UACE 2014

BOHISM

S.6 SEMINAR QUESTIONS 2014

General algebra and simultaneous equations

1. Solve $3(3^{2x}) + 2(3^x) - 1 = 0$ **(ILONGOLE EMMANUEL)** (5 marks)
2. Solve the inequality $\frac{x-1}{x-2} > \frac{x-2}{x+3}$ (5 marks)
3. Solve the simultaneous equations;

   $2x - 5y + 2y = 14$

   $9x + 3y - 4z = 13$

   $7x + 3y - 3z = 3$ **(TRUST PAUL)** (5 marks)
4. Given that the equation $2x^2 + 5x - 8 = 0$ has roots α and β. Find the equation whose roots are $\frac{1}{(\alpha+2)^2}$ and $\frac{1}{(\beta+2)^2}$ (5 marks)
5. Show that $Z = 1$ is a root of the equation $Z^3 - 5Z^2 + 9Z - 5 = 0$. Hence solve the equation for the other roots (5 marks)
6. Evaluate $\int_0^1 \frac{x^3}{x^2+1} dx$ (5 marks)
7. Solve the equation $2\cos\theta - \csc\theta = 0; 0^0 \le \theta \le 270^0$ (5 marks)

Curve sketching

8. a) Express $\frac{x^2}{(x+1)^2}$ in partial fractions.

   b) A curve is defined by $y = \frac{x^2}{(x+1)^2}$

   i. Determine the equations of all its asymptotes
   ii. Find its turning point and justify its nature
   iii. Sketch the curve showing clearly the features found in (i) and (ii).

   (12 marks)

9. A curve is given by $y = \frac{2(x-2)(x+2)}{2x-5}$
   i) Determine turning points on the curve and hence find the range of values of y for which the curve is Undefined.
   ii) Determine the asymptotes to the curve.
   iii) Sketch the curve. (12 marks)
10. A curve is given by
   $y = \frac{(x-1)(x-9)}{(x+1)(x+9)}$
   i) Determine the turning points on the curve
   ii) Determine the asymptotes to the curve
   iii) Sketch the curve. (12 marks)

(Rtr. AZOORA JAVIRA OBOT)

Trigonometry

11. a) Find all solutions of the equation $\tan\theta = 2 + \tan 3\theta$, where $0 < \theta < 360°$. (05 marks)

   b) Prove that $\tan[\theta + 60°]\tan[\theta - 60°] = \frac{tan^2\theta - 3}{1 - 3tan^2\theta}$.
   Hence solve $\tan[\theta + 60]\tan[\theta - 60] = 4sec^2\theta - 3$ where $0 < \theta < 360°$ (7 marks)
12. Prove that $\sin 3\theta = 3\sin\theta - 4sin^3\theta$
   b) Find all the solutions to $2\sin 3\theta = 1$ for θ between 0° and 360°. Hence, find the solutions of $8x^3 - 6x + 1$
13. a) Prove that $\frac{\cos A + \cos 2A + \cos 3A}{\sin A + \sin 2A + \sin 3A} = \cot 2A$ (04 marks)

   b) Express $3\sin x + 4\cos x = 2$ in the form $R\sin(x + \alpha)$ where $R > 0$ and α is acute. Hence:

   i) Solve the equation $3\sin x + 4\cos x = 2$ for $0 \le \theta \le 360°$.
   ii) Determine the maximum and minimum values of the expression $\frac{2}{4 + 3\sin x + 4\cos x}$ (8 marks)

15. a) Express $\sin\theta + \sin 3\theta$ in the form $m\cos\theta\sin n\theta$ where m and n are constants.

b) Find the general solution of $\cos 7\theta + \cos 5\theta = 2\cos\theta$.

c) Prove that $\frac{\sin A+\sin 4A+\sin 7A}{\cos A+\cos 4A+\cos 7A} = \tan 4A$. (12 marks)

(BWANA VICTOR)

16. Given that $y = \frac{\sin x}{1+\cos x}$, show that $\frac{dy}{dx} = \frac{1}{2}sec^2\left(\frac{x}{2}\right)$ (5 marks)

Indices and logarithms

17.

(a) Given that $\log_b a = x$ show that $b = a^x$ and deduce that $\log_a b = \frac{1}{\log_b a}$

(b) Find the values of x and y such that

i) $\log_{10} x + \log_{10} y = 1.0$

$\log_{10} x - \log_{10} y = \log_{10}(2.5)$

ii) $2^x.2^y = 432$

(c) Simplify $\frac{1+\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$ **(MAYIRANGA IVAN)**

18.

a) Find if $\log_x 8 - \log_{x^2} 16 = 1$

b) The sum of the p terms of an arithmetic progression is q and the sum of q terms is p. Find the sum of p + q terms.

Algebra and polynomial equations

19. Solve the simultaneous equations

$2^x + 4^y = 12$

$3(2)^x - 2(2)^{2y} = 16$

Hence show that $(4)^x + 4(3)^{2y} = 100$.

20. Given that α and β are roots of the quadratic equation $ax^2 + bx + c = 0$. Determine an equation whose roots are $\alpha + \beta$ and $\alpha^3 + \beta^3$. Hence or otherwise solve the equations

$\alpha + \beta = 2$

$\alpha^3 + \beta^3 = 26$

21.

a) Given that the roots of the quadratic $ax^2 + bx + c$ are in the ratio $p: q$. Show that $ac(p+q)^2 = b^2pq$.

b) The expression $x^7 - ax^3 + b$ is divisible $x - 1$ and has a remainder 8 when divided by $x - 2$. Find

(i) a and b

(ii) The remainder when divided by $x + 2$ using synthetic approach.

(ARINAITWE KETTY)

22.

a) Express $2x^2 + 11x + 6$ in the form $p(x+q)^2 + r$ and hence deduce its minimum value, where p, q and r are constants

b) If α and β are the roots of the equations $ax^2 + bx + c = 0$, express

$(\alpha - 2\beta)(\beta - 2\alpha)$ in terms of a, b and c. hence deduce the condition for one root to be twice the other.

c) Solve the simultaneous equations

$4p - q + 2r = 7$

$p + q + 6r = 2$

$8p + 3q - 10r = -3.$

Partial fractions and integration

23. Express $f(x) = \frac{2x^2 - x + 14}{(4x^2-1)(x+3)}$ in partial fraction

Hence evaluate $\int_1^3 f(x)dx$

i) Find $\int \frac{x^2}{(x^4-1)} dx$

ii) Evaluate $\int_0^1 \frac{x}{\sqrt{(1+x)}} dx$

24.

a) Intergrate $\frac{4x^2}{\sqrt{1-x^6}}$ with respect to x

b) Evaluate $\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}dx$ **(SSEMATO DENNIS)**

Series (Arithmetic Progression and Geometric Progression)

25.

(a) The first term of an arithmetic progression (A.P) is 73 and the ninth is 25. Determine
   (i) The common difference of the A.P
   (ii) The number that must be added to give a sum of 96

(b) A geometrical progression (G.P) and an arithmetic progression (A.P) have the same first term. The sum of their first, second, and third terms are 6, 10.5 and 18 respectively. Calculate the sum of their fifth terms.

(MATOVU BRIAN)

26. The sum to infinity of the terms of a G.P with common ratio r is S. the sum to infinity of cubes of the terms is $3S^3$. Determine the value of r. hence find S given that the sum of the first four terms is ${}^{45}/_{8}$.

b) A city tycoon invested £100,000 in a business at a compound interest rate at 8% per annum; determine

i) the amount after ten years

ii) The time corrected to the nearest years it takes to reach more than £300,000.

Vectors

27.

a) Given that $\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

Find the coordinates of the point R such that

$\overline{PR} : \overline{PQ} = 1 : 2$ and the points P, Q and R are collinear.

b) Show that the vector $5i - 2j - k$ is perpendicular to the line

$r = i - 4j + t(2i + 3j - 4k)$

c) Find the equation of the plane through the point with position vector $5i - 2j + 3k$ perpendicular to the vector $3i + 4j - k$. **(ODONGO DELEX)**

28. The points A, B and C have position vectors $(-2i + 3j), (i - 2j)\ and\ (8i - 5j)$ respectively.

(i) Find the vector equation of the line AC

(ii) Determine the coordinates of D if ABCD is a parallelogram

(iii) Write down the vector equation of the line through point B perpendicular to $\overrightarrow{AC}$ and find where it meets $\overrightarrow{AC}$.

Circles, parameters and parabolas

29. P is a variable point given by the parameter equations

$$x = \frac{a}{2}\left(t + \frac{1}{t}\right), y = \frac{b}{2}\left(t - \frac{1}{t}\right)$$

Show that the locus of P is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

State the asymptotes, state the coordinates of the point where the tangent from P meets the asymptotes. **(YIKI JOEL)**

30. a) Find the equation of the circle circumscribing the triangle whose vertices are A(1, 3), B(4, -5) and C(9, -1). Find also its centre and radius.

c) If the tangent to this circle at (1, 3) meets the x – axis at P(h, 0) and the y- axis at Q(0, k). Find the values of h and k.

Loci

31.

a) P is a variable point given by the parametric equations

$x = \frac{a}{2}\left[t + \frac{1}{t}\right]$ And $y = \frac{b}{2}\left[t - \frac{1}{t}\right]$

Show that the locus of P is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

b) State the asymptotes. Determine the coordinates of the points where the tangent from P meets the asymptotes. (12 marks)

Differential equations

32. a) Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$, where $x > 0$. (4 marks)

b) A chemical plant food loses its effectiveness at the rate proportional to the amount present in the soil. The amount M grams of plant food effective after t days satisfy the differential equation $\frac{dM}{dt} = kM$ where k is a constant.

i. Find the general solution for M in terms of t where the initial plant food is M grams
ii. Find the value of k if, after 30 days only half of initial amount of plant food is effective
iii. What percentage of the original amount of plant food is effective after 35 days?

(**MWESEZI MICHEAL)**

33. Mary starts walking at a speed of 3m/s to school 5Km away if she walks at a rate which is proportional to the distance, she still has to cover. How long will she take to cover ${}^{2}/_{3}$ of the journey?

Complex numbers

34.

a) Given that **Z** = $\sqrt{3} + i$, find the modulus and argument of
   i) $\boldsymbol{Z^2}$
   ii) $\boldsymbol{\frac{1}{Z}}$
   iii) Show in Argand diagram the points representing complex numbers **Z**, $\boldsymbol{Z^2}$ and $\boldsymbol{\frac{1}{Z}}$.

b) In an Argand diagram, P represent a complex number $\boldsymbol{Z}$ such that $\boldsymbol{2|Z - 2| = |Z - 6i|}$.

Show that P lies on a circle; find

i) The radius of the circle
ii) The complex number represented by its centre **(BUYINZA MICHEAL)**

35. Solve the simultaneous equations

$\boldsymbol{Z}_1 + \boldsymbol{Z}_2 = 8$

$4\boldsymbol{Z}_1 - 3\boldsymbol{i}\boldsymbol{Z}_2 = 26 + 8\boldsymbol{i}$

Using the values of $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$, find the modulus and argument of $\boldsymbol{Z}_1 + \boldsymbol{Z}_2 - \boldsymbol{Z}_1\boldsymbol{Z}_2$.

Permutation and combination

36. Price index

a). A student who offered mathematics, economics, geography and fine art (MEG/A) and obtained aggregates B, A, C and D respectively. The student wants to do statistics in Kyambogo University. MEG/A is weighed as follows, 3 for mathematics, 2 for economics, 1 for geography and 0.5 for art. Calculate his average weighted score.

b) The table below shows the consumption selected items at a certain home in 2003 and 2006.

	2003			2006
Item	Price(shs)	quantity	Price(shs)	Q_n
posho	700	50	1000	55
beans	600	25	900	30
rice	1100	15	1400	20
salt	350	12	500	16
meat	3500	14	4200	17

Probability density function (pdf) and cumulative density function (c.d.f)

37. a) a continuous random variable x has a p.d.f

$$f(x) = \begin{cases} \frac{1}{8}x, 0 \leq x \leq 4 \\ 0 \quad elsewhere \end{cases}$$

Find: (i) E(X) (ii) var (x) (iii) mode (iv) medium and interquatile range

(TUKASHABA FELIX)

38. A random variable x of a continuous p.d.f given by

$$f(x) = \begin{cases} k(x+1); & 0 \leq x \leq 4 \\ kx; & 4 \leq x \leq 6 \\ 0 & elsewhere \end{cases}$$

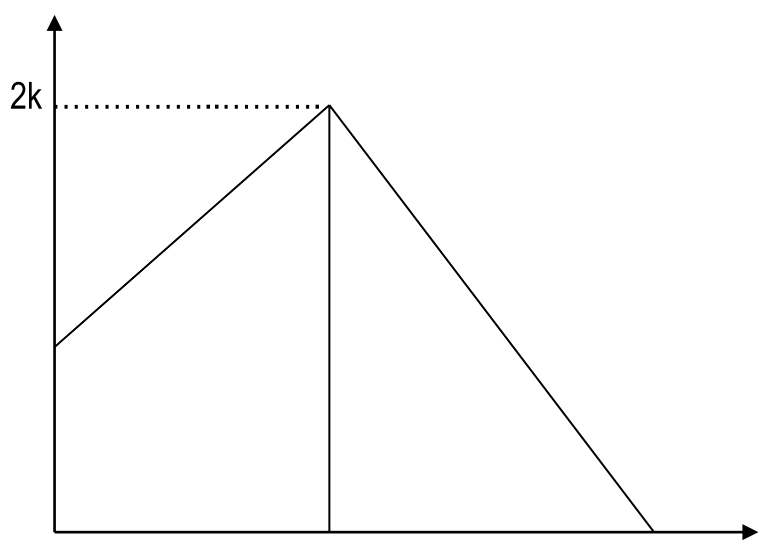
Find i) k and ii) E(x)

39. Three bags A,B and C contain 3 white and 2 red balls, 4 white and 4 red, 5 white and 2 red balls respectively. A ball is drawn unseen from A and placed in B. again a ball is then drawn from B and placed in C, what is the probability that if a ball is drawn from C, it will be red?

b). In a certain country 30% of the people are conservatives, 50% are liberals and 20% are independents. Records indicate that in election 65% of the conservatives voted, 85% of the liberals voted, 50% of the independents voted. A person in the city is selected at random.

I. Determine the probability that he voted.
II. Given that vote, determine the probability that he is conservative.

40. The distribution of a continuous random variable is represented graphically as shown



2 5

(i) Find the value of k
(ii) Calculate the $p(1 < x < 3/x < 2.5)$
(iii) Find var (x)

ERRORS

41. Two decimal numbers X and Y are recorded to give X and Y with the errors E_1 and E_2 respectively. Show that the maximum relative error recorded in approximating X^2y by x^2Y is given by $2\left|\frac{E_1}{X}\right| + \left|\frac{E_2}{Y}\right|$.

GENERAL MECHANICS

42. A small block of mass 3kg is suspended by two strings at lengths 0.6m and 0.8m from two points 1m apart on a horizontal rod. Find the values of the tensions in the strings.

(NIWAHABWE SAVIOUS)

43. A body consists of solid hemisphere of radius r joined to a right circular cone of base radius r and perpendicular height h plane surfaces of the cone and hemisphere coincide and both solids are made of the same uniform material

44. Show that the centre of gravity of the body lies on the axis of symmetry at a distance $\frac{3r^2-h^2}{\mu(h+2r)}$ from the base of the cone.

45. A spring AB of natural length 1.5m and modulus μN is fixed at A and hangs in a vertical position. The other end is joined to a second spring BC of natural length 1m and modulus 2μN. A particle of weight 15N is then attached to the end C of the second spring. When the system is hanging freely in equilibrium the distance AC is 4m. Find the value of μ.