pmatrix}$</p> <p>$\therefore C = \begin{pmatrix} 5 & 0 \\ 8 & -3 \end{pmatrix}$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	
	<p>(b) let $A = \begin{pmatrix} 2 & 5 \\ 3 & -2 \end{pmatrix}$</p> <p>Determinant = $(-2 \times 2) - (3 \times 5)$</p> <p>$= -4 - 15$</p> <p>$\det(A) = -19$</p> <p>Adjunct = $\begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix}$</p> <p>Inverse = $\frac{1}{-19} \begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix}$</p> <p>$A^{-1} = \begin{pmatrix} 2/19 & 5/19 \\ 3/19 & -2/19 \end{pmatrix}$</p>	<p>M1</p> <p>A1</p>	

8

NO.	SOLUTION	MKS	COMMENT
	<p>Hence $2x + 5y = 11$ $3x - 2y = 7$</p> $\begin{pmatrix} 2 & 5 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$ $\frac{1}{19} \begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19} \begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 11 \\ 7 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19} \begin{pmatrix} -22-35 \\ -33+14 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19} \begin{pmatrix} -57 \\ -19 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ <p>$\therefore x = 3$ $y = 1$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>05</p>	<p>$A^{-1} \cdot A = I$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$</p>
(C)	<p>let x denote biology book cost y denote cost of a chemistry book</p>		

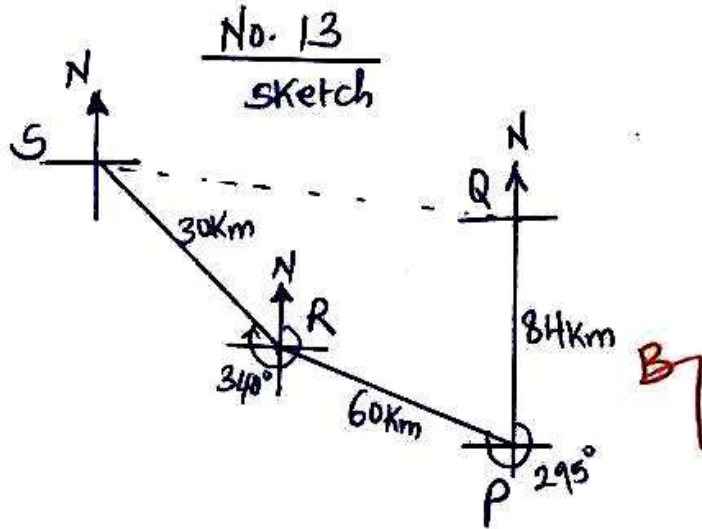
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NO. 12

NO.	SOLUTION	MKS	COMMENT
(c)	let B denote cost of a biology book C denote cost of a Chemistry book		
(i)	$5B + 6C = 2440$ $7B + 9C = 3560$ $\begin{pmatrix} 5 & 6 \\ 7 & 9 \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} 2440 \\ 3560 \end{pmatrix}$	B ₁	
(ii)	$\begin{pmatrix} 9 & -6 \\ -7 & 5 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 9 \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} 9 & -6 \\ -7 & 5 \end{pmatrix} \begin{pmatrix} 2440 \\ 3560 \end{pmatrix}$	M ₁	
	$\begin{pmatrix} 45 & -42 & 54 & -54 \\ -35 & 35 & -42 & 45 \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} 21960 & -21360 \\ -17080 & 17800 \end{pmatrix}$		
	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} 600 \\ 720 \end{pmatrix}$	M ₂	
	$3B = 600$		
	$\therefore B = 200$		
	$3C = 720$		
	$\therefore C = 240$		
	$\therefore \text{The price of one biology book is sh. } 200/-$	A ₁	
		04	

(10)

NO.	SOLUTION	MKS	COMMENT
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$$1 \text{ cm} : 10 \text{ km}$$

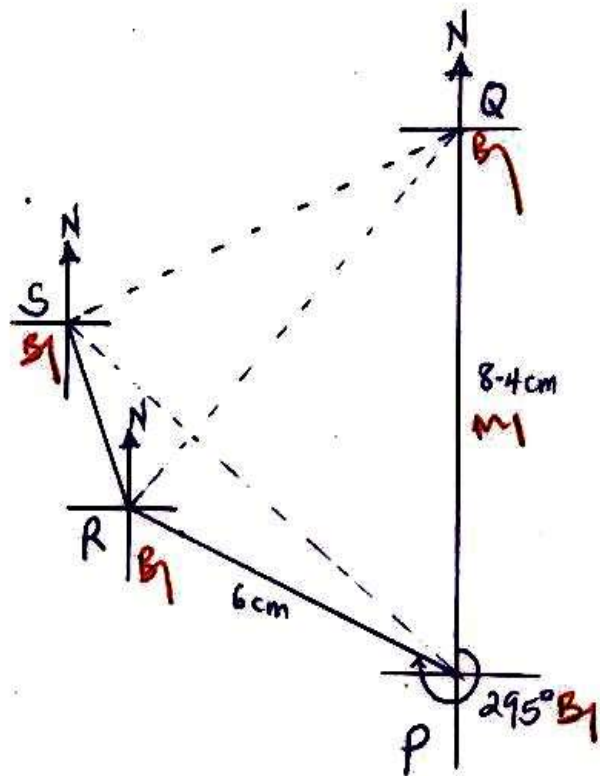
$$PR = \frac{84}{10} = 8.4 \text{ cm}$$

$$PR = \frac{60}{10} = 6 \text{ cm} \quad B7$$

$$RS = \frac{30}{10} = 3 \text{ cm}$$

No. 13

Accurate



- (b) (i) Distance QR = $8 \times 10 = 80 \text{ km} \pm 1 \text{ km}$
 Bearing of Q from R is $065^\circ \pm 1^\circ$
 ~~045°~~
- (ii) Distance SQ = $7.2 \times 10 = 72 \text{ km} \pm 1 \text{ km}$
 Bearing of S from Q = $180^\circ + 65^\circ = 245^\circ \pm 1^\circ$
- (iii) Bearing of P from S = $130^\circ \pm 1^\circ$

No.	SOLUTION	MKS	COMMENT
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14. (a) $y = 8 - 2x - x^2 \quad -5 \leq x \leq 3.$

x	-5	-4	-3	-2	-1	0	1	2	3
8	8	8	8	8	8	8	8	8	8
-2x	10	8	6	4	2	0	-2	-4	-6
-x ²	-25	-16	-9	-4	-1	0	-1	-4	-9
y	-7	0	5	8	9	8	5	0	-7

line $y = 4 - 2x$

x	0	2
y	4	0

$(0, 4) \quad (2, 0)$

from the graph, the solution
the simultaneous equations
 $y = 8 - 2x - x^2$ and
 $y = 4 - 2x$ are

for $x = -2, y = 8$ A1

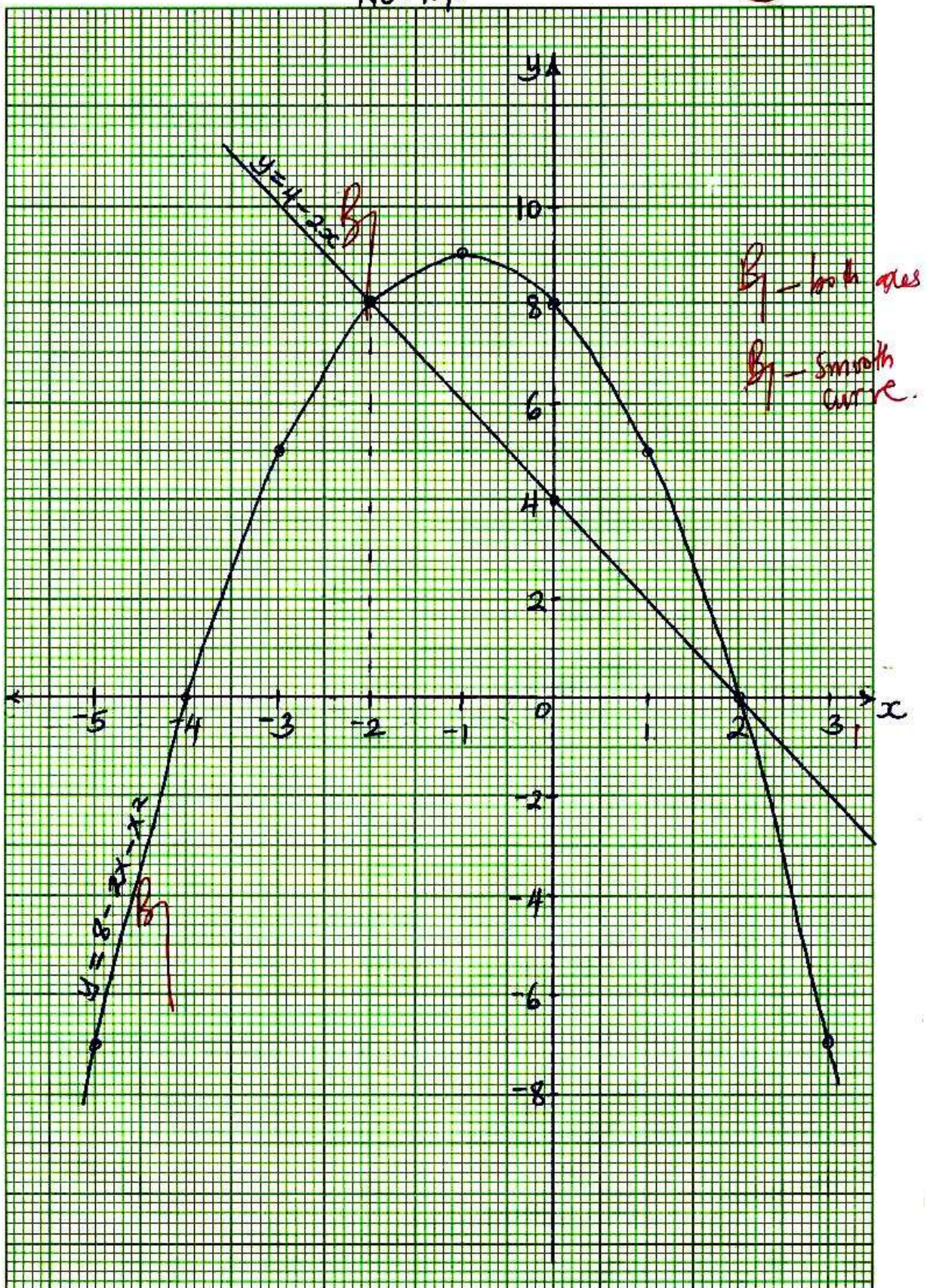
for $x = 2, y = 0$ A1

(b) from the graph the roots of
 $8 - 2x - x^2 = 0$ are

$x = -4$ A1 and $x = 2$ A1

NO-14

13



No. 15

Pg. 14

NO.	SOLUTION	MKS	COMMENT
15.	<p>(a). $p^2 - q^2 = (p+q)(p-q)$</p> $2557^2 - 2547^2 = (2557+2547)(2557-2547)$ $= (5104)(10)$ $= 51040$	<p>B₁</p> <p>M₁</p> <p>M₁</p> <p>A₁</p>	<p>- substit</p>
	<p>(b)</p> $\frac{x-y}{z} = \frac{8.52 \times 10^{-1} - 2.4 \times 10^{-2}}{4.0 \times 10^{-3}}$ $= \frac{85.2 \times 10^{-2} - 2.4 \times 10^{-2}}{0.4 \times 10^{-2}}$ $= \frac{(85.2 - 2.4) \times 10^{-2}}{0.4 \times 10^{-2}}$ $= \frac{82.8 \times 10}{0.4 \times 10} = \frac{828}{4}$ $= 207$ <p>$\therefore \frac{x-y}{z} = 2.07 \times 10^2$</p> <p>is standard form</p>	<p>M₁</p> <p>M₁</p> <p>M₁</p> <p>A₁</p>	<p>substit</p>

(15)

NO.	SOLUTION	MKS	COMMENT
15.	<p>(9) let Kamoga's net salary = y</p> <p>School fees = $\frac{1}{3}y$</p> <p>Rent = $\frac{1}{4}(y - \frac{1}{3}y) = \frac{1}{6}y$ ✓</p> <p>Transport = $\frac{1}{8}(y - \frac{1}{6}y) = \frac{5y}{48}$ ✓</p> <p>Balance left = $y - \left[\frac{y}{3} + \frac{y}{6} + \frac{5y}{48} \right]$</p> <p style="margin-left: 100px;">$= y - \frac{29y}{48}$</p> <p style="margin-left: 100px;">$= \frac{19y}{48}$</p> <p>$\Rightarrow \frac{19y}{48} = 4200$</p> <p style="margin-left: 40px;">$19y = 201600$</p> <p style="margin-left: 80px;">$y = 10610.53$</p> <p>\therefore Kamoga's net salary is sh. 10610.53</p> <p style="margin-left: 100px;">\approx sh. 10,611.</p>	<p style="text-align: center;">18</p> <p style="text-align: center;">17</p> <p style="text-align: center;">17</p> <p style="text-align: center;">17</p>	

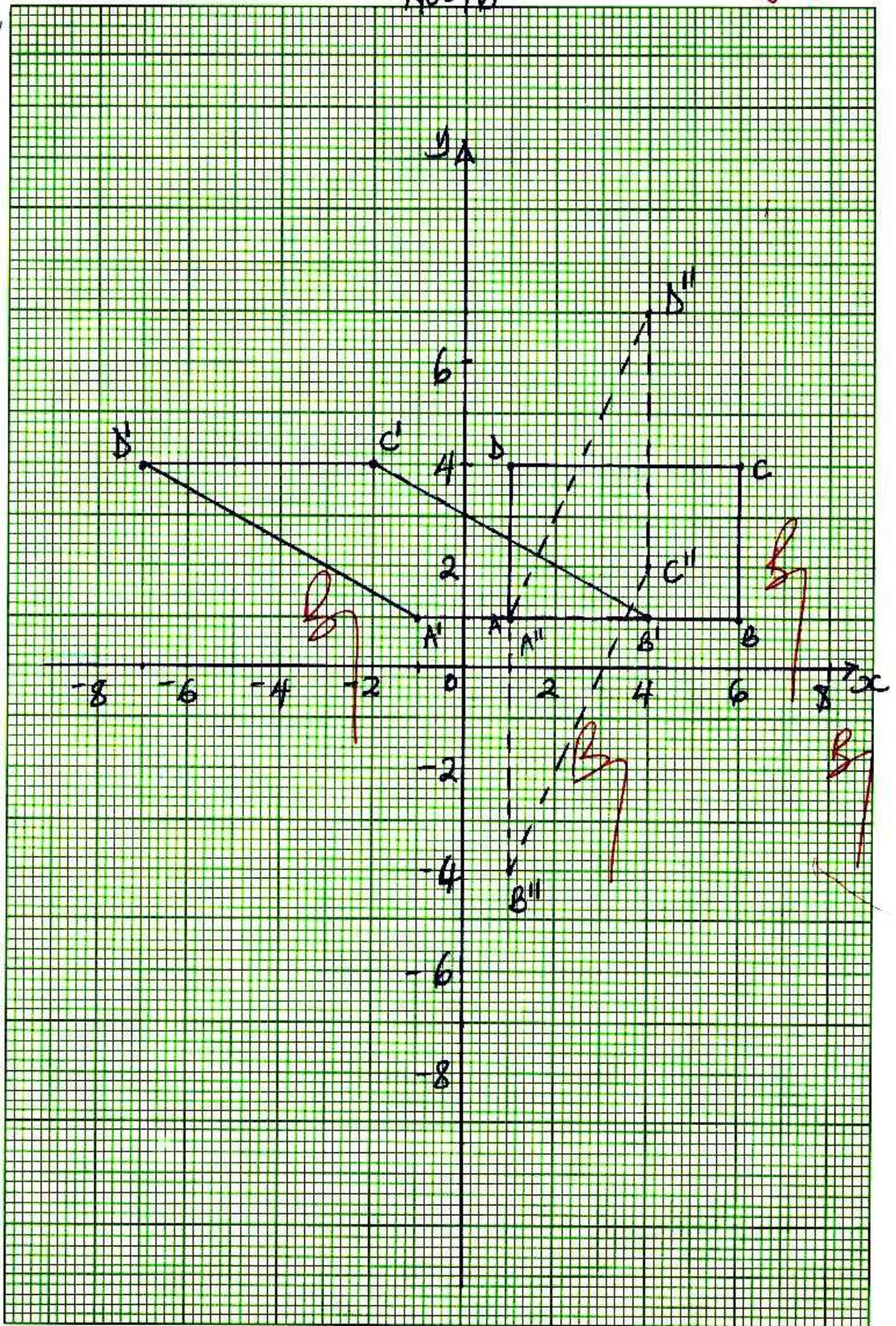
pg-16

NO.	SOLUTION	MKS	COMMENT
16	<p>(a) (i) Transformation \times Object = Image</p> $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 6 & 6 & 1 \\ 1 & 1 & 4 & 4 \end{pmatrix}$ $= \begin{pmatrix} A' & B' & C' & D' \\ 1+2 & 6+2 & 6+8 & 1+8 \\ 0+1 & 0+1 & 0+4 & 0+4 \end{pmatrix}$ $= \begin{pmatrix} -1 & 4 & -2 & -7 \\ 1 & 1 & 4 & 4 \end{pmatrix}$ <p>$\therefore A'(-1, 1), B'(4, 1), C'(-2, 4), D'(-7, 4)$</p>	<p>M₁</p> <p>B₁</p> <p>A₁</p> <p>03</p>	<p>All correct</p>
16	<p>(ii) Image of A''B''C''D'' of A'B'C'D' under $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$</p> $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} A' & B' & C' & D' \\ -1 & 4 & -2 & -7 \\ 1 & 1 & 4 & 4 \end{pmatrix}$ $= \begin{pmatrix} A'' & B'' & C'' & D'' \\ 1 & 1 & 4 & 4 \\ 1 & -4 & 2 & 7 \end{pmatrix}$ <p>A''(1, 1), B''(1, -4), C''(4, 2), D''(4, 7)</p>	<p>M₁</p> <p>A₁</p> <p>06</p>	

NO.	SOLUTION	MKS	COMMENT
16-	<p>(b) let $T = T_2 T_1$</p> $T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 0+0 & 0+1 \\ -1+0 & 2+0 \end{pmatrix}$ $T = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$ $\det(T) = 0 - (-1) = 1$ $T^{-1} = \frac{1}{1} \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ $T^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ <p>$\therefore \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ is a single matrix which would map A''B''C''D'' onto ABCD.</p>	<p>M</p> <p>M</p> <p>M</p> <p>A</p> <hr/> <p>03</p>	

No-16

Pg-18



No. 17

(a) x is the number of shoes of type A and y is the number of shoes of type B.

$$3x + 4y \leq 120$$

$$400x + 150y \leq 9000$$

$$8x + 3y \leq 180$$

$$x \geq 8$$

$$y > 12$$

$$3x + 4y = 120$$

x	0	40
y	30	0

$$8x + 3y = 180$$

x	0	22.5
y	60	0

Maximum profit when
 $x = 8$ and $y = 24$

$$\begin{aligned} \text{Profit} &= 8 \times 40 + 24 \times 70 \\ &= 320 + 1680 \\ &= \text{Sh } 2000 \end{aligned}$$

