



JINJA JOINT EXAMINATIONS BOARD
MOCK EXAMINATIONS 2019
MAKING GUIDE 2019 FOR
P425/1 PAPER 1 MATHEMATICS

SECTION A (40 MARKS)

$$1. \cos(45^\circ - x) = 2 \sin(30^\circ + x); -180^\circ \leq x \leq 180^\circ$$

$$\cos 45^\circ \cos x - 2 \sin 45^\circ \sin x = 2 \sin 30^\circ \cos x + 2 \cos 45^\circ \sin x \quad M1$$

$$\cos 45^\circ \cos x - 2 \sin 30^\circ \cos x = 2 \cos 30^\circ \sin x + 2 \sin 45^\circ \sin x$$

$$\cos x [2 \cos 45^\circ - 2 \sin 30^\circ] = \sin x [2 \cos 30^\circ + \sin 45^\circ] \quad x$$

$$\therefore \sin x [2 \cos 30^\circ + \sin 45^\circ] = \cos x [2 \cos 45^\circ - 2 \sin 30^\circ] \quad M1$$

$$\frac{\cos 45^\circ - 2 \sin 30^\circ}{2 \cos 30^\circ + \sin 45^\circ} \quad 0 \quad 0$$

$$\tan x = \frac{1}{\sqrt{2}} \quad M1$$

$$2 \cos 30^\circ + \sin 45^\circ$$

$$\tan x = -0.1201$$

$$x = \tan^{-1}(-0.1201) \quad M1$$

$$x = -6.8^\circ, 173.2^\circ \quad A1$$

05

$$2. \frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} > 2$$

$$\frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} - 2 > 0$$

$$\frac{4x^2 + 4x + 26}{3x^2 - 14x + 11}^2 < 0 \quad M1$$

$$\frac{(x-2)(4x-13)}{(x-1)(3x-11)} < 0$$

Critical values ;

$$x = 2, x = \frac{13}{4}, x = 4, x = \frac{11}{3}$$

B1

	$x < 1$	$1 < x < 2$	$2 < x < \frac{13}{4}$	$\frac{13}{4} < x < \frac{11}{3}$	$x > \frac{11}{3}$
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$(x - 2)$	-	-	+	+	+
$(4x - 13)$	-	-	-	+	+
$(x - 1)$	-	+	+	+	+
$(3x - 11)$	-	-	-	-	+
$(x - 2)(4x - 13)$	+	+	-	+	+
$(x - 1)(3x - 11)$	+	-	-	-	+
$\frac{(x - 1)(4x - 13)}{(x - 1)(3x - 11)}$	+	-	+	-	+

B1

The solution set is $1 < x < 2$ and $\frac{13}{4} < x < \frac{11}{3}$.

A1 A1

05

3. $\int_0^{\pi/2} x \cos x^2 dx$

$$\therefore x \cos x^2 = \frac{d}{dx} (\sin x^2) \quad M1$$

$$\Rightarrow \int_0^{\pi/2} x \cos x^2 dx = \int_0^{\pi/2} \frac{1}{2} \sin x^2 dx \quad M1$$

$$\begin{aligned} &= \frac{1}{2} \sin x^2 \Big|_0^{\pi/2} \\ &= \frac{1}{2} \sin \left(\frac{\pi}{2} \right)^2 - \frac{1}{2} \sin(0)^2 \end{aligned} \quad M1$$

$$\begin{aligned} &= \frac{1}{2} \sin \left(\frac{\pi}{2} \right)^2 - \frac{1}{2} \sin(0)^2 \\ &= \frac{1}{2} \times (4) \end{aligned} \quad M1$$

$$\therefore \int_0^{\pi/2} x \cos x^2 dx = \frac{1}{2} \quad A1$$

05

4. (i) $x^2 + y^2 - 2x - 8y - 8 = 0$

Let (a, b) be the centre

Comparing:

$$x^2 + y^2 + 2gx + 2ty + c = 0$$

x ;

$$2g = -2, \Rightarrow g = -1$$

But;

$$\begin{aligned} a &= -g \\ a &= -(-1) \\ a &= 1 \text{ either } \\ y; & \\ 2t &= -8 \\ t &= -4 \leftarrow \text{ or} \end{aligned} \quad \begin{aligned} & \\ & \\ & \\ & \\ & \quad \text{B1} \end{aligned}$$

But

$$b = -f \quad b = -(-4)$$

$$b = 4$$

\therefore centre is the point $(1, 4)$

B1

A1

(ii) Distance between centre and point A

$$t = \sqrt{(1+5)^2 + (4+4)^2} \quad \begin{aligned} & \\ \text{shortest distance, } d & \end{aligned} \quad \begin{aligned} & \\ & \quad \text{B1} \end{aligned}$$

$$d = |t - r|$$

$$d = |10 - 5|$$

$$d = 5 \text{ units}$$

M1

A1

06

5. Let x be the number of committees.

B1

B1

$$\Rightarrow x = 3c_3 \times 5c_3 + 3c^2 \times 5c_4 \quad \begin{aligned} & \\ \text{M1} & \end{aligned} \quad x = 10 + 15$$

$$x = 35 \text{ committees}$$

A1

04

6. $\cos 2x \frac{dy}{dx} = e^x \cos x + 3x; \quad y(\pi/2) = 3$

$$\frac{dy}{dx} = e^{x + 3x \sin x}$$

$$\int dy = \int (e^x dx + 3x \sin x dx)$$

$$dy = (e^x + 3x \sin x) dx$$

$$y = \int e^x dx + 3 \int e^x dx + 3 \int x \sin x dx$$

$$= e^x + 3 \int x \sin x dx$$

$$\int x \sin x dx$$

$$u = x, \quad v = \int \sin x dx$$

$$dy$$

$$\frac{du}{dx} = 1, \quad v = -\cos x$$

$$\int x \sin x dx = -x \cos x + \int \cos x dx$$

M1

$$\int x \sin x dx = -x \cos x + \sin x$$

$$y = e^x + 3(-x \cos x + \sin x) + c.$$

$$y = e^x + -3x \cos x + 3 \sin x + c$$

B1

$$\text{when } x = \pi/2, y = 3$$

$$\Rightarrow 3 = e^{\pi/2} - 3 \times \frac{\pi}{2} \cos \frac{\pi}{2} + 3 \sin \frac{\pi}{2} + c$$

M1

c

$$3 = e^{\pi/2} + 3 + c$$

$$c = e^{\pi/2}$$

$$\therefore y = e^{\pi/2} - 3x \cos x + \sin x$$

B1

A1 + 3

05

7. Cartesian equation of line: $\frac{x+4}{2} = \frac{y-2}{2} = \frac{z+3}{4}$, P(0, 6, 0)

$$\therefore \mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$$

$$\Rightarrow \overrightarrow{MP} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 + 2t \\ 3 - 4t \\ 0 \end{pmatrix}$$

B1

$$\begin{aligned} \overrightarrow{MP} &= \begin{pmatrix} -4 + 2t \\ 3 - 4t \\ 0 \end{pmatrix} \\ \text{But } \overrightarrow{MP} &\bullet b = 0 \end{aligned}$$

$$\begin{pmatrix} -4 + 2t \\ 3 - 4t \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = 0$$

M1

$$2(-4 + 2t) + -2(3 - 4t) + 4(0) = 0$$

$$-8 - 8t + 6 + 16t = 0$$

$$12 + 8t = 0$$

$$8t = -12$$

$$t = \frac{-12}{8}$$

$$t = -1.5$$

B1

$$\therefore \overrightarrow{MP} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix}$$

Distance of point C(0, 6, 0) from the line.

$$\Rightarrow |\overrightarrow{MP}| = \sqrt{(0)^2 + (2)^2 + (-5)^2}$$

$$|\overrightarrow{MP}| = \sqrt{29} \text{ units.}$$

A1

Q5

$$8. \quad x = 1 + \cos 2\theta \quad y = \sin \theta$$

$$x = 2\cos^2 \theta$$

$$dx \qquad \qquad dy$$

$$\frac{dy}{dx} = -4\cos \theta \sin \theta \quad \frac{dy}{dx} = \cos \theta$$

M1 M1 dθ

$$d\theta$$

Using:

$$\frac{dy}{dx} = \frac{dy}{d\theta} : \frac{d\theta}{dx}$$

$$dx \quad d\theta \quad dx$$

$$= \cos \theta : \frac{-1}{4\cos \theta \sin \theta} \quad -1$$

$$\frac{dy}{dx} = \frac{-1}{4\sin \theta}$$

B1

$$dx \quad 4\sin \theta$$

$$\text{But } d^2y = d\theta \frac{dy}{d\theta} d\theta$$

$$dx \quad d\theta \frac{dy}{d\theta} dx \quad d\theta$$

$$\frac{d^2y}{dx^2} = \frac{1}{4\sin \theta \cdot -4\sin \theta \cos \theta} \quad -1$$

$$= \frac{1}{16} (-\operatorname{cosec} \theta \cot \theta) \frac{1}{\sin \theta \cos \theta}$$

M1

$$= -100 - 100^3$$

$$16 \sin \theta$$

$$\frac{dx}{d\theta}$$

$$= 4 dy_3$$

B1

05

SECTION B

$$9. (a) \begin{array}{rcl} x - 10y + 7z = 13 & & (\text{i}) \\ -3z = -3 & & (\text{ii}) \\ -x + 2y - z = -3 & & (\text{iii}) \end{array}$$

Method: Elimination

$$\begin{array}{rcl} (\text{i}) & \underline{\quad} & (\text{ii}) \\ -14y + 6z = 16 & & \end{array}$$

$$7y - 5z = -8 \quad (\text{iv})$$

$$\begin{array}{rcl} (\text{i}) & + & (\text{ii}) \\ -8y + 6z = 16 & & \end{array}$$

M1

M1

$$4y + 3z = -5 \quad (\text{v})$$

$$\begin{array}{ll}
 & 3 \text{ (iv)} \quad 5 \text{(v)} \\
 y = +1 & \text{From (iv):} \\
 & \square 7y - 5z = -8 \\
 & \square 7(1) - 5z = -8 & \text{M1} \\
 & -z = 3 \\
 & \text{From (i)} \\
 & x - 10y + 7z = 13 \\
 & x - 10(1) + 7(8) = 13 & \text{M1} \\
 x = 2 & \\
 & x = 13 - 11 \\
 & x = 2 \\
 & & \text{A1 A1A1} \\
 & \square x = 2, y = 1, z = 3
 \end{array}$$

$$\begin{array}{ll}
 \text{(b)} & P(x) = ? \ g(x) = ? \ f(x) = x^2 - 5x - 14 \quad \text{Using:} \\
 & P(x) = g(x)f(x) + R(x)
 \end{array}$$

$$\begin{array}{ll}
 & \square P(x) = g(x)(x+2)(x-7) + 2x+5 & \text{M1} \\
 \text{(i)} & \text{Let } x = 7. \\
 & P(7) = g(7)(7+2)(7-7) + 2 \square 7 + 5 & \text{M1} \\
 & P(7) = 14+5 \\
 & P(7) = 19 \\
 & \square \text{The remainder is 19.} & \text{A1} \\
 \text{(ii)} & \text{Let } x = -2 \\
 & \square P(-2) = g(-2)(-2+2)(-2-7) + 2 \square 2 + 5 & \text{M1} \\
 & P(-2) = -4+5 \\
 & P(-2) = 1 \\
 & \square \text{The remainder is 1.} & \text{A1}
 \end{array}$$

12

10. (a) $4\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$

$$4\sin\theta - 3\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

Comparing:

$$\sin\theta;$$

$$R\sin\theta = 4 \quad \text{(i)}$$

$$R\cos\theta = 3 \quad \text{(ii)}$$

Value of R

$$(i)^2 + (ii)^2$$

$$(R\sin\theta)^2 + (R\cos\theta)^2 = (4)^2 + (3)^2$$

M1

$$R^2 \sin^2\theta + \cos^2\theta = 16 + 9$$

$$R^2 = 25$$

$$\theta = 5$$

B1

Size of angle, θ

$$(i) \div (ii)$$

$$\text{But } R(x) = 2x+5.$$

$$R\sin\theta = 4$$

$$R\cos\theta = 3 \tan\theta = \frac{4}{3}$$

$$\theta = \tan^{-1}\frac{4}{3} = 53.1^\circ$$

$$= 53.1^\circ$$

B1

$$\theta = 53.1^\circ$$

$$4\sin\theta - 3\cos\theta = 5\sin(\theta - 53.1^\circ)$$

Solving the equation $4\sin\theta - 3\cos\theta + 2 = 0$

$$5\sin(\theta - 53.1^\circ) + 2 = 0$$

M1

$$\theta - 53.1^\circ = \sin^{-1}\frac{-2}{5}$$

$$= 203.6^\circ, 336.4^\circ$$

M1

$$\theta = 256.7^\circ, 389.5^\circ$$

$$\square = 256.7 \square$$

A1

(b)

From the sine rule
LHS

M1

$$\frac{a + b - c}{a - b + c} = \frac{2R\sin A + 2R\sin B - 2R\sin C}{2R\sin A + 2R\sin B + 2R\sin C}$$

$$= \frac{\sin A + \sin B - \sin C}{\sin A - \sin B + \sin C}$$

M1

$$\begin{aligned} & \frac{2\sin \frac{A}{2} \cos \frac{A}{2} + 2\cos \frac{A}{2} \sin \frac{B}{2} + 2\cos \frac{A}{2} \sin \frac{C}{2}}{2\sin \frac{A}{2} \cos \frac{A}{2} - 2\cos \frac{A}{2} \sin \frac{B}{2} + 2\cos \frac{A}{2} \sin \frac{C}{2}} \\ &= \frac{\sin A \cos A + \cos A \sin B + \cos A \sin C}{\sin A \cos A - \cos A \sin B + \cos A \sin C} \end{aligned}$$

$$\sin(53.1^\circ) = \frac{-2}{5}$$

$$\begin{aligned} & \frac{\sin A \cos A + \cos A \sin B + \cos A \sin C}{\sin A \cos A - \cos A \sin B + \cos A \sin C} \\ &= \frac{\sin A \cos A + \cos A \sin B + \cos A \sin C}{\sin A \cos A - \cos A \sin B + \cos A \sin C} \end{aligned}$$

But $A + B + C = 180^\circ$

$$A = 180^\circ - (B + C)$$

$$\begin{aligned} \sin A &= \\ 90^\circ - B - C &= \\ 2 & 2 \end{aligned}$$

$$\sin A = \sin(90^\circ - B - C) = \cos(B + C) \quad \text{B1}$$

Also;

$$\begin{aligned} \sin A &= \cos(90^\circ - B - C) \\ &= \cos(B + C) \end{aligned}$$

$$\begin{aligned} \cos A &= \sin(90^\circ - B - C) \\ &= \sin(B + C) \end{aligned} \quad \text{B1}$$

$$\begin{aligned} \cos(B+C) \sin(B+C) + \cos(B+C) \sin(B-C) \\ = \frac{\cos(a+b-c)}{\cos^2 B - \cos^2 C} = \frac{\cos(a-b+c)}{\cos B + \cos C} = \frac{\cos(a-b+c)}{\cos B + \cos C} \\ = \frac{\cos(a-b+c)}{\cos^2 B - \cos^2 C} = \frac{\cos(a-b+c)}{\cos B \cos C - \cos B \cos C} = \frac{\cos(a-b+c)}{\sin B \sin C} \\ = \frac{\cos(a-b+c)}{\cos^2 B - \cos^2 C} = \frac{\cos(a-b+c)}{\cos^2 B - \cos^2 C} = \frac{\cos(a-b+c)}{\cos^2 B - \cos^2 C} \end{aligned}$$

$$\begin{aligned} \sin(B+C) + \sin(B-C) \\ = \frac{\sin(B+C)}{\sin^2 B - \sin^2 C} \end{aligned}$$

$$\begin{aligned}
 & \sin\left(\frac{B+C}{2}\right) - \sin\left(\frac{B-C}{2}\right) \\
 & = \frac{2\sin\frac{B}{2}\cos\frac{C}{2}}{2\cos\frac{B}{2}\sin\frac{C}{2}} \quad M1 \\
 & \boxed{\frac{a+b-c}{a-b+c} = \tan\frac{B}{2}\tan\frac{C}{2}} \quad B1
 \end{aligned}$$

12

11.

$$y^2 = 4ax$$

$$2y \frac{dy}{dx} = 4a \quad M1$$

$$\frac{2dy}{dx} = \frac{2a}{y}$$

At the point $P(at^2, 2at)$:

$$\frac{dy}{dx} = \frac{2a}{2at}$$

$$\frac{dy}{dx} = \frac{4}{t}$$

Using: $m_1 \cdot m_2 = -1$ M1

$$\frac{1}{t} \cdot m_2 = -1$$

$$m_2 = -t$$

B1

Equation of normal at the point $P(at^2, 2at)$

$$\begin{aligned}
 & \frac{y - 2at}{x - at} = -t \quad M1 \\
 & = -t(x - at^2) \quad y = -tx + at^2 + 2at \\
 & \qquad \qquad \qquad y = -tx + at(t^2 +
 \end{aligned}$$

- 2) Coordinates of the point G
 x - intercept occurs when $y = 0$.

$$\square 0 = -tx + at(t^2 + 2) \quad \text{M1} \quad x = a(t^2 + 2)$$

G is the point

$$y - \text{coordinate of } P \square a(t^2 + 2), 0 \quad \text{B1}$$

Let Q be the point (x, y) .

P is the midpoint of G and Q.

x - coordinate of P.

$$\square at^2 = \frac{1}{2}(x + a(t^2 + 2)) \quad \text{M1}$$

$$2at^2 = x + a(t^2 + 2)$$

$$2at^2 - at^2 - 2a = x$$

$$\square x = a(t^2 - 2) \quad \text{(i)} \quad \text{B1}$$

$$\square 2at = \frac{y + 0}{2} \quad \text{M1}$$

$$t = \frac{y}{4a} \quad \text{(ii)} \quad \text{B1}$$

Substitute (ii) in (i) for t

$$\square y^2 = 2x^2 \quad \text{M1}$$

$$\square x = a \quad \square -$$

$$\square 4a = \square$$

$$\square y^2 = 16a(x + 2a) \quad \text{B1}$$

12

12. (i)

$$Z_1 = \frac{1+i\sqrt{3}}{2}$$

$$r_1 = \sqrt{12^2 + 23^2} \quad \text{B1}$$

$r_1 = 1$ unit

Also;

$$\begin{array}{c} \boxed{3} \\ \boxed{\underline{1}} \end{array}$$

$$\theta_1 = \tan^{-1} \frac{\boxed{1}}{\boxed{2}}$$

$$= 2 \text{ radians}$$

$$\theta_1 = \tan^{-1} \left(\frac{\sqrt{3}}{3} \right)$$

$$\theta_1 =$$

$$3$$

B1

$$Z_1 = \cos \frac{-\pi}{3} + i \sin \frac{-\pi}{3}$$

$$Z_2 = \frac{1 - i\sqrt{3}}{2}$$

$$r_2 = \sqrt{1 \cdot 2 - 2 \cdot 2}$$

B1

$r_2 = 1$ unit

$$Q_2 = \begin{pmatrix} \frac{-\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \tan^{-1}(-1) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\pi}{6} \end{pmatrix}$$

$$Q_2 = \tan^{-1} \left(-\frac{\sqrt{3}}{1} \right)$$

$$\frac{Q_2}{3} = -\text{ or } 2 \frac{\pi}{3}$$

B1

$$Z_2 = \cos \frac{\pi}{3} - i \sin \frac{\pi}{3}$$

Or

$$Z_1 = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$$

A1

5

M1

(ii) $Z_{15} + Z_{25} = \cos \pi/3 + i \sin \pi/3 + \cos 5\pi/3 - i \sin 5\pi/3$

$$= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} + \cos \frac{5\pi}{3} - i \sin \frac{5\pi}{3}$$

$$= 2 \cos \frac{\pi}{3}$$

$$= 2 \cdot \frac{1}{2}$$

$$Z_1^5 + Z_2^5 = 1$$

A1

(b) $Z_1 = -4 - 3i$, $Z_2 = -4 + 3i$ is also a root.

A1

Using:

$$Z^2 - (-4 - 3i)Z - (-4 - 3i)(-4 + 3i) = 0$$

$$Z^2 + 8Z + 25 = 0$$

$Z^2 + 8Z + 25 = 0$ is a quadratic factor.

Solving for the roots.

$$\begin{array}{r} Z^2 - 12Z + 37 \\ \hline Z^2 + 8Z + 25 \end{array} \quad \begin{array}{r} Z^4 - 4Z^3 - 34Z^2 - 4Z + 925 \\ \hline Z^4 + 8Z^3 + 25Z^2 \end{array}$$

M1

$$- 12Z^3 - 59Z^2 - 4Z + 925$$

$$\hline - 12Z^3 - 96Z^2 - 300Z$$

$$+ 37Z^2 + 296Z + 925$$

$$\hline - 37Z^2 + 296Z + 925$$

0

Solving;

$$Z^2 - 12Z + 37 = 0$$

M1

$$Z = \frac{\sqrt{(-12)^2 - 4} \square 1 \square 37}{2(1)} - (-12) \square$$
$$\frac{12 \square \sqrt{144 - 148}}{2}$$

$$Z =$$

$$2$$

$$Z = 12 \square 2i$$

$$2$$

$$Z = 6 \square i$$

□ Other roots are; $-4-3i$, $6+i$ and $6-i$

A1 A1

$$\text{Let } \frac{5x^2 - 8x + 21}{2x(x-1)} = \frac{A}{2x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad M1$$

Let $x = 1$

$$\square 5(1)^2 - 8(1) + 1 = C \square 2(1)$$

$$\square C = -1$$

Let $x = 0$

$$\square 5(0)^2 - 8(0) + 1 = A(0 - 1)^2$$

□ A=1

Coefficient of x^2 :

$$5 = A + 2B$$

$$5 = 1 + 2B$$

$$4 = 2B, B= 2$$

$$= \boxed{} \sqrt{In\ 2\boxed{9} - In\ 2\boxed{4}\boxed{}} + \boxed{} \sqrt{In(9-1)^2 - In(4-1)^2} \boxed{} + \boxed{} \sqrt{\boxed{9^1} - 1} - \boxed{(4^1 - 1)} \boxed{}$$

M1

$$\boxed{9} \quad \boxed{2} \quad \boxed{3} \quad \boxed{-} \quad \boxed{24}$$

12

14. (a) $OA = \vec{3i} - j + 2k$

$$OB = -\mathbf{i} + \mathbf{j} + 9\mathbf{k}$$

$$= (-1 - 3)\mathbf{i} + (1 - -1) + (9 - 2)\mathbf{k}$$

$$AB = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$$

Using:

$$\mathbf{r} = OA + \square AB$$

$$\mathbf{r} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \square(-4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k})$$

Or

$$\begin{array}{l} \square 3 \square \quad \square -4\square \\ \square \quad \square \quad \square \\ \mathbf{r} = \square -1\square + \square \square 2 \square \\ \quad \square 2 \square \square \quad \square 7 \square \square \\ \quad \square \end{array}$$

(b) line L_1 :

$$\begin{array}{l} \square 3 \square \quad \square -4\square \\ \square \quad \square \quad \square \\ \mathbf{r}_1 = \square -1\square + \square \square 2 \square \\ \quad \square 2 \square \square \quad \square 7 \square \square \\ \quad \square \end{array}$$

Line L_2 :

$$\begin{array}{l} \square 8 \square \quad \square 1 \square \\ \square \quad \square \quad \square \\ \mathbf{r}_2 = \square 1 \square + \square \square - 2\square \\ \quad \square -6\square \square \quad \square -2\square \square \\ \quad \square \end{array}$$

At the point of intersection

$$\begin{array}{l} \mathbf{r}_1 = \mathbf{r}_2 \\ \square 3 \square \quad \square -4\square \quad \square 8 \square \quad \square 1 \square \\ \square \quad \square \quad \square \quad \square \quad \square \quad \square \\ \square -1\square + \square \square 2 \square = \square 1 \square + \square \square - 2\square \\ \quad \square 2 \square \square \square \quad \square 7 \square \square \quad \square -6\square \square \square - 2\square \square \\ \quad \square \end{array}$$

$$3 - 4\square = 8 + \square, \quad 4\square + \square = -5 \quad \text{--- (i)} \quad B1$$

$$-1 + 2\square = 1 - 2\square, \quad 2\square + \square = 1 \quad \text{--- (ii)} \quad B1$$

$$2 + 7\square = -6 - 2\square, \quad 7\square + 2\square = -8 \quad \text{--- (iii)} \quad \text{Solving (i) and (ii)} \quad B1$$

(i) ————— (ii)

M1

$$3\square = -6$$

$$\square = -2$$

From (i)

$$\square + \square = 1$$

M1

$$2 + \square = 1$$

$$\square = 3$$

From:

$$\square 3 \square \quad \square -4 \square \quad \square 8 \square \quad \square 1 \square$$

$$\square \quad \square \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$\square -1 \square + \square 2 \square = \square 1 \square + \square \square - 2 \square$$

$$\square \square 2 \square \square \square 7 \square \square \square - 6 \square \square \square - 2 \square \square$$

$$\square 3 \square \quad \square -4 \square \quad \square 8 \square \quad \square 1 \square$$

$$\square \quad \square \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$\square \square -1 \square + -2 \square 2 \square = \square 1 \square + 3 \square - 2 \square$$

$$\square \square 2 \square \square \square 7 \square \square \square - 6 \square \square \square - 2 \square \square$$

$$\square 11 \square \quad \square 11 \square$$

$$\square \quad \square \quad \square \quad \square$$

$$\square -5 \square = \square -5 \square$$

$$\square \square -12 \square \square \quad \square \square -12 \square \square$$

The lines intersect.

B1

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15.
$$y = \frac{x^2 - 2x + 1}{x^2 - 2x - 3} = 1 + \frac{-x + 3}{(x+1)(x-2)}$$

(a) (i) Horizontal asymptote.

A1

A1

□ $y = 1$ is a horizontal asymptote and $x = -1, x = 2$ are vertical asymptotes.

Vertical asymptote

For stationary points,

$$(ii) \frac{dy}{dx} = \frac{(2x-2)(x^2 - x-2) - (2x-1)(x^2 - 2x+1)}{(x_2 - x-2)^2} = 0 \quad B1$$

$$\begin{aligned} x^2 - 6x + 5 &= 0 \\ -(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5} \end{aligned}$$

$$x =$$

$$2(I)$$

$$x = 5 \text{ or } x = 1$$

Stationary points are;

$$(1, 0) \text{ and } (-5, -\frac{8}{9}) \quad A1 \quad A1$$

Nature of turning point. $dy = x_2$

$$\frac{-6x+5}{dx} \quad \begin{array}{c} \text{---} \\ (x - x-2) \end{array}$$

x	L	I	R	L	S	R
Sign of $\frac{dy}{dx}$	+	0	-	-	0	+



□ Point $(1, 0)$ is a maximum and $(5, \frac{8}{9})$ is a minimum. B1

B1

(b) Intercepts of the curve and axes

x - intercept occurs for $y = 0, x = 1$ either

B1

or

y - intercept occurs when $x = 0, y = \frac{-1}{2}$

Now As $x \rightarrow +\infty, y \rightarrow 1^-$

As $x \rightarrow -\infty, y \rightarrow 1^+$

□ $y = 1$ is a horizontal asymptote.

Intercept of curve and the line $y = 1$

$$\square 1 = \frac{x^2 - 2x + 1}{x - x - 2}$$

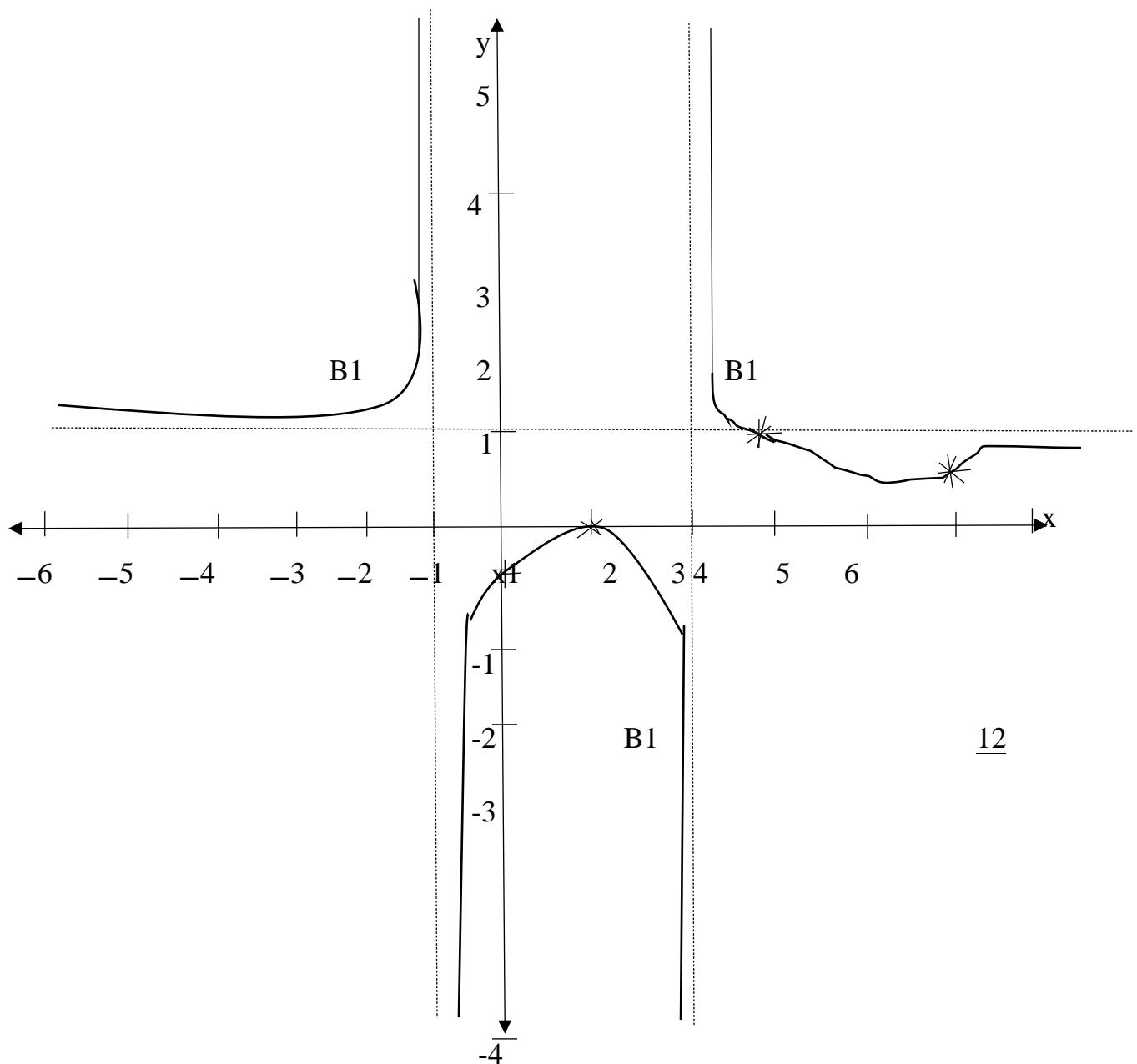
$$x^2 - x - 2 = x^2 - 2x + 1$$

$$x = 3$$

Point $(3, 1)$

B1

Sketch of the curve.



$$x = 1 \quad x = 2$$

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16. (a)

$$\begin{aligned}
 & \frac{dy}{dx} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
 & \frac{1}{2} = \frac{1}{x^2} \\
 & = \frac{(x + \Delta x) - x}{\Delta x} \\
 & = \frac{x^2 - (x + \Delta x)^2}{x^2 - (x + \Delta x)^2} \\
 & = \frac{x_2(x + \Delta x)_2 \Delta x}{B1} \\
 & = \frac{2x \Delta x + (fx)^2}{x^2(x + \Delta x)^2 \Delta x} \\
 & = \frac{-2x + (fx)}{x_2(x + \Delta x)_2} \\
 & = \frac{A1}{B1}
 \end{aligned}$$

$$\begin{aligned}
 & \text{As } \Delta x \rightarrow 0, \Delta y \rightarrow dy \\
 & \frac{\Delta x}{dx} \rightarrow \frac{dy}{dx} \\
 & B1 \quad A1 \quad \frac{dy - 2x - 2}{\Delta x} = \frac{dy}{x_2}
 \end{aligned}$$

(b)

$$e^x = \cos(x - y) \quad M1$$

$$e^x = \frac{1}{1 - \frac{dy}{dx} \sin(x - y)}$$

$$e^x = \sin(x - y) - \sin(x - y) \frac{dy}{dx} \quad M1$$

$$e^x = \sin(x - y) - \sin(x - y) \frac{dy}{dx}$$

$$\begin{aligned}
 & \frac{dx}{dy} = \frac{\sin(x-y) - e^x}{\sin(x-y)} \\
 \square \quad & \frac{dy}{dx} = \frac{\sqrt{1 - \cos^2(x-y)} - e^x}{\sqrt{1 - \cos^2(x-y)}} \quad \text{B1}
 \end{aligned}$$

Recall that:

$$\begin{aligned}
 \cos^2(x-y) + \sin^2(x-y) &= 1 \\
 \sin(x-y) &= \sqrt{1 - \cos^2(x-y)} \quad \text{B1}
 \end{aligned}$$

$$\square \quad \frac{dy}{dx} = \frac{\sqrt{1 - \cos^2(x-y)} - e^x}{\sqrt{1 - \cos^2(x-y)}}$$

$$\frac{dy}{dx} = \frac{\sqrt{1 - (e^x)^2} - e^x}{\sqrt{1 - (e^x)^2}} \quad \text{M1}$$

$$\square \quad \frac{dy}{dx} = \frac{\sqrt{1 - e^{2x}} - e^x}{\sqrt{1 - e^{2x}}} \quad \text{B1}$$

E N D

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