1. Solve the equation: $\tan 4x \square \tan 2x \square 0$ for $0^o \square x \square 2\square$

$\cancel{2\tan 2x}\,\cancel{3}\,2x \square 0$

$\square \tan 2x \square 0,\ 2\tan 2x \square \tan 2x \square \tan 1\square \tan^2 2x$

$\tan 2x\square 3 \square \tan^2 2x\square\square 0$ $\tan 2x \square 0,\ 2x \square 0_o, 180_o, 360_o,\ x \square$

$0_o, \square_2, \square$

$\sqrt{\ }$ $\tan 2x \square\square 3,\ 2x \square 60_o, 120_o, 240_o, 300_o,\ x \square \square 6, \square 3, 23\square$

$56\square$

ALTERNATIVELY

$\sin 4x \quad \sin 2x$ $\cos 4x \square \cos 2x \square 0$

$\sin 4x\cos 2x \square \cos 4x\sin 2x \square 0$ $\sin\square 4x \square 2x\square\square \sin 6x \square 0$

$6x \square 0^o, 180^o, 360^o, 540^o, 720^o, 900^o, 1080^o$ $x \square 0_o, \square 6, \square 3, \square 2, 23\square,$

$56\square, \square$

2. Find the coordinates of the circumscribing circle which passes through the points $A(1, 2)$, $B(2$ and $C(-3, 4)$.

Perpendicular bisectors of two chords intersect at the centre of the circle.

Midpoint of AB $\left(\frac{3}{2}, \frac{7}{2}\right)$, mid point BC $\left(\frac{-1}{2}, \frac{9}{2}\right)$

Gradient of AB $\frac{5}{2-1} = 3$, Gradient of BC $\frac{-4-5}{-3-2} = \frac{-1}{-5} = \frac{1}{5}$

Gradient of normal to AB $\frac{-1}{3}$, that to BC -5

Equation through $\left(\frac{3}{2}, \frac{7}{2}\right)$ is $y - \frac{7}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$, to get $3y = x + 8$…(i)

Equation through $\left(\frac{1}{2}, \frac{9}{2}\right)$ is $y - \frac{9}{2} = \frac{1}{2}(x - 5)$ to get $y = 5x + 2$..(ii)

Solve eqn(i) and eqn(ii) to get $x = -\frac{3}{7}$ and $y = \frac{29}{7}$

Thus centre is $C\left(-\frac{3}{7}, \frac{29}{7}\right)$

ALT: The general equation can be $x^2 + y^2 + 2gx + 2fy + c = 0$

$A(1, 2)$; $1 + 4 + 2g + 4f + c = 0$, $5 + 2g + 4f + c = 0$……..(i)

$B(2, 5)$; $4 + 25 + 4g + 10f + c = 0$ $29 + 4g + 10f + c = 0$…..(ii)

$C(-3, 4)$; $9 + 16 + 6g + 8f + c = 0$ $25 + 6g + 8f + c = 0$…..(iii)

Eqn(ii) – eqn(i): $2g + 6f = 24$, $g + 3f = 12$

Eqn(iii) –eqn(ii): $5g + f = 2$,

Solving: $g = -\frac{3}{7}$, $f = \frac{29}{7}$, $c = \frac{75}{7}$

Equation is; $x^2 + y^2 + \frac{6}{7}x + \frac{58}{7}y + \frac{75}{7} = 0$,

Thus centre is $C\left(-\frac{3}{7}, \frac{29}{7}\right)$

3.	If $y = e^{-x}\cos x$, prove that $\frac{dy}{dx} = 2e^{-x}\sqrt{\cos\left(x + \frac{\pi}{4}\right)}$. $y = e^{-x}\cos x$, $\frac{dy}{dx} = -e^{-x}(\cos x + \sin x)$, for t.p $\frac{dy}{dx} = 0$ $-e^{-x}(\cos x + \sin x) = 0$, for $-e^{-x} \neq 0$	

	$(\cos x+\sin x)=0$, let $(\cos x+\sin x)\equiv R\cos(x-\alpha)\equiv R\cos x\cos\alpha+R\sin x\sin\alpha$ $R\cos\alpha\equiv 1,\ R\sin\alpha\equiv 1$ thus $\tan\alpha=1,\ \alpha=\frac{\pi}{4}$ and $R=\ 2$ Therefore $\frac{dy}{dx}=-\sqrt{2}e^{-x}\cos\left(x-\frac{\pi}{4}\right)$ as required.	$\sqrt{}$
4.	Find the perpendicular distance from the point $A(1,\ 2,\ -4)$ to the plane which passes through the point $B(1,\ 4,\ 9)$ and is normal to the vector $3\mathbf{i}-\mathbf{k}$. Equation of the plane $\mathbf{r}.\begin{pmatrix}3\\0\\-1\end{pmatrix}=\begin{pmatrix}1\\4\\9\end{pmatrix}.\begin{pmatrix}3\\0\\-1\end{pmatrix}$ $3x-z=-6$ OR $3x-z+6=0$ $\Rightarrow$ perpendicular distance $d=\frac{3\times 1-1\times -4+6}{\sqrt{9+1}}=\frac{13}{\sqrt{10}}=\frac{13}{10}\sqrt{10}$	
5.	Given that $y=In\left(x+\sqrt{x^2+a^2}\right)$, where a is a constant, prove that $\frac{dy}{dx}=\frac{1}{\sqrt{x^2+a^2}}$ and hence evaluate $\int_0^4\frac{dx}{\sqrt{x^2+9}}$. $y=In\left(x+\sqrt{x^2+a^2}\right)$, $\frac{dy}{dx}=\frac{1+\frac{2x}{2\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$ $\frac{dy}{dx}=\frac{\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$ $\therefore\ \frac{dy}{dx}=\frac{1}{\sqrt{x^2+a^2}}$ $\int_0^4\frac{dx}{\sqrt{x^2+9}}$, since $\frac{d}{dx}\left(In\left(x+\sqrt{x^2+a^2}\right)\right)=\frac{1}{\sqrt{x^2+a^2}}$ $\Rightarrow\int_0^4\frac{dx}{\sqrt{x^2+9}}=\left[In\left(x+\sqrt{x^2+9}\right)\right]_0^4=(In9-In3)=In3$	

6.	If $z = 1 + 2i$ is a root of the equation $z^3 + az + b = 0$ where a and b are real, find the values of a and b. $(1 + 2i)^3 + a(1 + 2i) + b = 0$ $(11 + 2i) + a + 2ai + b = 0$, $(11 + a + b) + (2a + 2)i = 0$ Thus, $2a + 2 = 0$, $a = -1$ $(11 - 1 + b) = 0$, $b = -10$	
7.	Solve the equations: $\frac{x^2}{y} + \frac{y^2}{x} = 9$, $x + y = 6$ From eqn (1) we have $x^3 + y^3 = 9xy$, thus $(x + y)(x^2 - xy + y^2) = 9xy$ Thus $6(x^2 - xy + y^2) = 9xy$ $2x^2 - 5xy + 2y^2 = 0$, $x + y = 6$ so, $x = 6 - y$ $2(6 - y)^2 - 5y(6 - y) + 2y^2 = 0$ to get $9y^2 - 54y + 72 = 0$ $y^2 - 6y + 8 = 0$ $(y - 4)(y - 2) = 0$ so, $y = 2, 4$ and $x = 4, 2$	

8.	Evaluate: $\int_0^{1/4} \cos^{-1} 2x \, dx$ Let $u = \cos^{-1} 2x$, $\frac{du}{dx} = \frac{-2}{\sqrt{1-4x^2}}$ $\quad \frac{dv}{dx} = 1$, $v = x$ $\int_0^{1/4} x\cos^{-1} 2x \, dx = \left[x\cos^{-1} 2x\right]_0^{1/4} - \int_0^{1/4} \frac{-2x}{\sqrt{1-4x^2}} \, dx$ $= \left[x\cos^{-1} 2x\right]_0^{1/4} - \frac{1}{2}\left[\sqrt{1-4x^2}\right]_0^{1/4}$ $= \frac{1}{12} - \frac{\sqrt{3}}{4} + \frac{1}{2} = 0.3288$	

9a)	The remainder when the expression $x^3 \square 2x^2 \square ax \square b$ is divided by $x \square 2$ is five times the remainder when the same expression is divided by $x \square 1$, and 12 less than the remainder when the same expression is divided by $x \square 3$. Find the values of a and b . $x^3 \square 2x^2 \square ax \square b \square \square x \square 2 \square Q \square x \square \square 5R$ $x^3 \square 2x^2 \square ax \square b \square \square x \square 1 \square Q \square x \square \square R$ $x^3 \square 2x^2 \square ax \square b \square \square x \square 3 \square Q \square x \square \square 5R$ $\square 12$ When $x \square 2$, $2a \square b \square 5R \square 0$, When $x \square 1$, $2a \square b \square R \square 1$, When $x \square 3$, $3a \square b \square 5R \square 3$ Solve to get $a \square 3$, $b \square \square 1$	
b)	Given that the first three terms in the expansion in ascending powers of x of $\square 1 \square x \square x_2 \square_n$ are the same as the first three terms in the expansion of $\square \square \square \square 1$ $_____ 1 \square \square 3axax \square \square \square \square$, find the value of a and n. $_2 \square_n \square 1 \square n \square x \square x_2 \square \square n \square n \square 1 \square \square x \square x$ $\frac{\square 1 \square x \square x^2}{2!}$ $\square_2 \square \ldots$ $\square 1 \square nx \square nx^2 \square _ \frac{1}{2} n \square n \square 1 \square x^2 \square \ldots$	

	$\square\ 1\square\ ax\ \square^{3}$ $\square\square$ _____ $\square$ $\square\square\square$ $\square$ $\square 1\square\ ax\square^{3}\square 1\square\ 3ax\square^{\square 3}\ \square 1$ $3ax$	

	$\square\square 1\square\ 3ax\ \square\ 3a^2x^2\ \square\ldots\square\square 1\square\ 9ax\ \square 18a^2x^2\ \square\ldots\square$ $\square\ 1\square\ 9ax\ \square\ 54a^2x^2\ \square\ 3ax\ \square\ 27a^2x^2\ \square\ 3a^2x^2\ \square\ldots$ $\square\ 1\square 12ax\ \square\ 84a^2x^2\ \square\ldots$ Thus $1\square\ nx\ \square\ nx^2\ \square$ $\frac{1}{2}$ $n\square n\ \square 1\square x^2\ \square\ \ldots\ \square$ $1\square 12ax\ \square\ 84a^2x^2\ \square\ldots$ Equating coefficients, we get $n\ \square 12a$ .....$\square i\square$ n $84a^2\ \square\ n\ \square$ _____ $\square n\ \square 1\square$ .....$\square ii\square$ 2 $12a$	

	$84a^2 \square 12a \square$ ———— $\square_{12a \square 1}\square$, $84a^2 \square 12a \square 72a^2 \square 6a$ 2 $12a^2 \square 6a \square 0$, for $a \square 0, a \square \frac{1}{2}$ thus $n \square 6$	
10a)	Show that the lines $\mathbf{r} \square \square \square 2\mathbf{i} \square 5\mathbf{j} \square 11\mathbf{k}\square\square\square\square 3\mathbf{i} \square \mathbf{j} \square 3\mathbf{k}\square$, $\mathbf{r} \square \square 8\mathbf{i} \square 9\mathbf{j}\square\square$ $t\square 4\mathbf{i} \square 2\mathbf{j} \square 5\mathbf{k}\square$ intersect, hence, find the position vector of their point of intersection. Find also the Cartesian equation of the plane formed by these two lines. $\square\square 2\square \square 3\square\square \square 8\square \square 4t\square$ $\square \quad \square\square \quad \square\square\square\square\square$ $\square 5 \square\square\square\square\square\square\square 9\square\square\square 2t\square$ $\square\square 11\square\square \square\square 3\square\square\square \square\square 0\square\square \square\square 5t\square\square$ $\square$ $\square\square 2 \square 3\square\square 8 \square 4t$…..(i) $5 \square\square\square 9 \square 2t$ …(ii) $\square 11\square 3\square\square 5t$ …(iii) $\square 2 \square 3\square \square 8 \square 4t$	

	Eqn(i) – eqn(ii)x3 to get □17 □□19 □ 2*t* □□15 □ 3□□□ □□27 □ 6*t*□ □*t* □□1 then from eqn(i) □□ 2	

Substitute t &□in (i) , LHS. □11□ 6 □□5 □RHS

□ 4 □
□ □

Thus: position vector of point of intersection **r**□□ 7 □

□□5□□
□

□ 4 □ □3□ □4□
□ □ □ □ □ □

Vector equation is given by **r**□□ 7 □□□□1□□ t□2□-

□□5□□ □□3□□□□5□□
□

□ x □ 4 □ 3□□ 4t…(i), y □ 7 □□□ 2t …(ii) z □□5 □ 3□□ 5t …(iii)

Eqn(i) -3eqn(2)

x □ 4 □ 3□□ 4t

- to get x □ 3y □□17 □ 2t

3y □□□21□ 3□□ 6t□

x

□ 2 Thus t □ □ 3y □17 □ 3y □ x □17
2

Put t in eqn(i); x □ 4 □ 3□□ 2□3y □ x □17□

Thus □□ $\frac{3x - 6y + 30}{3}$ □ x □ 2y □10

Put t and □ in eqn(iii) to get:

z □□5 □ 3□x □ 2y □10□□$\frac{5}{2}$□3y □ x □17□

$$\square x \square 3y \square 2z \square 35\square 0$$

ALTERNATIVE:

$A\square\square 2,, 5, \square 11\square$, $B\square 8, 9, 0\square$, $C\square 4, 7, \square 5\square$ and let $p\square x, y, z\square$

$\square 10\square$ $\square 6\square$ $\square x\square 2\square$

$\square\square\square\square\square\square\square\square$ $\square$

$AB \square\square 4\square$, $AC \square\square 2\square$, $AP \square\square y\square 5\square$

$\square 11\square\square$ $\square\square 6\square\square$ $\square\square z\square 11\square\square$

$\square$

	□10□ □6□ □ x □2□ □ □ □ □ □ □ □ □ AP □□□ 4□□□□2□□□ y □5□ □ □11□□□6□□ □□z □11□□ □ Eqn(i) -3Eqn(ii) x □10□□ 6□□□ 2 to get x □ 3y □ 2□□□17.....iv 3y □12□□ 6□□15 3Eqn(ii) –Eqn(iii) 3y □12□□ 6□□15 to get 3y □ z □□□ 26.....v z □11□□ 6□□□11 FromEqn(iv) and 2Eqn(v) x □ 3y □ 2□□□17 6y □ 2z □ 2□□ 52 x □10□□ 6□□□ 2 i y □ 4□□ 2□□ 5 ii z □11□□ 6□□□11 .....iii to get x □ 3y □ 2z □ 35□ 0	
11a)	Differentiate with respect to x : i) y □ 2$x^{\cos x}$ $In y$ □cos$x$ $In 2x$ $\frac{1}{y}\frac{dy}{dx}$ □□ sin $x In 2x$ □ $\frac{2\cos x}{2x}$ $\frac{dy}{dx}$ □ $2x^{\cos x}$□cos$x$ □ sin $x In 2x$□ □ ii) y □ $e^{\sin x}$ / tan□1 x $In y$ □sin$x$□ $In \tan^{-1} x$ $\frac{1}{y}\frac{dy}{dx}$□ cos$x$ □ $\frac{1}{\square 1\square x^2 \square \tan^{-1} x}$ $\frac{dy}{dx}$ □ $\frac{e^{\sin x}\square}{\tan^{-1} x\square\square}$□cos$x$ $\frac{1}{\square 1\square x^2 \square \tan^{-1} x}$ □ □ □	

b)	$\int_{3}^{3} \sqrt{3 - x}\, dx$ (Use the substitution $x = 3\sin^2\theta$, $\theta = \cos^{-2}\theta$). Prove that $\int_{1}^{3} \sqrt{3 - x}\, dx = 1$	

	$\int_{1}^{3} \sqrt{3 - 3 + 2\cos^2\theta}\, dx$	$x = 3 + 2\cos^2\theta$
	$\int_{1}^{3} \sqrt{3 + 2\cos^2\theta}\, .4\sin\theta\cos\theta.d\theta$	$dx = 4\sin\theta\cos\theta d\theta$
	$= \int_{0}^{\frac{\pi}{2}} \sqrt{2\cos^2\theta}\, \sqrt{2\sin\theta}.4\sin\theta\cos\theta d\theta$	$x = 1, \theta = 0$ $x = 3, \theta = \frac{\pi}{2}$
	$= 4\int_{0}^{\frac{\pi}{2}} \cos^2\theta d\theta = 2\int_{0}^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta = 2\left[\theta + \frac{1}{2}\sin 2\theta\right]_{0}^{\frac{\pi}{2}}$	
	$= 2\left[\frac{\pi}{2} + \frac{1}{2} \times 0\right] - \left[0 + \frac{1}{2} \times 0\right]$	

12a	If A, B, C are angles of a triangle, prove that $\sin^2 A \square \sin^2 B \square \sin^2 C \square 2 \square 2\cos A\cos B\cos C$ From the L.H.S $\sin^2 A \square \sin^2 B \square \sin^2 C = \frac{1}{2}\square 1 \square \cos 2A \square \square \frac{1}{2}$ $\square 1 \square \cos 2B \square \square \sin^2 C$ $= 1 \square \frac{1}{2}\square \cos 2A \square \cos 2B \square \square \square 1 \square \cos^2 C \square$ $= 2 \square \frac{1}{2}\square 2\cos \square A \square B \square \cos \square A \square B \square \square \square \cos^2 C$ $:\cos(A \square B) \square \square \cos C$ $= 2 \square \square \square \cos C \cos \square A \square B \square \square \square \cos^2 C$ $= 2 \square \cos C \square \cos \square A \square B \square \square \cos C \square$ $= 2 \square \cos C \square \cos \square A \square B \square \square \cos \square A \square B \square \square$ $= 2 \square 2\cos A\cos B\cos C$	
b)	By expressing $6\cos^2 \square \square 8\sin \square \cos \square$ in the form $R\cos \square 2 \square \square \square \square$, find the maximum and minimum values of the function and the corresponding value of $\square$, hence solve $6\cos^2 \square \square 8\sin \square \cos \square \square 4$. $3(2\cos^2 \square) \square 4(2\sin \square \cos \square) = 3(1 \square \cos 2 \square) \square 4\sin 2 \square$	

	$= 3\cos 2\theta - 4\sin 2\theta - 3$ Let $3\cos 2\theta - 4\sin 2\theta - 3 = R\cos 2\theta\cos\alpha - R\sin 2\theta\sin\alpha$ So, $R\cos 2\alpha = 3$, $R\sin 2\alpha = 4$ $\tan\alpha = \dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{4}{3} \Rightarrow \alpha = 53.1^o$ $R^2\cos^2\alpha + R^2\sin^2\alpha = 3^2 + 4^2 \Rightarrow R = 5$ $\Rightarrow 3\cos 2\theta - 4\sin 2\theta - 3 = 5\cos(2\theta + 53.1^o) - 3$ Max. Value $= 8$ when $2\theta + 53.1^o = 180^o \Rightarrow \theta = 116.55^o$ Min. Value $= -2$ when $2\theta + 53.1^o = 270^o \Rightarrow \theta = 161.55^o$ Thus: $6\cos^2\theta - 8\sin\theta\cos\theta = 4$ $5\cos(2\theta + 53.1^o) - 3 = 4$ $\Rightarrow \cos(2\theta + 53.1^o) = \dfrac{1}{5}$ $2\theta + 53.1^o = 78.5^o, 281.5^o$ $\Rightarrow \theta = 65.8^o, 167.3^o$	

13.	The curve with the equation $y = \frac{ax}{x(x-2)} - b$ where a and b are constants has zero gradient at $(1, -2)$. (a) find the values of a and b. $y = \frac{ax}{x(x-2)} - b$, $\frac{dy}{dx} = \frac{a(x^2-2x)-(ax-2b)(2x-2)}{x^2(x-2)^2}$ For turning points, $\frac{dy}{dx} = 0$ so when $x = 1$, $\frac{dy}{dx} = \frac{3a-4(a-b)}{9} = 0$, so $a = -4b$……..(i)	

Curve passes through □1, □ 2□, so □ 2 □ ____ □b , so a □ b

□□6…….(ii) a

3

Thus a □□8 and b□2

(b) Find the equations for all the asymptotes to the curve, the turning points and sketch the curve.

y □

_____ __ _____□8x □ 2 , dy □ □8□x2 □ 2x□□□□8x 2□

2□□2x □ 2□ when x □ □ _12

x□x □ 2□ dx □x2 □ 2x□

dy □8 □1□□1 □□4□2□□□1□2□ □ 6□6 □0 as required.

dx □4 □ 2 9

□1 □

□ □□1□ 16

□4 □

Intercepts are: when y □ 0, x □ 1/4; □ 1/4 ,

0□

Vertical asymptotes; x □ 0, x □□2

Hence the turning points are □1, □ 2□ and □□ 1/2, □8□

	L	x □1	R	L	x □□ 1/2	R
Sign of dy/dx	-		+	+		-

		min			Max	

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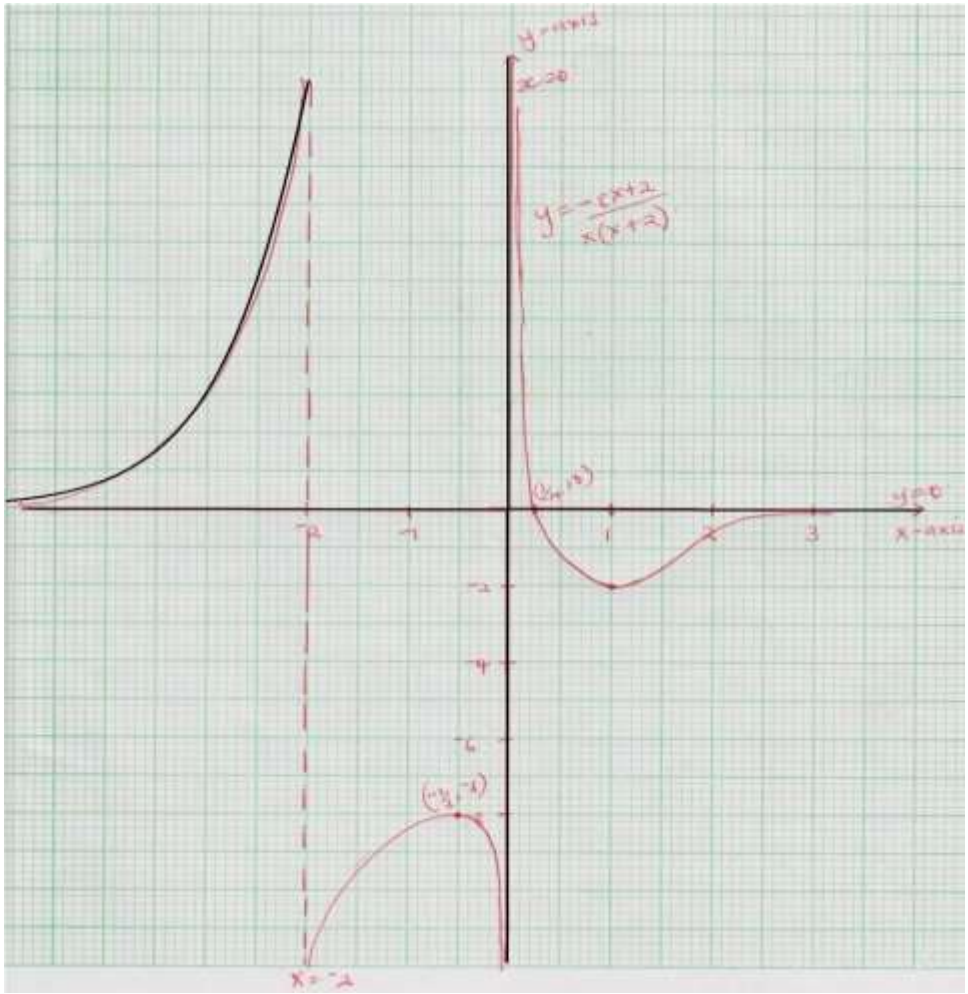

14a	Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$.	

$\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. $\arg(z-3)-\arg(z-2i)=\frac{\pi}{4}$, for $z=x+iy$,

	arg□□x □ 3□□ iy□□ arg□x □ i□y □ 2□□□□ □ _ 4 tan□1□□□ _ x □y 3□□□□□ tan□1□□□ y □x 2□□□□□4 , let □	

	tan□1□□□□ _ x □y 3□□□□□ A,tan□1□□□ _ y □x 2□□□□ B y y □2 _ □ _ tan□A □ B□□ tan□ _ □1, thus x□3 x □1 4 □ y □□ y □2□ 1□□□x□3□□□□□ x □□ □ xy □ xy □ 2x □3y □ 6 x² □ 3x □ y² □ 2y □ , x□x □ 3□ x□x □ 3□ x² □ y² □ 5x □ 5y □ 6 □ 0 □x □ 52 □2 □ □y □ 52 □2 □ 264, thus the locus is a circle with the centre □□ 5 , 5 □□ □ 2 2 □ √26 and radius units.	

	2	

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b)	Find the four roots of: $16i$. Let $z = 16i$, then $\lvert z \rvert = 16$, $\arg z = \tan^{-1}\left(\frac{16}{0}\right) = \frac{\pi}{2}$ So $z = 16i = 16\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ $z^{\frac{1}{4}} = 16^{\frac{1}{4}}\left(\cos\frac{\frac{\pi}{2} + 2k\pi}{4} + i\sin\frac{\frac{\pi}{2} + 2k\pi}{4}\right)$ for $k = 0,1,2,3$ For $k = 0$, $2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right) = 1.8478 + 0.7654i$ For $k = 1$, $2\left(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right) = -0.7654 + 1.8478i$ For $k = 2$, $2\left(\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right) = -1.8478 - 0.7654i$	

	For $k = 3$, $2\left(\cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}\right) = 0.7654 - 1.8478i$	

15a	Find the equation of the tangent to the curve $y = x^3$ at the point $P(t, t^3)$. Prove that this tangent cuts the curve again at the point $Q(-2t, -8t^3)$ and find the locus of the mid point of PQ. $\frac{dy}{dx} = 3x^2$, at $x = t$, the gradient of the tangent is $3t^2$ Thus the equation of the tangent at $P(t, t^3)$ is $\frac{y - t^3}{x - t} = 3t^2$, to get $y = 3t^2x - 2t^3$. For the tangent to meet $y = x^3$ again, we solve simultaneously. $x^3 = 3t^2x - 2t^3$, $x^3 - 3t^2x + 2t^3 = 0$ But $x = t$ is a root. Use long division $x - t \overline{)x^3 - 3t^2x + 2t^3}$ $x^2 + xt - 2t^2$ to get quotient as x^2 $+ xt - 2t^2 = 0$, $(x - t)(x + 2t) = 0$ Thus the other point is when $x = -2t$ and $y = -8t^3$ Mid point of PQ $\left(\frac{t - 2t}{2}, \frac{t^3 - 8t^3}{2}\right)$, we are required to eliminate the parameter t to find the Cartesian equation. $t = -2x$ and $y = \frac{7t^3}{2}$, the locus is $y = -28x^3$. So for $x = 0$	

b)	Given that the line $y = mx + c$ is a tangent to the circle $(x - a)^2 + (y - b)^2 = r^2$, show that $(1 + m^2)r^2 = (c - b - am)^2$.	

$$y = mx + c, \quad (x - a)^2 + (y - b)^2 = r^2$$

$$(x - a)^2 + (mx + c - b)^2 = r^2 \qquad x^2 - 2ax + a^2 + m^2x^2 + 2m(c - b)x + (c - b)^2 - r^2 = 0$$

$$(1 + m^2)x^2 + (2m(c - b) - 2a)x + a^2 + (c - b)^2 - r^2 = 0$$

For a tangent, $b^2 - 4ac = 0$,

$$(2m(c - b) - 2a)^2 - 4(1 + m^2)(a^2 + (c - b)^2 - r^2) = 0, \qquad m^2(c - b)^2 - 2am(c - b) + a^2 + m^2(c - b)^2 + m^2(a^2 - r^2) + (c - b)^2 + a^2 - r^2 = 0$$

$$-2am(c - b) + a^2m^2 + m^2r^2 + (c - b)^2 + r^2 = 0$$

$(1 + m^2)r^2 = (c - b)^2 - 2am(c - b) + a^2m^2 \quad (1 + m^2)r^2 = (c - b - am)^2$ as required.

16a	2 If $y = e^{\tan^{-1} 2x}$ show that $(1+4x^2)\frac{d^2y}{dx^2} + (2+4x)\frac{dy}{dx} = 0$ $y = e^{\tan^{-1} 2x}$, $\frac{dy}{dx} = \frac{2}{1+4x^2}.e^{\tan^{-1} 2x}$ so $\frac{dy}{dx}(1+4x^2) = 2$, $\frac{d^2y}{dx^2}(1+4x^2) + 8x\frac{dy}{dx} = 2$ $(1+4x^2)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 8x.\frac{dy}{dx} = 0$ 2 $(1+4x^2)\frac{d^2y}{dx^2} + (2+4x)\frac{dy}{dx} = 0$	
b)	The displacement of a particle at time t is x, measured from a fixed point and $\frac{dx}{dt} = c(a^2 - x^2)$, where a and c are positive constants, and $x = 0$ when $t = 0$. Prove that $x = \frac{a(e^{2act}-1)}{(e^{2act}+1)}$. If $x = 3$ when $t = 1$ and $x = \frac{175}{17}$ when $t = 2$, prove that $c = \frac{5}{2a}\ln e$	

and find the value of a.

$$\frac{\quad}{\quad} \; a\left(c^2 - x^2\right), \quad \frac{\quad}{\quad} c -$$

$$x\,dx\left(c - x\right) = a\,dt\,dx$$
$$dt$$

Let $1 = A\left(c - x\right) + B\left(c - x\right)$, thus $A = B = \frac{1}{2c}$

$$\frac{1}{2c}\int \frac{1}{\left(c - x\right)dx} + \frac{1}{2c}\int \frac{1}{\left(c - x\right)dx} = \int a\,dt$$

$$\frac{1}{2c}\ln\frac{c + x}{c - x} = at + k \text{, for } t = 0, x = 0, k = 0 \quad \frac{1}{2c}\ln\frac{c + x}{c - x} = at$$

$$c\left(e^{2act} - 1\right)$$

$c + x = e^{2act}\left(c - x\right)$, thus $x = \frac{e^{2act} - 1}{e^{2act} + 1}$ as required.

$$t = 1, x = 3 \quad \frac{1}{2c}\ln\frac{c + 3}{c - 3} = a \ldots \text{(i)}$$

$$t = 2, x = \frac{\quad}{\quad} \frac{\quad}{\quad} \frac{\quad}{\quad} \ln \frac{75}{17c} = 2a \ldots \text{(ii)} \quad \frac{75}{17} = \frac{1}{2c}\ln\frac{17c + 75}{17c - 75}$$

$$\frac{17c + 75}{17c - 75} = \frac{\left(c + 3\right)^2}{\left(c - 3\right)^2} = \frac{\left(c^2 + 6c + 9\right)}{\left(c^2 - 6c + 9\right)} = \frac{\left(17c + 75\right)}{\left(17c - 75\right)}$$

$$17c + 75 = \left(c + 3\right)^2$$

$$17c^3 + 27c^2 + 297c + 675 = 17c^3 - 27c^2 - 297c + 675$$

	$54c^2 \square 1350$ thus $c^2 \square 25$ so $c \square\square 5$ since it was positive then $c \square 5$	

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