P425/ 1
PURE
MATHEMATICS
PAPER 1
3hrs

# Uganda Advanced Certificate of Education PURE MATHEMATICS 

## PAPER 1

3hours

## Instructions

Attempt all the eight questions in Section A and Not more than five from Section $B$.

Any additional question(s) will not be marked

All working must be shown clearly

Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.

## SECTION A (40MARKS)

## Answer all questions in this section

1. Solve the simultaneous equations.

$$
\begin{aligned}
& P-2 q-2 r=0 \\
& 2 p+3 q+r=1
\end{aligned}
$$

$$
\begin{equation*}
3 p-q-3 r=3 \tag{5marks}
\end{equation*}
$$

2. Solve the equation, $5 \sin 2 x-10 \sin ^{2} x+4=0$ for $-180^{\circ} \leq x \leq 180^{\circ}$.
(5marks)
3. The position vectors of the points A and B are $3 i-j+2 k$ and $2 i+2 j+$ $3 k$ respectively. Find the acute angle between the line $A B$ and the line

$$
\begin{equation*}
1-x=\frac{y+3}{2}=\frac{4-z}{4} . \tag{5marks}
\end{equation*}
$$

4. Find $\int \frac{\cos x}{4+\sin ^{2} x} d x$
(5marks)
5. If $x^{2}+y^{2}=2 y$, show that $(1-y)^{3} \frac{d^{2} y}{d x^{2}}=1$
(5marks)
6. Find the equation of the tangent to the circle $(x-1)^{2}+(y+2)^{2}=8$ at the point $(3,-4)$.
(5marks)
7. Prove by induction that, $6^{n}-1$ is divisible by 5 for all positive integral values of $n$.
(5marks)
8. If $=\frac{1}{x^{2}}$, find $\frac{d y}{d x}$ from the first principles.
(5marks)

## SECTION B (60MARKS)

Answer any five questions from this Section.
9. a) i) Solve the simultaneous equations $Z_{1}+3 Z_{2}=8,4 Z_{1}-3 i Z_{2}=17+$ $9 i$.
(5marks)
ii) Given that $Z=\frac{7-i}{-4-3 i}$ find the modulus and argument of $Z$, Hence express $Z$ in the polar form.
(4marks)
b)
10. a) Solve the equation $2 \cos 2 \theta=7 \sin \theta$ for $0^{0} \leq \theta \leq 360^{\circ}$.
(6marks)
b) Prove that; $\frac{1+\cos \theta+\sin \theta}{1-\cos \theta+\sin \theta}=\cot \frac{\theta}{2}$
(6marks)
11. a) Find the Cartesian equation of a curve whose polar equation is given by $r=a \tan \theta$.
(3marks)
b) Obtain the equation of the tangent to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ at the point $(a \cos \theta, a \sin \theta)$. If the tangent cuts the $x$ and $y$ axes at points Q and R respectively, determine the locus of the midpoint of QR. (9marks)
12. a) Find, $\int \frac{x^{2}}{(x-1)^{2}\left(x^{2}+4\right)} d x$
(9marks)
b) A function is defined by $y=I_{0} \sin ^{2}(\omega t+\alpha)$ for $0 \leq t \leq \frac{2 \pi}{\omega}$ where $\alpha$, $\omega$ and $I_{0}$ are constants. Determine the mean value of the function.
(3marks)
13. a) Find the angles between the vectors $a=2 i+3 j-k$ and

$$
b=5 i+2 j+k .
$$

(4marks)
b) A plane has the points $A(2,1,3), B(0,-6,2)$ and $C(3,2,1)$ on it.

Determine the Cartesian equation of the plane. (4marks)
c) The normal to the plane in (b) above is a directional vector to the line passing through (1, 1, 5). Find in Cartesian form, the equation of the line.
(4marks)
14. a) Evaluate; $\int_{0}^{\frac{\pi}{2}} \frac{4 d \theta}{3+\cos \theta}$
(7marks)
b) Obtain the integral $\int x e^{-2 x} d x$
(5marks)
15. a) The ninth term of an AP is -1 and the sum of the first nine terms is 45. Find the common difference and the sum of the first twenty terms.
(6marks)
b) In a geometric progression the first term is 7 and the $n^{\text {th }}$ term is 448 . The sum of the first n terms is 889 , find the common ratio. (6marks)
16. a) Solve the differential equation, $x \frac{d y}{d x}=y+k x^{2} \cos x$ given that $y=2 \pi$ when $x=\pi$.
(5marks)
b) A certain chemical reaction is such that the rate of transformation of the reacting substance is proportional to its concentration. If initially the concentration of the reagent was 9.5 gm per litre and if after 5 minutes the concentration was 3.5 gm per litre, find what the concentration was after 2 minutes.
(7marks)
**END **

