P425/2
APPLIED
MATHEMATICS
Paper 2
August, 2019
3 HOURS

# UNNASE MOCK EXAMINATIONS <br> Uganda Advanced Certificate of Education 

## APPLIED MATHEMATICS

## PAPER 2

## 3 HOURS

## INSTRUCTIONS TO CANDIDATES

Answer all the eight questions in Section A and any Five from Section B.

All necessary working must be shown clearly.

Begin each answer on a fresh page.

In numerical work, take $\boldsymbol{g}$ to be $\mathbf{9} \cdot \mathbf{8 m s}^{-2}$.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

## SECTION A: (40 MARKS)

Answer all the questions in this section.

1. Given that $\mathbf{P}(\mathbf{A})=\mathbf{0 . 5 9}, \mathbf{P}(\mathbf{B})=\mathbf{0 . 4 5}$ and $\mathbf{P}(\mathbf{A} \mathbf{n} \mathbf{B})=\mathbf{0 . 1 5}$, find:
(i) $\mathbf{P}(\mathbf{A} \mathbf{U B})$
(ii) $\boldsymbol{P}(\overline{\boldsymbol{A}} / \overline{\boldsymbol{B}})$
(05 marks)
2. A particle moving with $\mathbf{S} \cdot \mathbf{H} \cdot \mathbf{M}$ has velocity $\boldsymbol{v}^{\mathbf{2}}=\mathbf{1 6}\left(\mathbf{9}-\boldsymbol{x}^{\mathbf{2}}\right)$ when at a distance $\mathbf{x}$ from the centre of its path $\mathbf{O}$. Find the
(i) amplitude and period of its motion
(ii) speed as it passes $\mathbf{O}$
3. Use the trapezium rule with 4 ordinates to evaluate the integral of $\boldsymbol{x} \boldsymbol{c o s} \boldsymbol{x}$ between $\mathbf{6 0 ^ { \circ }}$ and $\mathbf{9 0}^{\circ}$ correct to $\mathbf{4}$ decimal places
4. A uniformly distributed $\mathbf{r} \cdot \mathbf{v} \mathbf{X}$ on the interval $[\alpha, \beta]$ is illustrated as follows:


Given that $\mathbf{X}$ has a lower quartile of $\mathbf{5}$ and an upper quartile of $\mathbf{9}$, use a graphical procedure to find the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$
(05 marks)
5. Forces of magnitude 5N and PN are acting away from each other at an angle of $\mathbf{6 0 ^ { \circ }}$. Given that their resultant is $\mathbf{7 N}$, find the:
(i) value of $\mathbf{P}$
(ii) angle $\mathbf{P}$ makes with the resultant
6. The table below shows the prices of items for the years 2016 and 2017

| Item | PRICE (£) |  | Weights |
| :---: | :---: | :---: | :---: |
|  | IN <br> 2017 |  |  |
| A | 25 | 28 | 5 |
| B | $x$ | $y$ | 3 |
| C | 30 | 36 | 2 |

Given that the simple aggregate price index and weighted mean price index for 2017 based on 2016 are 120 and 119 respectively, find the values of $\boldsymbol{x}$ and $\boldsymbol{y}$.
(05 marks)
7. The iterative formula $\boldsymbol{x}_{\boldsymbol{n}+\boldsymbol{1}}=\frac{1}{\boldsymbol{x}_{\boldsymbol{n}}^{2}}-1$ or $\boldsymbol{x}_{\boldsymbol{n}+\boldsymbol{1}}=\frac{\boldsymbol{1}}{\sqrt{1+\boldsymbol{x}_{\boldsymbol{n}}}}$ is to be used as a solution to an equation. Using $\boldsymbol{x}_{\boldsymbol{o}}=\mathbf{0} \cdot \mathbf{7 5}$, show without iterating that one of the choices is not suitable
(05 marks)
8. At 10:30 am, the position vector of ship $\mathbf{P}$ relative to $\operatorname{ship} \mathbf{Q}$ at time $\mathbf{t}$ hours is $\boldsymbol{p}^{r} \boldsymbol{q}=(\mathbf{1 4}-3 \boldsymbol{t}) \boldsymbol{i}+(\mathbf{1 2}-5 t) \mathbf{j} \mathbf{k m}$
(i) Write down the velocity of $\mathbf{P}$ relative to $\mathbf{Q}$
(01 mark)
(ii) Find the time at which the ships are closest together.
(04 marks)

## SECTION B (60 Marks)

Answer any five questions in this section. All questions carry equal marks.
9. The weights in kg of $\mathbf{2 5}$ boys were as follows:

| Weights | $20-24$ | $25-29$ | 30 | $31-34$ | $35-49$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 3 | 5 | 2 | 6 | 9 |

(a) Calculate the:
(i) mean weight
(03 marks)
(ii) number of boys weighing between $\mathbf{2 6 . 5} \mathbf{k g}$ and $\mathbf{3 2 . 5} \mathbf{k g}$
(02 marks)
(b) Display the data on a histogram and use it to estimate the mode
(07 marks)
10. A car of mass $\mathbf{m k g}$ has a maximum speed of $\boldsymbol{u} \boldsymbol{k m} \boldsymbol{h}^{-\boldsymbol{l}}$ up a hill inclined at an angle $\theta$ to the horizontal. It attains a maximum speed of $\boldsymbol{v} \boldsymbol{k} \boldsymbol{m} \boldsymbol{h}^{-\boldsymbol{1}}$ when descending the same hill with the engine cut off. If the resistance to motion is proportional to the square of the speed,
(i) Show that the power output of the engine is $\frac{\mathbf{5 u m g}}{\mathbf{1 8} \boldsymbol{v}^{\mathbf{2}}}\left(\boldsymbol{u}^{\mathbf{2}}+\boldsymbol{v}^{\mathbf{2}}\right) \boldsymbol{\operatorname { s i n }} \boldsymbol{\theta}$
(ii) Find the power output of the engine if $\mathbf{m}=\mathbf{9 0 0} \mathbf{k g}, \boldsymbol{u}=\mathbf{3 6} \mathbf{k m} \boldsymbol{h}^{-1}$, $v=40 \mathrm{kmh}^{-1}$ and $\sin \theta=\frac{1}{21}$
(12 marks)
11. (a) The lower limit of a measurement is $\mathbf{4 . 0 5}$ and its upper limit is
6.75. Find the relative error of the measurement
(05 marks)
(b) A decimal number $\mathbf{x}$ was approximated with an error $\Delta \mathbf{x}$. Show
that the relative error in $\boldsymbol{x}^{p}$ is $\frac{|\boldsymbol{p} \| \Delta \boldsymbol{x}|}{|\boldsymbol{x}|}$. Hence if $\mathbf{x}=\mathbf{2 . 5 0}$, find the percentage error in $\boldsymbol{x}^{3}$
(07 marks)
12. A ball projected at an angle with a speed of $\mathbf{1 4} \sqrt{\mathbf{1 0}} \boldsymbol{m} \boldsymbol{s}^{\boldsymbol{1}}$ from the top of a tower $\mathbf{2 0 0} \mathbf{m}$ high hits the ground at a point $\mathbf{2 0 0 m}$ away from the foot of the tower.
(i) Show that the two possible directions of projection are at right angles to each other
(06 marks)
(ii) Find the two possible times of flight
13. A continuous $\mathrm{r} \cdot \mathrm{v} \mathbf{X}$ has the following $\mathrm{p} \cdot \mathrm{d} \cdot \mathrm{f}$

$$
f(x)=\left\{\begin{array}{cll}
\lambda x(x-2), & 2 \leq x \leq 3 \\
0, & \text { otherwise }
\end{array}\right.
$$

(a) Find the:
(i) Value of $\lambda \quad$ (04 marks)
(ii) Cumulative distribution function of $\mathbf{X}$ (04 marks)
(b) Show that the median of $\mathbf{X}$ lies between $\mathbf{2 . 7 0}$ and $\mathbf{2 . 7 5}$ (04 marks)
14. (a) Use Newton Raphson formula to show that the root of the equation

$$
\begin{equation*}
x^{3}+2^{x}=0 \text { is } x_{n+1}=x_{n}-\frac{x_{n}^{3}+2^{x_{n}}}{3 x_{n}^{2}+2^{x_{n} \operatorname{In2} 2}} \tag{02marks}
\end{equation*}
$$

(b) Draw a flow chart that:
(i) Reads the initial approximation $\boldsymbol{x}_{\boldsymbol{o}}$.
(ii) Computes and prints the root in (a) above correct to $\mathbf{3}$ decimal places
(c) Perform a dry run for your flow chart using $\boldsymbol{x}_{\boldsymbol{o}}=-\mathbf{0} \cdot 7$
(04 marks)
15. A uniform ladder $\mathbf{P Q}$ of length $\mathbf{2 a}$ and weight $\mathbf{w}$ is inclined at an angle of $\boldsymbol{t a n}^{-1} \mathbf{2}$ to the horizontal with its end $\mathbf{Q}$ resting against a smooth vertical wall and end $\mathbf{P}$ on a rough horizontal ground with which the coefficient of friction is $\frac{\mathbf{5}}{\mathbf{1 2}}$. If a boy of weight $\mathbf{W}$ can safely ascend a distance $\mathbf{x}$ up this ladder before it slips,
(i) show that $x=\frac{a(2 w+5 W)}{3 W}$
(ii) deduce that the boy can only reach the top of the ladder if $\mathbf{W}=\mathbf{2 w}$
16. (a) A family has 25 children. The probability of having a boy is $\mathbf{0 . 6 4}$. Find the probability of having more girls than boys
(05 marks)
(b) A random sample of 50 readings taken from a normal population gave the following data: $\sum x=163$ and $\sum x^{2}=548$. Calculate the:
(i) unbiased estimate for the population variance
(02 marks)
(ii) $\mathbf{9 9 \%}$ confidence interval for the population mean
(05 marks)
**** END ****

