STAHIZA S5 MTH(P1&P2)RECESS WORK 2020

PURE MATHEMATICS

1. Find the value of $\tan A$ when $tan(A-45)=\frac{1}{3}$

   (b) Prove that $tanA+tanB=\frac{sin(A+B)}{cosAcosB}$

2. (a) Prove that $sin3A=3sinA-4sin^3A$

   (b) Solve the equation $3cos2\theta+sin\theta=1$ for values of θ from 0^o to 360^o inclusive.

3. Find the maximum and minimum values of $2sin\theta-5cos\theta$ and state the corresponding values of θ between 0^o to 360^o inclusive.

4. If A, B, C are angles of a triangle, prove that

   (a) $CosA+CosB+CosC-1=4sin\frac{A}{2}\,sin\frac{B}{2}\,sin\frac{C}{2}$

   (b) $tanA+tanB+tanC=tanA\,tanB\,tanC.$

5. (a) Prove that $tan3A=\frac{3tanA-tan^3A}{1-3tan^2A}$

   (b) Eliminate θ from the equations $x=cos\theta+sin\theta,\ y=tan\theta$

6. Prove that: $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\dfrac{\theta}{2}$.

7. Solve the equation $\cos7\theta+\cos5\theta+\cos3\theta+\cos\theta=0$ for $0^o\le\theta\le360^o$.

8. If ABC is a triangle, prove that $\sin^2A+\sin^2B+\sin^2C=2+2\cos A\cos B\cos C$

9. Prove that $\sin\left(2\sin^{-1}x+\cos^{-1}x\right)=\sqrt{\left(1-x^2\right)}$

10. Prove that: $\tan^{-1}\dfrac{1}{2}-\operatorname{cosec}^{-1}\dfrac{\sqrt{5}}{2}=\cos^{-1}\dfrac{4}{5}$

11. Solve the equation: $5\sin^2 2x-3\sin2x\cos2x-14\cos^2 2x=0$, for $0^o\le x\le90^o$.

12. Solve the equation $2\cos^2\left(x-\dfrac{\pi}{2}\right)-3\cos\left(x-\dfrac{\pi}{2}\right)+1=0$ for $0\le x\le2\pi$.

13. Solve the equation. $3\sin^2x+2\cos^2x=\frac{5}{2}\tan x$, for $0^0\le\ x\ \le360^0$.

14. Solve the equation: $\tan4x+\tan2x=0$ for $0^o\le x\le2\pi$

15. If $2A+\mathbf{B}=\mathbf{45}$, show that $\tan B=\dfrac{1-2\tan A-\tan^2A}{1+2\tan A-\tan^2A}$

16. Given $x = \sec\theta + \tan\theta$, $y = \operatorname{cosec}\theta + \cot\theta$, show that $x + \frac{1}{x} = 2\sec\theta$ and $y + \frac{1}{y} = 2\operatorname{cosec}\theta$.

17. If $\tan\alpha = p$, $\tan\beta = q$, $\tan\gamma = r$, prove that $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$

18. The first, second, third and n^{th} terms of a series are 4, -3, -16 and $(an^2 + bn + c)$ respectively. Find a, b, c.

19. The sum of the first n terms of an A.P is $n^2 + 5n$. Find the first three terms of the series.

20. The first term of a geometric progression is A and the sum of the first three terms is $\frac{7}{4}A$. Show that there are two possible progressions.

21. Solve the equations:
$$2\log_y x + 2\log_x y = 5$$
$$xy = 27$$

22. Solve for n given that ${}^nC_4 = 5 \times {}^{n-2}C_3$.

23. Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$.

24. Solve $3^{2(x+1)} - 28(3^x) + 3 = 0$.

25. Given that $y = \sqrt{5x^2 + 7}$, show that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$

26. Solve the equation $Sin\ 2x + 1 = 2\ Cos^2 x$ for $0° \leq x \leq 360°$.

27, Give that $x^4 - 6x^3 + 10x^2 + ax + b$ is a perfect square, find a and b.

28. Solve the following simultaneous equations..

$3x - 2y + 4z = 7$, $x + y - 6z = -5$ and $2x + 3y + 3z = 5$

29. Show that $2x^3 + x^2 - 13x + 6$ *is divisible by* $x - 2$.

30. Find the equation of the tangent at the point (-2,-4) to the curve $y = 4 - 2x^2 - \frac{2}{3x^3}$

31. Simplify $$\frac{\frac{1}{2}x^{1/2}(1+x)^{-1/2} - \frac{1}{2}x^{-1/2}(1+x)^{1/2}}{x}$$

32. Solve the simultaneous equations

$2^x + 4^y = 12$ and $3(2^x) - 2(2^{2y}) = 16$ .Hence show that $4^x + 4(3^{2y}) = 100$

33. When the quadratic expression ap^2+ bp + c is divided by p-1, p-2 and p+1, the remainders are 1, 1 and 25 respectively. Determine the factors of the expression

34. Prove that,if one root of the equation $ax^2 + bx + c = 0$ is twice the other,

then $2b^2 = 9ac$.

35. Given that the roots of the equation $2x^2 - x - 3 = 0$ are α and β.Find the values of : (i) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (ii) $\alpha^3 - \beta^3$ iii $\alpha^2 - \beta^2$ iv $\alpha^2 + \beta^2$

36. Sketch the curve $y = (x^2 - 1)(2 - x)$,hence find the area enclosed by the curve and the x-axis

37. Show that $\cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

38. A polynomial P(x) is a multiple of x – 3 and the remainders when P(x) is divided by x + 2 and x-5 are 6 and-7 resp. Find the remainder when P(x) is divided by $x^3 - 5x^2 - 4x + 20$.

39. When P(x) = $x^3 + ax^2$ + bx + c is divided by $x^2 - 4$, the remainder is 2x + 11. Given that x + 1 is a factor of P(x), Find the values of a, b and c.

40. If the function P(x) = $x^4+ax^3+bx^2+6x-5$ is divisible by$(x-2)^2$.

Find the values of a and b.

41. Give that $x^4 - 6x^3 + 10x^2 + ax + b$ is a perfect square, find a and b.

42. The roots of the equation $ax^2+bx+c=0$ where a,b and c are non Zero constants are α and β,and the roots of equation $ax^2+2bx+c=0$ are θ and μ. Find the equation whose roots are αθ+βμ and αμ+βθ.

43. Solve the simultaneous equation. $\begin{matrix} \cos x + 4\sin y = 1 \\ 4\sec x - 3\operatorname{cosec} y = 5 \end{matrix}$ for $0^0 < x < 360^0$.

44. Show that in any triangle A B C, $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$. Then solve the triangle two sides 5 and 7 and the included angle of 45°.

45. Show that for allvalues of $\theta, cos\theta + \cos\left(\theta + \frac{2}{3}\pi\right) + \cos\left(\theta + \frac{4}{3}\pi\right) = 0$.

Hence show that $cos^2\theta + cos^2\left(\theta + \frac{2}{3}\pi\right) + c0s^2\left(\theta + \frac{4}{3}\pi\right) = \frac{3}{2}$.

46. Prove that $4\cos\theta 3\cos\theta + 1 = \frac{\sin 5\theta}{sin\theta}$

47. In any triangle ABC,Prove that TanBcotC=$\frac{a^2+b^2-c^2}{a^2-b^2+c^2}$.

48. Use the method of row-reduction to find values of a, b and c in the equations :

$$\begin{pmatrix} 2a + b = 3 + 3c \\ 3b - 5c = 1 - a \\ 6a - 2b = 9 - c \end{pmatrix}$$

49. A polynomial P(x) when divided by (x – 1) has a remainder of 3 and when divided by (x – 2) leaves a remainder of 1, find the remainder when P(x) is divided by (x – 1)(x – 2).

50. A curve has a gradient $\frac{dy}{dx} = 1 - bx$ with a turning point at $\left(\frac{1}{2}, 2\frac{1}{4}\right)$

(i) Find the value of b
(ii) Determine the nature of the turning point.
(iii) Find the equation of the curve and sketch the curve
(iv) Calculate the area bounded by the curve, the x-axis from x= -1 to x= 4.

APPLIED MATHEMATICS

51. The times corrected to the nearest seconds, taken by 100 athletes to cover a lap of running track were recorded as follows

Time(sec)	70- < 75	75 -< 80	80- < 85	85 - < 90	90 - < 95	95 -< 100
No of athletes	8	20	26	30	9	7

(a) Using a working mean of 87.5 , calculate the mean and the standard deviation
(b) Draw the cumulative frequency curve and use it to estimate ;
 (i) Median time
 (ii) Semi- interquartile range
 (iii) Number of athletes who used time below 86 seconds
 (iv) the 90^{th} and 60^{th} percentile range.

52. Events *A* and *B* are such that $P(A^I \cap B^I) = 0.1$, $P(A^I \cup B^I) = 0.8$ and $P(A only) = 0.3$. Find (i) *P(A)* (ii)*P(B)* (iii)*P(A* or *B* but not both) (iv) $P(A/B)$

53. In a large group of people, its known that 10% have a hot breakfast, 20% have a hot lunch and 25% have a hot lunch and a hot breakfast. Find the probability that a person picked at random from this group;

(i) has a hot breakfast and a hot lunch
(ii) has a hot lunch, given that he had a hot breakfast

54. A bag contains 4red and 5green balls. Another bag contains 3red and 3green balls. A bag is randomly selected and 2 balls are randomly picked from it without replacement. Find the probability that the balls picked are of different colors.

55. Events A and B are such that $P(A) = 0.4$, $P(A \cup B) = 0.9$ and $P(A \cap B) = 0.1$ obtain

(i) $P(A/B)$ (ii) $P(A \cup B^I)$

56. Given that $P(\bar{A}) = \frac{2}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{12}$, find $P(A \cup B)$.

57. The probability that a student X can solve a certain problem is $\frac{2}{5}$ and that student Y can solve it is $\frac{1}{2}$. Find the probability that the problem will be solved if both X and Y try to solve it independently.

58. Events A and B are such that $P(A)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{12}$. If A and B are independent events, find $P(A\cup B)$.

59. The average prices of a bunch of matooke in each third year over a period of $3\frac{1}{3}$ years are given in the table below

	1st third	2nd third	3rd third
1998	4500	5000	5200
1999	5500	5700	6000
2000	6200	6500	6800
2001	7000	X	

(a) calculate the 3-point moving averages
(b) On the same graph, show the raw data and the 3-point moving averages. Hence
(i) comment on the trend of the prices of matooke for this period
(ii) Estimate the value of X in the table.

60. The table below gives the number of bags of cement sold each week by a certain hardware shop in the first twelve weeks of a year

week	1	2	3	4	5	6	7	8	9	10	11	12
Bags sold	422	318	349	252	386	230	256	141	264	168	272	260

(a)Calculate the four point moving totals and hence the four point moving averages for the data
(b)On the same axes, draw a graph of moving averages and the actual sales
(c)Comment on the sales of cement over the twelve weeks period
(d)Estimate the probable sales in the thirteenth week.

61. The table below shows the number of boxes of pens sold by a certain wholesaler shop from the year 2009 to 2012.

	Quarter			
	1st	2nd	3rd	4th
2009	192	280	320	260
2010	300	360	380	270
2011	342	420	430	320
2012	424	480	510	412

a) Calculate the four-point moving averages for the data.

b) i) On the same axes, plot the original data and the five-point moving averages

ii) Comment on the trend of the number of boxes of pens sold over the five – year period.iii) Use your graph to estimate the number of boxes to be sold in the first quarter of 2013.

62 A particle is projected away from an origin O with an initial velocity of $0.25ms^{-1}$. The particle travels in a straight line and accelerates at $\frac{3}{2}ms^{-2}$. Find

(i)How far the particle is from O after 4seconds.
(ii)the distance travelled by the particle in the 4^{th} second

63. A stone is projected vertically upwards from the ground level at a speed of $24.5ms^{-1}$. Find how long is the stone atleast 19.6metres above the ground level.

64. A mass of 5kg is initially at rest at the bottom of a smooth slope which is inclined at $\sin^{-1}(3/5)$ to the horizontal . the mass is pushed up the slope by a horizontal force of 50N.

(i) Find the normal reaction between the mass and the plane
(ii)How far up the slope will the mass travel in the first 4 seconds

65. A car initially moving at a speed of $80m\,s^{-1}$ decelerates uniformly and attains a velocity of $40m\,s^{-1}$ for $20s$ and comes to rest in the next $30s$. Sketch a velocity – time graph and use it to calculate the average velocity.

66. When a horizontal force of 37N is applied to a body of mass 10kg which is resting on a rough horizontal surface, the body moves along the surface with an acceleration of $1.25ms^{-2}$. Find μ , the coefficient of friction between the body and the surface

67. Three forces qN, pN and 20N act on a particle in the directions, north,S050°W and S070°E respectively. If the system is in equilibrium, find the values of p and q.

68. ABCD is a rectangle. Forces of magnitude 8N, 4N, 10N and 2N act along AB, CB, CD and AD respectively, in the directions indicated by the order of the the letters. Find the magnitude and direction of the resultant.

69.. A block of 5000g lies on a rough horizontal floor. If the coefficient of friction between the block and the floor is $\frac{\sqrt{3}}{2}$;

(a.) Find the magnitude of the force ρ applied;
(i.) Horizontal to the plane
(ii.)At angle of 45° above the horizontal

70. Calculate the coefficient of friction when a mass of 2000g was added to the block and the force in (a.) (i.) above was applied.

71. A pharmacist had the following records of unit price and quantities of drugs sold for the years 2010 and 2011. Taking 2010 as the base year.

Drug	Unit price(@carton)		Quantities (@carton)	
	2010	2011	2010	2011
Asa	80	125	40	45
Pana	100	90	70	90
c/quinine	55	70	8	10
Caps	90	100	10	10

(i) Calculate the price relatives for each drug in 2011
(ii) Obtain the simple aggregate price index number for 2011
(iii) Calculate the weighted index number and comment on it

72. The marks scored by eight candidates in English and mathematics are as shown in the table below

Candidate	1	2	3	4	5	6	7	8
English(x)	50	58	35	86	76	43	40	60
Math(y)	65	72	54	82	32	74	40	53

a) Plot a scatter diagram for the data an draw the line of best fit
(b)Estimate the mark of the ninth candidate in mathematics if he had 65 in English
(i)Calculate the rank correlation coefficient between English and mathematics

73. 12 students were given a test at the beginning of of the course and their scores X_i were compared with their scores Y_i obtained in an examination at the end of the course

X_i	1	2	2	4	5	5	6	7	8	8	9	9
Y_i	3	4	5	5	4	8	6	6	6	7	8	10

(i)Calculate the rank correlation coefficient and comment on your result.
(ii) Plot a scatter diagram and comment on your graph

74. The table below shows the heights measured in cm for a group of senior six students;

Height	177-186	187-191	192-196	197-201	202-206	207-216
Frequency	12	8	8	9	7	6

a) Draw a histogram. Hence state the modal class.
b) Calculate the (i) mean (ii) standard deviation. (ii) Mode

75. The Maths and Physics examination marks of a certain school are given in the following table.

Maths (x)	28	34	36	42	52	54	60
Physics(y)	54	62	68	70	76	68	74

a) i Plot the marks on the scatter diagram and comment on the relationship between the two subjects.
ii) Draw a line of best fit and use it to predict the Physics mark of a student whose Maths

mark is 50.

b) Calculate the rank correlation coefficient between the marks. Comment on the significance of Maths on Physics performance based on 5% level of significance

76. The table below the number of reported accidents to workers in a certain factory over the past 12 years.

Age of Worker	15-	20-	25-	30-	35-	40- < 45
Number of employees	42	52	28	20	18	16

b) Plot a cumulative frequency curve and use it to estimate the;

(i) Proportion of employees who are not more than 27 years old. (ii) 80% central limits of the age of employees.

c) Find the (i) mean

(ii median,interquantile range and 60 th percentile range by graph and by calculation..

77. A survey of the mass (kg) of girls in the certain school in the final year was taken and the results are shown below;

Mass (kg)		50	60	75	80	85	90	100	105	110
Cumulative frequency	0	9	25	46	63	77	82	89	93	94

(a) Display the information on a histogram and hence find the modal mass

(b) Calculate the standard deviation of the mass.

78. Forces of magnitudes **6N, 6N, 4N, 10N** and **8N** act along **AB, BC, CD, DB** and **AD** respectively, in the directions indicated by the order of the letters of a rectangle **ABCD** of dimensions 4m by 3m. Find the;

(i) Magnitude of their resultant
(ii) Equation of the line of action of their resultant force
(iii) Distance from **B** where it cuts **AB.**

79. A particle is projected at an angle of 30º with a speed of $21\, m\, s^{-1}$. If the point of projection is 5m above the horizontal grounds, find the horizontal distance that the particle travels before striking the ground.

80. . A body of mass 0.2kg is acted upon by a force $\mathbf{F} = 8t\,\mathbf{i} - 4t^2\mathbf{j} + 2(3 - t^2)\mathbf{k}\ N$. Initially, the body is at rest at a point with position vector $-10\,\mathbf{i} + 12\mathbf{j} - 4\,\mathbf{k}$. Find the

(i) velocity after time t seconds,
(ii) Distance covered by the body in 2seconds.

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