P425/2

APPLIED MATHEMATICS

PAPER 2

June/July 2017

3 hours

UACE RESOURCE MOCK EXAMINATIONS – 2017

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and* **five** *questions from section* **B**

Any additional question(s) answered will **not** *be marked*

All *working* **must** *be shown clearly*

Begin each question on a fresh page

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** questions in this section

1. Events A and B are such that P(AUB) = 0.8, P(A/B) = 0.2 and P(A' ∩B) = 0.4. Find

   (i) P(B) (03 marks)

   (ii) P(A∩B) (02 marks)

2. A body of mass 20 kg rests on a horizontal floor. The coefficient of friction between the body and the floor is 0.5. Find the maximum force acting an angle of 30^{o} to the horizontal required to just move the body. (05 marks)

3. (a) Use trapezium rule with five sub-intervals to estimate $\int_0^1 \sqrt{(1+x)}\, dx$ (03 marks)

   (b) Find the error in your result. (02 marks)

4. An elastic string of natural length 60 cm is stretched to 70 cm by a stone of mass 1.5 kg hanging on it. Find the:

   (a) modulus of elasticity of the string.

   (b) energy stored in the string at equilibrium. (05 marks)

5. The following table summarises the distance to the nearest mile, travelled to work by a random sample commuters.

Distance (miles)	$0-9$	$10-19$	$20-29$	$30-39$	$40-49$	$50-59$
Number of commuters	15	38	22	15	8	2

Draw a cumulative frequency curve (Ogive) to represent the data and use it to estimate the median. (05 marks)

6. The formula based on Newton Raphson's method for determining the fourth root of number k is given by $x_{n+1} = \frac{3}{4}\left(x_n + \frac{k}{3\,x_n^3}\right)$, $n = 0,1,2, \ldots$

   Construct a flow chart that;

   - Reads the k and the initial approximation,x_o.
   - Computes, using the formula given and prints the fourth root of k, correct to 2 decimal places. (05 marks)

7. Given that $X{\sim}B(200{,}0.15)$, estimate $P(X > 21)$. (05 marks)

8. A particle is moving in straight line with simple harmonic motion with amplitude 50 cm and period time $\frac{\pi}{4}$ seconds. Find the:

   (a) velocity of the particle at a distance of 25 cm from centre. (03 marks)

   (b) maximum acceleration of the particle. (02 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section. All questions carry equal marks.

9. (a) Given that $y = \theta\cos\theta$ and $\theta = 120^o$. Calculate the

   (i) maximum possible error in y.

   (ii) limits within which y is expected to lie. Give your answer to three decimal points.

   (07 marks)

   (a) The table below gives values of a continuous function $f(x)$.

x	y	$2y$	$3y$
$f(x)$	0.16	0. 48	0.64

   Estimate:

   (i) $f(\frac{3}{2}y)$ (03 marks)

   (ii) n such that $f(ny) = 0.50$ (02 marks)

10. (a) The position vector of a ship P relative to Q at any time t is given by the expression

$\begin{pmatrix} 4 \\ 13 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \end{pmatrix} t$ metres

Find the:

(i) time when they are nearest together, (04 marks)

(ii) closest distance between the two ships. (02 marks)

(b) A projectile is fired from ground with initial velocity $3\boldsymbol{i} + 4\boldsymbol{j}\ ms^{-1}$. Find its:

(i) position vector at any time t. (03 marks)

(ii) horizontal range. (03 marks)

11. A continuous random variable X is defined by the p.d.f

$$f(x) = \begin{cases} kx & 0 < x < 1 \\ k & 1 < x < 2 \\ k(3-x) & 2 < x < 3 \\ 0 & elsewhere \end{cases}$$

(a) Sketch $f(x)$ hence determine

(i) the value of k. (04 marks)

(ii) median and expectation of X. (02 marks)

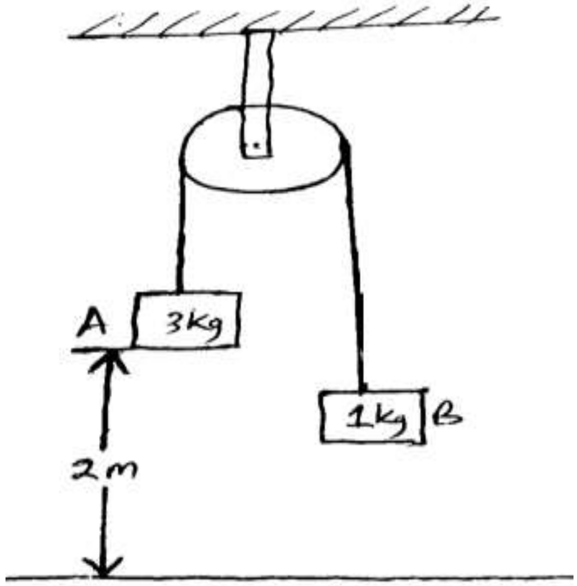
(b) Determine the cumulative distribution function, F(x). (03 marks)

(c) Find

(i) $P(|X - \mu| < 1)$. (01 mark)

(ii) the 90th percentile of X. (02 marks)

12. Particles A and B of masses 3 kg and 1 kg respectively as shown in the figure below hang at the ends of a light string passing over a smooth pulley.



A is released from rest when it is 2 m above the ground. Find

(a) the acceleration of the system. (06 marks)

(b) the tension in the string. (02 marks)

(c) the speed of B when A reaches the ground. (02 marks)

(d) how much higher B will travel afterwards. (Assume B did not reach the pulley).

(02 marks)

13. The table below shows the marks (in percentage) scored by ten students in physics and chemistry tests.

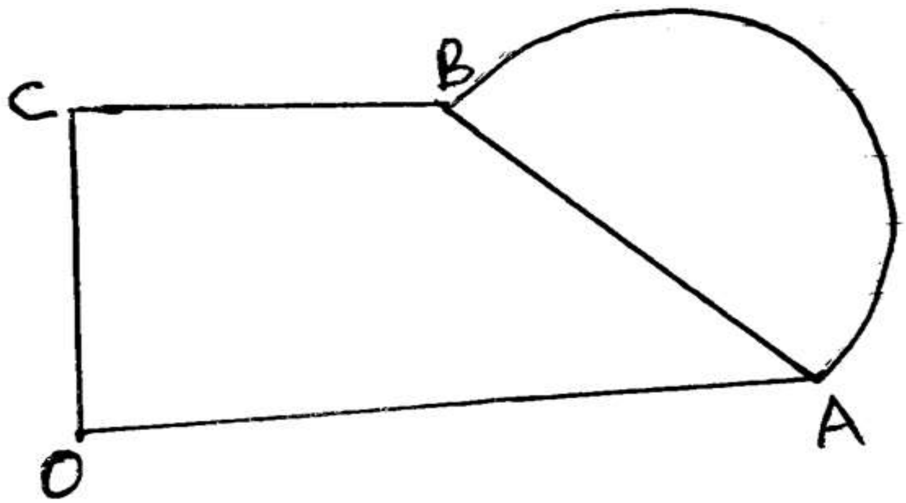
Physics(x)	80	75	65	90	95	98	78	65	54	60
Chemistry(y)	70	85	70	90	92	88	76	70	73	76

(a) (i) Plot a scatter diagram for the data. Comment on the relationship between the scores in physics and chemistry.
(ii) Draw a line of best fit through the points of the scatter diagram. (04 marks)

(b) Use your results to predict the scores of a student in
(i) chemistry when the score in physics is 76%. (01 mark)
(ii) physics when the score in chemistry is 84%. (01 mark)

(c) Calculate the rank correlation coefficient between the scores in chemistry and physics. Comment on your results. (06 marks)

14. The figure below shows a composite lamina made up of a trapezium OABC and a semi circle. Given that $\overline{OC} = \overline{BC} = 6\ cm$ and $\overline{OA} = 14\ cm$.



(a) Determine the position of the centre of gravity of the lamina taking OC as the y-axis and OA as the x-axis. Leave $\pi\ in$ your answer. (09 marks)

(b) The lamina is suspended from C, find the angle OC makes with the vertical at equilibrium. (03 marks)

15. (a)Show that the Newton-Raphson formula for finding the root of the equation $xe^{-x} = -1$ is given by $x_{n+1} = \frac{e^{x_n} + x_n^2}{x_n - 1}$, $n = 0,1,2,\ldots$ (04 marks)

(b) Show graphically that the real root of the equation $xe^{-x} = -1$ lies between 0 and 1. Hence use the formula in (a) above to determine the root to 3 decimal places.(08 marks)

16. The table below shows prices and quantities of four commodities A, B, C and D for assembling a radio for the years 2003 and 2005.

Commodity	**Price per unit(Ugsh)**		**Quantity**	
	2003	**2005**	**2003**	**2005**
A	1000	1200	36	42
B	1100	1000	96	88
C	500	650	10	12
D	800	850	11	10

Taking "2003 = 100"

(a) Calculate:

(i) the price relatives for 2005 for each commodity and hence the simple aggregative price index.

(ii) the weighted aggregative price index.

(iii) value index.

(10 marks)

(b) If a complete assembled radio costs Ugsh70,000 in 2005, using index in (a)(ii) above, estimate the price of a radio in 2003. (02 marks)

END