P425/1
PURE MATHEMATICS
PAPER 1
JULY 2017
3 HOURS

ST. JOSEPH OF NAZARETH HIGH SCHOOL
UGANDA ADVANCED CERTIFICATE OF EDUCATION
INTERNAL MOCK EXAMINATION 2017
PURE MATHEMATICS
PAPER 1
3 HOURS

INSTRUCTIONS:

- Answer all the questions in Section A and only five questions in Section B.
- Show all necessary working clearly.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formula may be used.

SECTION A (40 MARKS)

Attempt all questions from this section.

1. If $\frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}} = \boldsymbol{a} + \boldsymbol{b}\sqrt{2}$ Find the values of a and b. (5 marks)

2. The ninth term of an arithmetic progression is twice the third term, and the fifteenth term is 27. Evaluate the sum of the first 25 terms of the series. (5 marks)

3. Differentiate $x^{\cos x}$ with respect to x. (5 marks)

4. Evaluate the definite integral $\int_0^1 x\tan^{-1} x\, dx$(5 marks)

5. Solve the equation $3\cos 2\theta - 7\cos\theta - 2 = 0$for $0° \leq \theta \leq 360°$. (5 marks)

6. Find the equation of the circle which touches the line $3x - 4y = 3$ at the point $(5, 3)$ and passes through the point $(-2,4)$. (5 marks)

7. The roots of the equation $x^2 + px + 7 = 0$ are $\propto$ and β. Given that $\propto^2 + \beta^2 = 22$, find the possible values of p. (5 marks)

8. Prove that $\log_a x = \frac{1}{\log_x a}$. Hence solve the equation $\log_{10} x + \log_x 100 = 3$ (5 marks)

SECTION A (60 MARKS)

Answer any five questions from this section.

9. (a) If $z = x + iy$, determine the Cartesian equation of the locus given by
$$\left|\frac{(z-1)}{(z+1-i)}\right| = \frac{2}{5}$$ (6 marks)

   (b) Sketch the loci defined by the equations:

   (i) $\arg(z + 2) = \frac{-2\pi}{3}$

   (ii) $\arg\left(\frac{z-3}{z-1}\right) = \frac{\pi}{4}$

(6 marks)

10. (a) Prove that $\sin 4\theta = \frac{4tan\theta(1-tan^2\theta)}{(1+tan^2\theta)}$(6 marks)

(b) Solve the equation $\tan^{-1}(1+x) + \tan^{-1} 1 - x = \frac{\pi}{4}$ (6 marks)

11. Find the coordinates of any maxima, minima and points of inflexion of the function $y = \frac{3x-1}{(4x-1)(x+5)}$ that it may have. Hence sketch the curve $y = \frac{3x-1}{(4x-1)(x+5)}$ (12 marks)

12. (a) Find $\int x\sqrt{(1-x^2)}\, dx$

(b) Express $\int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)} dx = \frac{3}{4}\ln 2 + \frac{\pi}{8}$(9 marks)

13. (a) Find the particular solution of the differential equation $xy\frac{dy}{dx} = x^2 + y^2$,
Given that $y = 2,$ when $x = 1$(6 marks)

(b) A lump of radioactive substance is disintegrating. At time t days after it was first observed to have the mass of 10 grams and $\frac{dm}{dt} = -km$ where k is a constant.Find the time, in days for the substance to reduce to 1 gram in mass, given that its half –life is 10 days. (The half – life is the time in which half of any mass of the substance will decay.) (6 marks)

14. (a) Find the values of m for which the line $y = mx$ is a tangent to the circle $x^2 + y^2 + fy + c = 0$(3 marks)

(b) Find the points where the line $2y - x + 5 = 0$ meets the circle $x^2 + y^2 - 4x + 3y - 5 = 0$Obtain the equation of the tangents and normal to the circle at these points(6 marks)

15. (a) Show that the points A,B and C with position vectors $2\hat{\imath} + 3\hat{\jmath}, 4\hat{\imath} + 5\hat{\jmath}, 6\hat{\imath} + 9\hat{\jmath}$ respectively are the vertices of a triangle. Find the area of the triangle.
(5 marks)

(b) Find a vector $r$perpendicular to the vectors $s = 5\hat{\imath} + 3\hat{\jmath} + k$ and $t = -\hat{\imath} + 3\hat{\jmath} + 2\text{k}$.

Hence, find the equation of a plane passing through the point $A(5, -1, -2)$ and parallel to s and t. Find the angle between the plane and the line $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-2}{3}$(7 marks)

16. (a) If $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ show that $\frac{dy}{dx} = \frac{1}{1+x^2}$ (6 marks)

(b) Use the Maclaurin's theorem to find the first four terms of the expansion of $e^x \sin x$. (6 marks)

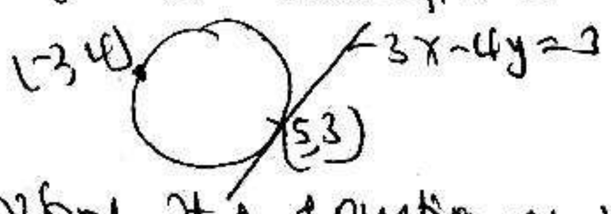
~END~

MARKING GUIDE FOR P425/1 INTERNAL MOCK 2017 ①

No.	SOLUTION ST. JONAHS	MKS	COMMENT
1.	$\frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}} = a + b\sqrt{2}$		
	Soln		
	$\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} + \frac{(1-\sqrt{2})(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = a+b\sqrt{2}$	M_1M_1	Rationalising / L.C.M
	$\frac{4+4\sqrt{2}+2}{4-2} + \frac{1-2\sqrt{2}+2}{1-2} = a+b\sqrt{2}$		
	$\frac{6+4\sqrt{2}}{2} + \frac{3-2\sqrt{2}}{-1} = a+b\sqrt{2}$		
	$\Rightarrow 3+2\sqrt{2}-3+2\sqrt{2} = a+b\sqrt{2}$	M_1	
	$4\sqrt{2} = a+b\sqrt{2}$		
	$\Rightarrow 0+4\sqrt{2} = a+b\sqrt{2}$		
	$\therefore a = 0$ and $b = 4$	A_1A_1 05	
2.	9th term $= a+8d$		
	3rd term $= a+2d$		
	15th term $= a+14d$		
	$\Rightarrow a+8d = 2(a+2d)$		
	~~$a+14d = 27$~~		
	$a-4d = 0$ --- ①	B_1	
	$\Rightarrow a+14d = 27$ --- ②		
	from ① $a = 4d$		
	substituting for a in ②		
	$4d+14d = 27$	M_1	subst. $a=4d$ in ②
	$18d = 27$		
	$d = \frac{27}{18} = \frac{3}{2}$	A_1	for both d and a
	$a = 4\times\frac{3}{2} = 6$		
	$S_n = \frac{n}{2}[2a+(n-1)d]$		
	$S_{25} = \frac{25}{2}[2\times6+24\times\frac{3}{2}]$	M_1	Subst
	$= \frac{25}{2}[12+36]$		
	$= \frac{25}{2}\times48$		
	$S_{25} = 600$	A_1	

No.	SOLUTION	MKS	COMMENT
3.	Let $y = x^{\cos x}$		
	Taking logarithms to base e		
	$\ln y = \ln x^{\cos x}$		
	$\Rightarrow \ln y = \cos x \ln x$	M1	
	$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\cos x \ln x)$		
	$\frac{1}{y}\frac{dy}{dx} = -\sin x \ln x + \frac{\cos x}{x}$	M1	
	$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x - x\sin x \ln x}{x}$		
	$\frac{dy}{dx} = y\frac{(\cos x - x\sin x\ln x)}{x}$	M1	
	$\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x - x\sin x \ln x}{x}\right)$	B1	
	$\frac{dy}{dx} = (\cos x - x\sin x\ln x)x^{\cos x - 1}$	A1	
		05	
4.	$\int_0^1 x\tan^{-1}x\,dx$		
	Let $u = \tan^{-1}x$		
	$\Rightarrow \tan u = x$		
	$\sec^2 u\frac{du}{dx} = 1$		
	$\frac{du}{dx} = \frac{1}{\sec^2 u}$		
	$\frac{du}{dx} = \frac{1}{1+x^2}$		
	Let $\frac{dv}{dx} = x$		
	$v = \int x\,dx$		
	$v = \frac{x^2}{2}$	B1	for $\frac{du}{dx}$ & v

SOLUTION	MKS	COMMENT
Using integration by parts		
$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$		
$\Rightarrow \int x\tan^{-1}x\,dx = \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int\frac{x^2}{1+x^2}dx \quad (i)$	M1	substn
To $\int\frac{x^2}{1+x^2}dx$ Let $x^2=\tan^2\theta$, $x=\tan\theta$, $dx=\sec^2\theta d\theta$		
$\Rightarrow \int\frac{\tan^2\theta}{1+\tan^2\theta}\cdot\sec^2\theta\,d\theta$		
$= \int\tan^2\theta\,d\theta$		
$= \int(\sec^2\theta - 1)\,d\theta$		
$= \tan\theta - \theta + c$		
$\Rightarrow \int\frac{x^2}{1+x^2}dx = x - \tan^{-1}x + c$ ✓	B1	for $\int\frac{x^2}{1+x^2}dx$
Substituting in (i)		
$\Rightarrow \int x\tan^{-1}x\,dx = \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}[x - \tan^{-1}x] + c$		
$= \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + c$		
$\int_0^1 x\tan^{-1}x\,dx = \left[\frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right]_0^1$	M1	
$= \left(\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}\right) - (0)$		
$\Rightarrow \int_0^1 x\tan^{-1}x\,dx = \frac{\pi}{4} - \frac{1}{2} = 0.285$	A1	
ALT		
To $\int\frac{x^2}{1+x^2}dx$ By long division: $x^2+1 \overline{)\,x^2\,}$ quotient 1, x^2+1, remainder -1		
$\Rightarrow \int\frac{x^2}{1+x^2}dx = \int 1\,dx + \int\frac{1}{x^2+1}dx$		
$= x + \tan^{-1}x + c$		

No.	SOLUTION	MKS	COMMENT
5.	$3\cos 2\theta - 7\cos\theta - 2 = 0$		
	$\Rightarrow 3(2\cos^2\theta - 1) - 7\cos\theta - 2 = 0$	B1	
	$6\cos^2\theta - 7\cos\theta - 2 = 0$		
	Let $x = \cos\theta$		
	$\Rightarrow 6x^2 - 7x - 5 = 0$		
	$\Rightarrow 6x^2 - 10x + 3x - 5 = 0$		
	$\Rightarrow 2x(3x-5) + 1(3x-5) = 0$	M1	
	$(2x+1)(3x-5) = 0$		
	$2x+1 = 0$ & $3x-5 = 0$		
	$x = -\frac{1}{2}$ $\quad x = \frac{5}{3}$		
	For $\cos\theta = -\frac{1}{2}$		
	$\theta = \cos^{-1}(-\frac{1}{2})$		
	$\theta = 120^0, 240^0$	A1 A1	
	For $\cos\theta = \frac{5}{3}$		
	θ is undefined.	B1	
6.			
	Using the equation of the circle		
	$x^2 + y^2 + 2gx + 2fy + c = 0$		
	At point $(5,3)$		
	$\Rightarrow 10g + 6f + c = -34$ --- (1)		
	At point $(-2,4)$		
	$-4g + 8f + c = -20$ --- (2)	B1	for (1) & (2)
	Using (1) − (2)		
	$14g - 2f = -14$		
	$\Rightarrow 7g - f = -7$ --- (3)		

(3)

SOLUTION	MKS	COMMENT
For line $3x-4y=3$ its gradient $= 3/4$ For $x^2+y^2+2gx+2fy+c=0$ $2x+2y\frac{dy}{dx}+2g+2f\frac{dy}{dx}=0$ $(y+f)\frac{dy}{dx}=-x-g$ $\frac{dy}{dx}=\frac{-x-g}{y+f}$	B1	
At point $(5,3)$ $\Rightarrow \frac{3}{4}=\frac{-5-g}{3+f}$ $\Rightarrow 9+3f=-20-4g$ $\Rightarrow 4g+3f=-29$ --- (4) using $3(3)+(4)$ $21g-3f=-21$ $+\ 4g+3f=-29$ $25g=-50$ $g=-2$ Subst $g=-2$ in (3) $-14-f=-7$ $f=-7$	M1	
Substituting for g and f in (1) $10(-2)+6(-7)+c=-34$ $\Rightarrow c=28$	A1	for g, f & c
∴ The eqn of the circle is $x^2+y^2-4x-14y+28=0$	B1	

No.	SOLUTION	MKS	COMMENT
7.	$x^2 + px + 7 = 0$		
	$\alpha + \beta = -p$ & $\alpha\beta = 7$	B1	
	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	M1	
	$\Rightarrow 22 = p^2 - 14$	M1	Subst
	$p^2 = 36$		
	$p = \pm 6$	A1A1	(values of P)
8.	Let $y = \log_a x$		
	$\Rightarrow a^y = x$		
	$\log_x a^y = \log_x x$	M1	Taking logs to base x
	$y \log_x a = 1$		
	$y = \frac{1}{\log_x a}$		
	$\therefore \log_a x = \frac{1}{\log_x a}$	B1	
	$\log_{10} x + \log_x 100 = 3$		
	$\Rightarrow \log_{10} x + \log_x 10^2 = 3$		
	$\log_{10} x + \frac{2}{\log_{10} x} = 3$		
	Let $p = \log_{10} x$		
	$\Rightarrow p + \frac{2}{p} = 3$		
	$p^2 - 3p + 2 = 0$		
	$p^2 - 2p - p + 2 = 0$		
	$p(p-2) - 1(p-2) = 0$		
	$(p-1)(p-2) = 0$	M1	
	$\Rightarrow p = 1$ & $p = 2$		
	$\Rightarrow \log_{10} x = 1$		
	$\Rightarrow x = 10$	A1	
	$\log_{10} x = 2$		
	$x = 100$	A1	

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No.	SOLUTION	MKS	COMMENT
10	a) $\sin 4\theta = 2\sin 2\theta \cos 2\theta$	M1	
	$= 2(2\sin\theta\cos\theta)(2\cos^2\theta - 1)$		
	$= 8\sin\theta\cos^3\theta - 4\sin\theta\cos\theta$	M1	
	Divide thru by $\cos^4\theta$		
	$= \dfrac{\dfrac{8\sin\theta\cos^3\theta}{\cos^4\theta} - \dfrac{4\sin\theta\cos\theta}{\cos^4\theta}}{\sec^4\theta}$	M1	Dividing thru by $\cos^4\theta$
	$= \dfrac{8\tan\theta - 4\tan\theta\sec^2\theta}{(1+\tan^2\theta)^2}$	M1	
	$= \dfrac{8\tan\theta - 4\tan\theta(1+\tan^2\theta)}{(1+\tan^2\theta)^2}$		
	$= \cancel{8\tan\theta - 4\tan\theta - 4}$		
	$= \dfrac{4\tan\theta(2-(1+\tan^2\theta))}{(1+\tan^2\theta)^2}$	M1	
	$= \dfrac{4\tan\theta(1-\tan^2\theta)}{(1+\tan^2\theta)^2}$.	B1	

No.	SOLUTION	MKS	COMMENT
b	$\tan^{-1}(x+1) + \tan^{-1}(1-x) = \frac{\pi}{4}$		
	Let $A = \tan^{-1}(1+x) \Rightarrow \tan A = (1+x)$	B1	
	$B = \tan^{-1}(1-x) \Rightarrow \tan B = (1-x)$	B1	
	$\Rightarrow A + B = \frac{\pi}{4}$		
	$\tan(A+B) = \tan\frac{\pi}{4}$		
	$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$	M1	
	$\frac{1+x+1-x}{1-(1+x)(1-x)} = 1$	M1	
	$\Rightarrow \frac{2}{1-(1-x^2)} = 1$		
	$\frac{2}{x^2} = 1$	M1	
	$x^2 = 2$		
	$x = \pm\sqrt{2}$		
	$x = \pm 1.414$	A1	

No.	SOLUTION No - 11	MKS	COMMENT
11	$y = \frac{3x-1}{(4x-1)(x+5)} = \frac{3x-1}{4x^2+19x-5}$		
	Let $u = 3x-1$, $v = 4x^2+19x-5$		
	$\frac{du}{dx} = 3$ $\quad \frac{dv}{dx} = 8x+19$		
	$\frac{dy}{dx} = \frac{3(4x^2+19x-5)-(3x-1)(8x+19)}{(4x^2+19x-5)^2}$		
	$= \frac{12x^2+57x-15-(24x^2+49x-19)}{(4x^2+19x-5)^2}$		
	$= \frac{-12x^2+8x+4}{(4x^2+19x-5)^2}$ ✓	M1	$\frac{dy}{dx}$
	At turning points, $\frac{dy}{dx} = 0$		
	$\Rightarrow \frac{-12x^2+8x+4}{(4x^2+19x-5)^2} = 0$		
	$\Rightarrow -12x^2+8x+4 = 0$		
	$3x^2-2x-1 = 0$		
	$3x^2-3x+x-1 = 0$		
	$3x(x-1)+1(x-1) = 0$		
	$(x-1)(3x+1) = 0$		
	$x = 1$ or $x = -\frac{1}{3}$ ✓	M1	
	when $x=1$, $y = \frac{3\times1-1}{(4\times1-1)(1+5)} = \frac{1}{9} = 0.11$		
	when $x = -\frac{1}{3}$, $y = \frac{3(-\frac{1}{3})-1}{(4\times-\frac{1}{3}-1)(-\frac{1}{3}+5)}$		
	$= \frac{9}{49} = 0.18$		

No.	SOLUTION	MKS	COMMENT
	value of x | L | $-\frac{1}{3}$ | R | L | 1 | R sign of $\frac{dy}{dx}$ | $-$ | 0 | $+$ | $+$ | 0 | $-$		
	$\Rightarrow (-\frac{1}{3}, \frac{9}{49})$ is a minimum point	A1	
	$\Rightarrow (1, \frac{1}{9})$ is a maximum point	A1	
	For $y = \frac{3x-1}{(4x-1)(x+5)}$ when $x=0$, $y = \frac{-1}{-1\times5} = \frac{1}{5}$ $\therefore$ The curve cuts the y-axis at $(0, \frac{1}{5})$.	A1	
	when $y=0$, $\frac{3x-1}{(4x-1)(x+5)} = 0$ $\Rightarrow 3x-1=0$, $x=\frac{1}{3}$ $\therefore$ The curve cuts the x-axis at $(\frac{1}{3}, 0)$.	A1	
	For vertical asymptotes $(4x-1)(x+5)=0$ $\Rightarrow x=\frac{1}{4}$ and $x=-5$ are equations for vertical asymptotes.	B1	For vertical asymptotes
	For horizontal asymptotes $y = \frac{3x-1}{4x^2+19x-5} = \frac{\frac{3}{x} - \frac{1}{x^2}}{4 + \frac{19}{x} - \frac{5}{x^2}}$ as $x \to \pm\infty$, $y \to 0$ $\therefore y=0$ is the equation for horizontal asymptote.	B1	

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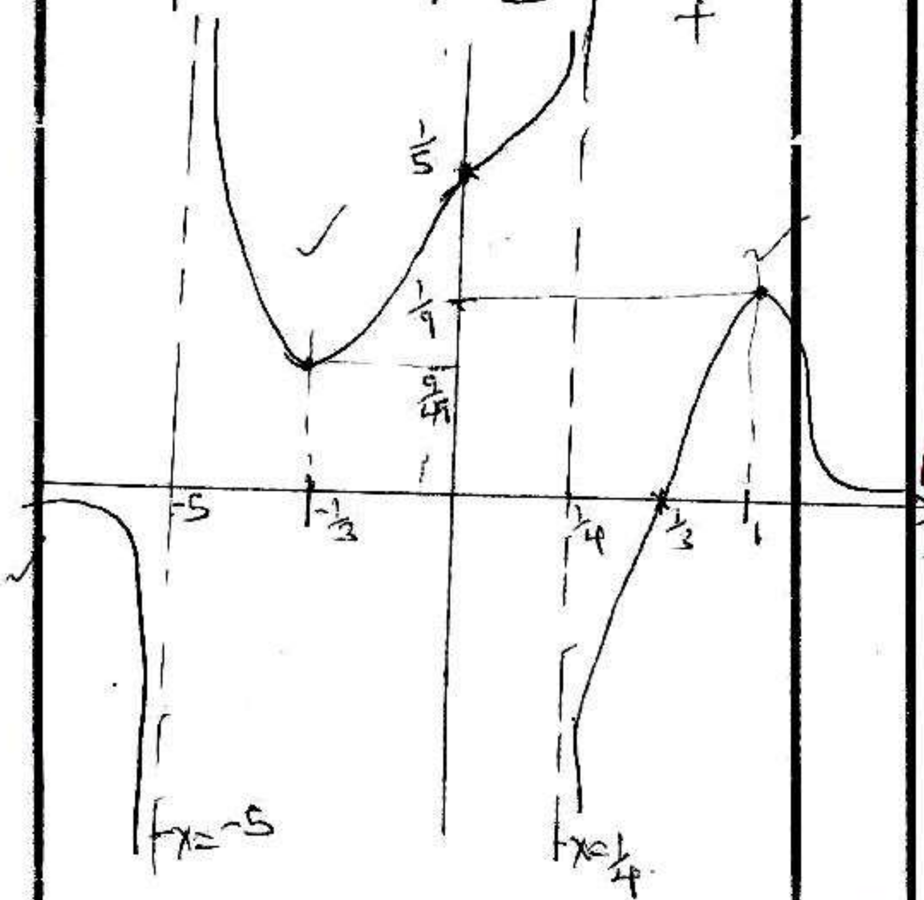
SOLUTION	MKS	COMMENT
- Check for sign of y b/n critical values		

	$x<-5$	$-5<x<\frac{1}{4}$	$\frac{1}{4}<x<\frac{1}{3}$	$x>\frac{1}{3}$
$3x-1$	$-$	$-$	$-$	$+$
$4x-1$	$-$	$-$	$+$	$+$
$x+5$	$-$	$+$	$+$	$+$
y	$-$	$+$	$-$	$+$

B_1



B_3 for the three regions with asymptotes properly indicated.

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No.	SOLUTION	MKS	COMMENT
12	$\int x\sqrt{1-x^2}\,dx$		
	Let $u = 1-x^2$		
	$\frac{du}{dx} = -2x$		
	$dx = -\frac{du}{2x}$	M1	
	$\Rightarrow \int x\sqrt{1-x^2}\,dx = \int x\sqrt{u}\,\frac{du}{-2x}$		
	$= -\frac{1}{2}\int u^{\frac{1}{2}}\,du$		
	$= -\frac{2}{3}\left(\frac{1}{2}\right)u^{\frac{3}{2}} + c$		
	$= -\frac{1}{3}u^{\frac{3}{2}} + c$	M1	
	$= -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + c$	B1	
	Alternative		
	$\int x\sqrt{1-x^2}\,dx$ let $x^2 = \sin^2 u$		
	$x = \sin u$		
	$dx = \cos u\,du$	M1	
	$\Rightarrow \int x\sqrt{1-x^2}\,dx = \int \sin u\sqrt{1-\sin^2 u}\,\cos u\,du$		
	$= \int \sin u \cdot \cos^2 u\,du$		
	Let $t = \cos u$		
	$\frac{dt}{du} = -\sin u$		
	$du = -\frac{dt}{\sin u}$		
	$\int \sin u\cos^2 u\,du = \int \sin u\, t^2\,\frac{dt}{-\sin u}$		
	$= -\int t^2\,dt$		
	$= -\frac{1}{3}t^3 + c$	M1	
	$= -\frac{1}{3}\cos^3 u + c$		
	But $\sin u = x$		
	$\Rightarrow \cos u = \sqrt{1-\sin^2 u}$		
	$= \sqrt{1-x^2}$		
	$\int x\sqrt{1-x^2}\,dx = -\frac{1}{3}\sqrt{(1-x^2)^3} + c$	B1	

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SOLUTION	MKS	COMMENT
(b) $\frac{x^2+x+1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$	M1	
$\Rightarrow x^2+x+1 = A(x^2+1)+(Bx+C)(x+1)$		
put $x=-1 \Rightarrow 1 = 2A$		
$\Rightarrow A = \frac{1}{2}$	A1	
put $x \to 0 \Rightarrow 1 = A + C$		
$1 = \frac{1}{2} + C$		
$C = \frac{1}{2}$	A1	
put $x=1 \Rightarrow 3 = 2A+(B+C)\times 2$		
$3 = 2\times\frac{1}{2} + 2(B+\frac{1}{2})$		
$2 = 2B+1$		
$2B = 1$		
$B = \frac{1}{2}$	A1	
$\therefore \frac{x^2+x+1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{x+1}{2(x^2+1)}$	A1	
$\int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)}dx = \int_0^1\left(\frac{1}{2(x+1)} + \frac{x+1}{2(x^2+1)}\right)dx$		
$= \frac{1}{2}\left[\ln(x+1)\right]_0^1 + \frac{1}{2}\int_0^1 \frac{x+1}{x^2+1}dx$	M1	
$= \frac{1}{2}\ln 2 + \frac{1}{2}\int_0^1\left(\frac{x}{x^2+1} + \frac{1}{x^2+1}\right)dx$		
$= \frac{1}{2}\ln 2 + \frac{1}{4}\left[\ln(x^2+1)\right]_0^1 + \frac{1}{2}\left[\tan^{-1}x\right]_0^1$		
$= \frac{1}{2}\ln 2 + \frac{1}{4}\ln 2 + \frac{1}{2}\tan^{-1}1$	M1	
$= \frac{1}{4}(2\ln 2 + \ln 2) + \frac{1}{2}\times\frac{\pi}{4}$		
$= \frac{1}{4}(\ln 4 + \ln 2) + \frac{\pi}{8}$	B1	
$= \frac{1}{4}\ln 8 + \frac{\pi}{8}$		
$= \frac{1}{4}\ln 2^3 + \frac{\pi}{8}$		
$= \frac{3}{4}\ln 2 + \frac{\pi}{8}$ $\boxed{7}$	A1	

o.	SOLUTION	MKS	COMMENT
3.	$xy\frac{dy}{dx} = x^2+y^2$		
	Let $y = vx$		
	$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$	M_1	
	$\Rightarrow x \cdot vx\left(v + x\frac{dv}{dx}\right) = x^2 + v^2x^2$		
	$\Rightarrow x^2v\left(v + x\frac{dv}{dx}\right) = x^2(1+v^2)$		
	$\Rightarrow v\left(v + x\frac{dv}{dx}\right) = 1+v^2$		
	$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{v}$		
	$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{v} - v$		
	$x\frac{dv}{dx} = \frac{1+v^2-v^2}{v}$		
	$x\frac{dv}{dx} = \frac{1}{v}$	B_1	for reduced/simplified D.E
	$\Rightarrow v\,dv = \frac{dx}{x}$		
	$\int v\,dv = \int \frac{dx}{x}$		
	$\frac{1}{2}v^2 = \ln x + c$	M_1	
	But $y = vx$		
	$\Rightarrow v = \frac{y}{x}$		
	$\frac{1}{2}\frac{y^2}{x^2} = \ln x + c$	B_1	Substitution for v in general form.
	when $y = 2, x = 1$		
	$\frac{1}{2}\left(\frac{4}{1}\right) = \ln 1 + c$	A_1	
	$\Rightarrow c = 2$		
	$\Rightarrow \frac{y^2}{2x^2} = \ln x + 2$		
	$\Rightarrow y^2 = 2x^2(\ln x + 2)$	B_1	

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SOLUTION	MKS	COMMENT
b) $\frac{dm}{dt} = -Km \Rightarrow \frac{dm}{m} = -K\,dt$		
$\int \frac{dm}{m} = \int -K\,dt$		
$\Rightarrow \ln m = -Kt + C$	M1	
when $t=0$, $m=10$, $\Rightarrow C = \ln 10$		
$\therefore \ln m = -Kt + \ln 10$ --- (1)	M1	Substituting for C
Half life is 10 days		
$\Rightarrow$ when $t=10$, $m = \frac{1}{2} \times 10 = 5$ grams		
when $t = 10$, $m = 5g$		
eqn i becomes		
$\ln 5 = -10K + \ln 10$		
$10K = \ln 10 - \ln 5$		
$10K = \ln 2$		
$K = \frac{1}{10} \ln 2$	A1	
Eqn i becomes		
$\ln m = -\frac{t}{10} \ln 2 + \ln 10$ --- (2)	M1	Substituting for K
when $m=1$, $t = ?$		
Eqn (2) becomes		
$\ln 1 = -\frac{t}{10} \ln 2 + \ln 10$	M1	Substituting $m=1$
$\frac{t}{10} \ln 2 = \ln 10 - \ln 1$		
$\frac{t}{10} \ln 2 = \ln 10$		
$\frac{t}{10} = \frac{\ln 10}{\ln 2}$		
$t = 10 \times \frac{2.3026}{0.693}$		
$t = 33.2$ days	A1	

No.	SOLUTION	MKS	COMMENT
14	a $x^2+y^2+fy+c=0$ --- (1) $y=mx$ --- (2) Substitute for y $\Rightarrow m^2x^2+x^2+f(mx)+c=0$ $(m^2+1)x^2+mfx+c=0$ ✓	M1	
	For a line to be a tangent $b^2=4ac$ $4(m^2+1)c=(mf)^2$ $4m^2c+4c=m^2f^2$	B1	
	$4m^2c-m^2f^2=-4c$ $m^2(4c-f^2)=-4c$ $m^2=\frac{-4c}{4c-f^2}$ $m=\pm\sqrt{\frac{-4c}{4c-f^2}}$ $m=\pm\sqrt{\frac{4c}{f^2-4c}}$	A1	
	b $2y-x+5=0$ --- (1) $x^2+y^2-4x+3y-5=0$ --- (2) From (1) $x=2y+5$ $\Rightarrow (2y+5)^2+y^2-4(2y+5)+3y-5=0$ ✓	M1	
	$4y^2+20y+25+y^2-8y-20+3y-5=0$ $5y^2+15y=0$ $5y(y+3)=0$ $y=0$ or $y=-3$ ✓ ∴ points of contact are $(5,0)$ and $(-1,-3)$ ✓	A1 A1	
	Differentiating $x^2+y^2-4x+3y-5=0$ $\Rightarrow 2x+2y\frac{dy}{dx}-4+3\frac{dy}{dx}=0$	M1	

o.	SOLUTION	MKS	COMMENT
	$(2y+3)\frac{dy}{dx} = 4-2x$		
	$\frac{dy}{dx} = \frac{4-2x}{2y+3}$ ✓	B1	
	At the point $(5,0)$, $\frac{dy}{dx} = \frac{4-2\times5}{2\times0+3} = -2$		
	Eqn of the tangent		
	$(y-0) = (x-5)\cdot -2$		
	$y = -2x+10$ ✓	A1	
	Eqn of the normal		
	$y-0 = (x-5)\frac{1}{2}$		
	$2y = x-5$ ✓	A1	
	At the point $(-1,-3)$, $\frac{dy}{dx} = \frac{4+2}{2\times-3+3} = -2$		
	Eqn of the tangent		
	$y+3 = (x+1)\cdot -2$		
	$y+3 = -2x-2$		
	$y+2x+5 = 0$ ✓	A1	
	Eqn of the normal		
	$y+3 = (x+1)\cdot\frac{1}{2}$		
	$2y+6 = x+1$		
	$2y-x+5 = 0$ ✓	A1	

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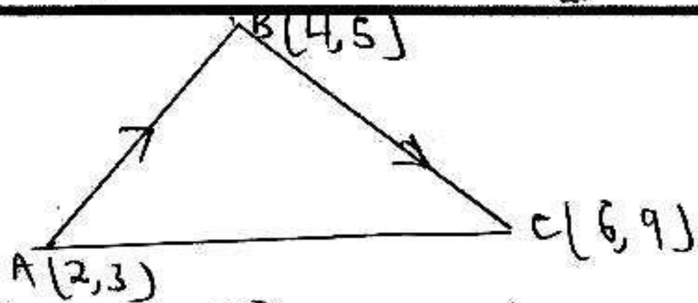
No.	SOLUTION	MKS	COMMENT
5	a) 		
	$\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix}4\\5\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$	B1	
	$\vec{BC} = \vec{OC} - \vec{OB} = \begin{pmatrix}6\\9\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$	B1	
	Since $\vec{AB} \neq \lambda \vec{BC}$, therefore the points A, B and C are vertices of a triangle.	B1	
	Area of triangle $= \frac{1}{2}\lvert \vec{AB} \times \vec{BC} \rvert$ $\vec{AB} \times \vec{BC} = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 2 & 4 & 0 \end{vmatrix}$		
	$= 0i - 0j + 4k$	M1	
	Area, A $= \frac{1}{2} \times \sqrt{4^2}$ $= \frac{1}{2} \times 4 = 2$ sq units	A1	
	b) $r = s \times t$ $r = \begin{vmatrix} i & j & k \\ 5 & 3 & 1 \\ -1 & 3 & 2 \end{vmatrix}$		
	$r = (3\times2 - 3\times1)i - (5\times2 - {-1}\times1)j + (5\times3 - {-1}\times3)k$	M1	
	$r = 3i - 11j + 18k$	A1	

No.	SOLUTION	MKS	COMMENT
	$\underset{\sim}{a} \cdot \underset{\sim}{n} = \underset{\sim}{p} \cdot \underset{\sim}{n}$		
	$\begin{pmatrix}5\\-1\\-2\end{pmatrix} \cdot \begin{pmatrix}3\\-11\\18\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}3\\-11\\18\end{pmatrix}$	M1	
	$15+11-36 = 3x-11y+18z$		
	$-10 = 3x-11y+18z$		
	$\Rightarrow 3x-11y+18z+10 = 0$	A1	
	Normal vector $\underset{\sim}{r} = 3i-11j+18k$		
	Let d be directional vector for the line		
	$\Rightarrow \underset{\sim}{d} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$		
	$\underset{\sim}{r} \cdot \underset{\sim}{d} = \lvert \underset{\sim}{r} \rvert \lvert \underset{\sim}{d} \rvert \sin\theta$		
	$\begin{pmatrix}3\\-11\\18\end{pmatrix} \cdot \begin{pmatrix}1\\2\\3\end{pmatrix} = \sqrt{3^2+-11^2+18^2} \cdot \sqrt{1^2+2^2+3^2} \sin\theta$	M1	
	$3-22+54 = \sqrt{454} \cdot \sqrt{14} \sin\theta$	M1	
	$35 = \sqrt{6,356} \sin\theta$		
	$\sin\theta = \frac{35}{\sqrt{6356}}$		
	$\theta = \sin^{-1} \frac{35}{\sqrt{6356}}$		
	$\theta = \sin^{-1}(0.439012)$		
	$\theta = \underline{26.04^{\circ}}$	A1	

No.	SOLUTION No-16	MKS	COMMENT
16.	$y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$		
	$\Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}$	M1	
	$\cos y \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right)$		
	$\cos y \frac{dy}{dx} = \dfrac{\sqrt{1+x^2}\frac{d}{dx}(x) - x\frac{d}{dx}(1+x^2)^{\frac{1}{2}}}{1+x^2}$		
	$\cos y \frac{dy}{dx} = \dfrac{\frac{\sqrt{1+x^2}}{1} - \frac{\frac{1}{2}x(\cancel{2}x)}{\sqrt{1+x^2}}}{1+x^2}$	M1	
	$\cos y \frac{dy}{dx} = \left(\frac{1+x^2-x^2}{\sqrt{1+x^2}}\right)\cdot\frac{1}{1+x^2}$		
	$\cos y \frac{dy}{dx} = \frac{1}{(1+x^2)^{\frac{3}{2}}}$ — *	B1	for $\frac{d}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{(1+x^2)^{\frac{3}{2}}}$
	But $\cos y = \sqrt{1-\sin^2 y}$ $= \sqrt{1-\frac{x^2}{1+x^2}}$ $= \sqrt{\frac{1+x^2-x^2}{1+x^2}}$ $= \frac{1}{\sqrt{1+x^2}}$	B1	for $\cos y = \frac{1}{\sqrt{1+x^2}}$
	Substituting in * $\frac{1}{\sqrt{1+x^2}}\frac{dy}{dx} = \frac{1}{(1+x^2)^{\frac{3}{2}}}$	M1	Substituting
	$\frac{dy}{dx} = \frac{1}{(1+x^2)^{\frac{3}{2}}}\cdot\sqrt{1+x^2}$		
	$\frac{dy}{dx} = \frac{1}{1+x^2}$	B1	

No.	SOLUTION	MKS	COMMENT
b	$f(x) = e^x \sin x$		
	$\Rightarrow f(0) = e^0 \sin 0 = \underline{0}$		
	$f'(x) = e^x \sin x + e^x \cos x$ (by pdt rule)		
	$\Rightarrow f'(0) = e^0 \sin 0 + e^0 \cos 0 = \underline{1}$	B1	
	$f''(x) = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x$		
	$= 2e^x \cos x$		
	$\Rightarrow f''(0) = 2e^0 \cos 0 = \underline{2}$	B1	
	$f'''(x) = 2e^x \cos x - 2e^x \sin x$		
	$\Rightarrow f'''(0) = \underline{2}$	B1	
	$f^{iv}(x) = 2e^x \cos x - 2e^x \sin x - 2e^x \sin x - 2e^x \cos x$		
	$= -4e^x \sin x$		
	$\Rightarrow f^{iv}(0) = \underline{0}$	B1	
	$f^{v}(x) = -4e^x \sin x - 4e^x \cos x$		
	$\Rightarrow f^{v}(0) = \underline{-4}$	B1	
	$\therefore f(x) = x + \frac{2x^2}{2!} + \frac{2x^3}{3!} - \frac{4x^5}{5!}$		
	$\therefore e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{1}{30}x^5$	B1	