# UGANDA ADVANCED CERTIFICATE OF EDUCATION MOCK EXAMINATION <br> PURE MATHEMATICS 

PAPER P425/1
Time: 3 hours

## INSTRUCTIONS TO CANDIDATES:

- Attempt ALL the EIGHT questions in section A and any FIVE from section B.
- All working must be clearly shown.
- Clearly indicate the questions you have attempted and where necessary, begin a question on a fresh sheet of paper.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using CAL for calculator and TAB for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts. DONOT

| Qn |  |  |  |  |  |  |  |  |  |
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| Marks |  |  |  |  |  |  |  |  |  |

SECTION A (40 MARKS)

1. The points A, B and C have coordinates $(0,6),(1,3)$ and $(4,6)$ respectively. Find the coordinates of the point D , the foot of the perpendicular from A to BC .
2. The $n t h$ term of an A.P is $\frac{3 n-1}{6}$, prove that the sum of $n$ terms is $\frac{n}{12}(3 n+1)$. ( 05 marks)
3. The volume of a sphere is increased by $3 \%$. Find the percentage increase in the radius.
(05 Marks)
4. In a triangle $\mathrm{OAB}, \mathbf{O A}=\mathbf{a}$ and $\mathbf{O B}=\mathbf{b}$. Given that E divides OA in the ratio $6: 1, \mathrm{D}$ divides AB in the ratio $2: 1$ and point C is on OB produced such that $\mathbf{O C}: \mathbf{O B}=3: 2$, find the ratio ED : DC.
(05 marks)
5. Find the area bounded between the curve $y=x^{2}-8 x$ and the x - axis.
6. Given that $\alpha$ and $\beta$ are the roots of the equation $x^{2}-2 x+6=0$, find the quadratic equation with roots $\frac{\alpha}{1-\alpha^{2}}$ and $\frac{\beta}{1-\beta^{2}}$.
7. Prove that $4 \cos 3 \theta \cos \theta+1=\frac{\sin 5 \theta}{\sin \theta}$.
8. By differentiating and eliminating the constants A and B of $x=e^{-4 t}(A+B t)$, find the final expression of the function

## SECTION B (60 MARKS)

9a) The third term of an arithmetic progression is 3 and the seventh term exceeds three times the third term by 2 . Find the:
(i) common difference
(ii) sum of the first 20 terms.
(06 marks)
b) Using the binomial theorem, expand $\sqrt{\frac{1+x}{1-x}}$ as far as the term in $x^{2}$. Hence, find an approximate value of $\sqrt{1.5}$.
(06 marks)
10. Sketch the curve $y=\frac{2 x^{2}-9 x-18}{x^{2}-x-2}$ by clearly finding the turning points and the asymptotes.
(12 marks)
11a) Find the magnitude and argument of the complex number $w=\frac{2}{1-3 i}+\frac{i}{(2-i)^{2}}$. (06 marks)
b) If k is a variable and $z=4 k+3(1-k) i$, find:
i) the locus of a point $P(x, y)$ representing $Z=x+y i$. (03 marks)
ii) the minimum value of $|Z|$.

12a) Evaluate: $\int_{0}^{\pi / 2} \sin 2 x \cos x d x$
b) Determine the values of $\mathrm{P}, \mathrm{Q}$ and R such that $\frac{x^{2}+2 x-4}{x^{2}+2 x-3}=P+\frac{Q}{x+3}+\frac{R}{x-1}$. Hence, evaluate $\int_{2}^{4} \frac{x^{2}+2 x-4}{x^{2}+2 x-3} d x$.

13a) The position vectors of points $P$ and $Q$ are $2 \mathbf{i}-3 \mathbf{j}+4 \mathbf{k}$ and $3 \mathbf{i}-7 \mathbf{j}+12 \mathbf{k}$ respectively. Determine;
i) the size of PQ .
ii) The Cartesian equation of PQ .
(03 marks)
b) Find the equation of the plane containing the points P and Q and the line $\mathbf{r}=\mathbf{i}-4 \mathbf{j}+t(2 \mathbf{i}+\mathbf{j}-3 \mathbf{k})$.
14. Prove that the equation of the normal at the point $P\left(a t^{2}, 2 a t\right)$ on the parabola $y^{2}=4 a x$ is $t x+y=2 a t+a t^{3}$ and that it meets the parabola again at $Q\left(a T^{2}, 2 a T\right)$ where $T=-t-\frac{2}{t} . \quad$ The tangents at P and Q meet at R . Prove that
if P is a variable point on the parabola, the locus of R is $y^{2}(x+2 a)+4 a^{3}=0 . \quad(12$ marks $)$
15a) Solve the equation: $\cos ^{-1} 2 x-\cos ^{-1} x=\frac{\pi}{3}$.
(05 marks)
b) Find the maximum and minimum values of the function $\frac{1}{3+\sin x-2 \cos x}$ stating clearly the values of $x$.
(06 marks)
16a) Solve the differential equation $\frac{d y}{d x}+y=e^{-x} \cos \frac{1}{2} x$ given that $y=-1$ when $x=0$. (05 marks)
b) A kettle of hot water is cooling in a room where the room temperature is $15^{\circ} \mathrm{C}$. The rate of cooling is proportional to the difference between the temperature of the water and the room temperature. Given that the water takes 10 minutes to cool from $75^{\circ} \mathrm{C}$ to $45^{\circ} \mathrm{C}$, find the temperature of the water after 20 minutes.

