P425/1

Pure Mathematics

Paper 1

3Hours

Uganda Advanced Certificate of Education

End of Term Two Examinations, 2018

Senior Five

Pure Mathematics

Paper 1

3 Hours

Instructions:

(i) *Answer* **all** *questions in* **Section A**, *and* **NOT** *more than* **FIVE** *from* **Section B**.

(ii) *Any additional questions will* **NOT** *be marked.*

(iii) *All necessary working* **must** *be shown clearly.*

(iv) *Begin* ***each*** *Section B solution on a* ***new PAGE***

(v) *Silent non–programmable calculators and a list of mathematical tables may be used.*

SECTION A (40 MARKS)

1. Solve simultaneously:

   $p^3 + q^3 = 26$

   $p + q = 2$ (05mks)

2. The roots of the quadratic equation $x^2 - x - 6 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Find a quadratic equation with integral coefficients whose roots are α^2 and β^2. (05mks)

3. Show that $\frac{d}{dx}\left\{\frac{\sqrt{4+x^2}}{4x}\right\} = -\frac{1}{x^2\sqrt{4+x^2}}$ (05mks)

4. Prove that $4\cos\theta\cos 3\theta + 1 = \frac{\sin 5\theta}{\sin\theta}$. (05mks)

5. Given that $y = x(x^2+4)^{\frac{1}{2}}$, find $\frac{dy}{dt}$ when $x = 2$ and $\frac{dx}{dt} = 3$. (05mks)

6. Given that $x = 2 + \cos\theta$, and $y = \sin\theta - 2$, show that $x^2 + y^2 - 4x + 4y + 7 = 0$. (05mks)

7. Solve the equation $(n+2)! = 42n!$ where n is a positive integer. (05mks)

8. Given that, $\frac{dy}{dx} = 4x^3 - 3x^2 + 2x - 5$ find the equation of the curve $y = f(x)$ given that it crosses the y–axis at the point $(0,2)$ (05mks)

SECTION B (60 MARKS)

9. a) Solve the equation; $\cos 5x + 1 = 2\sin^2 x$ for $0^0 \le x \le 180^0$ (05mks)

b) Given that $f(x) = 4\cos x + 5\sin x$,

(i) express $f(x)$ in the form $A\sin(x+\beta)$; where A is positive and β is an acute angle.

(ii) determine the maximum value of $5 - f(x)$ and hence the value of x for which it occurs.

(iii) solve the equation $f(x) = 0$ for $0^0 \le x \le 360^0$. (07mks)

10. a) Given the curve $y = 3x^2 - 2x - 5$ find the;

(i) intercepts of the curve on both axes.

(ii) coordinates of the turning point of the curve, and distinguish it. (05mks)

b) (i) Sketch the curve (03mks)

(ii) Find the area between the curve and the $x-$axis from $x = -1$ to $x = 3$ (04mks)

11. a) The roots of the equation $ax^2 + bx + c = 0$ are such that the sum of their squares is

1. Show that $b^2 = a^2 + 2ac$. (06mks)

b) Solve the equation $\log_4 x^2 - 6\log_4 x - 1 = 0$ leaving surds in your answer. (06mks)

12. Given the lines L_1 $3x - 4y + 6 = 0$ and L_2 $5x + y + 13 = 0$;

(i) find the equation of the line through $(2,4)$ and the point P, where P is the intersection of the lines L_1 and L_2. (05mks)

(ii) Find the equation of the line through the point P and inclined at an angle $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ to the positive horizontal. (04mks)

(iii) Find the acute angle between the lines in (i) and (ii) above. (03mks)

13. a) In the triangle PQR; prove that $\dfrac{1-\cos P+\cos Q+\cos R}{1-\cos R+\cos P+\cos Q}=\dfrac{\tan\frac{P}{2}}{\tan\frac{R}{2}}$ (06mks)

b) Solve the equation $5\sin(\beta+60^0)-3\cos(\beta+30^0)=4$, for $0^0\le\beta\le360^0$ (06mks)

14. a) Solve simultaneously; $x\log_4 128-y\log_8 2=6$ and $\log_2 x+\frac{1}{3}\log_2 y^3=2\log_4 6$ (07mks)

b) The polynomials x^2+ax+b and $3x^2+b$ have a common factor $x-c$. Show that if a,b and c are non–zero constants, then $3a^2+4b=0$. (05mks)

15. a) In how many ways can the letters of the word **NASASIRA** be arranged in a row without restriction? (02mks)

b) In how many of these arrangements

(i) are all the **A**'s together?

(ii) are all the **S**'s together?

(iii) is the **R** between the **S**'s? (05mks)

c) Opio wishes to invite 6 friends of his to a house party, and dinner is to be served at a round table. In how many ways can he and his friends sit around this table if;

(i) there's no restriction on their sitting positions.

(ii) one of the guests is his twin brother and insists that he must sit next to him.

(iii) in addition to (ii), two of the guests are a couple and must also sit together. (05mks)

16. a) Find the range of values of the constant k for which the quadratic equation $x^2 + kx = 3x - k$ has real roots (06mks)

b) Show that for real values of x, $\frac{x+2}{x^2+3x+6}$, cannot lie between $-\frac{1}{5}$ and $\frac{1}{3}$. (06mks)

END.