## S. 6 P425/1 QUESTIONS 2020 (STAHIZA)

1. (a) Prove that the number $a_{n}=4^{n}+5$ is divisible by 3 for all positive integral values of $n$.
(b) The sum of $\mathbf{n}$ terms of the progression in $\mathbf{S}_{\mathbf{n}}=\mathbf{2} \mathbf{n}^{\mathbf{2}} \mathbf{+ 3}$. Find the tenth term.
(c) Prove by induction that.
(i) $\quad \sum_{r=1}^{n} 3^{\mathrm{n}-1}=\frac{1}{2}\left(3^{\mathrm{n}}-1\right)$
(ii) $\quad \sum_{r=1}^{n} \frac{1}{(2 r+1)(2 r+3)}=\frac{n}{3(2 n+3)}$
iii) $\sum_{r=1}^{n} \operatorname{Sin} r \theta=\frac{\operatorname{Sin}\left(\frac{n+1}{2}\right) \theta \operatorname{Sin}\left(\frac{n \theta}{2}\right)}{\operatorname{Sin}^{\frac{\theta}{2}}} \frac{2}{2}$
(d) DON operate an account with Cairo bank which pays a compound interest rate of $12.5 \%$ per annum. They opened the account at the beginning of the year with Shs800,000 and deposit the same amount of money at the beginning of every year. Calculate how much they will receive at the end of 10 years. After how long will the money have accumulated to 3.32 million.
2. (a) A polynomial $P(x)$ is a multiple of $x-3$ and the remainders when $P(x)$ is divided by $x+2$ and $x-5$ are 6 and- 7 resp. Find the remainder when $P(x)$ is divided by $x^{3}-5 \mathrm{x}^{2}-4 \mathrm{x}+20$.
(b) When $\mathrm{P}(\mathrm{x})=x^{3}+a x^{2}+\mathrm{bx}+\mathrm{c}$ is divided by $x^{2}-4$, the remainder is $2 \mathrm{x}+11$. Given that $x+1$ is a factor of $P(x)$, Find the values of $a, b$ and $c$.
(c) ) If the function $P(x)=x^{4}+a x^{3}+b x^{2}+6 x-5$ is divisible by $(x-2)^{2}$.

Find the values of $a$ and $b$.
(d) Give that $x^{4}-6 x^{3}+10 x^{2}+\mathrm{ax}+\mathrm{b}$ is a perfect square, find a and b .

The roots of the equation $a x^{2}+b x+c=0$ where $a, b$ and $c$ are non Zero constants are $\alpha$ and $\beta$, and the roots of equation $a x^{2}+2 b x+c=0$ are $\theta$ and $\mu$. Find the equation whose roots are $\alpha \theta+\beta \mu$ and $\alpha \mu+\beta \theta$.

$$
x^{2}-x
$$

3. (a) Express in partial fractions; $\mathrm{f}(\mathrm{x})=(\ldots x 2+3)(x 2+2)$ Hence $\int$ $f(x) \mathrm{dx}$
(b) $\int \frac{x^{6}-x^{5}-4 x^{2}+x}{x^{4}+3 x^{2}+2} d x$

$$
4 x+5
$$

(c) Express $\mathrm{f}(\mathrm{x})=\frac{}{(x+1)(2 x+3)}$ in partial fractions; hence find $f^{\prime}(x)$ and $f^{\prime \prime}(x)$.
(d) Express $\mathrm{y}=\frac{2 x^{2}+3 x+5}{(x+1)\left(x^{2}+3\right)}$ in partial fractions and hence show that $\frac{d y}{d x}=-\frac{2}{3}$ when $\mathrm{x}=0$ and evaluate $\int_{0}^{\sqrt{3}} y d x$, state the mean value of y in the interval $0 \leq x \leq \sqrt{3}$
4. Differentiate;
(a) $\sqrt[3]{\frac{x^{2}-2}{x^{2}+4}}$
(b) $\sqrt{\frac{\operatorname{Sin} 2 x}{\operatorname{Cos}^{3} x}}$
(d) $e^{-3 x} \sin ^{2} 2 x \cos 3 x$
(e) $\frac{e^{2 x} \tan x}{\left(x^{2}-2\right) \sin x \cos x}$
(c) (i) If $\mathrm{y}=t^{2}-\cos \mathrm{t}$ and $\mathrm{x}=\sin \quad \frac{d y}{d x^{2}}$. t, find $\quad$ (ii )Find $\frac{d y}{d x}$ of $\tan 4 \mathrm{x}^{0}$ from1st principles.
5. Integrate the following;
(a) $\int_{1}^{3} \frac{2 x^{2}+3 x}{2 x-1} \mathrm{dx}$
(b) $\int \frac{x+1}{125 x^{3}-8} \mathrm{dx}$
(c) $\int \frac{1}{(1+\sqrt{x}) \sqrt{x}} \mathrm{dx}$
(d) $\quad \int_{0}^{\pi / 2} \operatorname{Sin}^{3} \mathrm{x} \operatorname{Cos}^{2} x_{\mathrm{dx}}$ (e) $\int_{0}^{\pi / 2} \operatorname{Sin} 2 x \operatorname{Sin} 5 x \mathrm{dx}$
(f) Show that $\int_{0}^{\Pi / 4}(1+\tan x)^{2} \mathrm{dx}=1+\ln 2$
(g) $\int \frac{1}{x^{4}-9} d x$
(g) Show that $\int_{0}^{1} \frac{1}{(x+1)\left(x^{2}+2 x+2\right)} \mathrm{dx}=\frac{1}{2} \ln \frac{8}{5}$
(h) Show that $\int_{0}^{\Pi / 2} \frac{\operatorname{Sin}^{3} x}{2-\operatorname{Sin}^{2} x} \mathrm{dx}=\frac{\Pi}{2}-1$
(i) Show that $\int_{0}^{\Pi / 2} \frac{4}{3+5 \cos \theta} \mathrm{~d} \theta=\ln 3$
(j) $\int_{0}^{\Pi / 4} \frac{1}{5 \operatorname{Cos}^{2} \theta-1} \mathrm{~d} \theta$
(k) $\quad \int_{0}^{3} \frac{x+3}{x^{2}+3} \mathrm{dx}$
(1) $\int x \ln x d x \quad$ (m) $\int x \tan ^{2} x \mathrm{dx}$
(n) $\int x^{3} \operatorname{Sec} x^{2} \mathrm{dx}$
(o) $\int \frac{6(x-3)}{x\left(x^{2}+9\right)} \mathrm{dx}$
(p) $\int \frac{e^{x}+e^{2 x}}{1+e^{2 x}} \mathrm{dx}$
q) $\int_{0}^{2} \frac{x^{4}+2 x}{(x-1)\left(x^{3}-1\right)} d x$
(r) $\quad \int_{0}^{\frac{\pi}{6}} \sec x d x$ Evaluate
using $\sin \mathbf{x}=\mathbf{u}$. Hence show that $\int_{0}^{\overline{6}} \sec ^{3} x d x=\frac{1}{3}+\frac{1}{4} \ln 3$
6. (a) If $\mathrm{y}=e^{x} \sin \mathrm{x}$, Show that $\frac{d^{2} y}{d x^{2}}=2\left(\frac{d y}{d x}-\mathrm{y}\right)$. By further differentiation of this result, find the Maclaurin's expansion as far as the term in $x^{6}$.
(b) Show that using maclaurins theorem, $\ln \sqrt{\frac{1+x}{1-x}}=\mathrm{X}+\frac{x^{3}}{3}+\frac{x^{5}}{5}+\frac{x^{7}}{7}$.
(c) Show that if $\mathrm{y}=e^{3 x} \sin 4 x, \frac{d^{2} y}{d^{2} x}-6 \frac{d y}{d x}+25 y=0$.
7. (a) Expand $(8+3 x)^{1 / 3}$ in ascending powers of x as far as the term in $x^{3}$, stating the values of $x$ for which the expansion of $x$ is valid. Hence obtain an approximate value for $\sqrt[3]{ } 8.72$
(b) Expand $\left(1+16 x^{2}\right)^{1 / 2}$ in descending powers of x in the term as far as the third term.
(c) ) Prove that if x is so small that its cube and other higher powers are neglected,then

$$
\sqrt{\frac{1+x}{1-x}}-1+x+\frac{1}{2} x^{2} \cdot \text { By taking } x=\frac{1}{16}, \text { Prove that } \sqrt{17}=4 \frac{33}{128}
$$

8 (a) According to Newton's law, the rate of cooling of a body in air is proportional to the difference between the temperature T of the body and $\mathrm{T}_{\mathrm{o}}$ of the air. If the air temp is kept constant at $20^{\circ} \mathrm{c}$ and the body cools from $100^{\circ} \mathrm{C}$ to $60^{\circ} \mathrm{C}$ in 20 minutes, in what further time will the body cool to $30^{\circ} \mathrm{C}$.
(b) A liquid is being heated in an oven maintained at constant temperature of $180^{\circ} \mathrm{C}$.

It is assumed that the rate of increase in temperature of the liquid is proportional to $(180-\theta)$, where $\theta^{\circ} \mathrm{C}$ is the temperature of the liquid at time $t$ minutes. If the temperature of the liquid rises from $0^{\circ} \mathrm{C}$ to $120^{\circ} \mathrm{C}$ in 5 minutes, find the temperature of the liquid after a further 5 minutes.
(c) The rate of speed of bush fire spreading is proportional to the area of unburnt Bush .At a certain moment 0.6 of the bush had been burnt, 2hrs later 0.65 of The bush was bunrt.Find the fraction remaining unburnt after 5hours.
9. (a) (i) If $Z_{1}=6\left(\operatorname{Cos} \frac{5}{12} \Pi+\mathrm{i} \operatorname{Sin} \frac{5}{12} \Pi\right)$ and $Z_{2}=3\left(\operatorname{Cos} \frac{1}{4} \Pi+\mathrm{i} \operatorname{Sin} \frac{1}{4} \Pi\right)$. Find $Z_{1} Z_{2}$ and $\frac{Z_{1}}{Z_{2}}$ in the form $a+i b$.
(ii) Express each of the complex numbers in a polar form if
(b) Find the locus of the complex number $|\mathrm{Z}-1-\mathrm{i}|<3$, represent it on the Argand diagram and shade the region representing it.
(c) (i) Solve the equations $Z^{3}-8 i=0$ and $z^{3}+i=0$.
(ii) Find in the form $a+i b$, the roots of the equation $Z^{3}-26-18 i=0$
(d) If z is a complex number, describe and illustrate on the argand diagram this locus

$$
\text { Given by each; (i) }\left|\frac{z-2 i}{z-i}\right|=2 \quad \text { (ii) } \arg ^{(z+2+3 i)-\frac{\pi}{3}}
$$

(e) Show that the locus of the complex number Z of $|z+1|=2|z-1|$ is an ellipse and Find its equation, hence find its focus,eccentricity and equation of directrix.

10 Solve the following by row reduction to echelon form.
(b) $3 x-2 y+4 z=-7, x+y-6 z=-10$ and $2 x+3 y+2 z=3$
(c) By using the method of synthetic division, divide

$$
6 x^{4}-17 x^{3}+22 x-9 \text { by } 2 x-3
$$

(d) Find the equation of circle that touches line $y=x$ at $p t(3,3)$ and passes thru $B(5,9)$.
(e) Solve the simultaneous equations

$$
2^{x}+4 y=12 \text { and } 3\left(2^{x}\right)-2\left(2^{2 y}\right)=16 . \text { Hence show that } 4^{x}+4\left(3^{2 y}\right)=100
$$

(f) When the quadratic expression $a p^{2}+\mathrm{bp}+\mathrm{c}$ is divided by $\mathrm{p}-1, \mathrm{p}-2$ and $\mathrm{p}+1$, the remainders are 1,1 and 25 respectively. Determine the factors of the expression.
11. (a) Find the term independent of x in the expansion of $\left(2 x-\frac{1}{x^{2}}\right)^{12}$ DOWNLOAD MORE RESOURCES LIKE THIS ON ECOLEBOOKS.COM
(b) Expand $\left(\frac{1+x}{1+3 x}\right)^{1 / 3}$ up to the term. Hence by taking $\mathrm{x}=1 / 125$, use your result to calculate the cube root of 63 correct to four decimal places.
12. (a) Show that $\cos ^{-1}\left(\frac{63}{65}\right)+2 \tan ^{-1}\left(\frac{1}{5}\right)=\sin ^{-1}\left(\frac{3}{5}\right)$
(b) Solve the equation (i)Sin $3 x+1 / 2=2 \operatorname{Cos}^{2} \mathrm{x}$ for $0^{\circ} \leq \mathrm{x} \leq 360^{\circ}$

$$
\text { (ii) } 4 \sin x \cos 2 x \sin 3 x=1 \text { for } 0^{\circ} \leq x \leq 180^{\circ}
$$

(c) Show that in any triangle A B C, $\tan \left(\frac{B-C}{2}\right)=\frac{b-c}{b+c} \cot \left(\frac{A}{2}\right)$. Then solve the triangle two sides 5 and 7 and the included angle of $45^{\circ}$.
(d) Show that for allvalues of $\theta, \cos \theta+\cos \left(\theta+\frac{2}{3} \pi\right)+\cos \left(\theta+\frac{4}{3} \pi\right)=0$.

Hence show that $\cos ^{2} \theta+\cos ^{2}\left(\theta+\frac{2}{3} \pi\right)+c 0 s^{2}\left(\theta+\frac{4}{3} \pi\right)=\frac{3}{2}$.
(e) (i) Prove that $4 \cos \theta 3 \cos \theta+1=\frac{\sin 5 \theta}{\sin \theta}$
(ii) In any triangle ABC, Prove that $\operatorname{TanB} \cot \mathrm{C}=\frac{a^{2}+b^{2}-c^{2}}{a^{2}-b^{2}+c^{2}}$.
13. (a) Solve the inequality,
(i) $\frac{x-1}{x-2}>\frac{x-2}{x+3}$ (ii) $2|x-1|=|x+3|$ (iii) $|5 x-6|=x^{2}$

2
(b) Sketch the curve .

$$
f(x)=\frac{x+2 x+3}{x^{2}+3 x+2}
$$

(c) If $\mathrm{y}=\frac{(x-1)^{2}}{(x+1)(x-3)}$. Find the values of y for which the curve is not defined, Hence find the nature of the turning points. sketch the curve.
(d) Find the co-ordinate of intersection of $\mathrm{y}=\frac{x^{2}+1}{x} \quad$ and $\mathrm{y}=\frac{x}{x+3}$.

Sketch both curves on Same axes and show that the area of finite region in the first quadrant enclosed by two curves is $\frac{7}{2} \ln 5-3 \ln 3-2$.
(e) The domain of the function defined by $\mathrm{f}(\mathrm{x})=\frac{4(2 x-7)}{(x-3)(x+1)}$ is the set of all real

Values of $x$ other than 3 and -1.Express $f(x)$ in partial fractions, hence or
Otherwise, show that $\mathrm{f}^{1}(\mathrm{x})$ is zero for two +ve integral values of x . Find the Turning points of this function. Hence sketch $\mathrm{f}(\mathrm{x})$. Dertermine the area of the Region bounded by the cyrve,the $x$-axis and the lines $x=4, x=6$.
( Leave your answer in logarithmic form.)
14. (a) Show that at any point on the hyperbola

$$
\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1_{\text {may be written as }(\mathrm{a} \sec \theta, \mathrm{~b} \tan \theta)}
$$

(b) Find c in terms of $\mathrm{a}, \mathrm{b}, \mathrm{m}$ if $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ is a tangent to the hyperbola $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$.Find the asymptotes.
(c ) The normal at the point $\mathrm{P}(5 \operatorname{Cos} \theta, 4 \sin \theta)$ on the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ meets the X and Y axes at Land M respectively. Show that the locus of R ,the midpoint of LM is an ellipse having the same eccentricity as the given ellipse.
(d ) The curve $b^{2} x^{2}+a^{2} y^{2}=a^{2} b^{2}$ intersects the positive X -axis at
A and the Y -axis at B .
(i) Determine the equation of the perpendicular bisector of AB .
(ii ) Given that this line intersects the X -axis at P and that M is the bisection of $b\left(a^{2}+b^{2}\right)$
AB.Show that the area of triangle PMA is $8 a$
(e ) (i) State the vertex, focus and directix of $\mathbf{8} \mathbf{y}^{\mathbf{2}} \mathbf{+ 6} \mathbf{- 9}=\mathbf{4 x}$ and $\mathbf{2} \mathbf{x}^{\mathbf{2}} \mathbf{+ 3} \mathbf{y}^{\mathbf{2}}-\mathbf{4} \mathbf{x}+\mathbf{9} \mathbf{y}+\mathbf{5}=\mathbf{0}$.
(ii ) Show that the two tangents of gradients $m$ to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ are $\mathrm{Y}=\mathrm{mx} \pm \sqrt{ }\left(a^{2} m^{2}+b^{2}\right)$.find eqnsof tangents to ellipse $\frac{x^{2}}{6}+\frac{y^{2}}{3}=1$ at $(-2,5)$.
15. (a) Find the angle between $\frac{x-3}{2}=\frac{2 y-1}{4}=\frac{3 z-1}{4} \quad$ and $-4 x+3 y+2 z=7$
(b) Find the angle between the line of intersection of the planes $2 x+y+3 z=4$ and

$$
3 \mathrm{x}+2 \mathrm{y}+2 \mathrm{z}=7 \text { and the line } \frac{1-x}{1}=\frac{y-2}{2}=\frac{z-3}{4}
$$

(c) Find the distance between the planes $2 x-3 y+4 z=7$ and $8 x-12 y+16 z=6$.
(d) Find the equation of the plane through the ( $1,0,-1$ ) and containing the
line $X=-y=\frac{z}{2}$.
(e) Find the equation of the plane containing the lines $\frac{x-3}{5}=\frac{y+1}{2}=\frac{z-3}{1}$ and

$$
\frac{3-x}{-2}=\frac{y+1}{4}=\frac{z-3}{3} .
$$

(f) Find the equation of the plane through the point ( $1,-2,1$ ) which also

Contains the line of intersection of the planes $x+y+z+6=0$ and

$$
\mathrm{x}-\mathrm{y}+\mathrm{z}+5=0
$$

(g) Show that vectors $i-2 k,-2 i+j+3 k$ and $-\mathrm{i}+\mathrm{j}+\mathrm{k}$ form a triangle .

Hence or otherwise find the area of the triangle.
(h) Show that the vectors $\mathbf{a}=\mathbf{3 i}+\mathbf{j}-\mathbf{4 k}, \mathbf{b}=\mathbf{4 i} \mathbf{i} \mathbf{j} \mathbf{- 3 k}, \mathbf{c}=\mathbf{5 i} \mathbf{- 3 j} \mathbf{- 2 k}$ are coplanar.
(I) The equations $\frac{x-3}{2}=\frac{y-5}{1}=\frac{z-2}{-3}$ and $\frac{x+1}{3}=\frac{y+4}{1}=\frac{z-2}{-2}$ represent pipes

A and B in a chemical plant. Find the the length of the shortest pipe that can be fitted at its end points.

- TO THE PROBLEMS OF YOUR LIFE,YOU'RE THE SOLUTION, AND TO THE THE QUESTIONS OF YOUR LIFE ,YOU'RE THE SOLUTION.
- IF YOU ARE GOING TO ACHIEVE EXCELLENCE IN BIG THINGS, YOU DEVELOP THE HABIT IN LITTLE MATTERS.
- WHEN SPIDER WEBS UNITE, THEY CAN TIE UP A LION.
- PRAYER AND HARD WORK GO TOGETHER WITH DISCPLINE.
- THINGS DO NOT CHANGE; WE CHANGE

```
GOOD LUCK AND SUCCES
```


## ©stahiza

 'ויוניוניו He who thinks that he can make it,makes it",". $\qquad$