P425/2

## APPLIED MATHEMATICS

AUGUST 2016
3 HOURS

## UGANDA ADVANCED CERTIFICATE OF EXAMINATION APPLIED MATHEMATICS <br> TIME: 3 HOURS

## INSTRUCTIONS TO CANDIDATES

- Attempt ALL the eight questions in section $\boldsymbol{A}$ and any five from section $\boldsymbol{B}$.
- All necessary calculations must be done on the same page as the rest of the answers. Therefore there is no paper for rough work.
- Mathematical tables with a list o formulae and squared papers are provided.
- Silent non-programmable scientific calculators may be used.
- In numerical work, take $g$ to be $9.8 \mathrm{~ms}^{2}$


## SECTION A: (40 Marks)

1. Two hunters A and B shoot at an animal. The probability that A hits the animal is $1 / 2$ and the probability that B does not hit the animal is $1 / 3$. A shoots at the animal first, and then B. find the probability that:
i) Both A and B hit the animal
ii) only one hits the animal.
2. A car approaching a town does two successive half-kilometers in 16 and 20 seconds respectively. Assuming the retardation is uniform. Find the further distance the car runs before stopping.
3. Given that $f(0.9)=0.2661, f(1.0)=0.2420$ and $f(1.1)=$.0.2179 , estimate:
i) $\quad \mathrm{f}(0.96)$
ii) $\quad \mathrm{f}^{-1}(0.2372)$ using linear interpolation or extrapolation
4. A machine manufacturing nails make approximately $85 \%$ that are within the set tolerance limits. If a random sample of 200 nails is taken, find the probability that more than 21 nails will be outside the tolerance limits.
5. An elastic string of natural length 60 cm is stretched to 70 cm by a stone of mass 1.5 kg hanging on it. Find the;
a) modulus of elasticity of the string
b) Energy stored in the string at equilibrium.
6. Using a Trapezium rule with six ordinates estimate the value of $\int_{0}^{\pi} x^{2} e^{x} d x$ to $2 d . p$
7. The mock examination and average final examination marks are given in the table below:

| Mock marks | 28 | 34 | 36 | 42 | 48 | 52 | 54 | 60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Average final marks | 54 | 62 | 68 | 70 | 76 | 66 | 76 | 74 |

Calculate the rank correlation coefficient between the marks and comment on your result.
8. To a motor cyclist travelling due North at $50 \mathrm{kmh}^{-1}$, the wind appears to come from North West at $60 \mathrm{kmh}^{-1}$. Find the true velocity of the wind.

## SECTION B: (60 Marks)

## Attempt five questions

9. The following table summarizes the distance to the nearest mile, travelled to work by a random sample of commuters

| Distance (miles) | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of commuters | 15 | 38 | 22 | 15 | 8 | 2 |

a) Calculate the:
i) Mean
ii) Median
iii) Standard deviation
b) Draw a histogram to represent the data and use it to estimate the mode.
10. a) Given the equation $\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}=0$, show that the Newton Raphson method for solving the above equation is given by $X_{n+1}=\frac{a x_{n}^{2}-c}{2 a x_{n}+b}, n=0,1,2, \ldots \ldots \ldots$.
b) Construct a flow chart that reads $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and the initial approximation $\alpha$, computes and prints the root and number of iterations with an error of less than 0.0001.
c) Use your diagram to calculate the positive square root of 20 , for $\alpha=4$.
11.a) Forces of the magnitude $4 \mathrm{~N}, 1 \mathrm{~N}, 2 \mathrm{~N}$ and 3 N act along sides
$\overrightarrow{A B}, \overrightarrow{B C}, \overrightarrow{C D}$ and $\overrightarrow{A D}$ of a rectangle ABCD in which $\mathrm{AB}=4 \mathrm{~m}$ and $\mathrm{BC}=3 \mathrm{~m}$. Given the direction of the forces is indicated by the order of the letters and $A B$ is horizontal, determine;
i) magnitude of the resultant force
ii) length AT, where $T$ is the point where the resultant cuts $A B$
b) A non-uniform ladder AB of weight 78.4 N ad length 5 m is freely suspended horizontally by two light inelastic strings AC and BD that make angles $30^{\circ}$ and $40^{\circ}$ respectively with the vertical; find the distance from $A$, where weight of the ladder acts.
12. A continuous random variable $x$ is defined by the p.d.f
$f(x)=\left\{\begin{array}{lc}\frac{1}{5}(x+2), & -2<x<0 \\ \left.\left.\frac{2}{15}\right) 3-x\right), & 0<x<3 \\ 0, & \text { elsewhere }\end{array}\right.$
a) sketch $f(x)$
b) Determine the
i) expectation of $x$
ii) Cumulative distribution function, $\mathrm{F}(\mathrm{x})$ hence find $P(-1 \leq x<1)$

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13. a) The position vector of ship P relative to Q at any time t is given by $\operatorname{Pr} \boldsymbol{Q}=$ $\binom{4}{13}+\binom{-2}{-4} t$ meters. Find the:
i) Time when they are nearest together
ii) Closest distance between the two ships
b) Projectile is fired from the ground with initial velocity $3 \boldsymbol{i}+4 \boldsymbol{j} \mathrm{~ms}^{-1}$. Find its;
i) Position vector at any time t
ii) Horizontal range
14. Strips of a metal are cut to length (cm) which follows a normal distribution with mean 50 cm and variance $0.25 \mathrm{~cm}^{2}$. find the;
i) Probability that the length selected at random will lie between 49.25 cm and 50.75 cm .
ii) Length range of the middle $70 \%$ of the cut of the strips.
15. a) Two decimal number X and Y are rounded to give $x$ and $y$ with errors $\boldsymbol{e}_{1}$ and $\boldsymbol{e}_{2}$ respectively. Show that maximum absolute relative error committed when $x^{2} y$ is used instead of $X^{2} Y$ is given by $2\left|\frac{\mathrm{e}_{1}}{x}\right|+\left|\frac{\mathrm{e}_{2}}{y}\right|$
b) Determine the limits within which the exact value of $\frac{6.042-3.21}{12.1}$ lie.
16. A body of mass mkg lies on a rough plane inclined at $\theta^{0}$ to the horizontal. When a force of $\frac{m g}{2} N$ parallel to and up the plane is applied to the body, it is just about to move up the plane. When a force of $\frac{m g}{4} N$ parallel to and down the plane is applied to the body, it just about to move down the plane. Calculate correct to two decimal places the value of:
i) $\theta$
ii) The coefficient of friction between the body and the plane.
