P425/1

## PURE MATHEMATICS

## PAPER 1

June/July 2017
3 hours

# UACE RESOURCE MOCK EXAMINATIONS 2017 <br> PURE MATHEMATICS 

## Paper 1

3 hours

## INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section $\mathbf{A}$ and five questions from section $\mathbf{B}$
Any additional question(s) answered will not be marked
All working must be shown clearly

Begin each question on a fresh page

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

## SECTION A (40 MARKS)

1. (a) Simplify $\qquad$ $\left.4.2 n_{n}+{ }^{1}{ }_{1}-2 n_{n}+2\right)$
marks)

$$
-2)(b) \text { Show that } \log \left(100-y^{2}\right)=2+\log \left(1-\_^{y 2}\right)
$$

(03 marks)
2. By using $y=m x$, solve the equations: $x^{2}-y^{2}=3,2 x^{2}+x y-2 y^{2}=4 \quad$ ( 05 marks)
3. Prove that $4 \tan ^{-1} \square \square-{ }^{1 \square} \square-\tan ^{-1} \square_{\square}^{1} \square^{\square}={ }^{\square}$.
(05 marks)
-5
ㅁ239—4
4. Differentiate $x \log _{e} x$ with respect to $x$ hence evaluate $\int_{1}^{2} \log _{e} x d x \quad$ (06 marks)
5. The first three terms of in the expansion of $(1+k x)^{n}$ in ascending powers of $x$ are $1-6 x+\frac{33}{2}-x^{2}$, find the values of $k$ and $n$. (04 marks)
6. Find a vector perpendicular to the vectors $\boldsymbol{a}=\mathbf{2 i}-\boldsymbol{j}+\mathbf{3} \boldsymbol{k}$ and $\boldsymbol{b}=\boldsymbol{i}+\mathbf{2} \boldsymbol{j}+\boldsymbol{k}$. (05 marks)
7. Water is emptied from a cylindrical tank of radius 20 cm at the rate of 2.5 litres per second and fresh water is added at the rate of 2 litres per second. Determine the rate at which the water level in the tank is changing.). (05 marks)
8. A is a point $(0,4)$. P is a variable point such that it's distance from A is twice it's distance from the line $3 x=4 y$.Find the locus of P .

## SECTION B ( 60 MARKS)

9. (a) Show that $i^{9}+2 i^{11}+i^{13}=0$. (04 marks)
(b) If $z_{1}=1+i \sqrt{3}$ and $z_{2}=\sqrt{3}+i$, represent $z_{1}$ on an argand diagram.
(03 marks)
(c) Given that $Z=1+i$ is a root of the equation $z^{4}-4 z^{3}+3 z^{2}+2 z-6=0$. Find the other roots.
10. (a)Angles A and B are both obtuse angles. Given that $\sin A=\frac{5}{13}$ and $\cos B=-\frac{3}{5}$, find $\tan (A-B)$. (05 marks)
(b) If $\tan \square=\square \tan (A-\square)$ show that $(\lambda-1) \sin A=(\lambda+1) \sin (2 \theta-A) .(07$ marks $)$
11. The points $P(1,3), R(4,-5)$ and $Q(9,-1)$ are on the vertices of a triangle $P Q R$.

Find the equation of the
(a) circle and hence state its radius and the coordinates of the centre.
(08 marks)
(b) tangent to the circle at the point Q .
(04 marks)
12. (a) Differentiate the following functions with respect to $x$.
(i) $\frac{\sqrt{x}}{}$
(03 marks) $x-2$
(ii) $2 x^{x 2}$
(04 marks)

13. (a) Find the point of intersection between the lines $x-2=2 y+1=3-z$ and the plane $x+2 y+$ $z=3$. (04 marks)
(b) Show that the points with position vectors $\boldsymbol{O A}=4 \boldsymbol{i}-8 \boldsymbol{j}-13 \boldsymbol{k}, \boldsymbol{O B}=3 \boldsymbol{i}-2 \boldsymbol{j}-3 \boldsymbol{k}$ and $\boldsymbol{O C}=$ $3 \boldsymbol{i}+\boldsymbol{j}-2 \boldsymbol{k}$ are vertices of a triangle ABC .
(04 marks)
(c) Find the equation of a plane through the origin parallel to the lines
$\boldsymbol{r}_{\mathbf{1}}=\mathbf{3 i}+\mathbf{3} \boldsymbol{j}-\boldsymbol{k}+m(\boldsymbol{i}-\boldsymbol{j}-\mathbf{2 k})$ and $\boldsymbol{r}_{\mathbf{2}}=\mathbf{4 i} \mathbf{- 5} \mathbf{j}-\mathbf{8 k}+t(\mathbf{i} \boldsymbol{i}+\boldsymbol{j}-\mathbf{2 k}) ; \mathrm{m}$ and t are scalars.
14. (a) Express ${ }^{2}$ $\qquad$ $x_{4}-3 x 3+{ }_{2} 7 x_{2} 2-8 x+5$ into partial fractions. marks)

$$
(x-1)(x+2)
$$

(b) Hence find $\int\left(\frac{2 x^{4}-3 x^{3}+7 x^{2}-8 x+5}{(x-1)^{2}\left(x^{2}+2\right)}\right) d x$
15. A curve is given by $y=\frac{( }{x(x-2)}^{x+1)(x-3)}$.
(i) Show that for real $x, y$ cannot be between 1 and 4 .
(ii) Hence determine the turning points and distinguish them.
(iii) State the asymptotes and the intercepts of the curve.
(iv) Hence sketch the curve.
(03 marks)
16. (a)Solve the differential equation: $x^{2 d y} \ldots=x^{2}+x y+y^{2}$. (04 marks)
$d x$
(b) When a murder is committed, the body originally at $37^{\circ} \mathrm{C}$, loses heat at a rate proportional to the difference between the body temperature, H and the surrounding temperature, $\mathrm{H}_{\mathrm{o}}$. Suppose that after two hours the temperature is $35^{\circ} \mathrm{C}$, and that the temperature of the surrounding air is a constant $20^{\circ} \mathrm{C}$. If the body is found at $4: 00 \mathrm{pm}$ having a temperature of $30^{\circ} \mathrm{C}$, estimate the when the murder was committed.
(08 marks)

END

