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## DEPARTMENT OF MATHEMATICS

S. 6 PURE MATHEMATICS-2020

## PAPER 1 WEEK 1

## 3 HOURS

- Answer all the eight questions in section $A$ and any five from section B.
- Any additional question(s) answered will not be marked.


## SECTION A: (40 MARKS)

1. Solve the equation ${ }^{\mathrm{p}}(6 x+1)-\mathrm{p}(2 x-4)=3$.
2. Solve $\cos \theta=\sin \left(\theta+30^{\circ}\right)$ in the range $0^{\circ} \leq \theta<360^{\circ}$.
3. Show that down the first three terms in the
binomial expansion of $\left(1-\frac{x}{4}\right)^{\frac{-1}{2}}$ in ascending powers of $x$.
(05 marks)
4. Evaluate $\int_{0}^{\frac{\pi}{8}} \frac{e^{\tan 2 x}}{\cos ^{2} 2 x} d x$. (05 marks)
5. Find the shortest distance from point $P(11,-5,-3)$ to the line $l$ with

$$
\text { equation } \mathbf{r}\left(\begin{array}{c}
1 \\
0 \\
0
\end{array}\right)^{+\lambda}\left(\begin{array}{c}
-3 \\
1 \\
4
\end{array}\right)
$$

6. An open box with a square base has a total surface area of $300 \mathrm{~cm}^{2}$. Find the greatest possible volume. (05 marks)
7. Find the area of the largest square contained within the circle

$$
\begin{equation*}
x^{2}+y^{2}-2 x+4 y+1=0 \tag{05marks}
\end{equation*}
$$

8. The graph of $y=\frac{x+2}{x^{2}+4 x+3}$ is shown below.


Find the area of the shaded region.
(05 marks)

## SECTION B: (60 MARKS)

9. (a) Prove that $\frac{d}{d x}(u v)=v \frac{d u}{d x}+u \frac{d v}{d x}$.
(b) Differentiate the following.
(i) $(x-1)^{\frac{1}{3}}(x-2)^{3}$.
(ii) $\frac{2 x^{2}-3 x}{(x+4)^{2}}$.
10. (a) Find the equation of the circle passing through the points $A(3,2)$,

$$
B(-1,0) \text { and } C(5,-2)
$$

(b) Show that the locus of the point $P$ with co-ordinates $(1+2 \cos \theta, 2+2 \sin \theta)$ is a circle and find its radius and centre.
11. (a) Find $\frac{d^{2} y}{d x^{2}}$ when $y=\sin ^{-1} x-x^{\mathrm{p}}\left(1-x^{2}\right)$, expressing your answer as simple as possible.
(b) Use Maclaurin's theorem to express $\ln \sqrt{\frac{1+x}{1-x}}$ as a power series up to the term in $x^{3}$.
12. The diagram shows an extension to a house. Its base and walls are rectangular and the end of its roof, $E P F$, is sloping, as illustrated.

(a) Write down the co-ordinates of $A$ and $F$.
(b) Find, using vector methods, the angle EPF.
(c) The owner decorates the room with two streamers which are pulled taut. One goes from $O$ to $G$, the other from $A$ to $H$. She says that they touch each other and that they are perpendicular to each other. Is she right? (12 marks)
13. Bacteria in a culture increase at a rate proportional to the number of bacteria present. If the number increases from 3000 to 4000 in one hour,
(a) how many bacteria will be present after $2 \overline{2}$ hours. (09 marks)
(b) how long will it take for the number of bacteria in the culture to become 6000?
(03 marks)
14. (a) Solve the simulateous equations

$$
(x+3)(y+3)=10 \text { and }(x+3)(x+y)=2
$$

(05 marks)
(b) Use the substitution ${ }^{y=x+\frac{2}{x}}$ to solve

$$
x^{4}-5 x^{3}+10 x^{2}-10 x+4=0 .
$$

(07 marks)
15. (a) Use the substitution $t=\tan \theta$ to solve $\sin 2 \theta+2 \cos 2 \theta=1$ for $0<\theta \leq 2 \pi$.
(05 marks)
(b) The equation $1+\sin ^{2} \theta^{0}=a \cos 2 \theta^{\circ}$ has a root of 30 . Find the value of $a$ and all the roots in the range 0 to 360. (07 marks)
16. (a) Prove by induction that

$$
\sum_{r=1}^{n} r(r+1)=\frac{1}{3} n(n+1)(n+2) \cdot(05
$$ marks)

(b) Aisha opens an account with a saving scheme which offers a 12.9\% compound interest per annum. The scheme does not allow any withdrawal until after a period of 5 years. A customer is required to deposit a half the amount of money he/she opens the account with every beginning of other years for a period of 4 years. If Aisha started with shs 600,000, calculate how much;
(i) money she will earn from the scheme after 5 years.
(ii) interest she earns from the saving scheme. (07 marks)

