

P425/1

PUREMATHEMATICS

PAPER1

June/July2017

3hours

UACERESOURCEMOCKEXAMINATIONS2017

PUREMATHEMATICS

Paper1

3hours INSTRUCTIONSTOCANDIDATES:

Answer all the eight questions in section A and five questions from section B

Any additional question(s) answered will not be marked

All working must be shown clearly

Begin each question on a fresh page

Silent, non-programmable scientific calculators and mathematical tables with lists of formulae may be used.

SECTION A (40 MARKS)

1.(a) Simplify $(4(2^{n+1} - 2^n) + 2)$ (03 marks)

(b) Show that $\log(100-y^2) = 2 + \log(1-100^{-y^2})$ (03 marks)

2. By using $y=mx$, solve the equations: $x^2 - y^2 = 3$, $2x^2 + xy - 2y^2 = 4$ (05 marks)

3. Prove that $4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$ (05marks)

4. Differentiate $x \log_e x$ with respect to x hence evaluate $\int_1^2 \log_e x dx$ (06marks)

5. The first three terms of the expansion of $(1+kx)^n$ in ascending powers of x are

$1 - 6x + \frac{33}{2}x^2$, find the values of k and n . (04marks)

6. Find a vector perpendicular to the vectors $a = 2i - j + 3k$ and $b = i + 2j + k$. (05marks)

7. Water is emptied from a cylindrical tank of radius 20 cm at the rate of 2.5 litres per second and freshwater is added at the rate of 2 litres per second. Determine the rate at which the water level in the tank is changing.). (05marks)

8. A is a point (0,4). P is a variable point such that it's distance from A is twice its distance from the line $3x=4y$. Find the locus of P. (05marks)

SECTION B (60 MARKS)

9.(a) Show that $i^9 + 2i^{11} + i^{13} = 0$. (04marks)

(b) If $z_1 = 1+i$ and $z_2 = 3+i$, represent z_2 on an Argand diagram. (03marks)

(c) Given that $Z = 1+i$ is a root of the equation $z^4 - 4z^3 + 3z^2 + 2z - 6 = 0$. Find the other roots. (05marks)

10.(a) Angles A and B are both obtuse angles. Given that $\sin A = \frac{5}{13}$ and $\cos B = -\frac{3}{5}$, find $\tan(A-B)$. (05marks)

(b) If $\tan = \frac{\lambda}{1-\lambda}$ show that $(\lambda^2+1)\sin A = (\lambda+1)\sin(2\theta-A)$. (07marks)

11. The points P(1,3), R(4,-5) and Q(9,-1) are on the vertices of a triangle PQR.

Find the equation of the

(a) circle and hence state its radius and the coordinates of the centre. (08marks)

(b) tangent to the circle at the point Q. (04marks)

12. (a) Differentiate the following functions with respect to x.

(i) $\sqrt{x-2}$ (04marks)

(ii) $2x^{x_2}$ (04marks)

— (03marks)

(b) Find the first three terms of the expansion $x + \frac{1}{1}$ using Maclaurin's theorem. (05marks)

13. (a) Find the point of intersection between the lines $x-2=2y+1=3-z$ and the plane $x+2y+z=3$. (04marks)

(b) Show that the points with position vectors $OA = 4i - 8j - 13k$, $OB = 3i - 2j - 3k$ and $OC =$

$3i + j - 2k$ are vertices of a triangle ABC. (04marks)

(c) Find the equation of a plane through the origin parallel to the lines $r_1 = 3i + 3j - k + m(i - j - 2k)$ and $r_2 = 4i - 5j - 8k + t(3i + j - 2k)$; m and t are scalars.

(04marks)

14.(a) Express $\frac{2x^4-3x^3+7x^2-8x+5}{(x-1)^2(x^2+2)}$ into partial fractions. (07marks)

(b) Hence find $\int \frac{2x^4-3x^3+7x^2-8x+5}{(x-1)^2(x^2+2)} dx$ (05marks)

$$= \frac{(x+1)(x-3)}{x(x-2)}$$

15. A curve is given by $y = \frac{(x+1)(x-3)}{x(x-2)}$.

- (i) Show that for real x , y cannot be between 1 and 4. (04marks)
- (ii) Hence determine the turning points and distinguish them. (04marks)
- (iii) State the asymptotes and the intercepts of the curve. (03marks)
- (iv) Hence sketch the curve. (03marks)

16.(a) Solve the differential equation: $x^2 dx - dy = x^2 + xy + y^2$. (04marks)

(b) When a murder is committed, the body originally at 37°C , loses heat at a rate proportional to the difference between the body temperature, H and the surrounding temperature, H_0 . Suppose that after two hours the temperature is 35°C , and that the temperature of the surrounding air is a constant 20°C . If the body is found at 4:00 pm having a temperature of 30°C , estimate the when the murder was committed.

(08marks)

END