P425/1

## MATHEMATICS

## PAPER ONE

3 HOURS

## UGANDA ADVANCED CERTIFICATE OF EDUCATION <br> S. 6 INTERNAL MOCK 2016 <br> 3 HOURS <br> INSTRUCTIONS

Answer all questions in Section A and Not more than five in section B All working must be shown clearly.

Silence, non-programmable calculators may be used.

## SECTION A (40 MARKS)

1. Solve the equation $5 \operatorname{Sin} 2 x+4=10 \operatorname{Sin}^{2} x$ for $-180 \leq \mathrm{x} \leq 180^{\circ} \quad$ ( 5 mks )
2. The second and third terms of a geometrical progression are 24 and $12(d+1)$ respectively. Find $d$ if the sum of the first three terms of the progression is 76 .
3. Points A and B have position vectors $2 \mathbf{i}-5 \mathbf{j}+3 \mathbf{k}$ and $7 \mathbf{i}-2 \mathbf{k}$ respectively. Find the coordinates of the point c which divides AB internally in the ratio $2: 3$ and point D which divided AB externally in the ratio $3: 8$.
4. Given that $\mathrm{y}=\frac{\sin x}{x}$, show that $x \frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}+x y=0$
5. Find the coordinates of the points of intersection of the curve with parametric equations $\mathrm{x}=2 \mathrm{t}^{2}-1, \mathrm{y}=3(\mathrm{t}+1)$ and the line $3 \mathrm{x}-4 \mathrm{y}=3$.
6. Find $\int \frac{\cos x}{4+\sin ^{2} x} d x$
7. Solve the equation $\log _{x} 32+\log _{1 / x} 16=1$ (5mks)
8. A spherical water container of internal radius 10 cm has water to a maximum depth of 18 cm . Find the volume of the water in the container.
(5mks)

## SECTION B (60 MARKS)

9. a) Use the binomial theorem to obtain the first four terms of the expansion of $(1-16 x)^{1 / 4}$. by taking the first two terms, find $4 \sqrt{39}$.
(6mks)
b) Use manclaurin's theorem to find the expansion of $e^{x} \sin x$ in ascending powers of $x$ as far as the term $x^{3}$.
(6mks)
10. By splitting the numerator, find;
a) $\int \frac{2 x-1}{4 x^{2}+3} d x$
b) $\int \frac{\cos \theta-2 \sin \theta}{3 \cos \theta+4 \sin \theta} \mathrm{~d} \theta$
11. a) If $0^{\circ} \leq \theta \leq 90^{\circ}, x>y>0$ and $\cot \theta=\frac{x^{2}-y^{2}}{2 x y}$, find the value of $\sec \theta$ in the simplest form. ( 6 mks )
b) Solve $2 \tan x \sin x+\sin x=\tan x+1$ for $0^{\circ} \leq \mathrm{x} \leq 360^{\circ}$ ( 6 mks )
12. a) By using a linear combination, find the Cartesian equation of the plane which passes through the point $(1,2,3)$ and which is parallel to the vectors $2 \hat{\imath}+4 \hat{\jmath}-10 \hat{\mathrm{k}}$ and $6 \hat{\imath}-4 \hat{\jmath}+2 \hat{k}$.
c) Find the equation of the line of intersection of the planes
$4 x+3 y+z=10$ and $x+y+z=6$. Find the angle between the planes above.
13. Sketch the curve $y=x^{3}-3 x^{2}+2$ and find the area enclosed by the curve and the x -axis between $x=0$ and $x=4$. If this area is now rotated about the x -axis through $2 \pi$ radians, determine the volume of the solid generated, correct to three significant figures.
14. a) Show that $z=2+3 i$ is a root of the equation $z^{4}-5 z^{3}+18 z^{2}-17 z+13=0$, hence find the other roots.
b) If $\mathrm{Z}=1+\operatorname{Cos} 2 \theta+i \sin 2 \theta$, where $\left(\frac{-\pi}{2}<\theta<\frac{\pi}{2}\right)$
prove that $|Z|=2$ and $\arg (z)=\theta$.
15. a) $\mathrm{P}\left(a p^{2}, 2 a p\right)$ is any point on the parabola $\mathrm{y}^{2}=4 \mathrm{ax}$ and the chord from P passing through the focal point meets the parabola again at $\mathrm{Q}\left(a q^{2}, 2 a q\right)$ Show that the locus of the mid-point $M$ of $P Q$ is $y^{2}=2 a(x-a)$.
(6mks)
(b)Find the equation of the normal at $\mathrm{R}(a \operatorname{Cos} \theta, b \operatorname{Sin} \theta)$ to the eclipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$. If the normal at R to the eclipse meets the x -axis at N and the $y$-axis at $S$, find the area of triangle NOS where $O$ is the centre of the eclipse.
(6mks)
16. a) Find the particular solution of the equation. $\frac{d y}{d x}=x-\frac{2 y}{x}$; given $y(2)=4$.
b) The rate of increase of the population, P , of baboons in Busitema forest reserve is proportional to the number present in the forest at any time, $t$ years. On first June 2010, there were 300 baboons in the forest and a year later they were found to be 380 .
i) Form an expression for P in terms t .
ii) Predict the population of baboons by, first June, 2018.
