P425/2
APPLIED MATHEMATICS
PAPER 2
3HRS

SECTION A (40 MARKS)

1. X is a discrete random variable such that

$$P(X=x) = \begin{cases} \lambda \log_{10} x & ; \lambda = 2,4,5,10,25 \\ o & ; \text{elsewhere} \end{cases}$$

   i) Show that the value of constant λ is ¼ (3mks)
   ii) Find the expected value of X (2mks)

2. The resistance of a certain metal at temperatures of 80^0C, 160^0C and 240^0C are 100,200 and 800 ohms respectively. using linear/extra –polation estimate
   i) the resistance at 100^0C (2mks)
   ii) the temperature at a resistance of 100 ohms. (3mks)

3. A car of mass 200kg accelerates at $1.5ms^{-2}$ while developing 28kw of power. The resistance to motion is constant and equal to 500N. Calculate the speed of the car at the given acceleration. (5mks)

4. Thirty six percent of all the cars in Uganda are brand new. What is the probability that, in a batch of 400 randomly cars, at least 154 are brand new? (5mks)

5. ABCD is a square of length 2m x 2m. Forces of magnitude 5, P, Q, 2 and 3√2N act along AB, BC, CD, DA and CA respectively as shown in the figure. If the system reduces to a couple, find P and Q and compute the moment of the couple. (5mks)

6. An approximate value, rounded off to two decimal places, has a maximum relative error of 0.02.
   i) State the maximum error in the approximation. (1mk)

ii) Find the approximate value, and deduce the limits within which the exact value lies. (4mks)

7. Two markers at UNEB marking centre separately assessed six candidates and gave them the following grades

Marker 1	A	C	D	B	F	E
Marker 2	D	A	B	C	E	F

Compute a rank correlation coefficient for the two assessments and on this basis whether these markers need close supervision or not. (5mks)

8. The position vector of a particle at anytime t seconds is $r = 6t^2\ \mathbf{i} + (42t + t^2)\ \mathbf{j}$ m. Calculate the speed of the particle at t = 3s . (5mks)

SECTION B (60 MARKS)

9. The continous random variable X has a cumulative distribution function (c.d.f) given by

$$F(x) = \begin{cases} o & ;\ \lambda \le -2 \\ \alpha(\lambda+\beta)^2 & ;\ -2 \le x \le 0 \\ \lambda(x^2+2x+2) & ;\ 0 \le x \le 2 \\ 1 & ;\ x \ge 2 \end{cases}$$

a) Find the non-zero constants α ,β and λ. (5mks)

b) Calculate P(X < 1/X > -1) (4mks)

c) Find f(x); the p.d.f of X, (3mks)

10. At ship a certain instant a ship travelling at speed of $45kmh^{-1}$ due $Ntan^{-\,4}/_3E$ is initially 50km, on a bearing of 300^0, from a motor boat. The boat is travelling Northwards with a speed, Vmh^{-1}

a) if the vessels collide, show that $V= 3(4\sqrt{3}+9)\ kmh^{-1}$. (5mks)

b) calculate the shortest distance between the vessels if at that instant the boat is instead North westwards with the same speed, $V\ kmh^{-1}$ (7mks)

11. a) Given that x = 5.0 + 0.02 ; y =1.25 + 0.001 calculate maximum error in $\frac{x+y}{x-y}$ (5mks)

b) Use the trapezium rule with six ordinates to estimate $\int_0^{\pi/2} (x+\sin x)\,dx$, correct to four decimal places (7mks)

12. The temperatures in a certain laboratory in a period of 50 days were recorded in the table below;

Temperate /^{0}C	10-<15	15-<20	20-<25	25-<35	35-<45	45-<50
Number of days	8	10	14	6	7	5

a) calculate the i) mean,
ii) median temperatures. (6mks)

b) Plot an Ogive for this information, and use it to estimate the interval of values of temperature containing the middle 68% of the observations. (6mks)

13. An elastic string of natural length 0.5m and modulus of elasticity 49N is fixed to a point, and its free end carries a particle of mass 2kg. The particle is depressed from its equilibrium position by 0.14, and then released to perform vertical oscillations.

a) Show that the oscillations are simple harmonic, with a period of $^2/_7\pi$ seconds. (6mks)

b) Another particle of mass 0.5kg is attached to the first particle just as it passes through its equilibrium position. calculate the new
i) maximum velocity,
ii) Amplitude of motion. (6mks)

14. a) (i) show that the equation $x^2 = \sin x$ has a root lying between 0.5 and 1 (2mks)
ii) Use linear interpolation to estimate the first approximation, λ_0 of the root in (a)(i). (2mks)

c) Construct a flow chart that

i) reads the first approximation x_0,

ii) computes the root, correct to three decimal places, using the Newton-Raphson method,

iii) Prints the root. Hence perform a dry run for your flow chart. (8mks)

15. The marks obtained by the one hundred candidates of a certain school in the 2011 UNEB mathematics paper 2 were normally distributed with a mean of 57% and a standard deviation of 13%

    a) In this school 15 candidates failed, and 20 candidates obtained distinctions in, this paper. calculate, to the nearest whole number,
    the i) pass mark.
    ii) lowest mark for a distinction.

    b) Using this school's result construct the 99% confidence limits for the mean mark obtained, in this paper, by all candidates in the country.
    (12mks)

16. A body of mass m kg is just prevented from sliding down a rough inclined plane by a horizontal force of ½ mg Newton's. The coefficient of friction between the body and the plane is μ.

    a) prove that the angle of inclination of the plane to the horizontal is

    $\tan^{-1}\left({}^{1+2\mu}/_{2-\mu}\right)$

    b) given that μ = ½ show that the magnitude of the least force parallel to the plane that will just move the body up the incline is ${}^{11}/_{10}$mg Newton's

END